Baryon Spectroscopy in Coupled Channels

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JÜLICH
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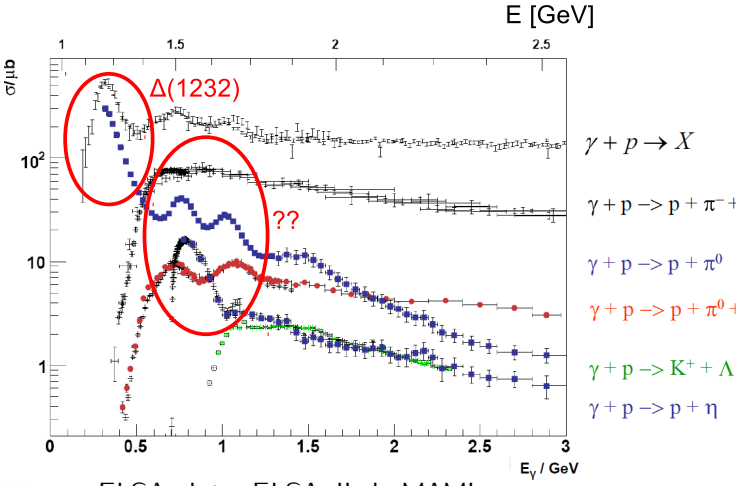
Hadron 2015, Sept. 17, 2015

The excited hadron spectrum:
Connection between experiment and QCD in the non-perturbative regime

Excited hadron spectrum: testing ground for theories of the strong force at low and medium energy

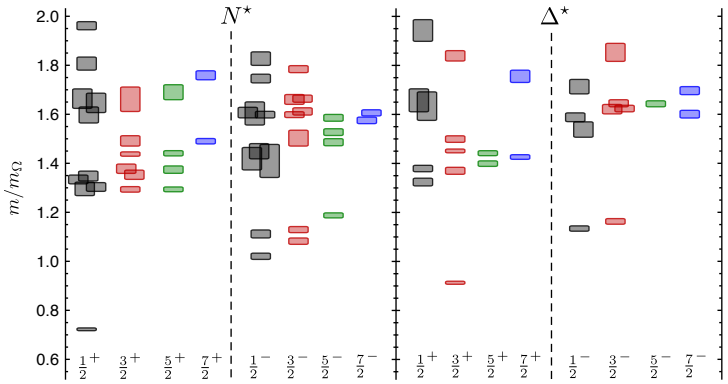
Experimental study of hadronic reactions



source: ELSA; data: ELSA, JLab, MAMI

$\Longleftrightarrow$

Theoretical predictions of excited hadrons
e.g. from lattice calculations:



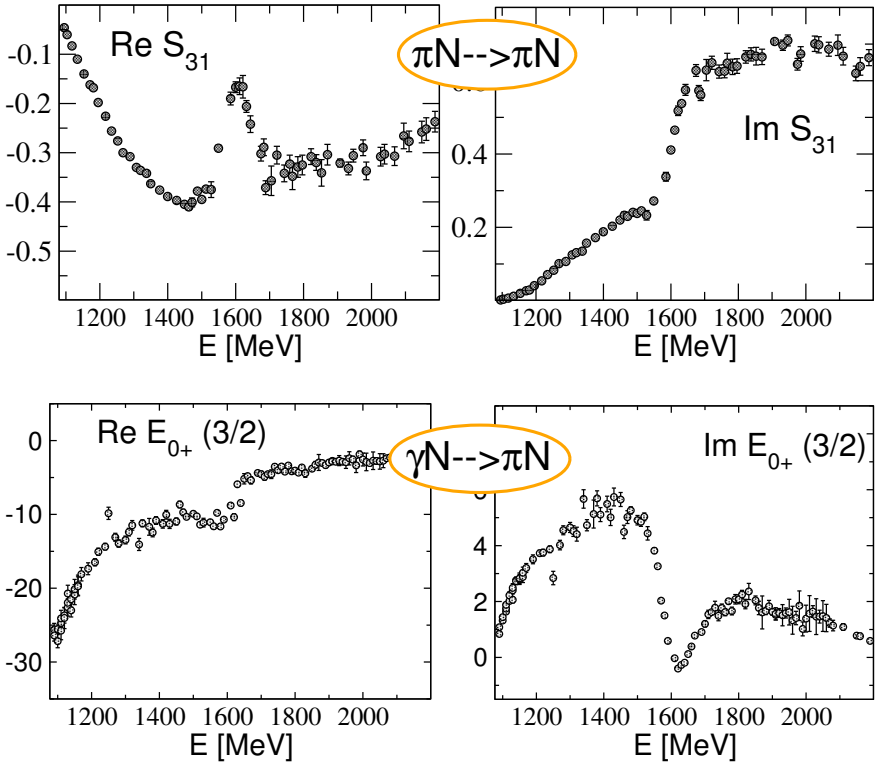
$m_\pi = 396$ MeV [Edwards *et al.*, Phys.Rev. D84 (2011)]

$\Rightarrow$ Partial wave decomposition:
decompose data with respect to a conserved quantum number: J^P

Missing resonance problem

Resonances

$J^P = 1/2^+, I = 3/2$



Points: SAID 2006 and CM12

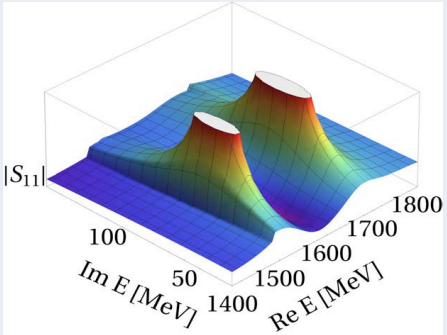
Breit-Wigner parameterization:

$$\mathcal{M}_{ba}^{Res} = -\frac{g_b g_a}{E^2 - M_{BW}^2 + iE\Gamma_{BW}}$$

- M_{BW}, Γ_{BW} channel dependent
- background? overlapping resonances? thresholds?

Resonances: poles in the T-matrix

- Pole position E_0 is the same in all channels
- thresholds: branch points



Re(E_0) = "mass"

-2Im(E_0) = "width"

residues→ branching ratios

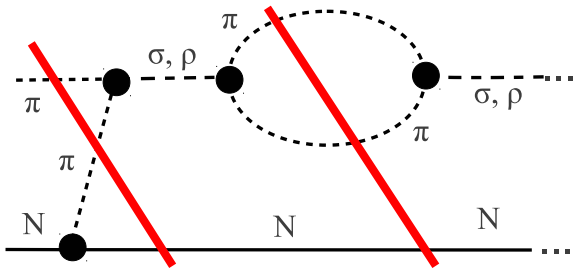
- How to connect lattice eigenvalues to resonance poles?
- But first: How to construct a suitable amplitude to determine resonance poles?

The scattering matrix

$$S = \mathbb{1} + iT$$

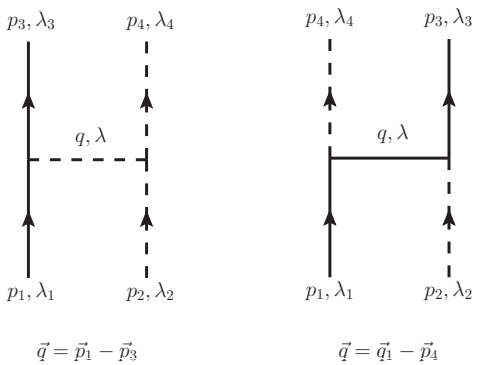
Unitarity: $SS^\dagger = 1 \Leftrightarrow -i(T - T^\dagger) = T\,T^\dagger$

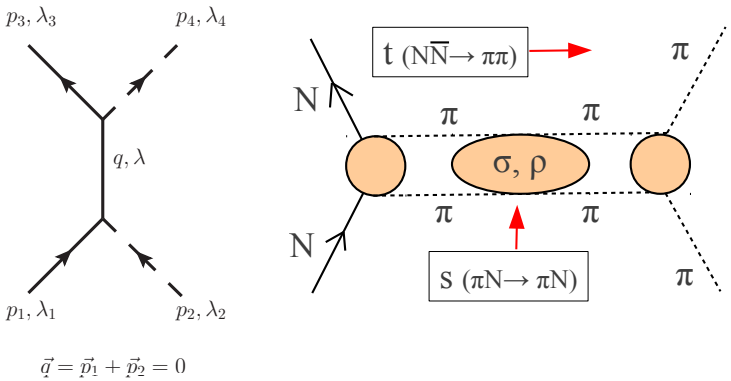
- 3-body unitarity:
  discontinuities from t-channel exchanges
  $\rightarrow$ Meson exchange from requirements of the S-matrix [Aaron, Almado, Young, Phys. Rev. 174, 2022 (1968)]

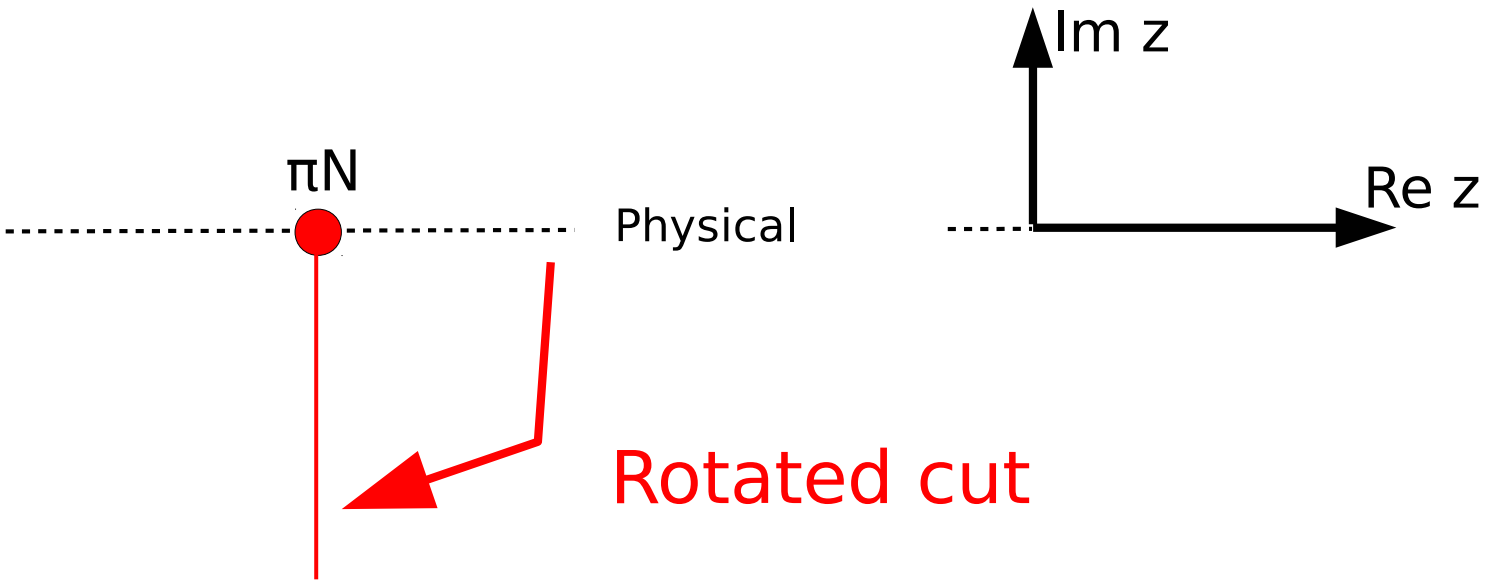


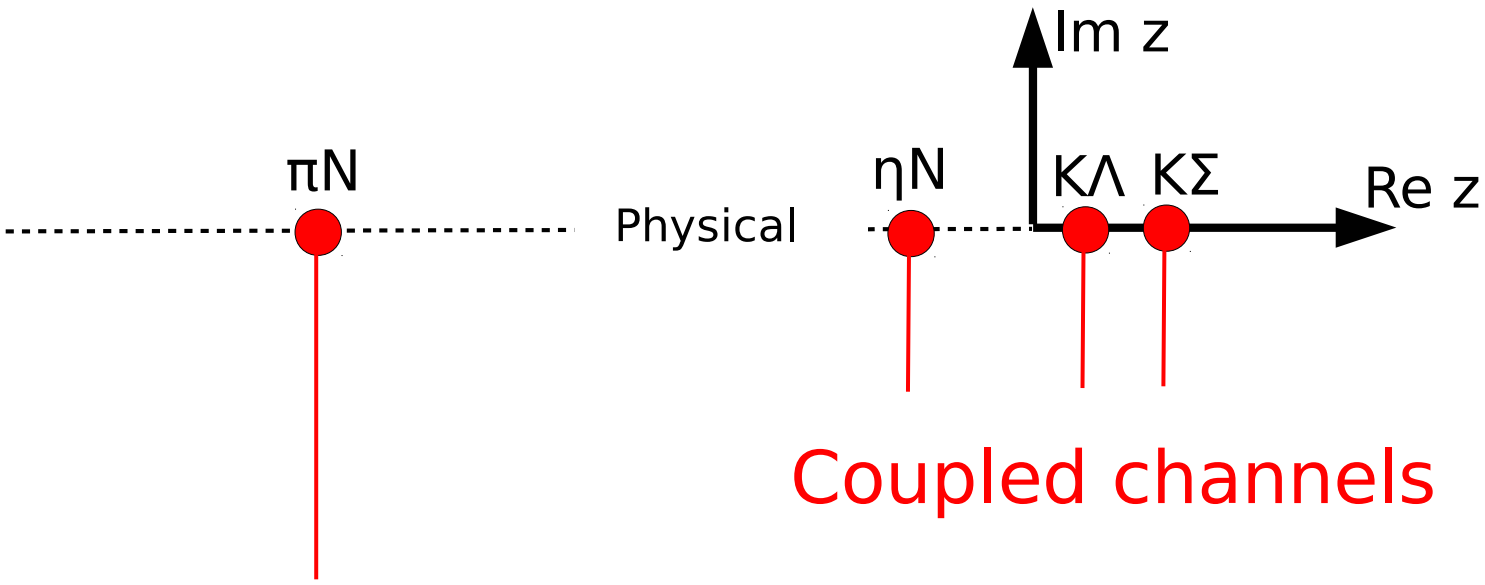
Other cuts

- to approximate left-hand cut $\rightarrow$ Baryon u-channel exchange
- σ, ρ exchanges from crossing plus analytic continuation.

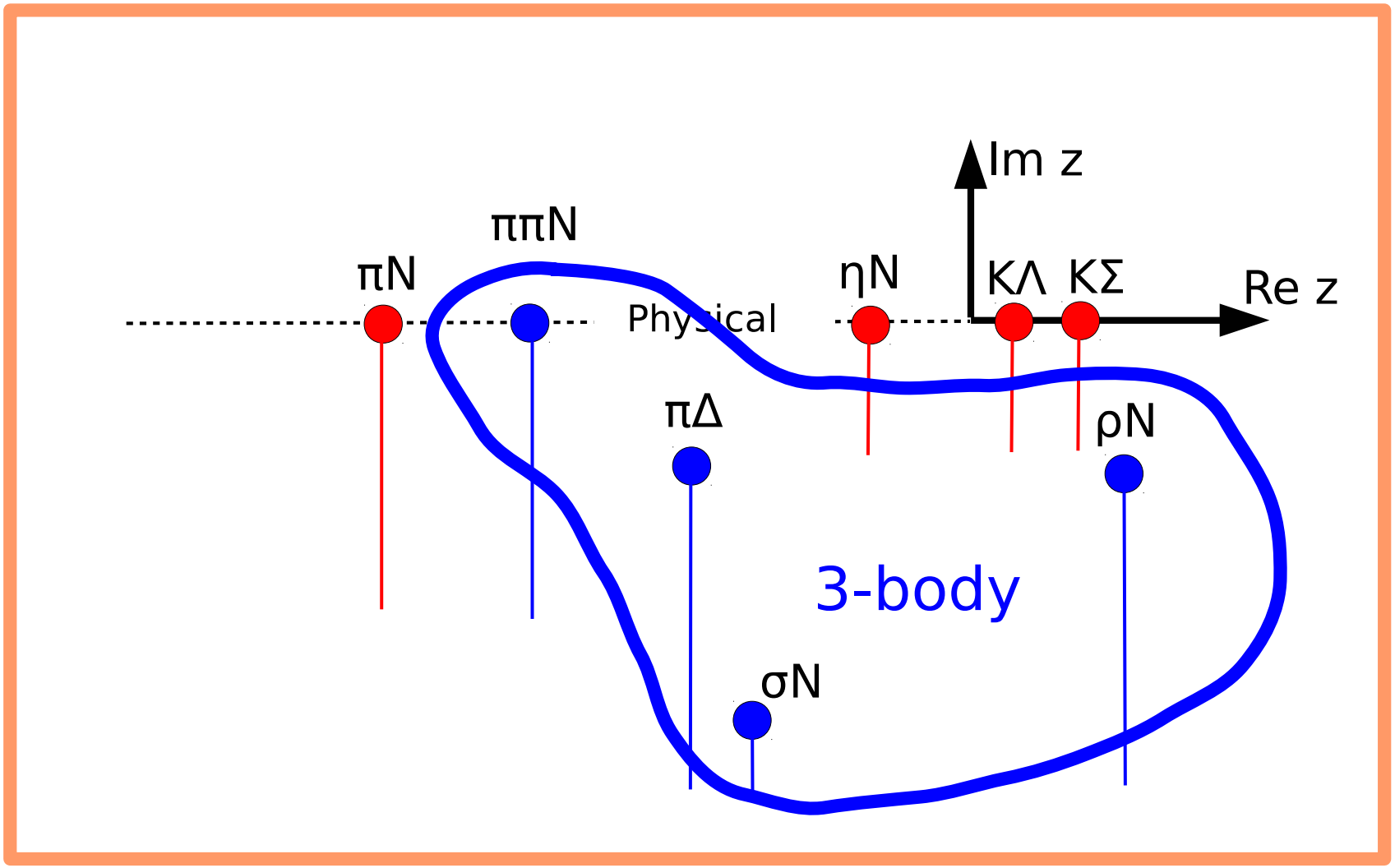


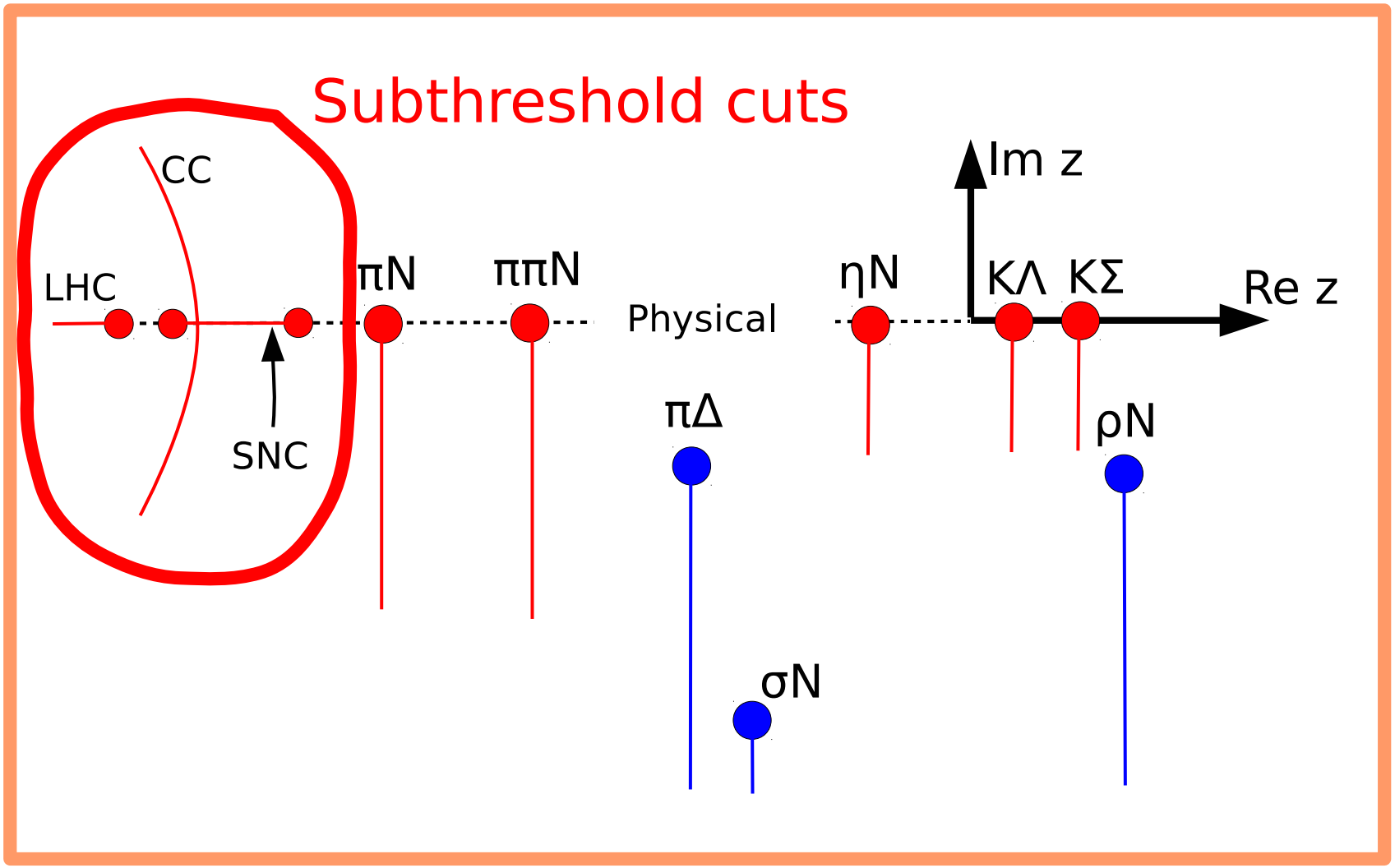


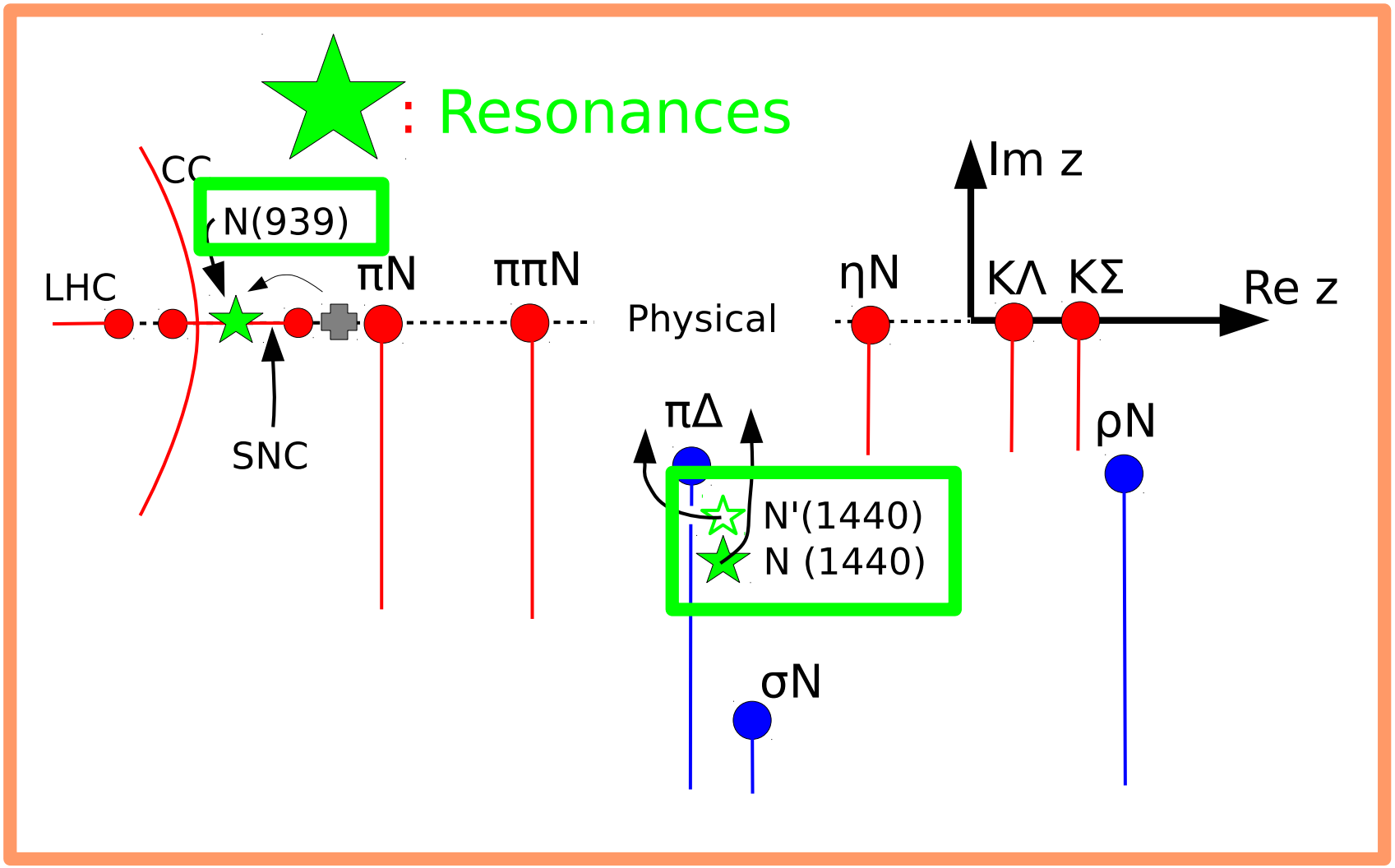

πN
Physical
Im z
Re z
Rotated cut

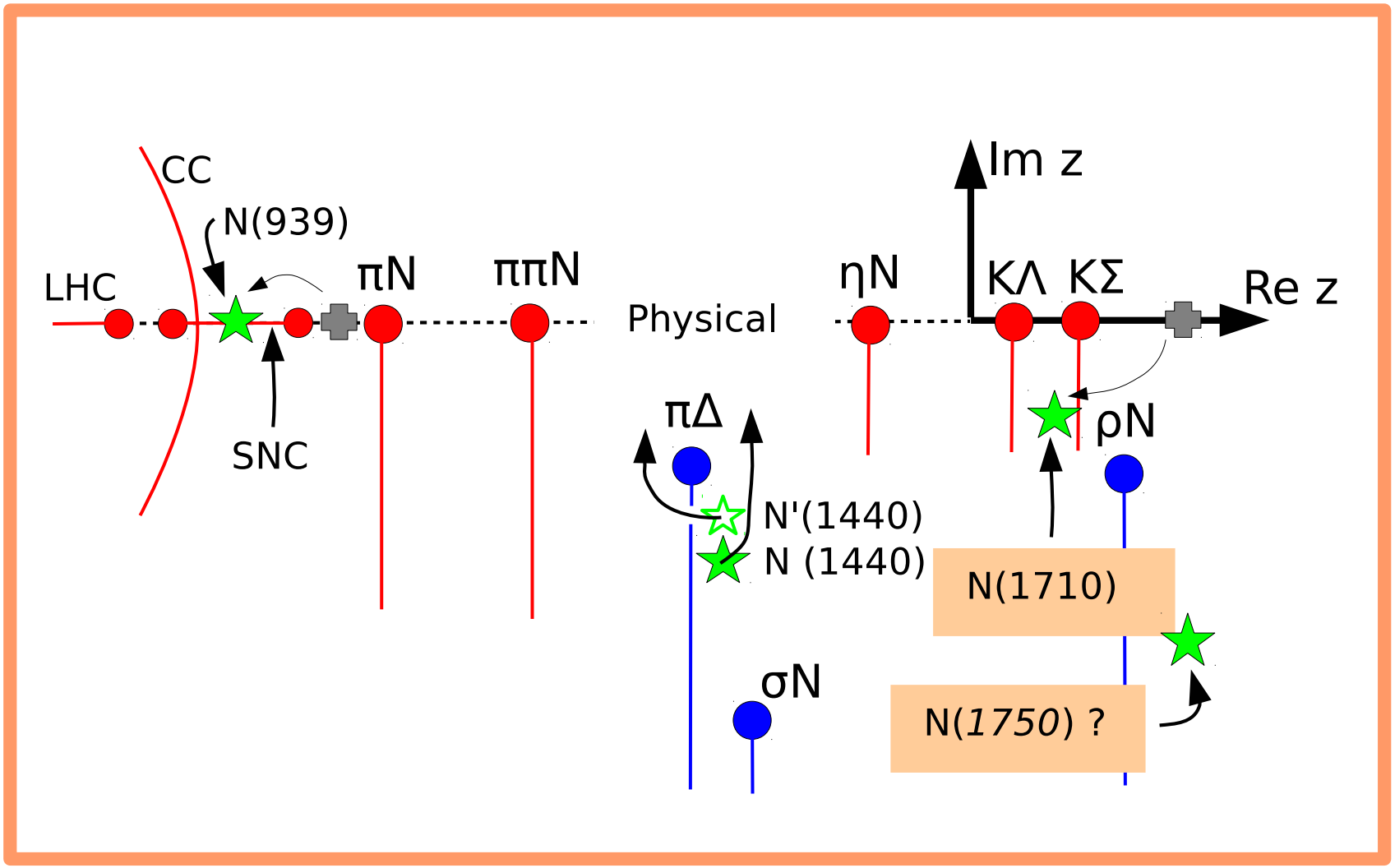


Coupled channels

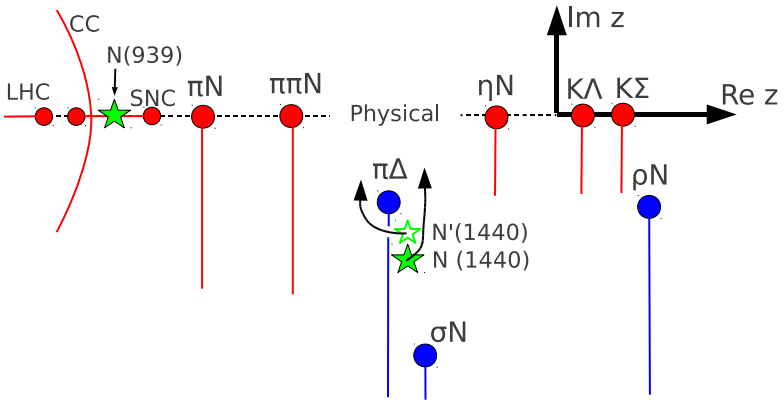

Im z
ππN
πN
ηN
KΛ KΣ
Re z
Physical
πΔ
ρN
3-body
σN


Subthreshold cuts
CC
LHC
SNC
πN
ππN
Physical
πΔ
σN
ηN
KΛ
KΣ
Im z
Re z
ρN


: Resonances
CC
N(939)
LHC
πN
ππN
Physical
ηN
Im z
KΛ KΣ
Re z
SNC
πΔ
ρN
N'(1440)
N (1440)
σN

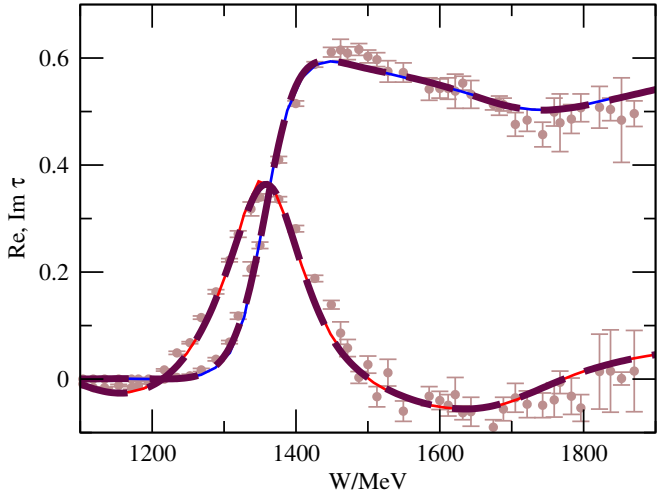

CC
N(939)
LHC
πN
ππN
SNC
Physical
ηN
Im z
KΛ KΣ
Re z
πΔ
ρN
N'(1440)
N (1440)
N(1710)
σN
N(1750) ?

Relevance of three-body dynamics



- Roper pole + $\pi\Delta$ branch point $\rightarrow$ non-standard resonance shape.
- See results by GWU/SAID data analysis center.

- Where is the 3∗ N(1710)? [S. Ceci, M.D. et al, PRC84, 2011]



Fit of a model without ρN branch point (CMB type) [solid lines] to the Jülich amplitude [dashed lines]

- CMB fit to JM has pole at $1698 - 130\,i$ MeV, simulates missing branch point.

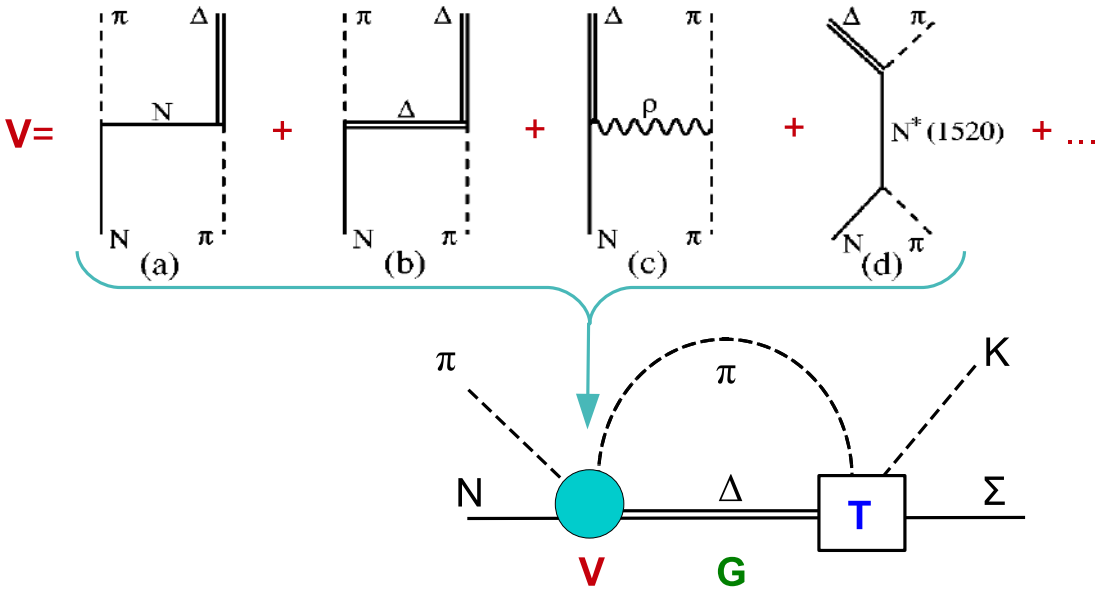
- Inclusion of full analytic structure important to avoid false pole signals in baryon spectroscopy.

A dynamical coupled-channel approach: the hadronic Jülich model

Dynamical coupled-channels (DCC): **simultaneous** analysis of different reactions

The scattering equation in partial wave basis

$$\langle L'S'p'|T^{IJ}_{\mu\nu}|LSp\rangle = \langle L'S'p'|V^{IJ}_{\mu\nu}|LSp\rangle + \sum_{\gamma,L''S''}\int_0^\infty q^2\, dq \langle L'S'p'|V^{IJ}_{\mu\gamma}|L''S''q\rangle \frac{1}{E - E_\gamma(q) + i\epsilon} \langle L''S''q|T^{IJ}_{\gamma\nu}|LSp\rangle$$



- t- and u-channel: T^{NP}
  dynamical generation of poles
  partial waves strongly correlated
- potentials V constructed from effective $\mathcal{L}$
- bare s-channel resonance states; Meaning? → Talk by W. Schweiger

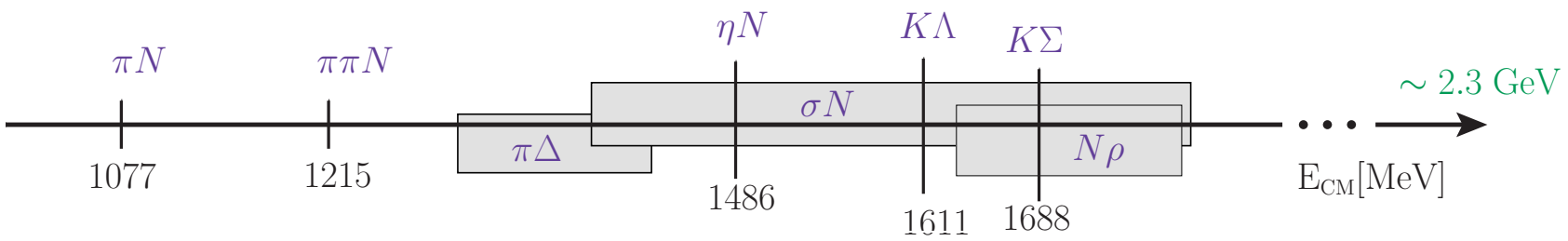
A dynamical coupled-channel approach: the hadronic Jülich model

Dynamical coupled-channels (DCC): **simultaneous** analysis of different reactions

The scattering equation in partial wave basis

$$\langle L'S'p'|T^{IJ}_{\mu\nu}|LSp\rangle = \langle L'S'p'|V^{IJ}_{\mu\nu}|LSp\rangle + \sum_{\gamma,L''S''}\int_0^\infty q^2\, dq \langle L'S'p'|V^{IJ}_{\mu\gamma}|L''S''q\rangle \frac{1}{E - E_\gamma(q) + i\epsilon} \langle L''S''q|T^{IJ}_{\gamma\nu}|LSp\rangle$$

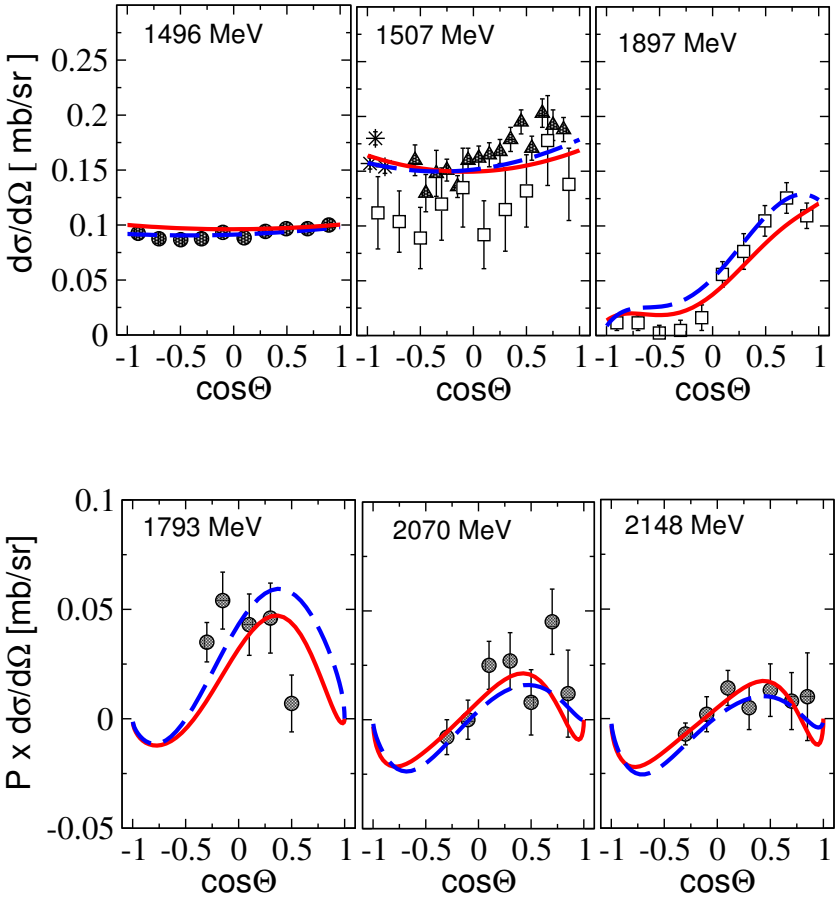
- **Analyticity** is respected (correct structure of branch points and cuts)
  $\hookrightarrow$ reliable extraction of resonance parameters
- **Unitarity**
- $J \leq 9/2$



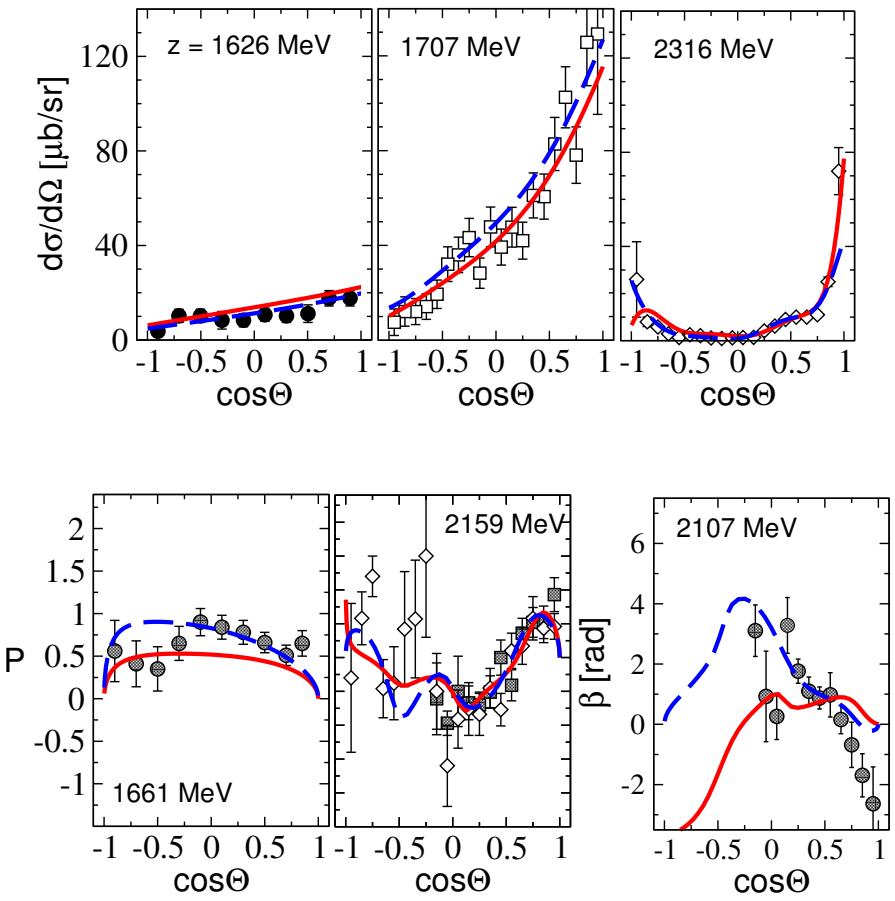
$\pi N \to \eta N, K\Lambda$ selected results

D. Rönchen, M.D., et al., Eur.Phys.J. A49 (2013) 44, Nucl.Phys. A851 (2011) 58-98

$\pi^- p \to \eta N$



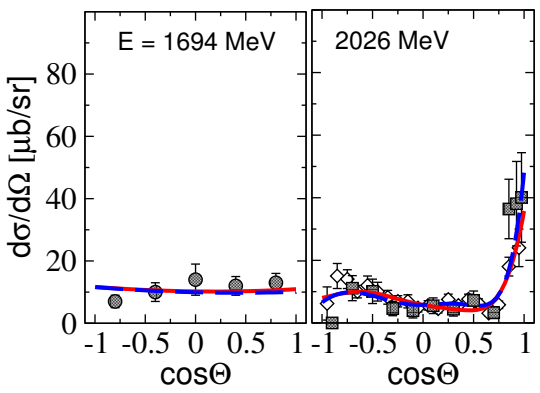
$\pi^- p \to K^0 \Lambda$



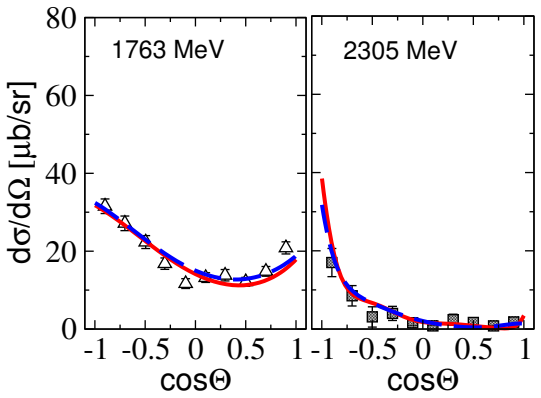
Full results

$\pi N \to K\Sigma$ selected results

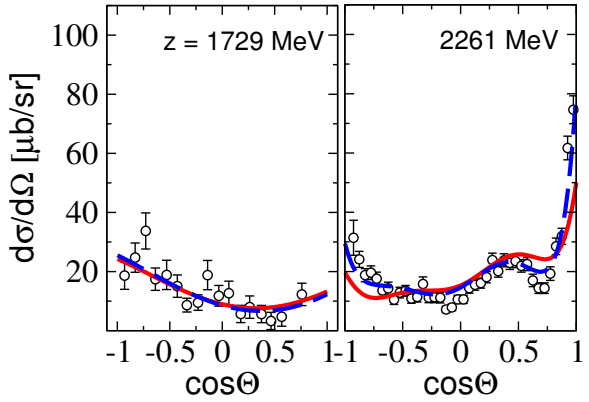
$\pi^- p \to K^0 \Sigma^0$

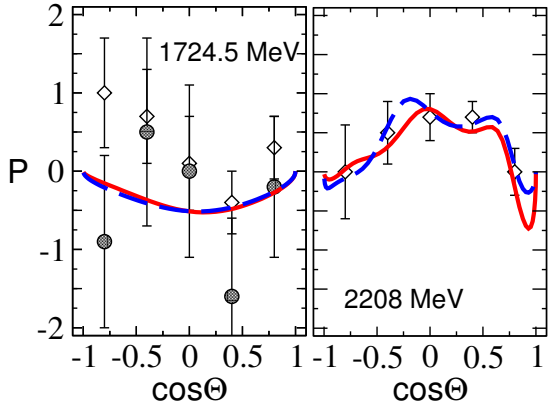


$\pi^- p \to K^+ \Sigma^-$

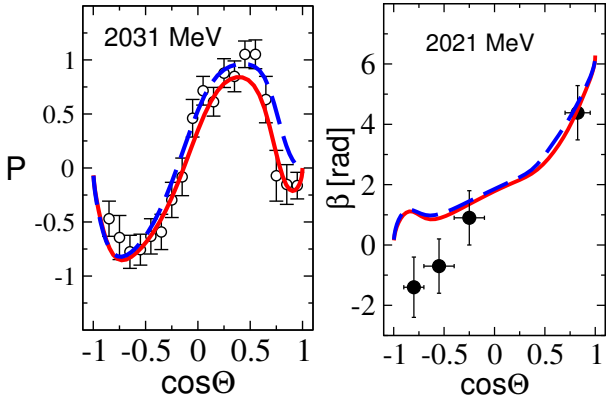


$\pi^+ p \to K^+ \Sigma^+$





No polarization data!

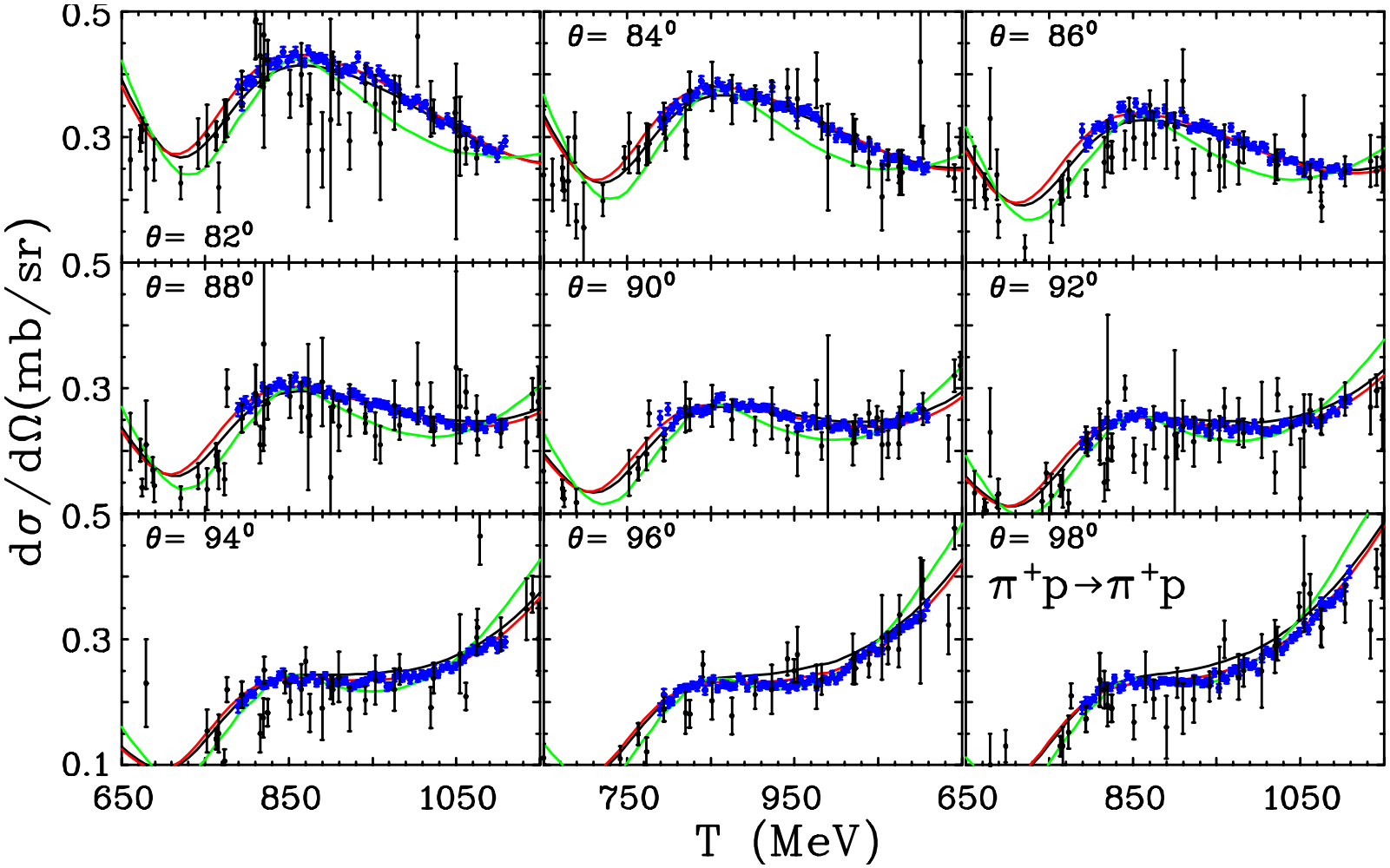


Full results

Physics Opportunities with Hadron Beams, arXiv:1503.07763

Improvement in Modern Experimental Facilities: $\pi N \to \pi N$

EPECUR & GWU/SAID, Alekseev *et al.*, PRC91, 2015



Black: WI08 prediction; Red: WI14 fit; green: KA84.

Data base of $\gamma p \to \pi^0 p,\ \pi^+ n$

D. Rönchen, M.D. *et al.*, EPJA50, 2014

	Fit 1	Fit 2
Line style		
# of data	21,627	23,518
Excluded data	$\underline{\pi^0 p}$: $E > 2.33$ GeV and $\theta < 40°$ for $E > 2.05$ GeV $\underline{\pi^+ n}$: $E > 2.26$ GeV and $\theta < 9°$ for $E > 1.60$ GeV	
$ds/d\Omega$, P, T	included	included
Σ	included (CLAS 2013 predicted)	included
$\Delta\sigma_{31}$, G, H	predicted	included
E, F, $C_{x'L}$, $C_{z'L}$	predicted	predicted

Complete experiment

one possibility:

σ, Σ, T, P, E, G, C_x, C_z

$\Rightarrow$ Influence of new double polarization observables on resonance parameters

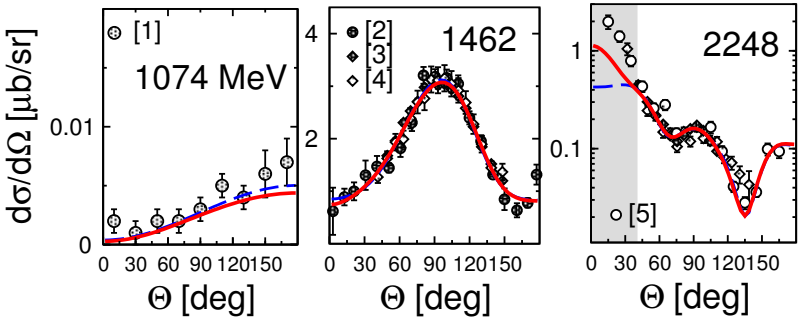
Minimization using parallelized code on JUROPA (grant from Jülich Supercomputing Centre)

Single polarization observables

selected results

- $d\sigma/d\Omega$

$\gamma p \to \pi^0 p$

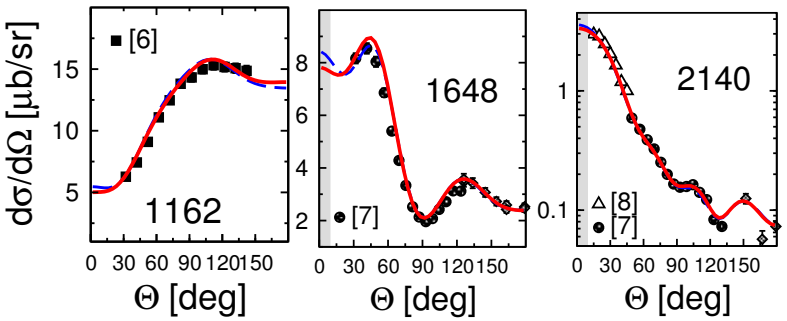


[1] Schmidt 2001 (MAMI) [2] Beck 2006 (MAMI)
[3] Dugger 2007 (JLab) [4] Bartholomy 2005 (ELSA)
[5] Credé 2011 (ELSA)

Polarization:

Beam	Target	Recoil
0	0	0

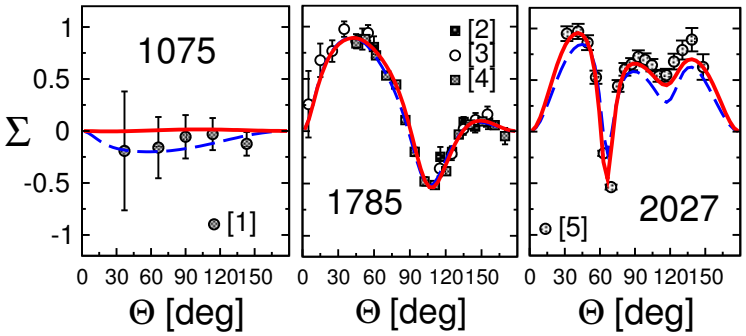
$\gamma p \to \pi^+ n$



[6] Ahrens 2004 (MAMI) [7] Dugger 2009 (JLab) [8] Buschhorn 1966

- Beam asymmetry Σ

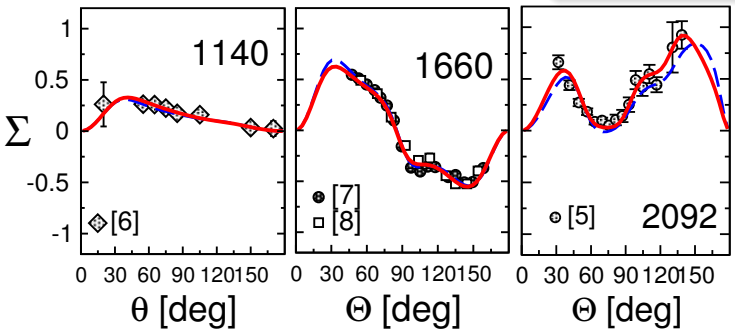
$\gamma p \to \pi^0 p$



Polarization:

Beam	Target	Recoil
⊥	0	0
∥	0	0

$\gamma p \to \pi^+ n$



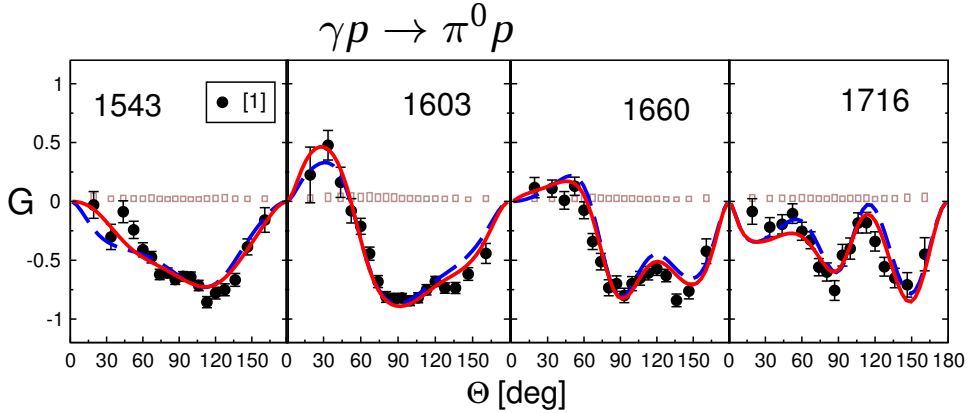
[1] Hornidge 2013 (MAMI)
[2] Elsner 2009 (ELSA)
[3] Sparks 2010 (ELSA)
[4] Bartalini 2005 (GRAAL)
[5] Dugger 2013 (JLab)
[6] Blanpied 2001 (LEGS)
[7] Bartalini 2002 (GRAAL)
[8] Ajaka 2000 (GRAAL)

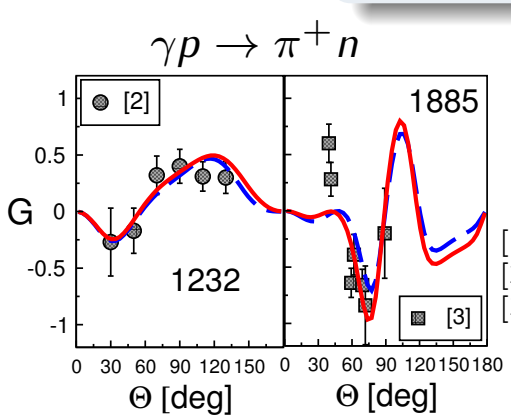
Double polarization observables

selected results

- G

Beam	Target	Recoil
$\perp'$	$-z$	0
$\|\|'$	$-z$	0



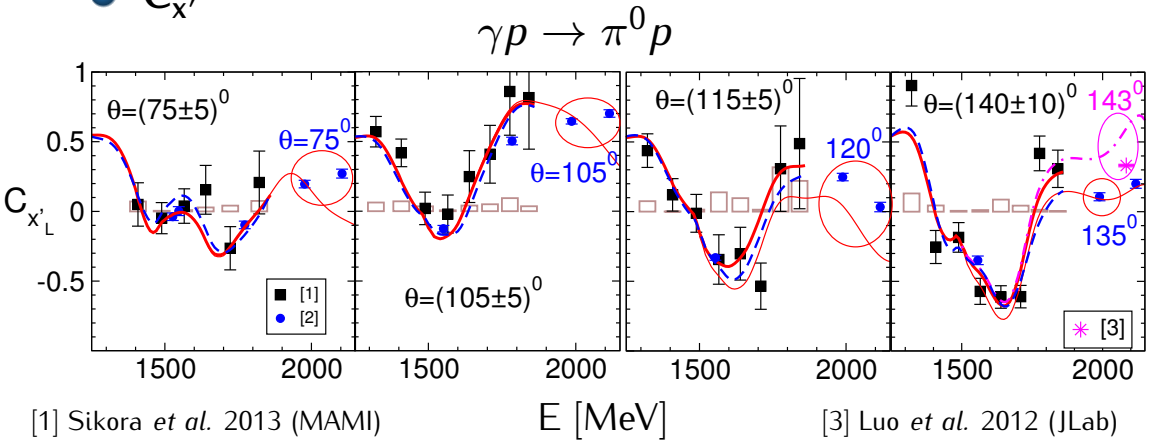


prediction
fit

[1] Thiel *et al.* 2012 (ELSA)
[2] Ahrens *et al.* 2005 (MAMI)
[3] Bussey *et al.* 1980

- $C_{x'}$

Beam	Target	Recoil
$+1$	0	$-x'$
-1	0	$-x'$



[1] Sikora *et al.* 2013 (MAMI)
[2] Wijesooriya *et al.* 2002 (JLab)
[3] Luo *et al.* 2012 (JLab)

Predictions of

- fit 1, angle averaged
- fit 2, angle averaged
- fit 2, **not** angle averaged
- fit 2, **not** angle averaged

Photocouplings at the pole

selected results

$$\tilde{A}^h_{pole} = A^h_{pole} e^{i\vartheta^h}$$
$$h = 1/2,\ 3/2$$

$$\tilde{A}^h_{pole} = I_F \sqrt{\frac{q_p}{k_p} \frac{2\pi\,(2J+1)\mathrm{E}_0}{m_N\ \mathrm{r}_{\pi\mathrm{N}}}}\ \mathrm{Res}\, A^h_{L\pm}$$

I_F: isospin factor
q_p (k_p): meson (photon) momentum at the pole
$J = L \pm 1/2$ total angular momentum
E_0: pole position
$r_{\pi N}$: elastic πN residue

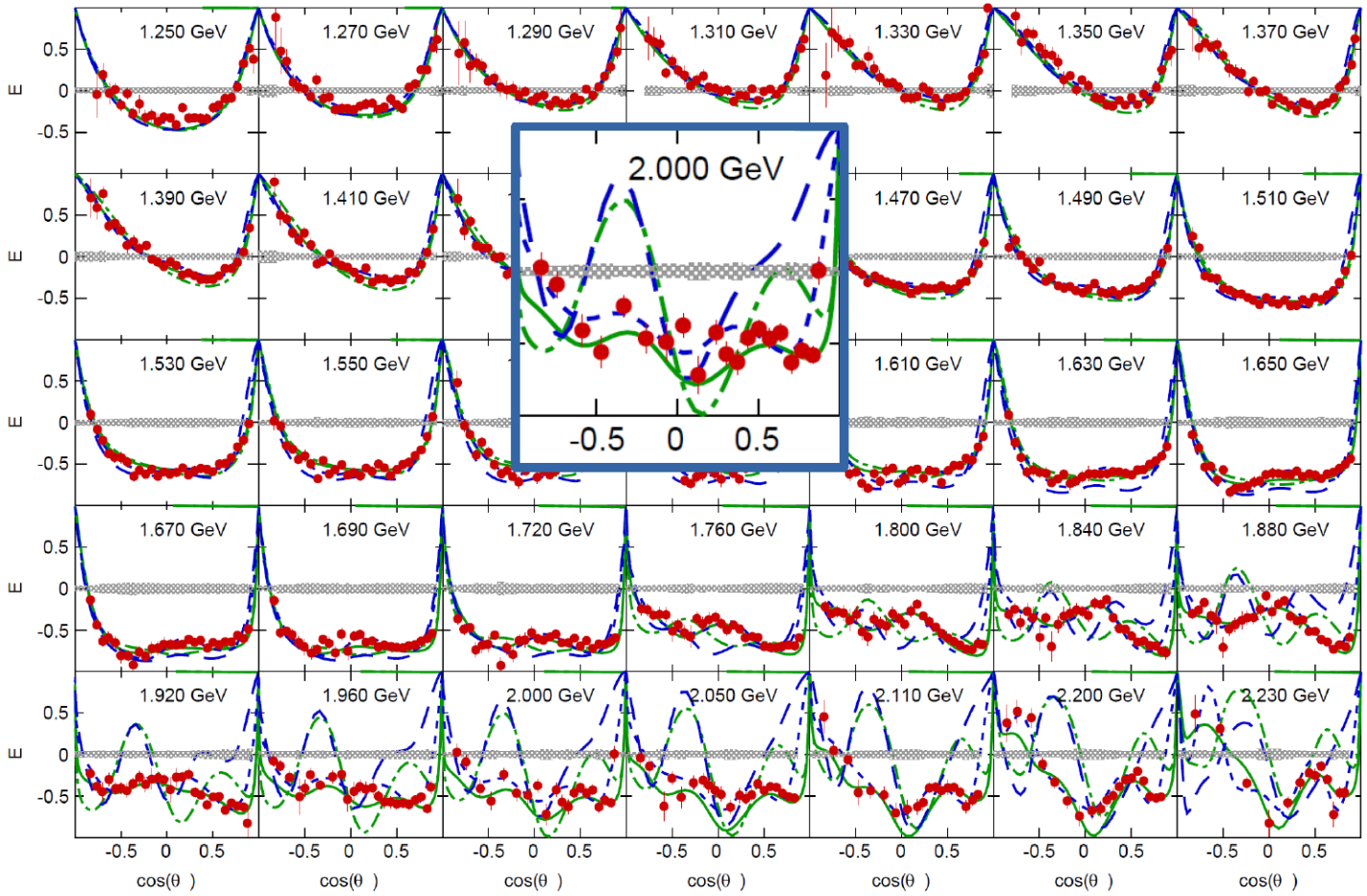
	$A^{1/2}_{pole}$ [10^{-3} GeV$^{-1/2}$]		$\vartheta^{1/2}$ [deg]		$A^{3/2}_{pole}$ [10^{-3} GeV$^{-1/2}$]		$\vartheta^{3/2}$ [deg]	
fit→	1	2	1	2	1	2	1	2
$N(1710)\ 1/2^+$	15	28^{+9}_{-2}	13	77^{+20}_{-9}				
$\Delta(1232)\ 3/2^+$	-116	-114^{+10}_{-3}	-27	-27^{+4}_{-2}	-231	-229^{+3}_{-4}	-15	$-15^{+0.3}_{-0.4}$

Fit 1: only **single** polarization observables included

Fit 2: also **double** polarization observables included

First FROST results: E in $\vec{\gamma}\vec{p} \to \pi^+ n$

PLB 750 (2015); data not preliminary any more.



Data: CLAS (USC/Strauch et al.); preliminary
SAID analysis (prediction and re-analysis)
Jülich Athens Washington (prediction and re-analysis)
→ Significant changes of helicity amplitudes $A^{1/2}$, $A^{3/2}$

Impact on resonance parameters

- A [10^{-3} GeV$^{-1/2}$]; ϑ [deg].
- Pole values (P) or Breit-Wigner Parameters (BW).
- ";" separates before and after new FROST E-measurement.

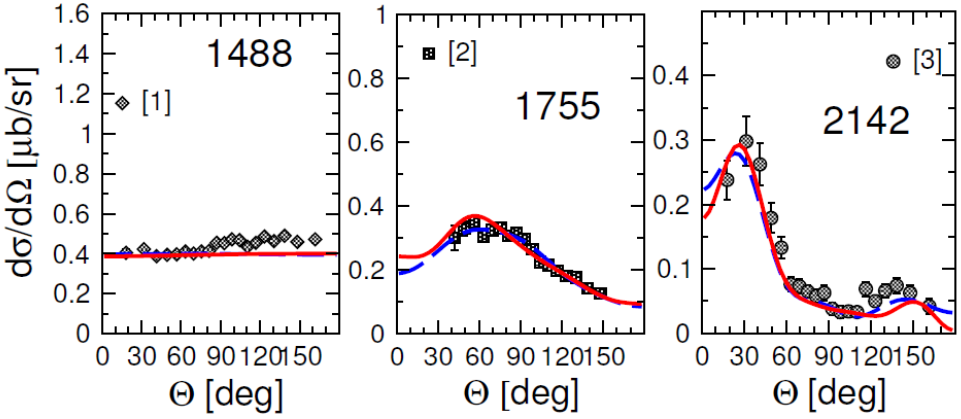
		$A^{1/2}_{pole}$	$\vartheta^{1/2}$	$A^{3/2}_{pole}$	$\vartheta^{3/2}$
$N(1440)\ 1/2^+$					
SAID	BW	**-60** ± 5; -56 ± 1			
Jülich	P	$-$**52**; -54	$-$**51**; -43		
BnGa	P	-44 ± 7	-38 ± 5		
ANL-Osaka	P	49	-10		
WTS	P	-66 ± 5	-38		
PDG	BW	-60 ± 4			
$N(1675)\ 5/2^-$					
SAID	BW	**10** ± 3; 13 ± 1		**16** ± 4; 16 ± 1	
Jülich	P	**28**; 22	**40**; 38	**63**; 36	$-$**19**; -41
BnGa	P	24 ± 3	-16 ± 5	26 ± 8	-19 ± 6
ANL-Osaka	P	5	-22	33	-23
PDG	BW	19 ± 8		15 ± 9	

Fit results $\gamma p \to \eta p$

selected results, preliminary

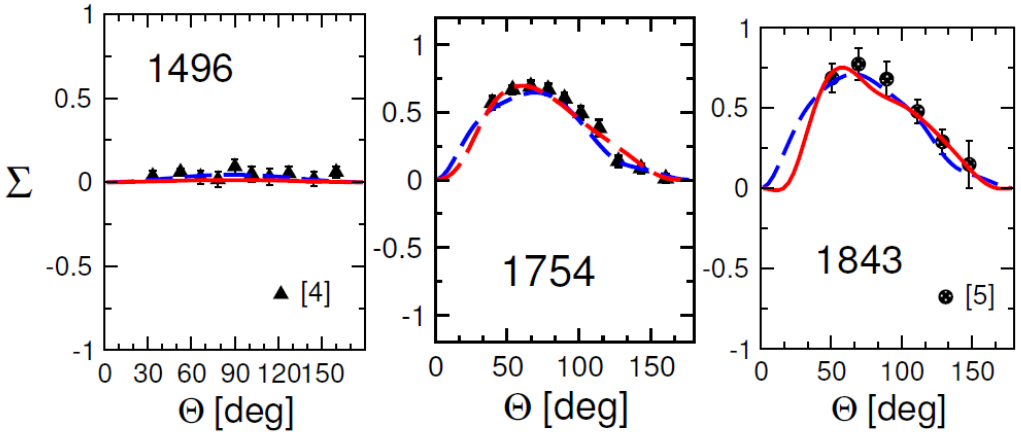
T, F not included — T, F included

- Differential cross section



[1] McNicoll *et al.* 2010 (MAMI), [2] Williams *et al.* 2009 (JLab), [3] Credé *et al.* 2009 (ELSA)

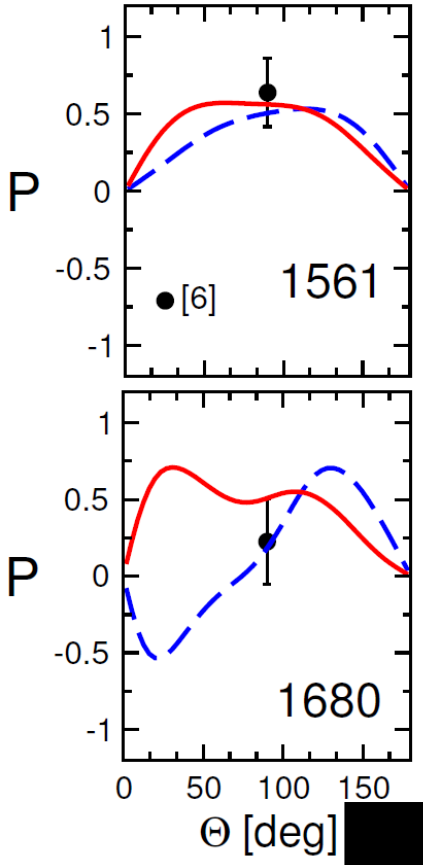
- Beam asymmetry



[4] Bartalini *et al.* 2007 (GRAAL), [5] Elsner *et al.* 2007 (ELSA)

- Recoil polarization

– only 7 data points in total –



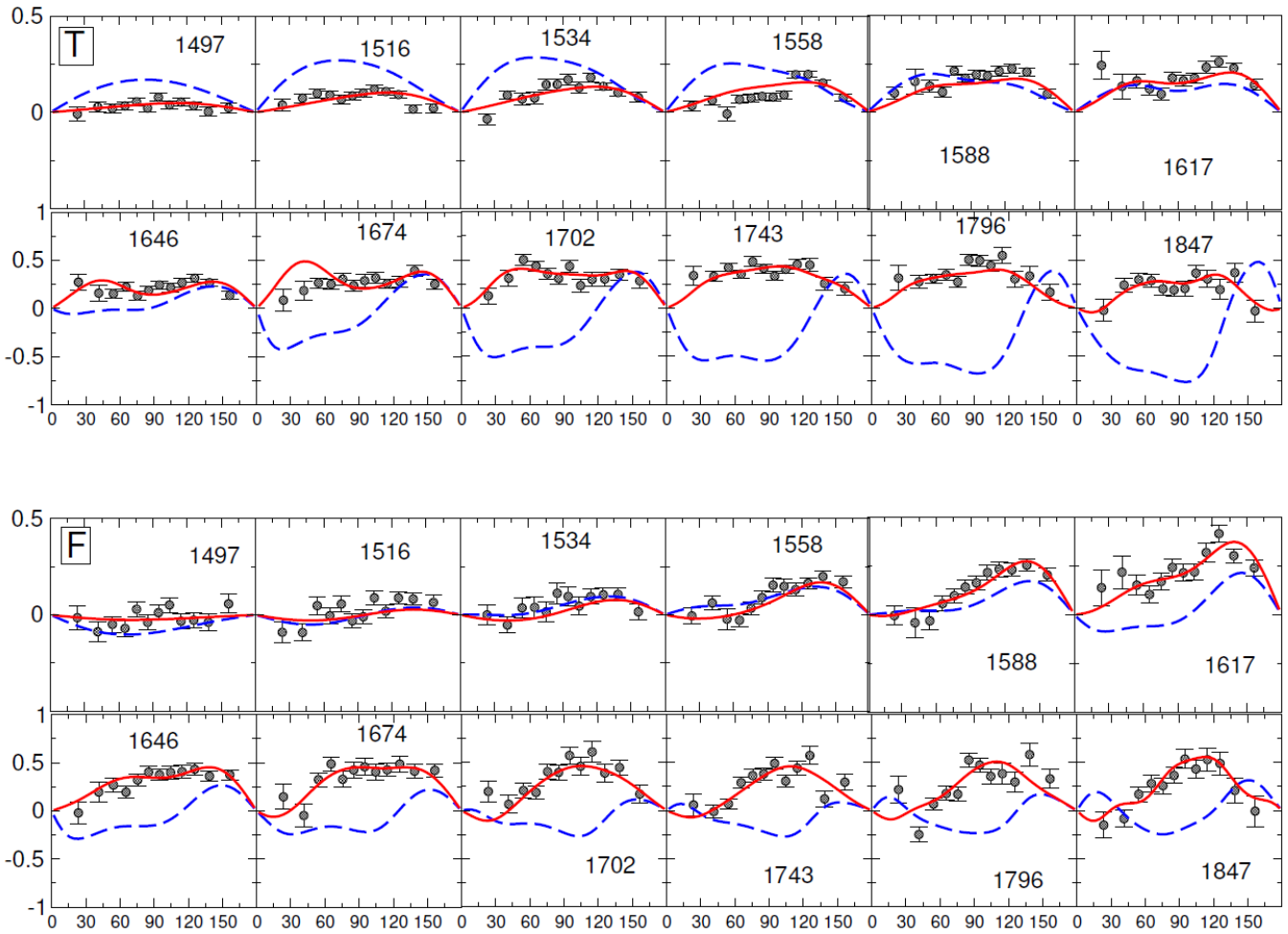
[6] Heusch *et al.* (Caltech), PRL25, 1381 (1970)

[redacted]

T and F in $\gamma p \to \eta p$ (MAMI)

EPJA 51, 6 (2015)

Data: Akondi *et al.* (A2 at MAMI) PRL 113, 102001 (2014)



prediction

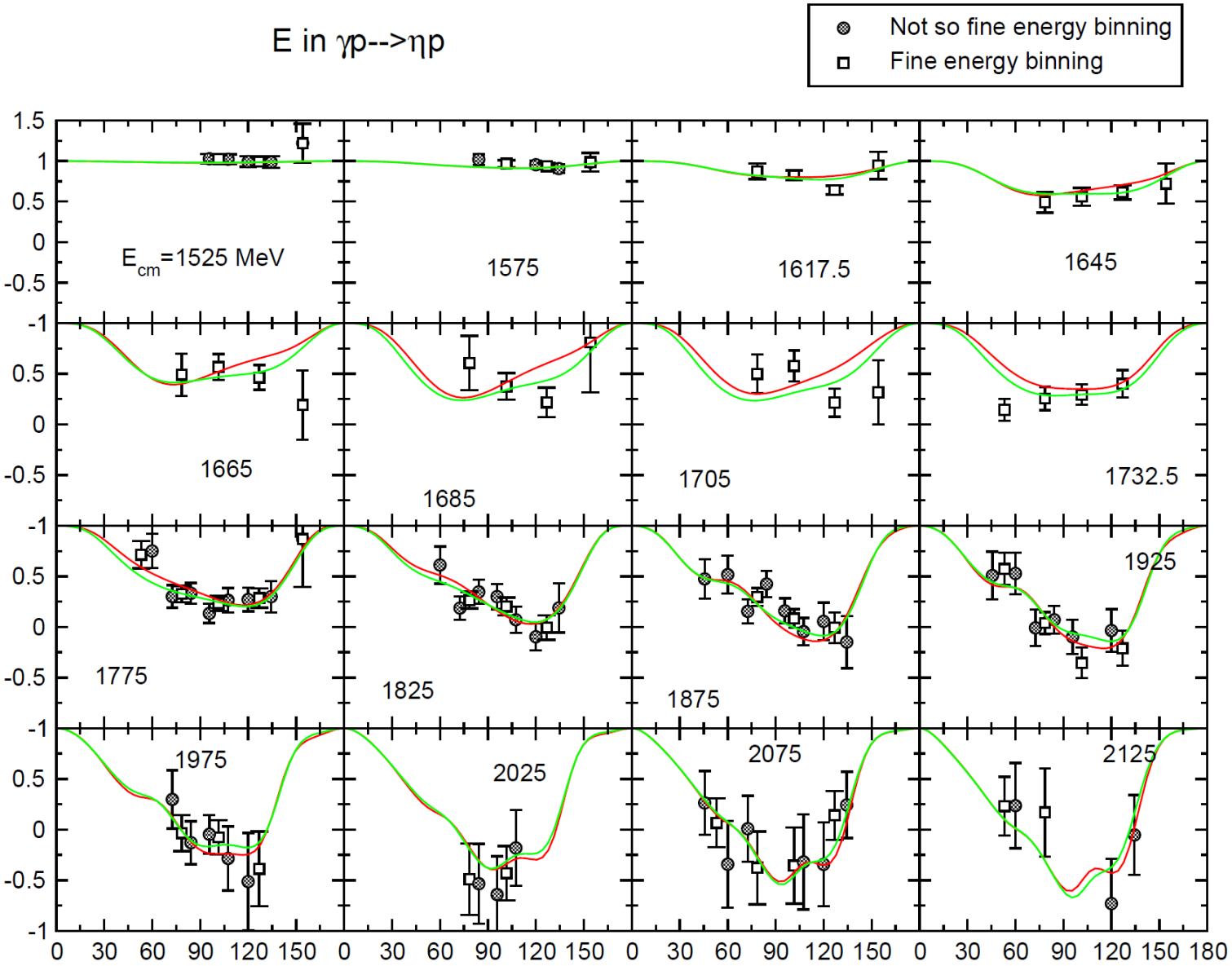
fit

Beam	Target	Recoil
0	$+y$	0
0	$-y$	0

Beam	Target	Recoil
$+1$	$+x$	0
-1	$+x$	0

FROST E in $\gamma p \to \eta p$

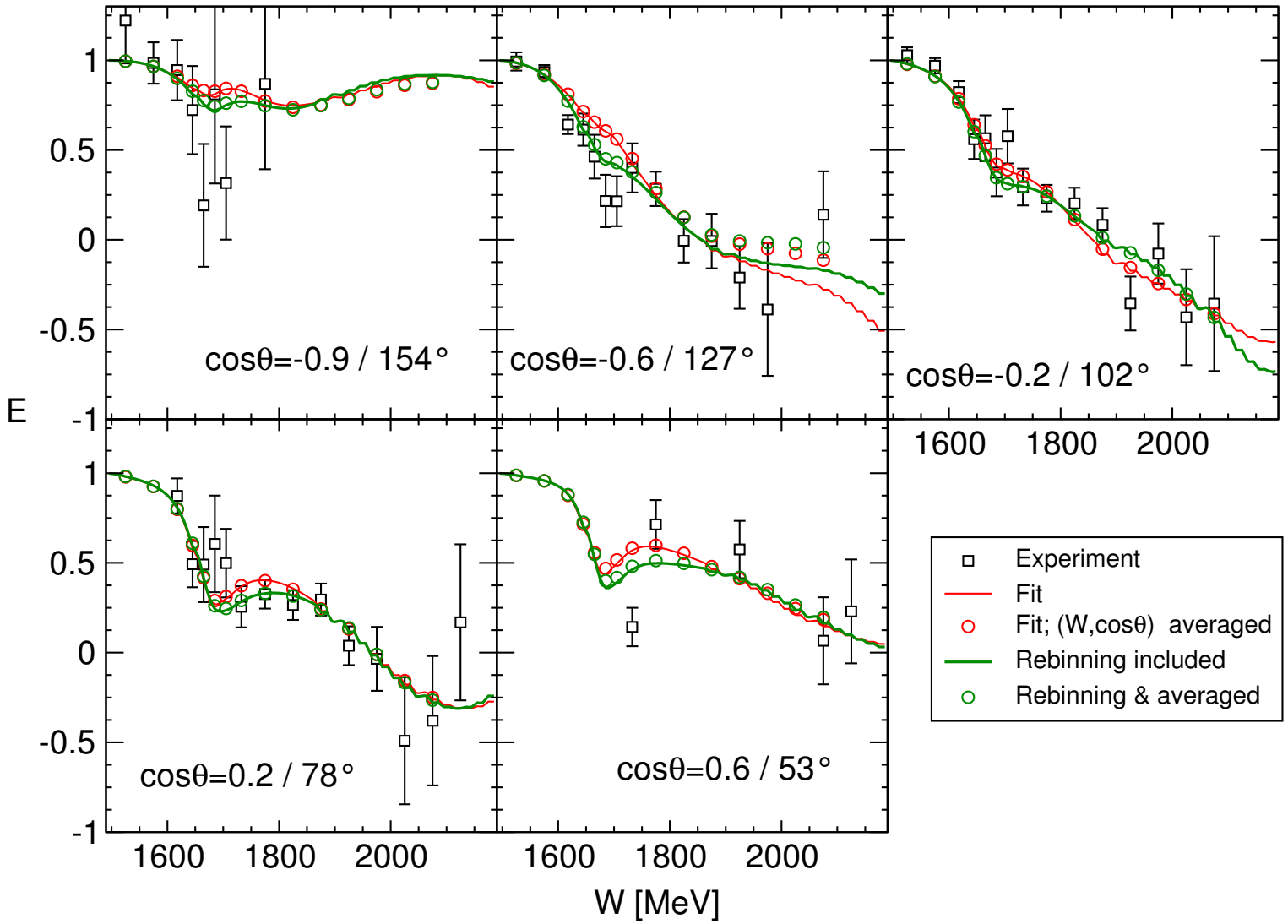
arXiv: 1507.00325



FROST E in $\gamma p \to \eta p$: Excitation function

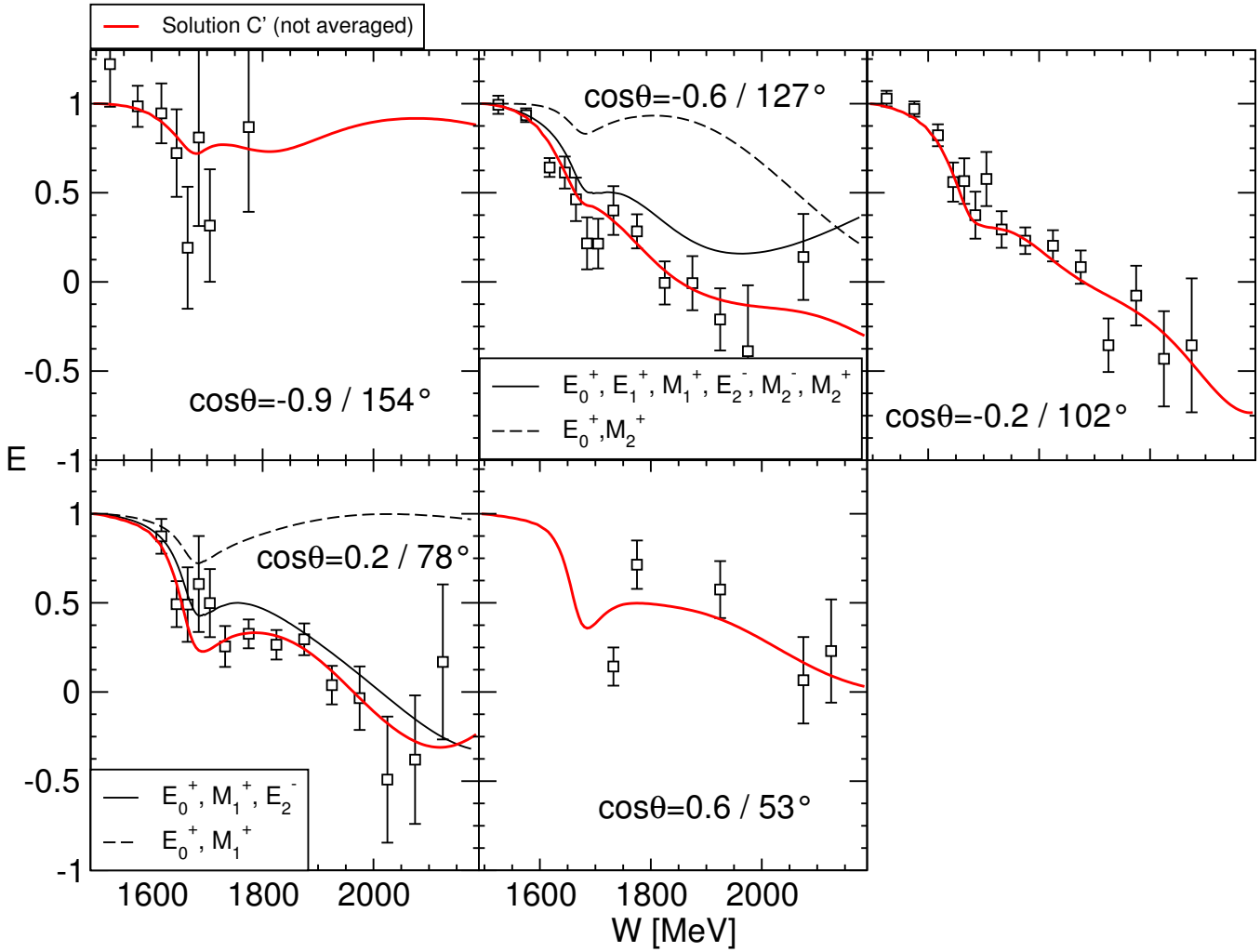
arXiv: 1507.00325

preliminary



Structure at $W = 1.68$ GeV: Conventional Resonances

arXiv: 1507.00325

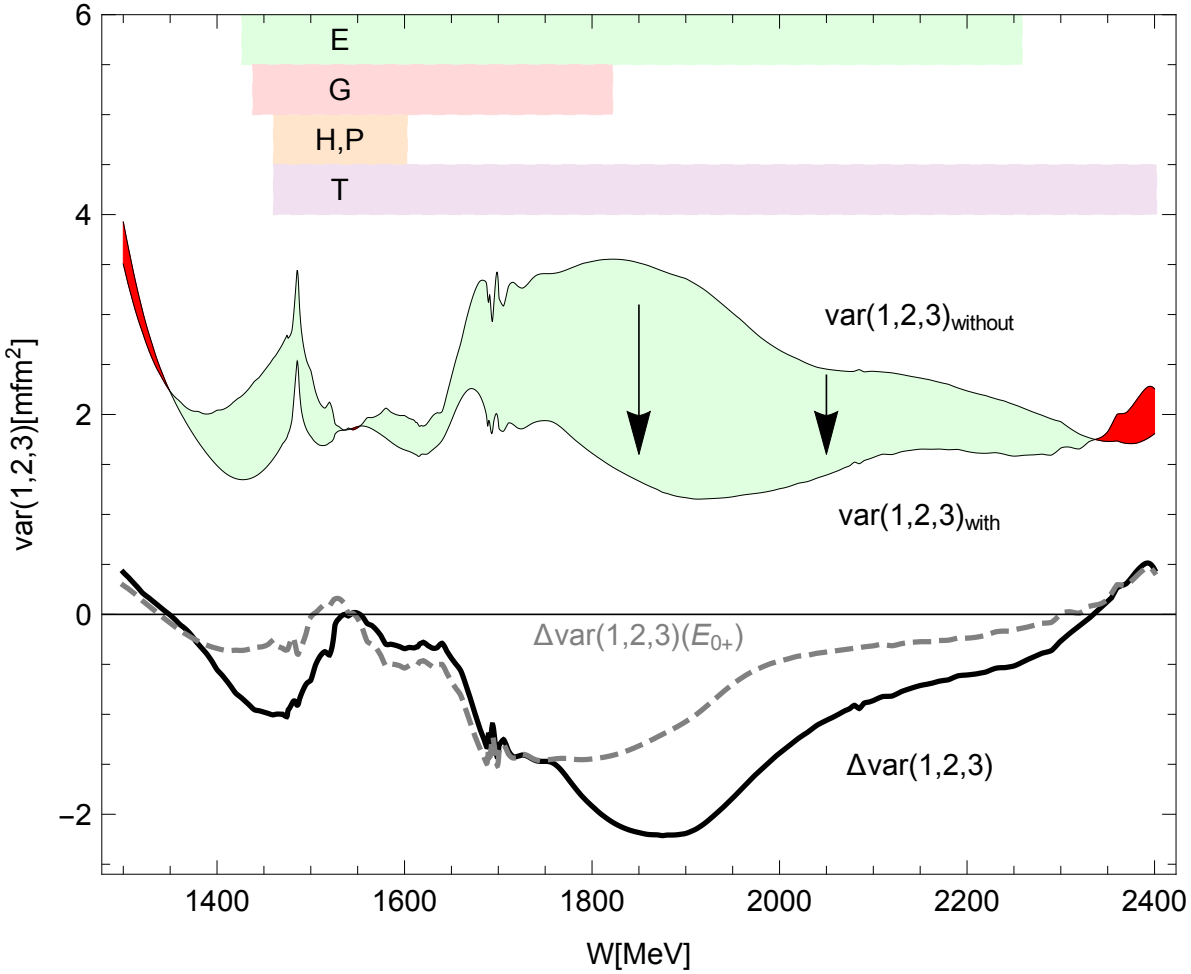


NO narrow state at $W = 1.68$ GeV needed.

Partial convergence of Partial Wave Analyses

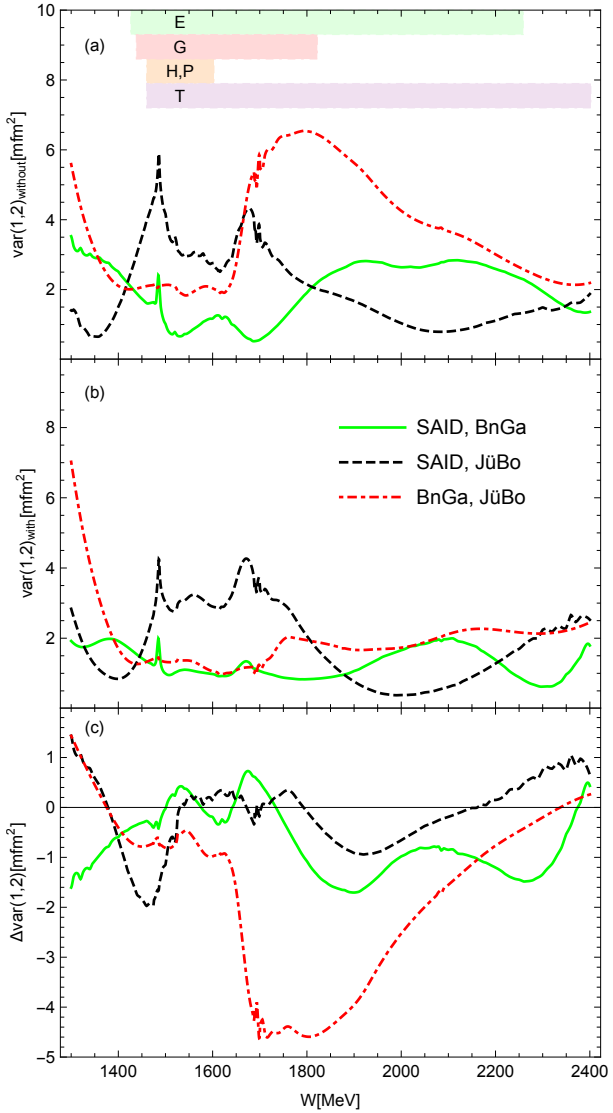
Before-after scenario for Jülich/Bonn, SAID, Bonn-Gatchina

Change in overall variance after including latest $\pi^0 p$ (and other) polarization measurements from ELSA (M. Gottschall, J. Hartmann, A. Thiel, 2015), MAMI (Hornidge 2015), JLab (Dugger, Luo, 2015).



Partial convergence of Partial Wave Analyses

Pairwise variances: (a) before, (b): after, (c): difference



Some questions for baryon resonance analysis

- How well does a certain observable constrain resonance properties? (taking into account data accuracy, and measured energy/angular range)
- How well would the measurement of a new observable further constrain and disentangle (correlations!) resonance properties?
- Are polarization measurements useful, despite the fact that much less accurate than cross section measurements?

Can these questions be addressed quantitatively?

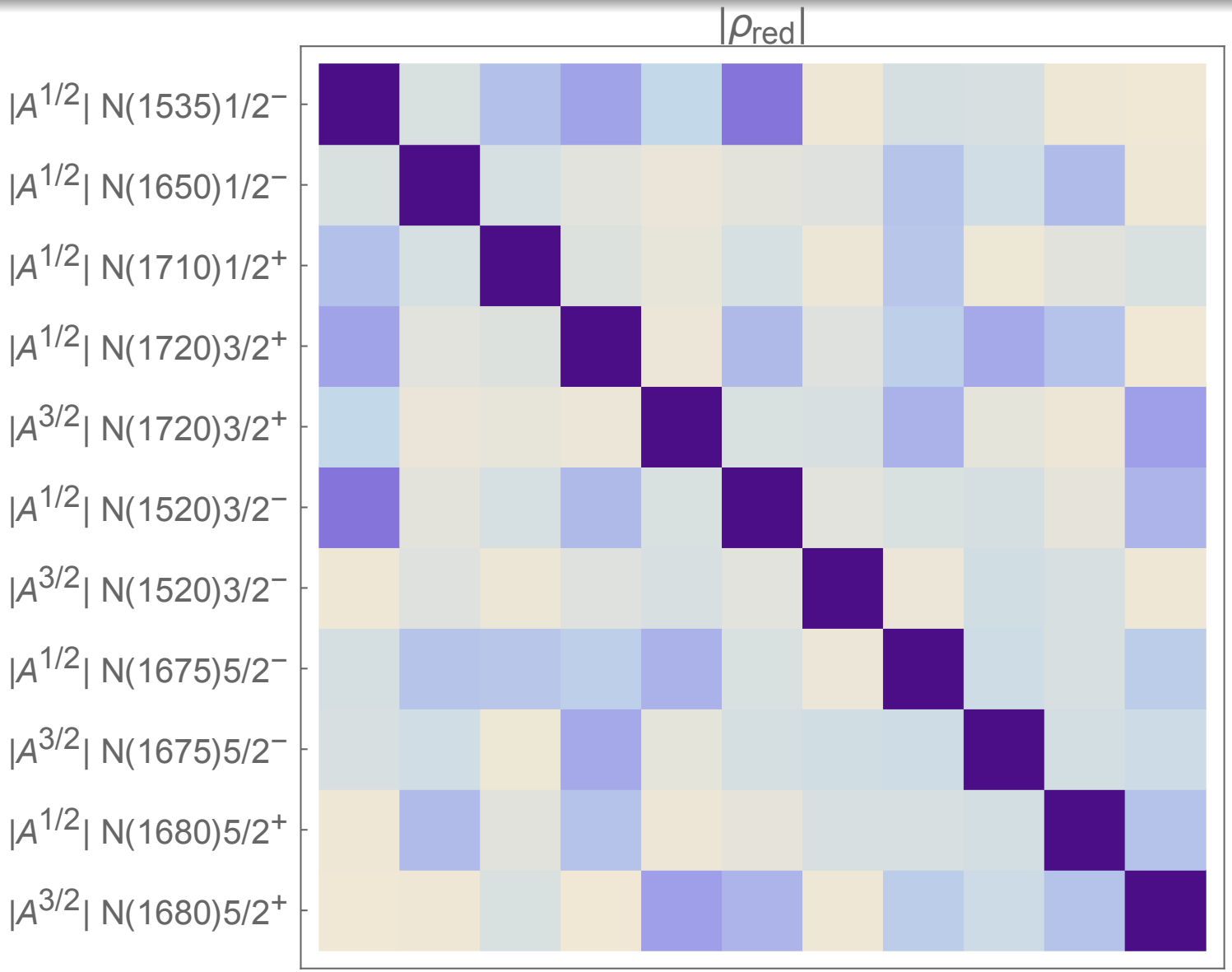
- Uncertainties in EM resonance couplings $\rightarrow$ entire covariance matrix to get full & meaningful picture,

$$\Sigma_{ij} = \frac{\partial^2 \chi^2}{\partial A_i \partial A_j}$$

  - A_i: coefficient of Laurent expansion around resonance pole (helicity coupling, modulus and phase, ...)
  - χ^2: the chi-square of a certain data set, e.g., E in $\gamma p \rightarrow \eta p$, or: the chisquare of entire class of final states (is η photoproduction needed, compared to pion photoproduction?), or: simulated data with realistic errors, angular, and energy coverage (e.g.: would C_X in $\gamma p \rightarrow \eta p$ be a good observable to disentangle resonance properties?)

Example: correlation matrix for E in $\gamma p \to \eta p$

based on discussed FROST data

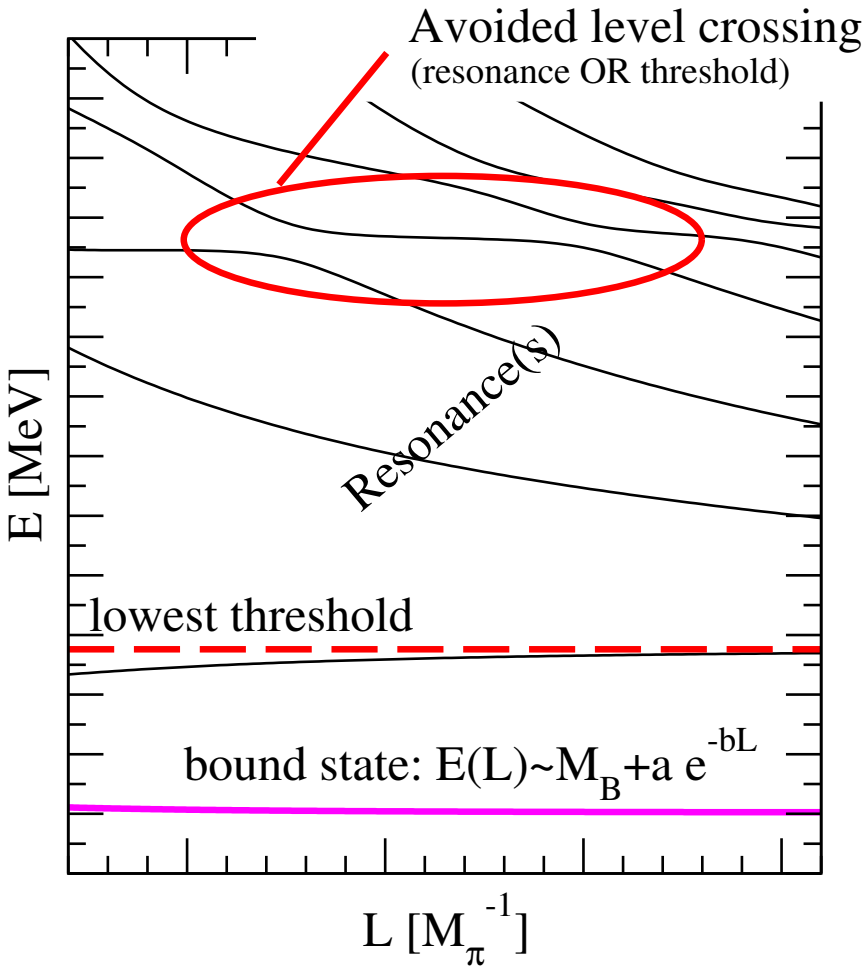


Figures of Merit (world data in $\gamma p \to \eta p$)

- Multicollinearity (resonance entanglement): $|\rho|$
  Comparison (smaller is better):
  $E : -\log|\rho| = 7.3$,
  $F : -\log|\rho| = 10.1$,
  $T : -\log|\rho| = 12$,
  $\Sigma : -\log|\rho| = 13$,
  $d\sigma/d\Omega : -\log|\rho| = 11$,
  all: $-\log|\rho| = 6.6$,
  $C_x : -\log|\rho| = 8.8$ (simulated).
- Overall quality: differential entropy (volume of the hyper-error-ellipse) $|\Sigma|$
  Comparison (smaller is better)
  $E : \log[|\Sigma|] = 98$,
  $F : \log[|\Sigma|] = 88$,
  ...
  $d\sigma/d\Omega$: $\log[|\Sigma|] = 11$,
  all: $\log[|\Sigma|] = 0.015$,
  $C_x : \log[|\Sigma|] = 62$ (simulated).
- Generalized variance,...

Resonances decaying on the lattice

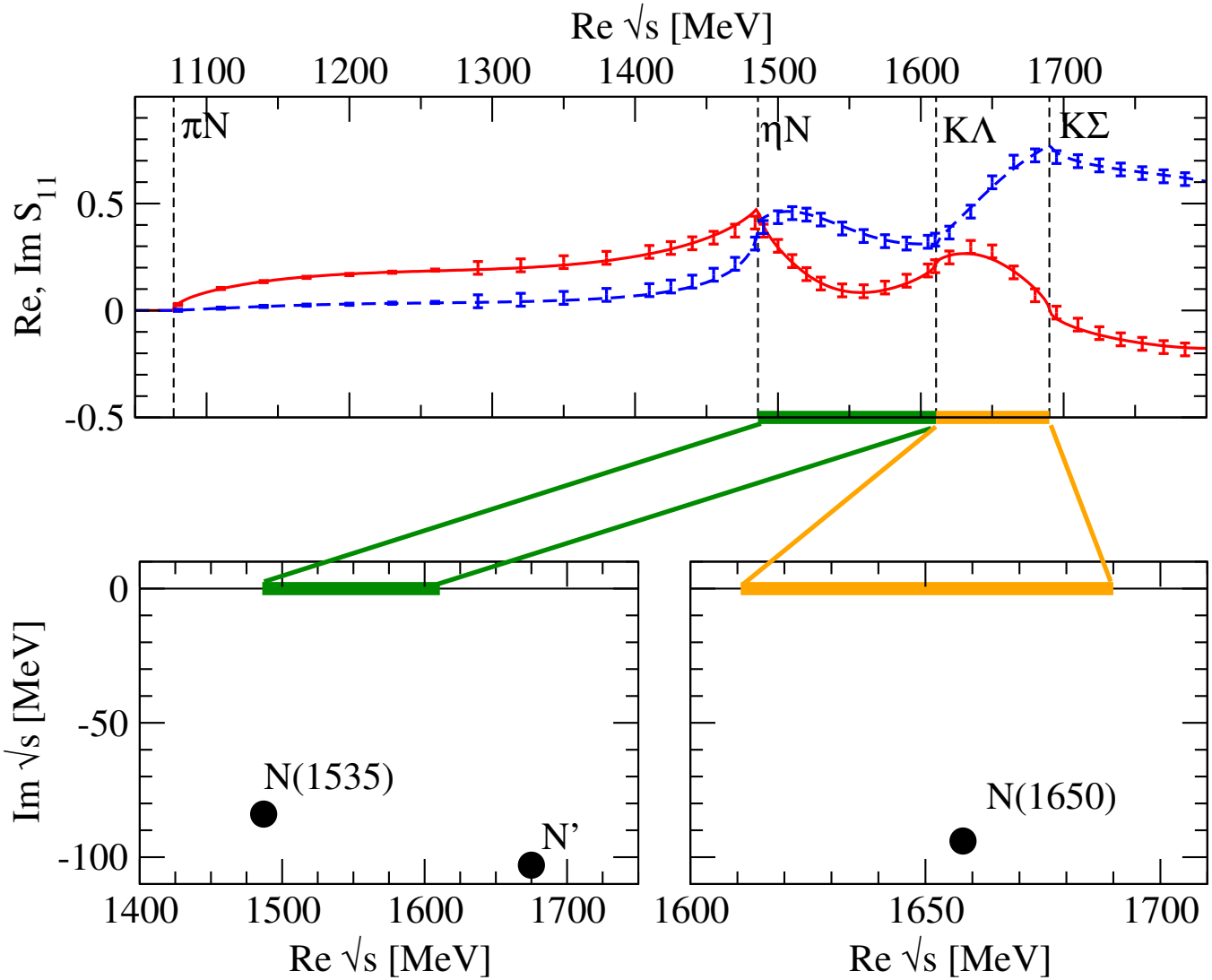
Eigenvalues in the finite volume



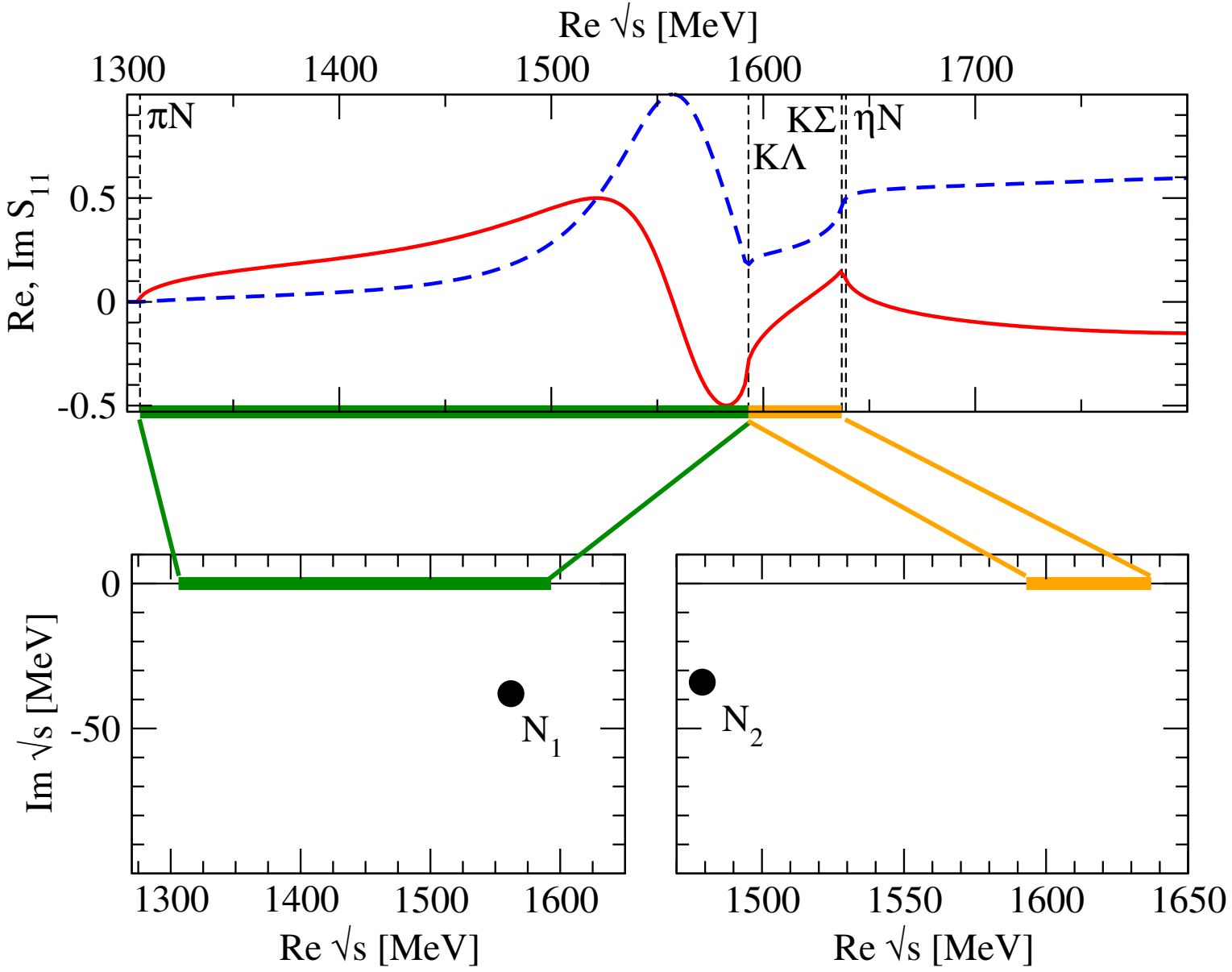
$J^P = 1/2^-$ in the finite volume

[M.D., Mai, Meißner, EPJA (2013)

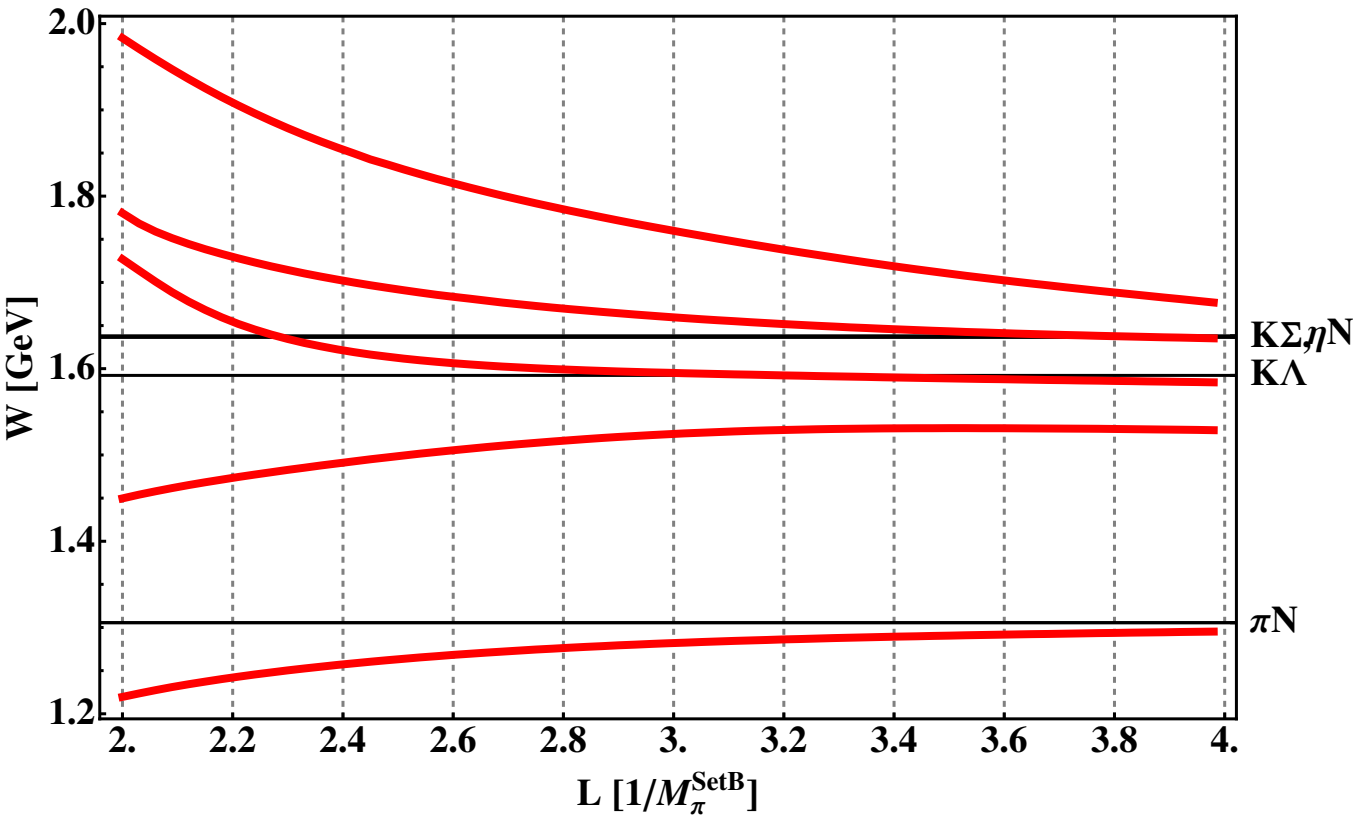
- Unitarized chiral interaction with NLO contact terms



Chiral extrapolation to a QCDSF lattice setup



Prediction of the lattice spectrum

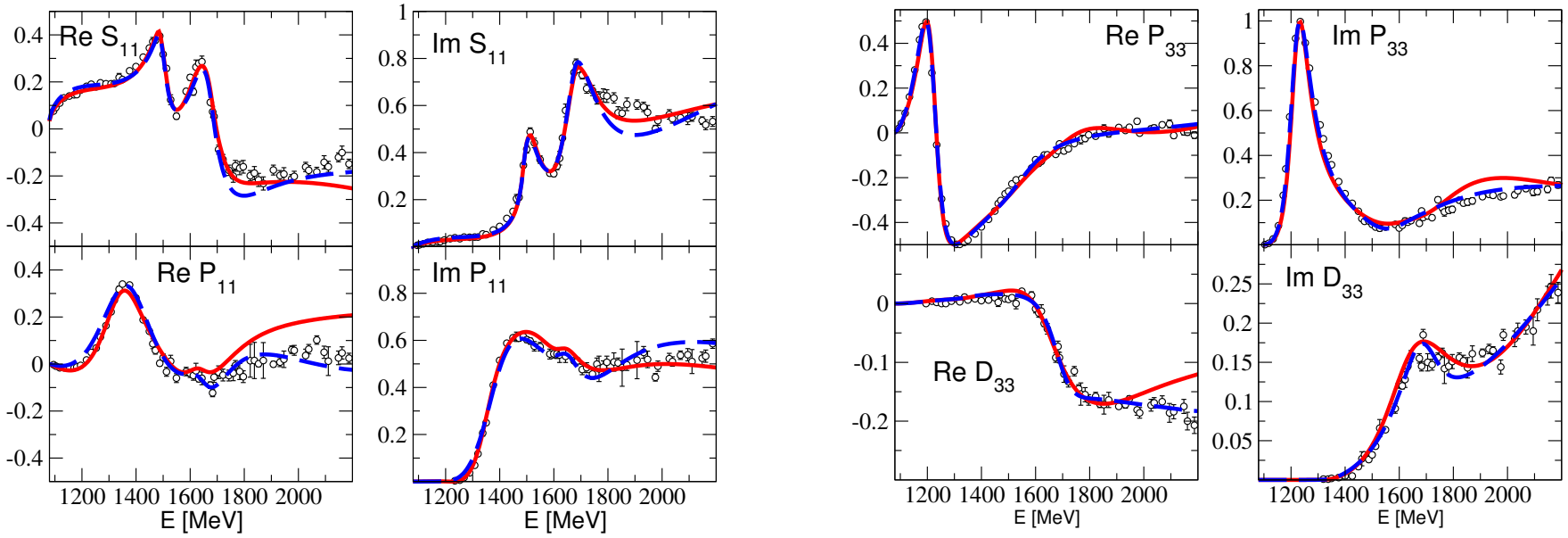


- No one-to-one mapping of levels to resonances → coupled channel analysis; hidden poles appear. Tomorrow: R. Molina on the Λ(1405) in the finite volume and comparison to recent lattice data.

Summary and Outlook

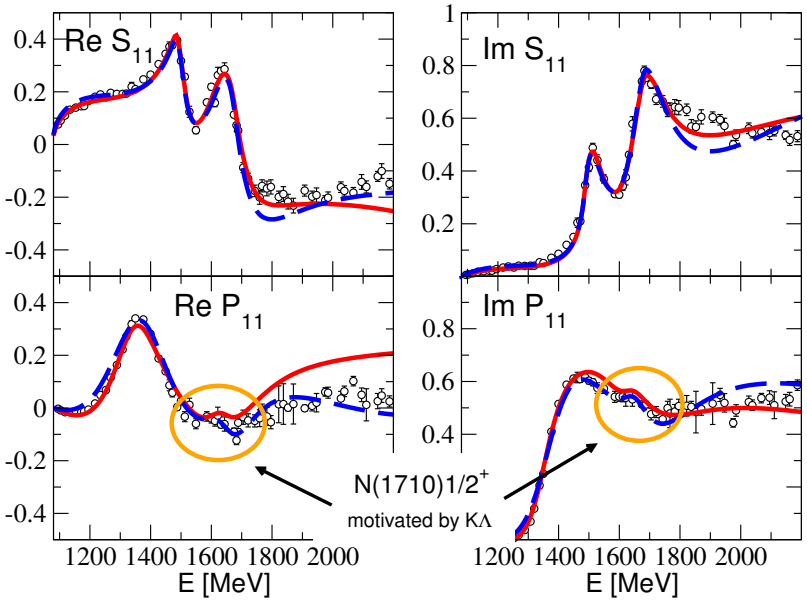
- Amplitude construction along general properties of the scattering amplitude
- Analysis of $\pi N \to \pi N$, ηN, $K\Lambda$ and $K\Sigma$; $\gamma N \to \pi N$, ηN.
- High-quality polarization observables from FROST@JLab start appearing; large impact on resonance spectrum and properties.
- Three-body effects cannot be ignored.
- Partial convergence between different PWAs observed as data become more complete.
- Correlations between electromagnetic resonance couplings can quantitatively address questions:
  - How well does a certain observable constrain resonance properties? (taking into account data accuracy, and measured energy/angular range)
  - How well would the measurement of a new observable further constrain and disentangle (!) resonance properties?
  - Are polarization measurements useful, despite the fact that much less accurate than cross section measurements? (Yes.)
- Connections to ab-initio Lattice QCD through extensions of the Lüscher formalism.

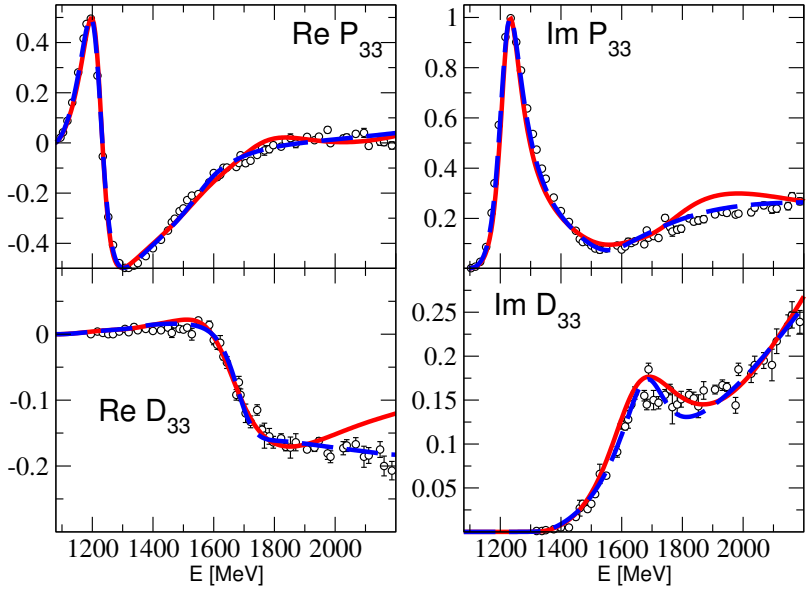
$\pi N \to \pi N$ partial wave amplitudes



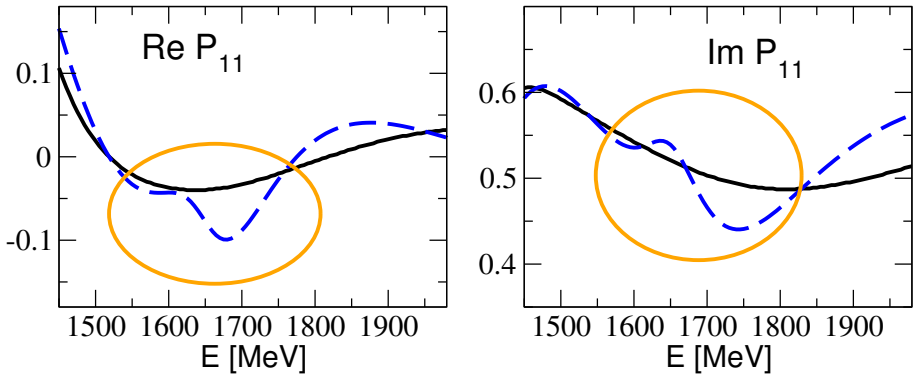
- Notation: $L_{2I\,2J}$
- Input to fit: energy-dependent partial wave analysis, GWU/SAID 2006
  up to $J = 9/2$ ($\sim H_{39}$)

$\pi N \to \pi N$ partial wave amplitudes



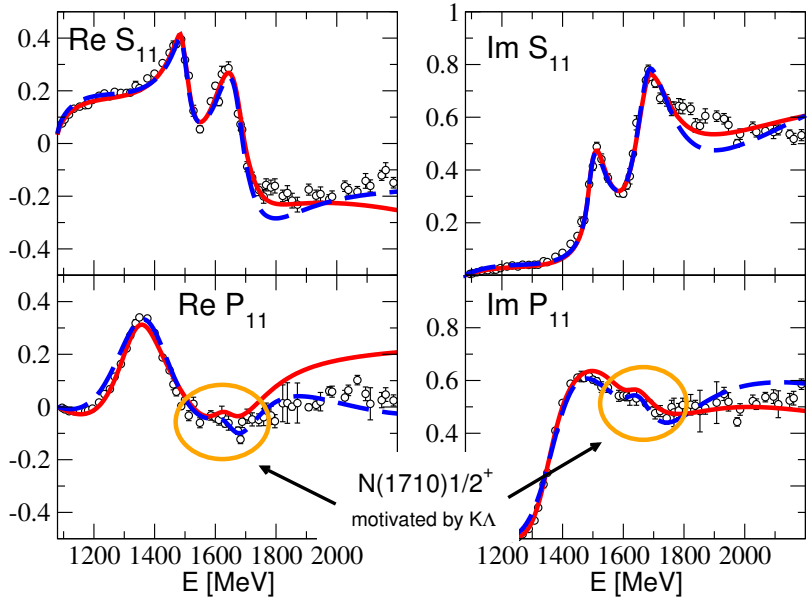


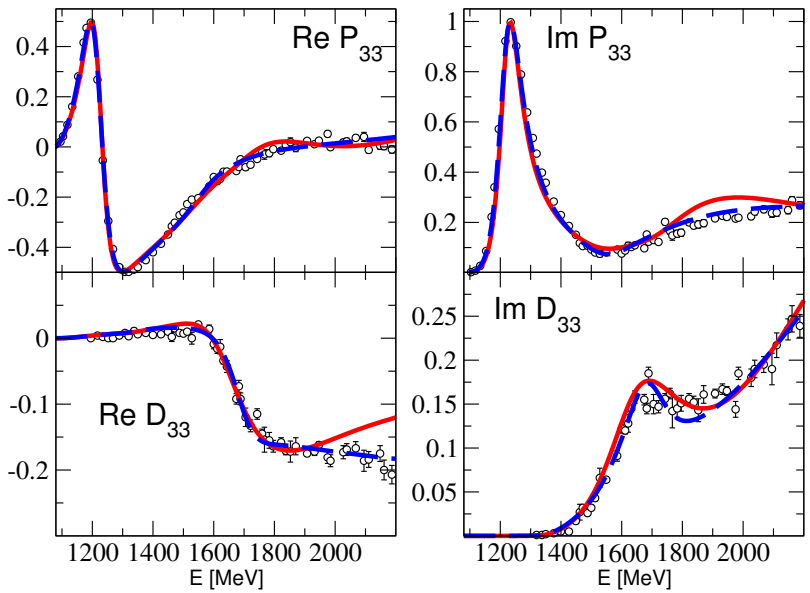
Genuine $N(1710)1/2^+$:



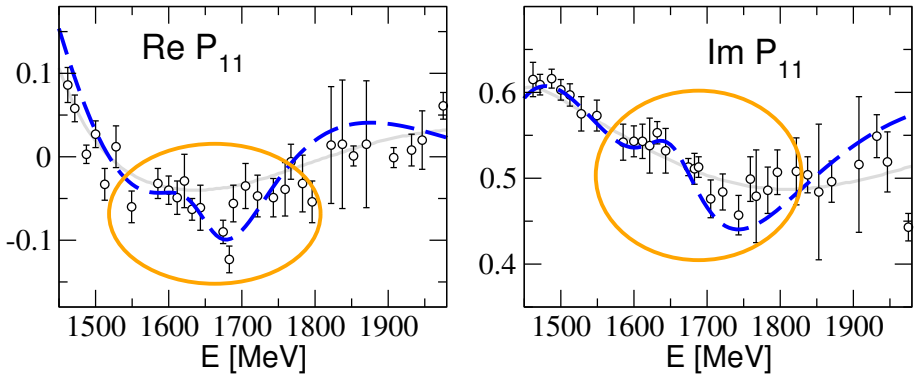
- Black line: input to fit (energy-dependent solution)
- Inclusion of $P_{11}(1710)$ necessary to improve $K\Lambda$
- Coupled-channels essential (Fit to πN observables)

$\pi N \to \pi N$ partial wave amplitudes





Genuine $N(1710)1/2^+$:



- Black line: input to fit (energy-dependent solution)
- Inclusion of $P_{11}(1710)$ necessary to improve $K\Lambda$
- Coupled-channels essential (Fit to πN observables)

Photoproduction of pseudoscalar mesons [D. Rönchen, M.D., *et. al.*, Eur. Phys. J. A (2014) 50: 101]

- Photocouplings of resonances
- high precision data from ELSA, MAMI, JLab...$\rightarrow$ resolve questionable/find new states

Photoproduction amplitude of pseudoscalar mesons:

$$\hat{\mathcal{M}} = F_1 \vec{\sigma} \cdot \vec{\epsilon} + iF_2 \vec{\epsilon} \cdot (\hat{k} \times \hat{q}) + F_3 \vec{\sigma} \cdot \hat{k}\hat{q} \cdot \vec{\epsilon} + F_4 \vec{\sigma} \cdot \hat{q}\hat{q} \cdot \vec{\epsilon}$$

$\vec{q}$: meson momentum
$\vec{k}$ ($\vec{\epsilon}$): photon momentum (polarization)
F_i: complex functions of the scattering angle

$\Rightarrow$ 16 polarization observables:
asymmetries composed of **beam**, **target** and/or **recoil** polarization measurements

$\Rightarrow$ 8 carefully selected observables: *complete experiment*
$\hookrightarrow$ Caveat: in reality more observables needed (data uncertainties)

Focus of the present analysis:

extraction of electromagnetic resonance parameters

$\Rightarrow$ flexible, "model-independent" parameterization of photo excitation

- Advantage: easy to implement, analyze large amounts of data
- Disadvantage: no information on microscopic reaction dynamics

$\Rightarrow$ intermediate step towards a combined DCC analysis of pion- and photon-induced reactions

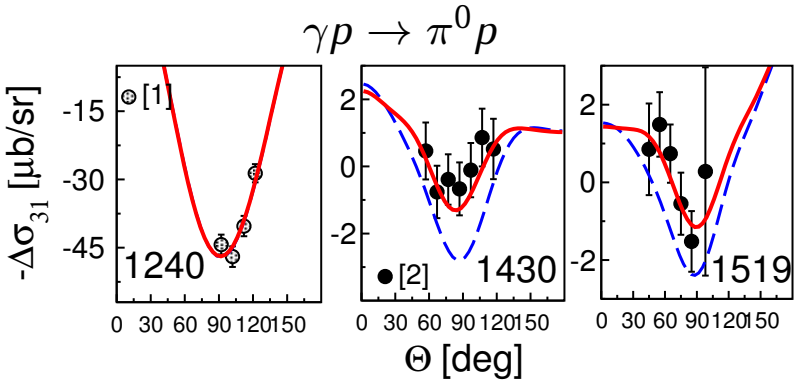
Double polarization observables

selected results

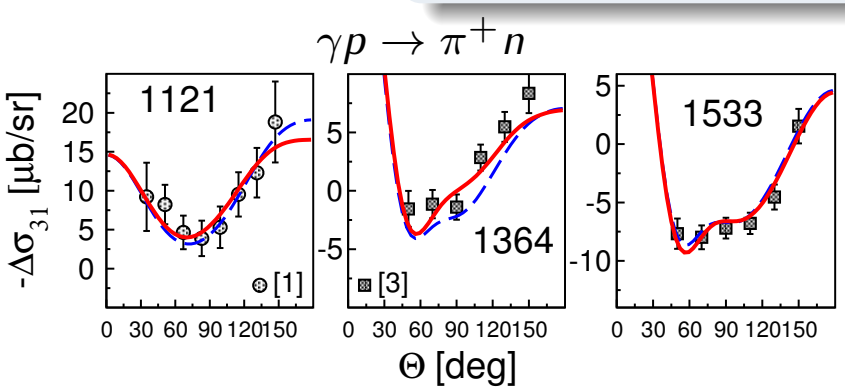
- $\Delta\sigma_{31}$ $\qquad \Delta\sigma_{31} = -2\frac{d\sigma}{d\Omega}E$

prediction
fit

Beam	Target	Recoil
$+1$	$-z$	0
-1	$-z$	0



[1] Ahrens *et al.* 2004 (MAMI) [2] Ahrens *et al.* 2002 (MAMI)

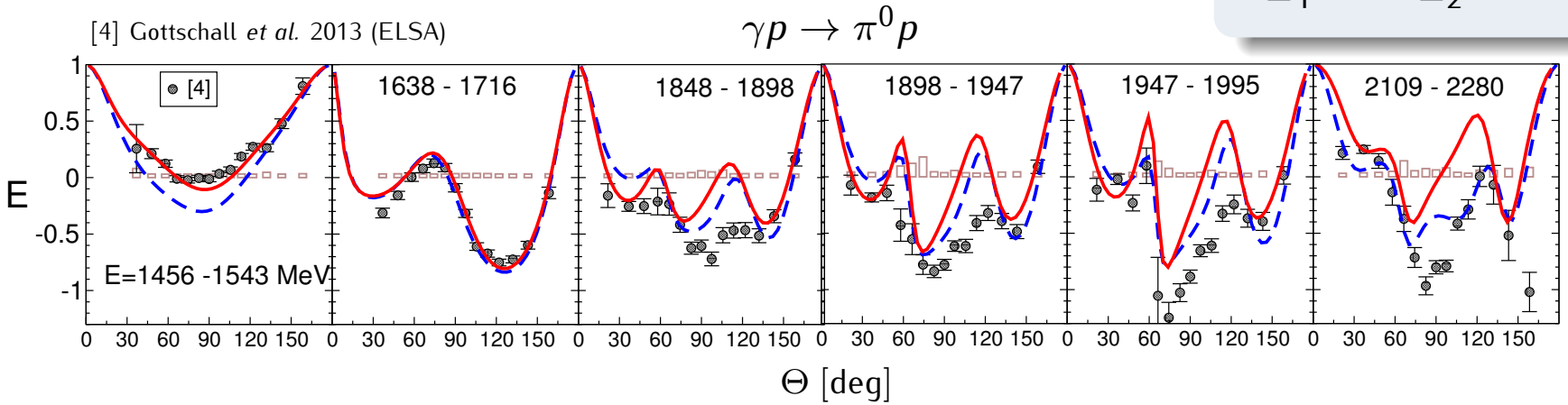


[3] Ahrens *et al.* 2006 (MAMI)

- Predictions of E

Beam	Target	Recoil
$+1$	$-z$	0
-1	$-z$	0

[4] Gottschall *et al.* 2013 (ELSA)



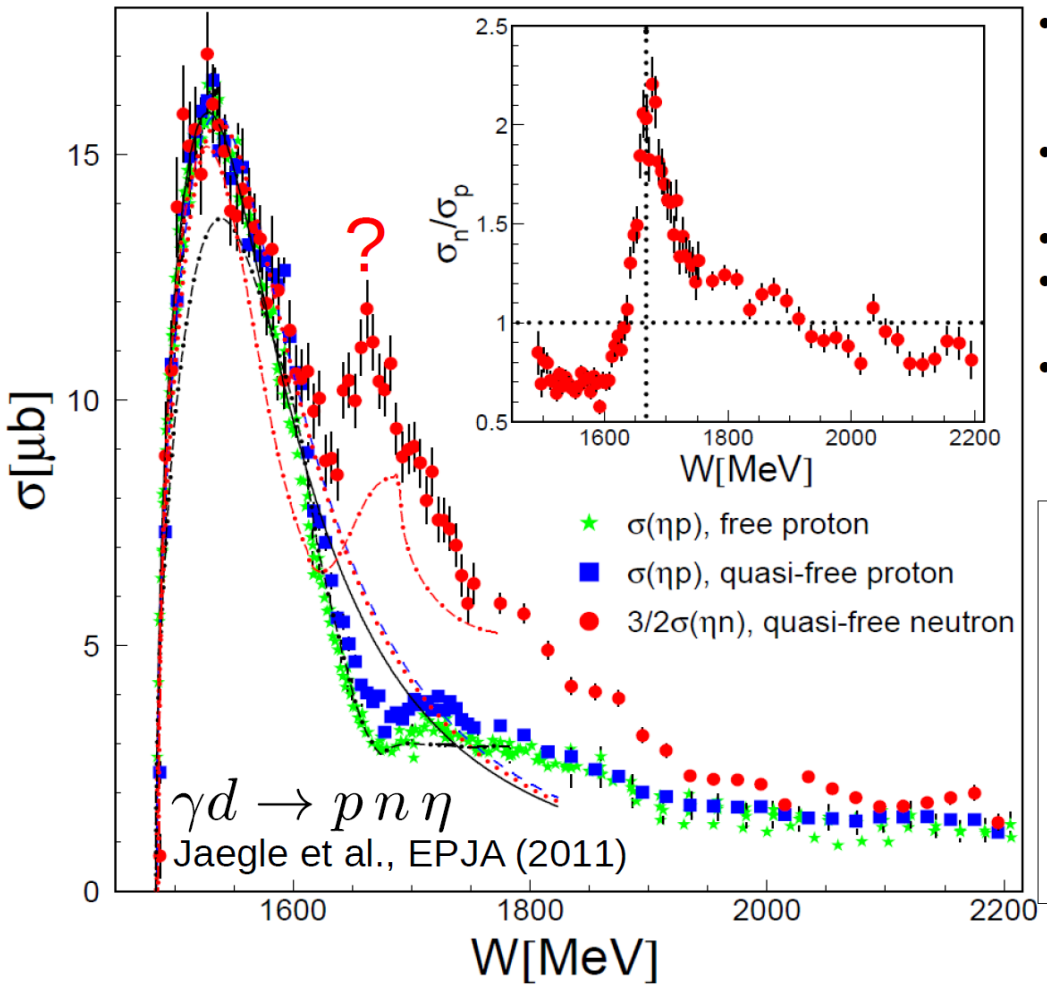
Challenges in Baryon Analysis

- Set common ground for different analyses - definite answers needed!
- Better $\pi N \to \pi N, \pi\pi N\, \eta N$, KY data to improve analysis of photo- and electroproduction.
- Consistent data $\to$ Get rid of weighting data sets in fits (otherwise all statistical meaning lost).
- GWU/SAID: Provide correlations so other group can carry out statistically meaningful analyses.
- Answer the question: How sensitive are helicity couplings to the new set of XY data? Maybe: Eigenvectors ordered by eigenvalues of the inverse correlation matrix

$$\frac{1}{2}\left(\frac{\partial^2 \chi^2_{XY}}{\partial A_i \partial A_j}\right) \tag{1}$$

  A_i: i-th helicity coupling $|A|$ or complex phase θ ($A = |A| \exp(i\theta)$) at the pole.
- Approaching the precision frontier in baryon analysis:
  Joint effort by GWU/INS SAID, Jülich, JPAC.

Phenomenological challenges: bump in "$\gamma n \to \eta n$" (MAMI)

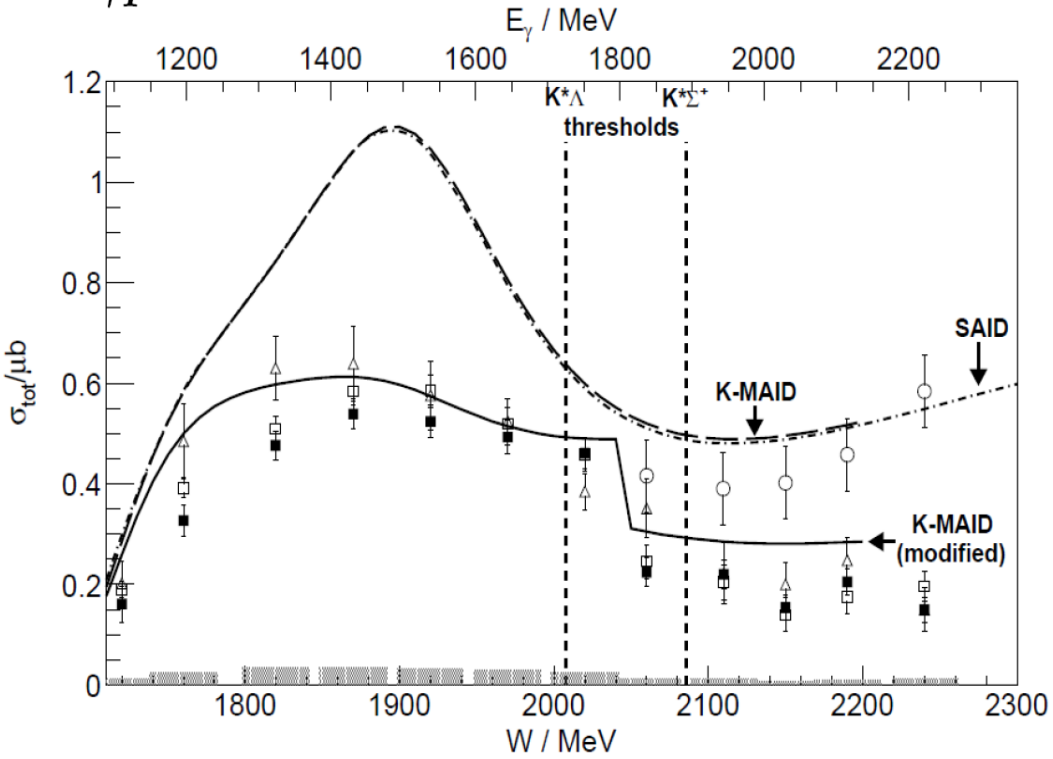


- "*N(1685)*": large couplings to γn, ηn
- Pentaquark? [Prediction Polyakov et al.]
- Unusual combination of neutron
- helicity couplings? [BnGa]
- Coupled channel chiral dynamics from $K\Lambda$, $K\Sigma$? [M.D. et al.]

- MAMI: Especially designed for neutral final states.
- Excellent energy resolution & angular coverage
- $\pi^0 p$ final state needed for isospin decomposition in PWA; complementary to CLAS ($\pi^+ n$).
- Strong **involvement** of US physicists (GWU, KSU,...)

Phenomenological challenges: $K\Sigma$ photoproduction (ELSA)

$$\gamma p \to K^0 \Sigma^+$$



CBELSA/TAPS [PLB 2012]

- Sudden drop at W~2.05 GeV
- Dynamic effect
  [Ramos/Oset, PLB 2013]?
- CLAS, MAMI, ELSA: Strong complementary experimental program
- Joint data essential for partial wave analysis
- Still surprises in meson photoproduction.

Analysis of pion-induced reactions

Eur.Phys.J. A49 (2013) 44, Nucl.Phys. A851 (2011) 58-98

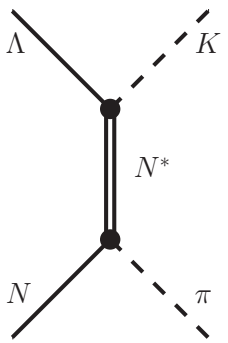
- calculate observables from T-matrix
- fit **free parameters** of T to data or partial wave amplitudes

$$\sigma = \frac{1}{2}\frac{4\pi}{p^2}\sum_{JLS,L'S'} |\tau_{LS}^{JL'S'}|^2$$

with $\tau_{fi} = -\pi\sqrt{\rho_f \rho_i}\, T_{fi}$

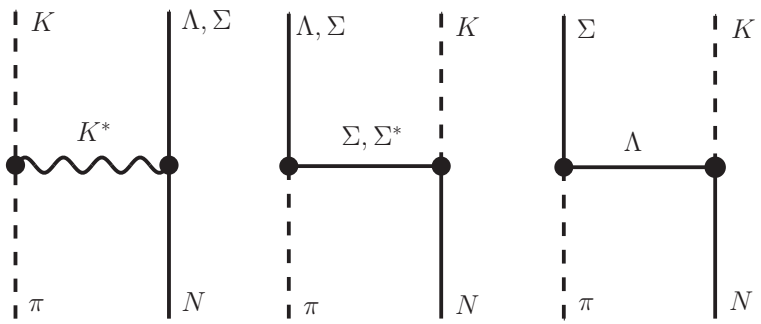
ρ: phase factor

s-channel: resonances (T^P)



$m_{bare} + f_{\pi NN^*}$

t- and u-channel exchange: "background" (T^{NP})



cut offs Λ in form factors $\left(\frac{\Lambda^2 - m_{ex}^2}{\Lambda^2 + \vec{q}^2}\right)^n$

(couplings fixed from SU(3))

$\Rightarrow$ search for poles in the complex energy plane of T

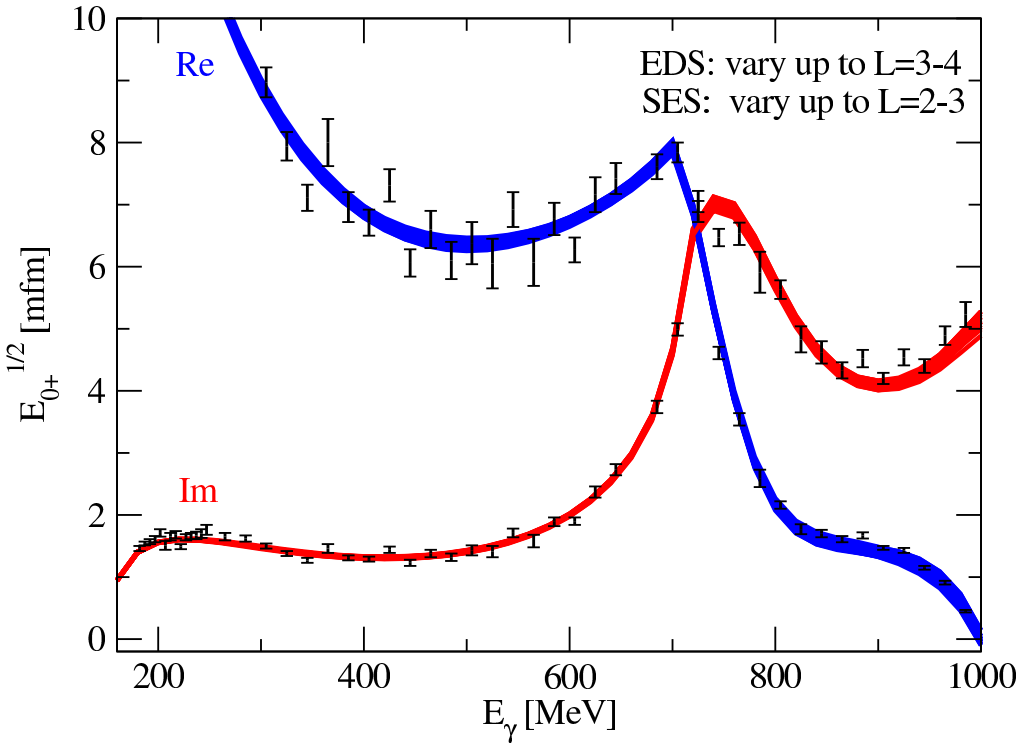
All exchange diagrams | data base

Putting different analyses on common ground

... and have eventually convergence of results among different groups (?)

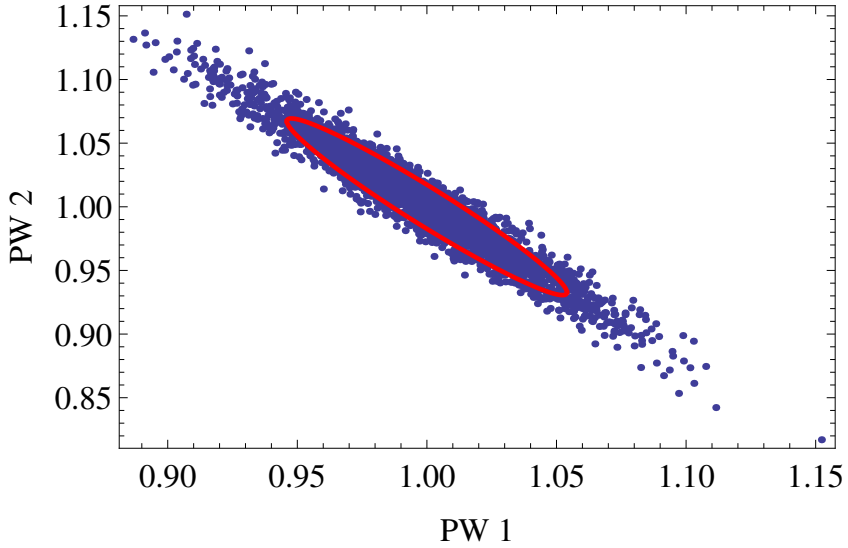
- BnGa, Jülich, MAID, Zagreb...fit to SAID elastic πN partial waves or use it as FSI.
- Single-energy solutions of SAID: narrow structures & consistency, but little statistical meaning.
- No control of the statistical impact of elastic πN data on multi-reaction fits, performed by other groups.
- Q: is it possible to provide a simple to use interface, such that other groups can fit to SAID elastic πN PWs and get the same χ^2 as if they had fitted to the elastic πN data?

Work in progress: Monte-Carlo error propagation



- Bands: Monte-Carlo error propagation using bootstrap (energy-dependent fit).
- Statistically meaningful
- but: correlations between different partial waves/multipoles are also there!
- How to provide? Solution: Instead of PWs, we plan to provide correlation matrices for the SES.

Work in progress: Correlated χ^2 fits



Fit "observable" O(E) with

$$O(E) = (\text{PW } 1)E + (\text{PW } 2)\,\frac{E^2}{100}$$

Data generated around

$$O(E) = E + \frac{E^2}{100}$$

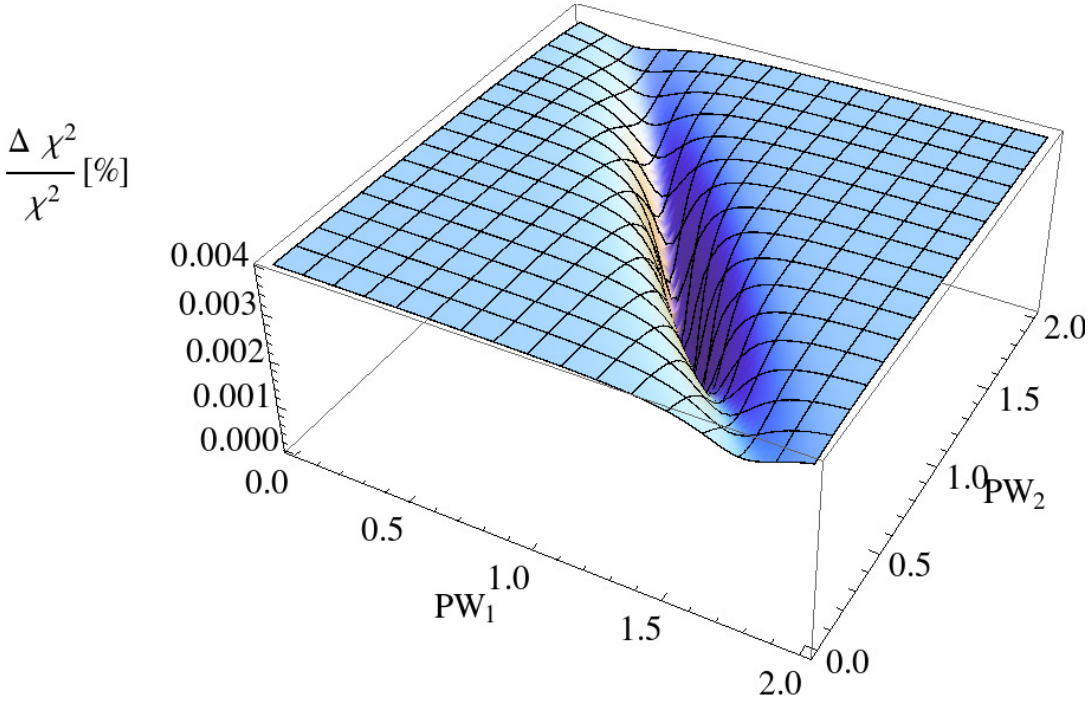
- Stochastic estimate of correlation matrix.
- Correlated χ^2: $\chi^2(x) = (x - \bar{x})^T\, C^{-1}(x - \bar{x})$, $x = (S_{11}, S_{31}, \dots)$
- but does this provide the same χ^2 as a fit to the original data?

Work in progress: Correlated χ^2 fits

- ... almost; the function

$$\chi^2 = (x - \bar{x})^T C^{-1} (x - \bar{x}) + \chi^2_{\text{best}} + \text{corrections}$$

  does.
- Difference between correlated χ^2 and actual χ^2 of the data themselves (toy model):



Data base and fit

Data base:

- elastic πN PWA SAID 2006 [Arndt *et al.*, *PRC 74 (2006)*]
- $\pi N \to \eta N$, KY observables ($d\sigma/d\Omega$, P, β)

$\Big\}$ $\sim$ 6000 data points

Fit parameters:

- T^P: 128 free parameters
  11 N^* resonances $\times$ (1 m_{bare}+ couplings to πN, ρN, ηN, $\pi\Delta$, $K\Lambda$, $K\Sigma$))
  + 10 Δ resonances $\times$ (1 m_{bare}+ couplings to πN, ρN, $\pi\Delta$, $K\Sigma$)
- T^{NP}: 68 free parameters

$\Big\}$ 196 free parameters

$\Rightarrow$ Simultaneous fit of T^P and T^{NP} using MINUIT on the JUROPA supercomputer (FZJ)

Sensitivity of results to starting conditions?

$\Longrightarrow$ Perform 2 fits, start from different scenarios in parameter set

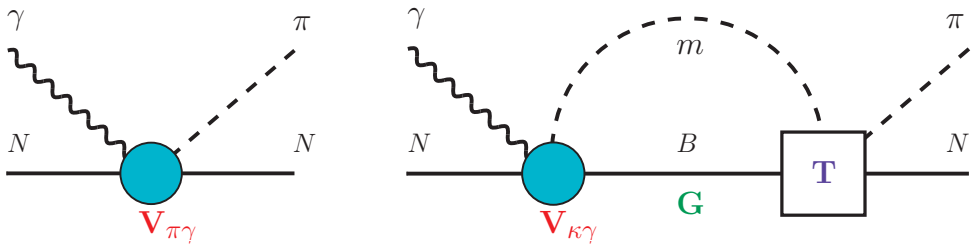
- Fit A: start from parameter set of a preceding Jülich model [Gasparyan *et al.* PRC68 , (2003)]
- Fit B: start from a new scenario

Photoproduction in a semi-phenomenological approach

Multipole amplitude

$$M^{IJ}_{\mu\gamma} = V^{IJ}_{\mu\gamma} + \sum_{\kappa} T^{IJ}_{\mu\kappa} G_{\kappa} V^{IJ}_{\kappa\gamma}$$

(partial wave basis)

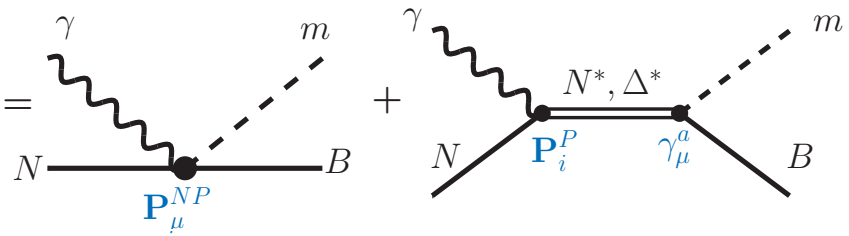


$m = \pi, \eta$, $B = N, \Delta$

$T_{\mu\kappa}$: Jülich hadronic T-matrix → Watson's theorem fulfilled by construction
→ **analyticity of T**: extraction of resonance parameters

Phenomenological potential:

$\mathbf{V}_{\mu\gamma}(E,q) =$



$$= \frac{\tilde{\gamma}^a_{\mu;i}(q)}{m_N} P^{\mathrm{NP}}_{\mu}(E) + \sum_i \frac{\gamma^a_{\mu;i}(q) P^{\mathrm{P}}_i(E)}{E - m^b_i}$$

$\tilde{\gamma}^a_{\mu;i}$, $\gamma^a_{\mu;i}$: hadronic vertices → correct threshold behaviour

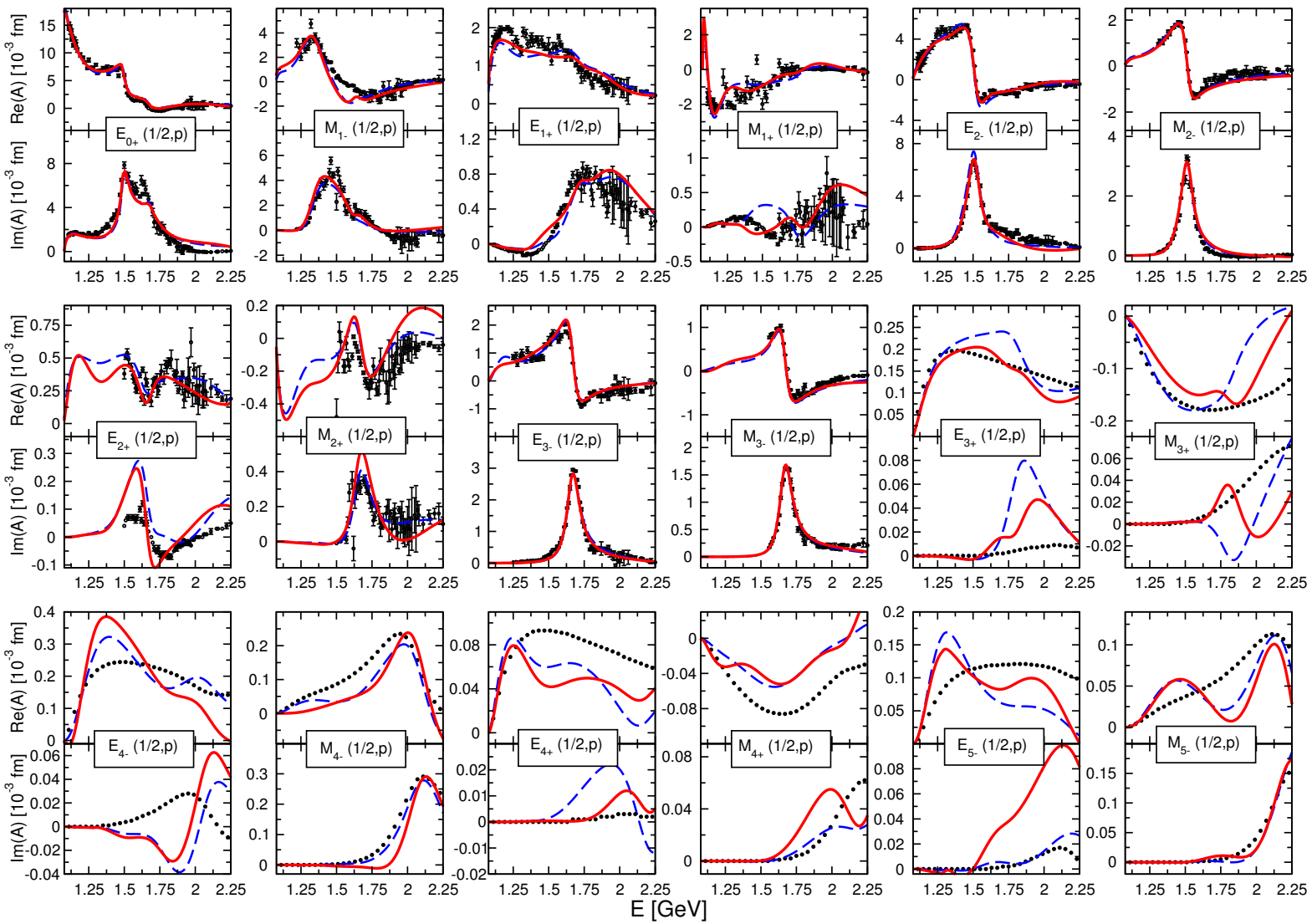
i: resonance number per multipole; μ: channels πN, ηN, $\pi\Delta$

Polynomials

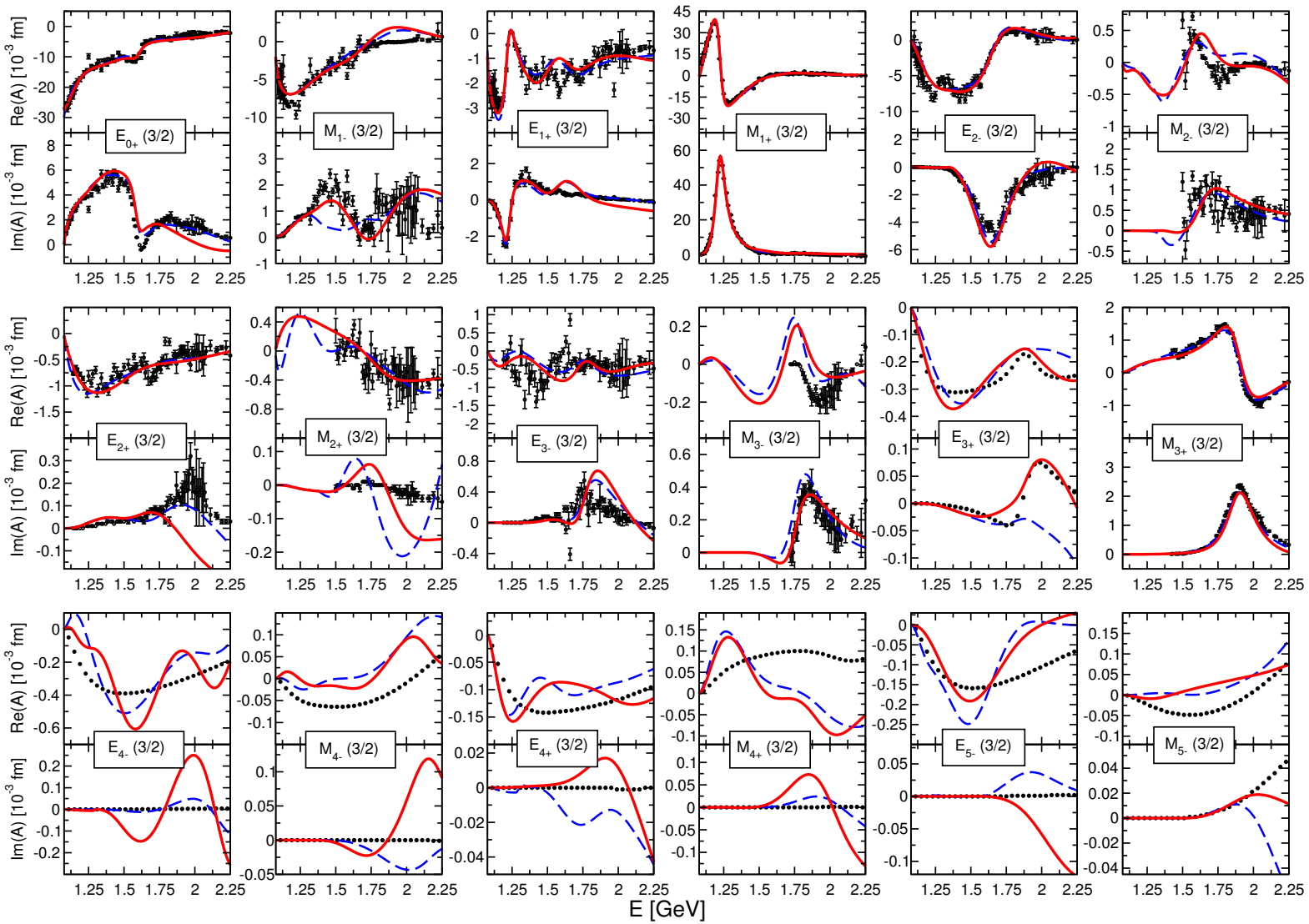
Method inspired by SAID

but different

Multipoles: Comparison with GWU/SAID CM12

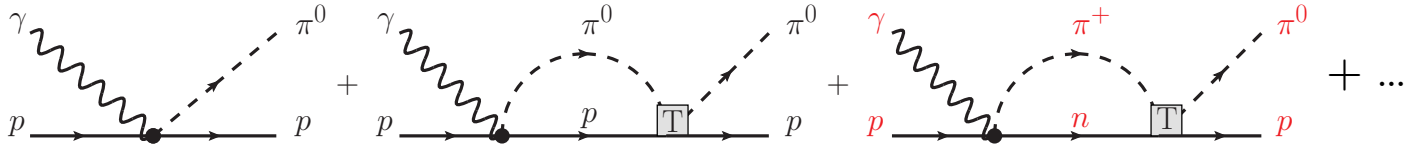


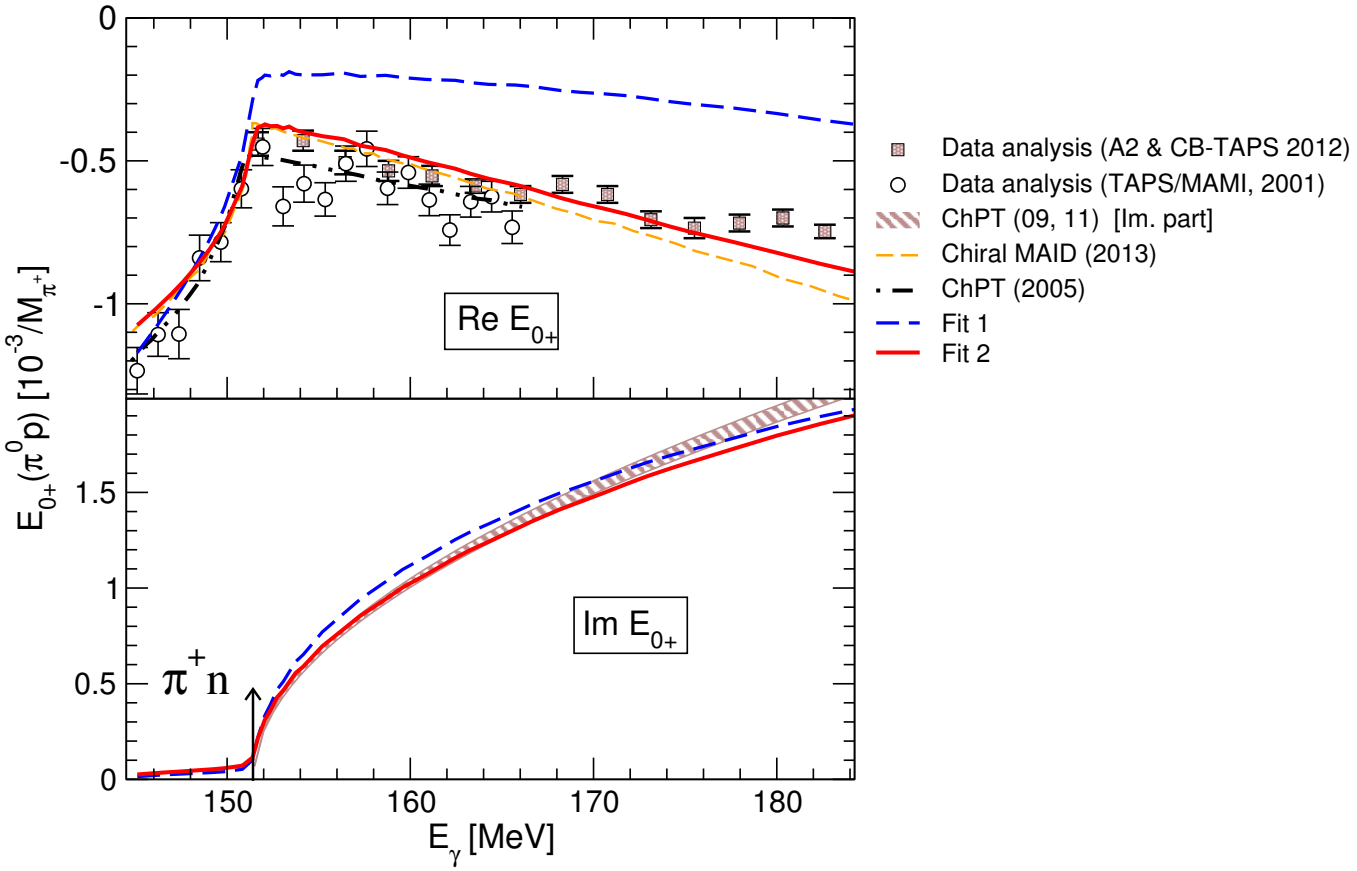
Multipoles: Comparison with GWU/SAID CM12



Multipoles: E_{0+} $(\pi^0 p)$ at threshold

$M^{\gamma}_{\pi^0 p} =$





Different π & N masses
$\rightarrow$ cusp effect

Details of the formalism

Polynomials:

$$P_i^{\mathrm{P}}(E) = \sum_{j=1}^{n} g_{i,j}^{\mathrm{P}} \left(\frac{E - E_0}{m_N} \right)^j e^{-g_{i,n+1}^{P}(E-E_0)}$$

$$P_\mu^{\mathrm{NP}}(E) = \sum_{j=0}^{n} g_{\mu,j}^{\mathrm{NP}} \left(\frac{E - E_0}{m_N} \right)^j e^{-g_{\mu,n+1}^{\mathrm{NP}}(E-E_0)}$$

back

- $E_0 = 1077$ MeV
- $g_{i,j}^{\mathrm{P}}$, $g_{\mu,j}^{\mathrm{NP}}$: fit parameter
- $e^{-g(E-E_0)}$: appropriate high energy behavior
- $n = 3$

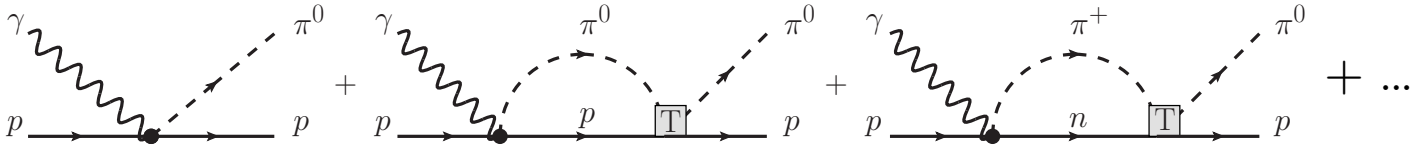
Isospin breaking in the πN channel

- Hadronic reactions: data well above threshold $\rightarrow$ **isospin averaged** masses
- Pion-photoproduction: data near threshold $\rightarrow$ include **isospin breaking**

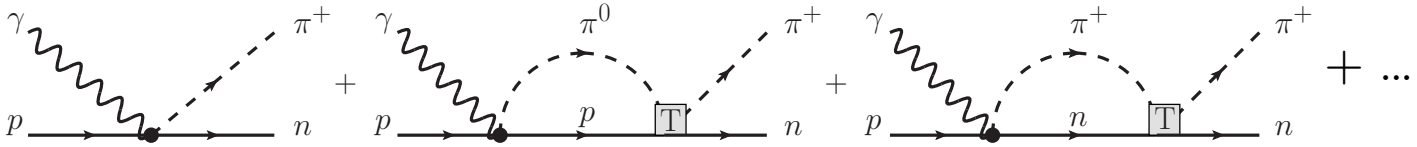
  $\hookrightarrow$ use exact π^0 (π^+) and p (n) mass

$E_{cm} < 1140$ MeV:

$M^{\gamma}_{\pi^0 p} =$



$M^{\gamma}_{\pi^+ n} =$



Isospin breaking in the πN channel

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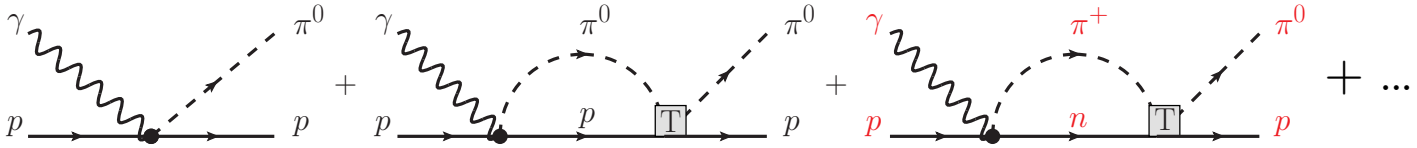
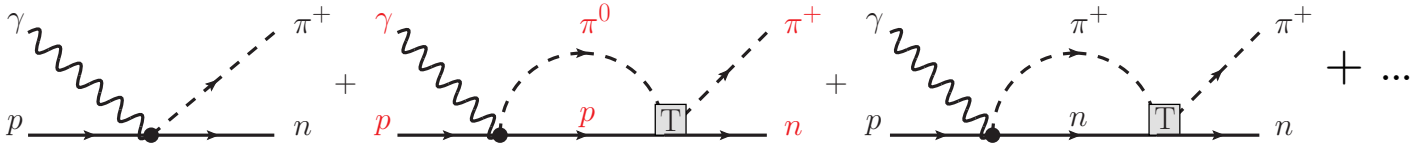
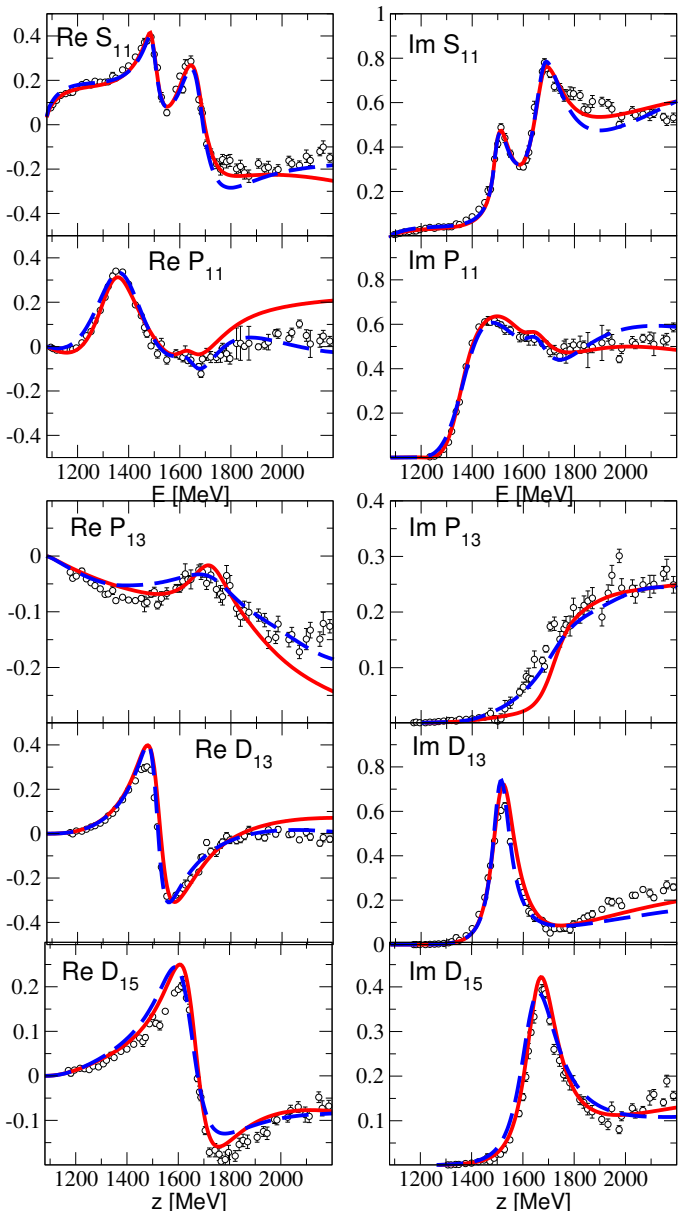
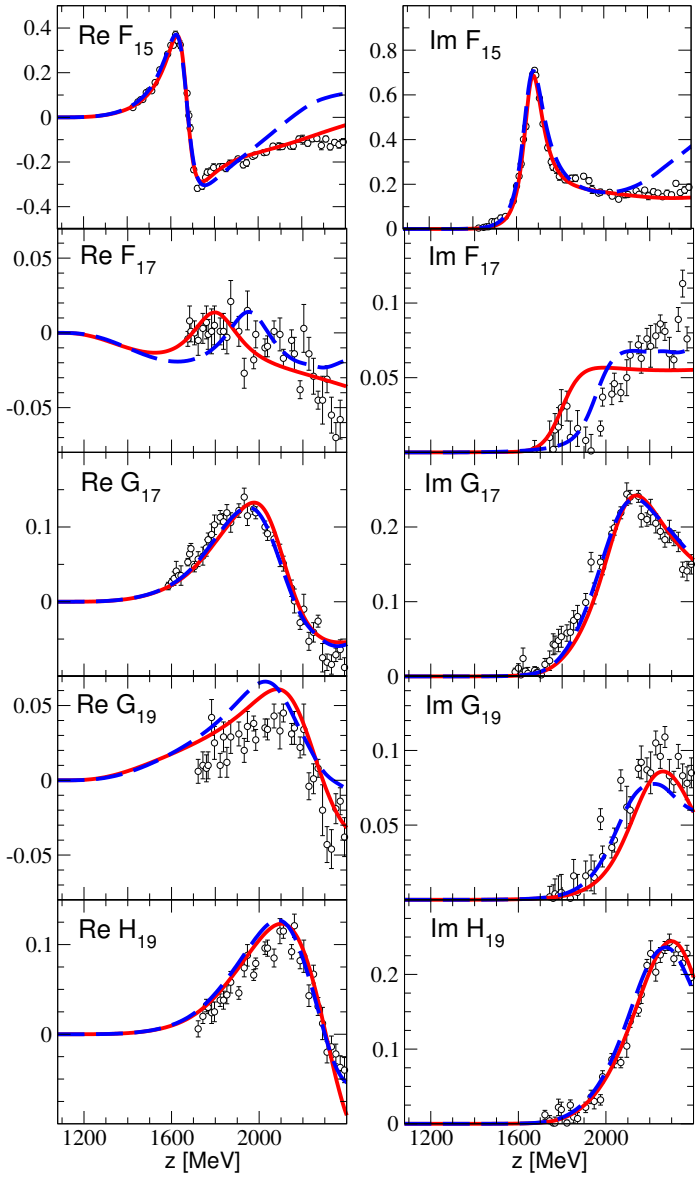


$M^{\gamma}_{\pi^+ n} =$

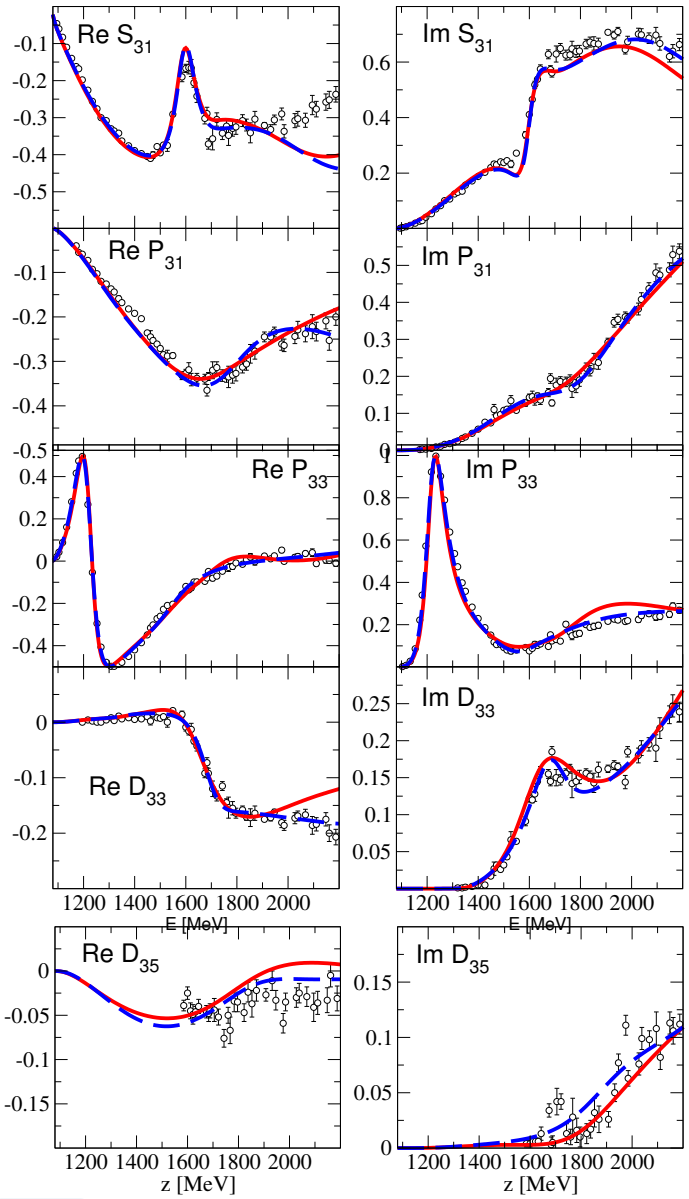


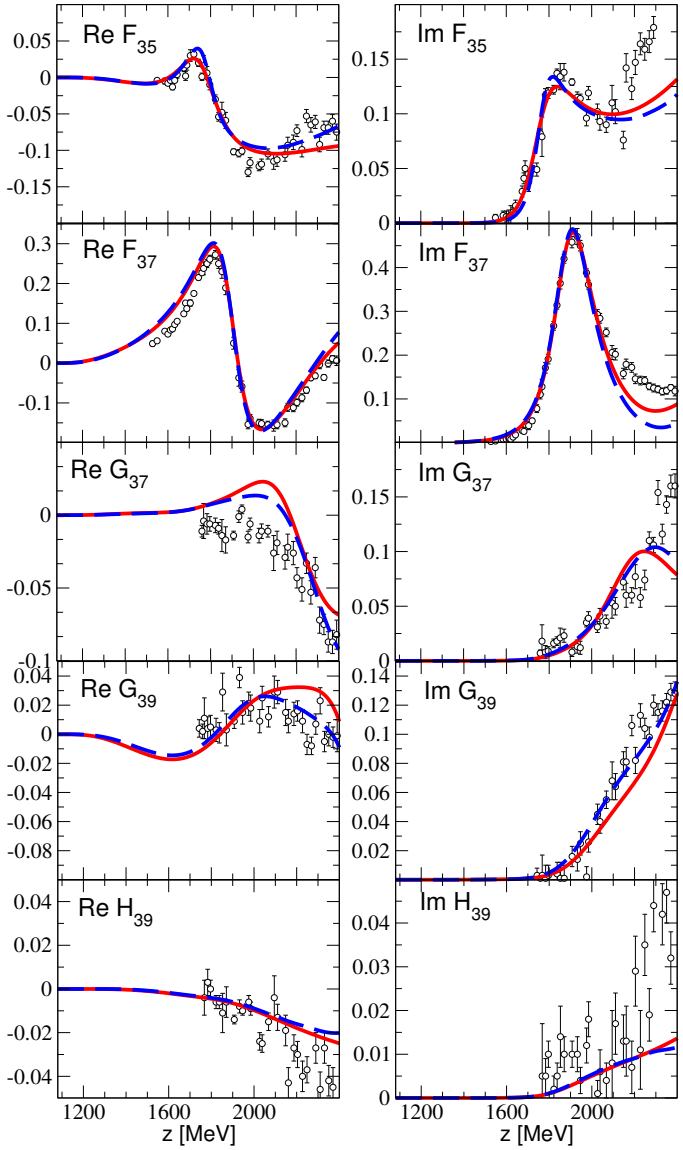
$\pi N \to \pi N$: Partial wave amplitudes I=1/2



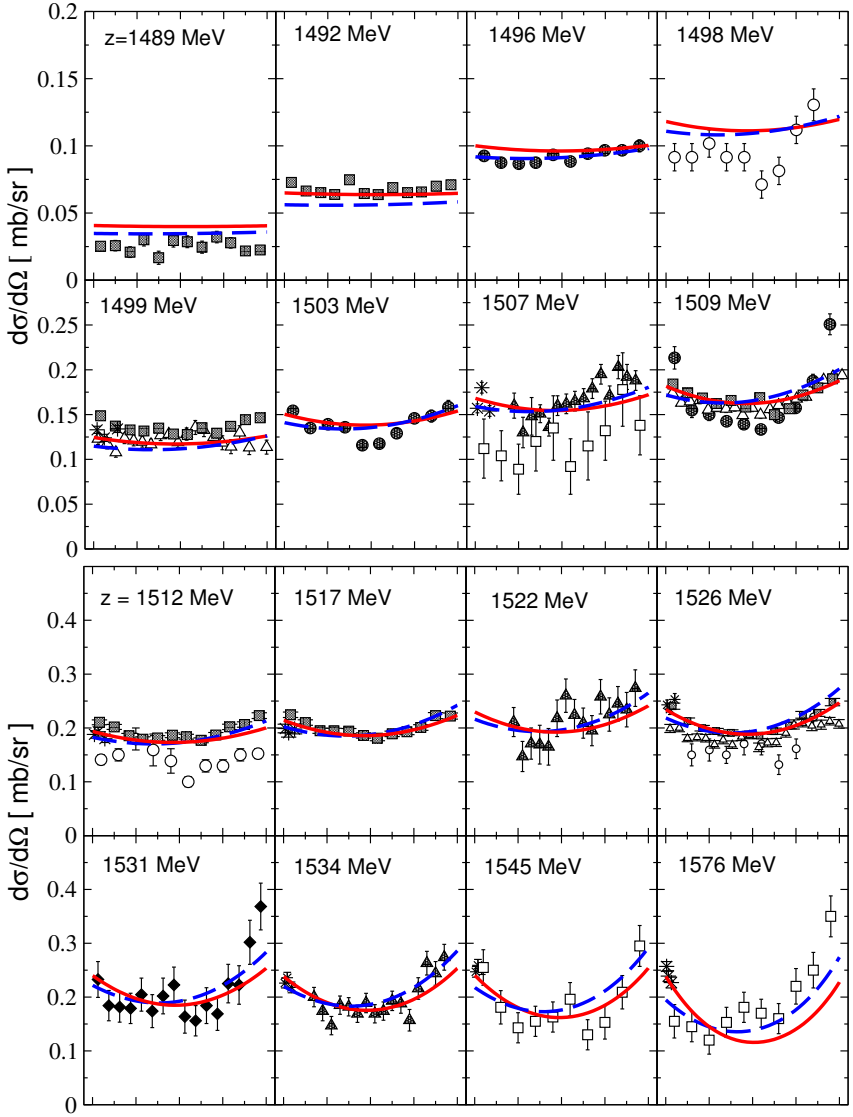


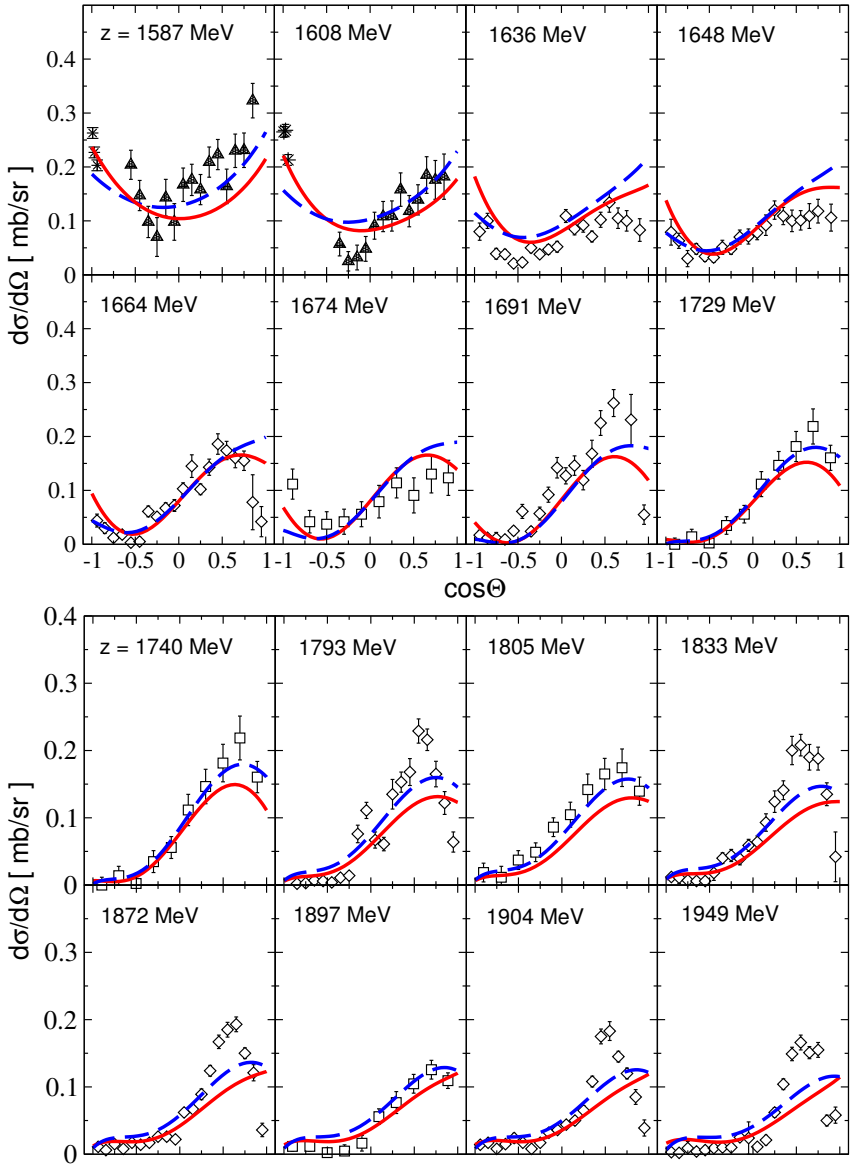
$\pi N \to \pi N$: Partial wave amplitudes I=3/2





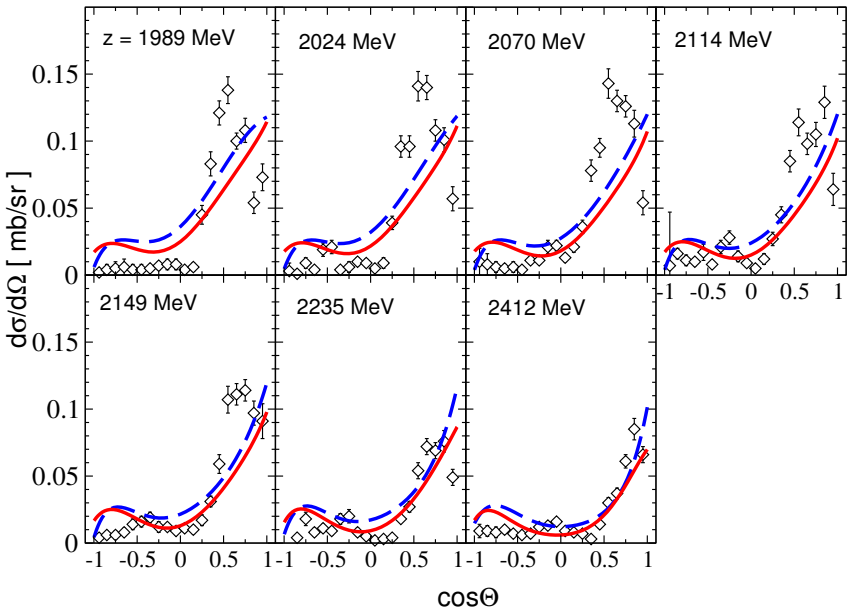
Full results ηN: $d\sigma/d\Omega$

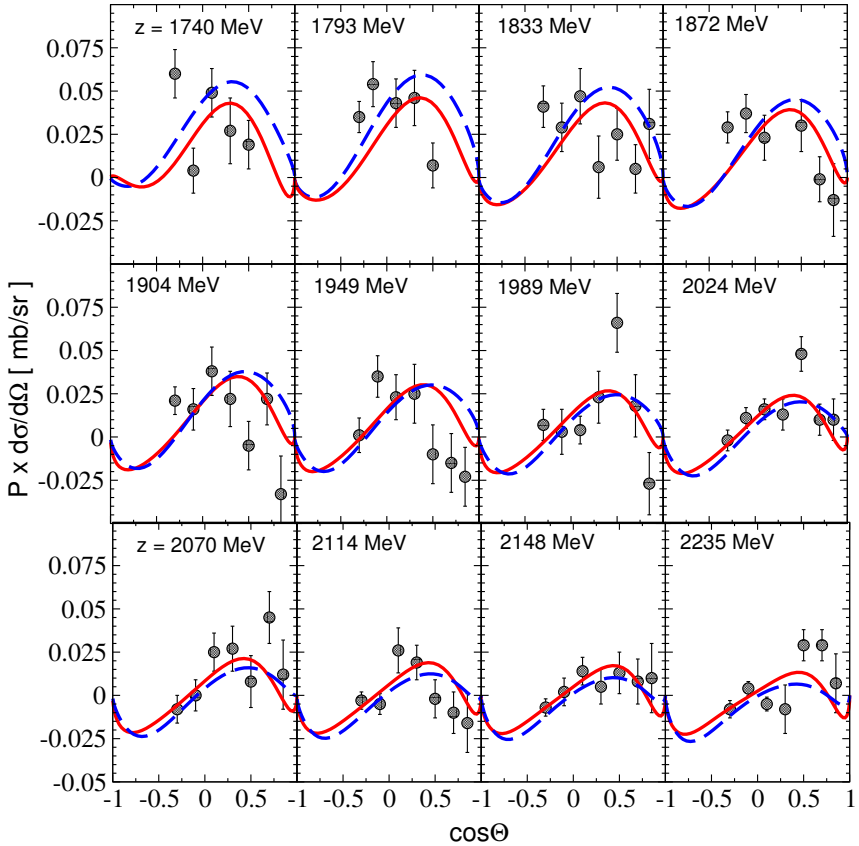




back

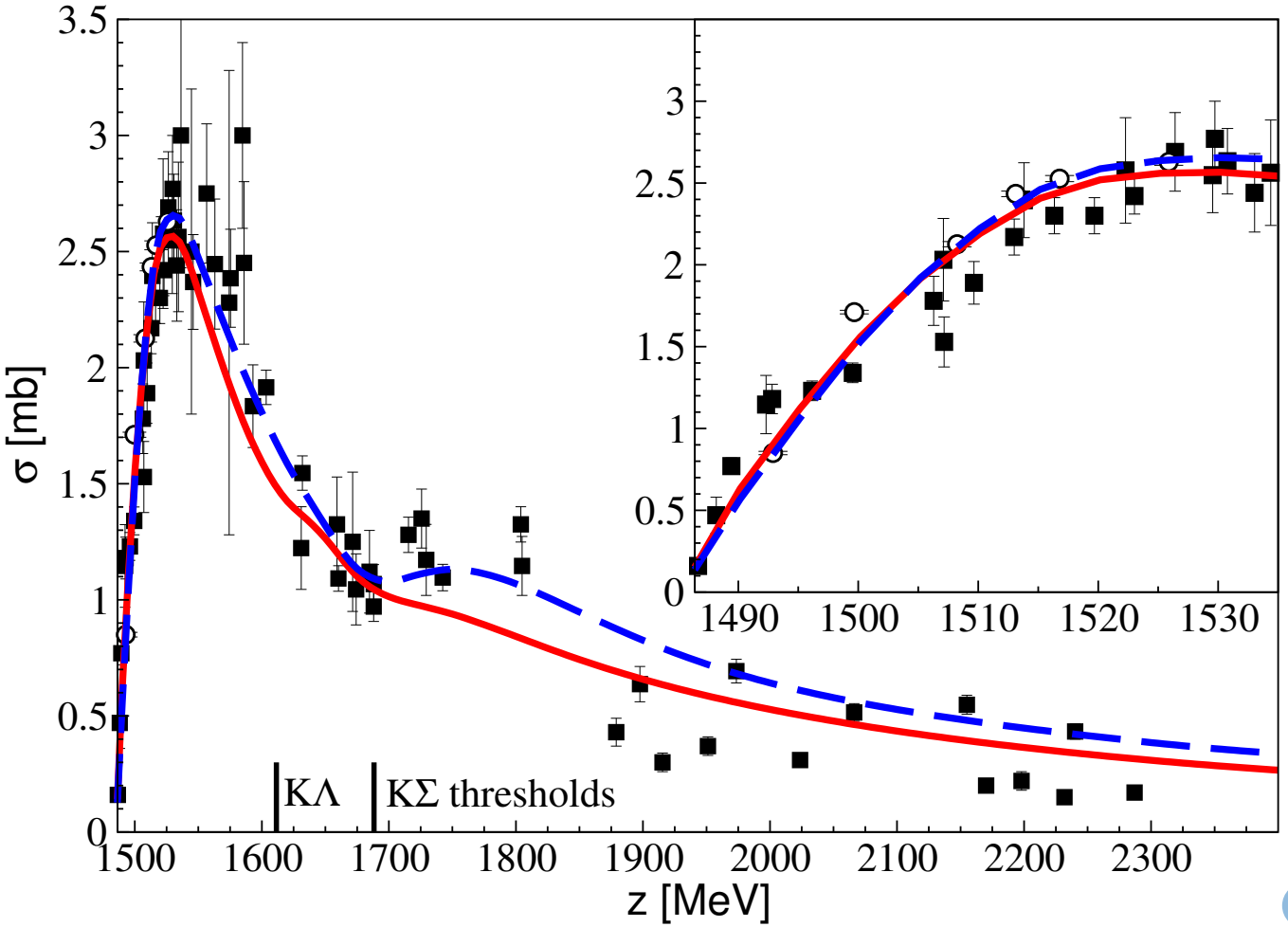
Full results ηN: $d\sigma/d\Omega$ & Polarization





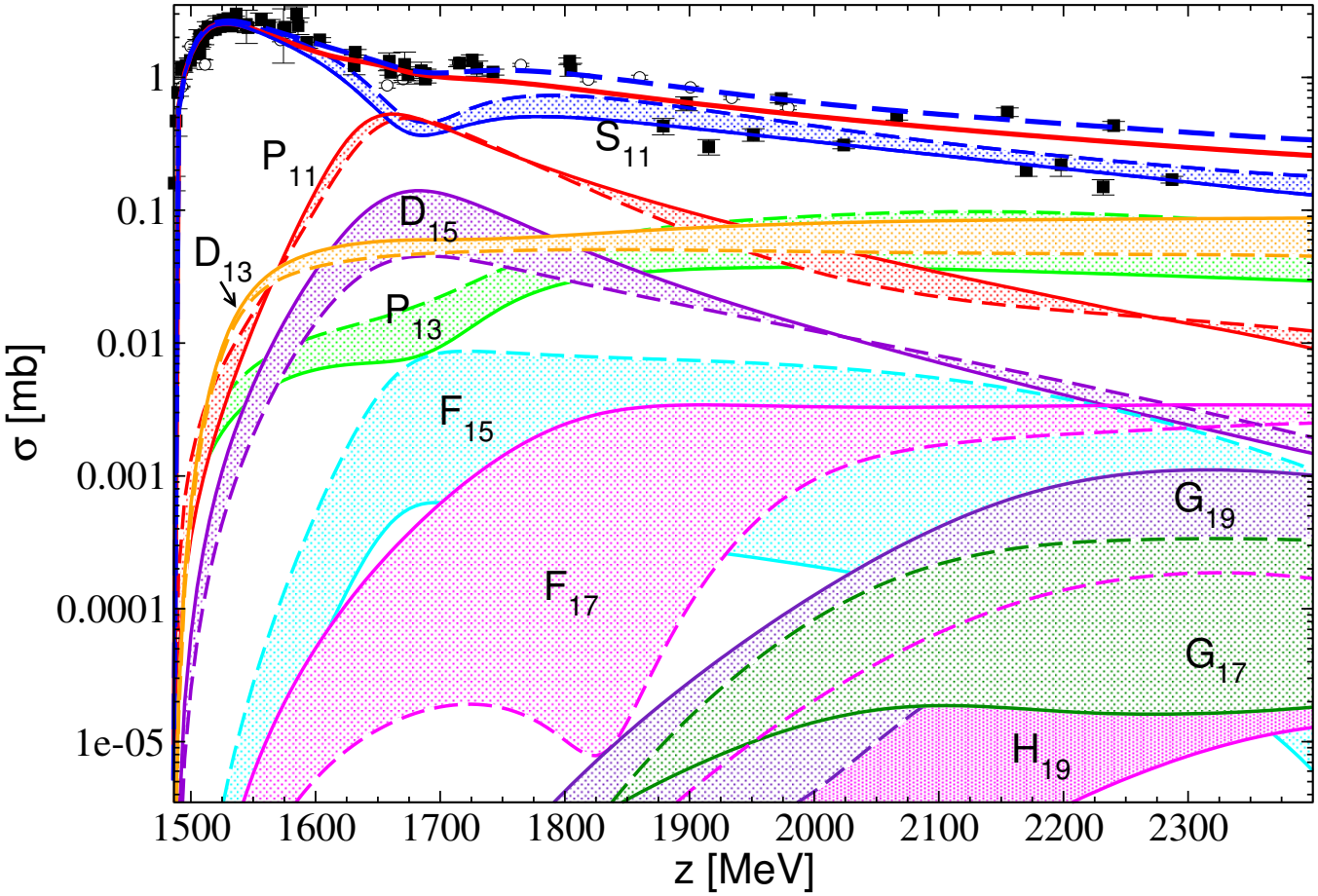
back

$\pi^- p \to \eta N$: Total cross section



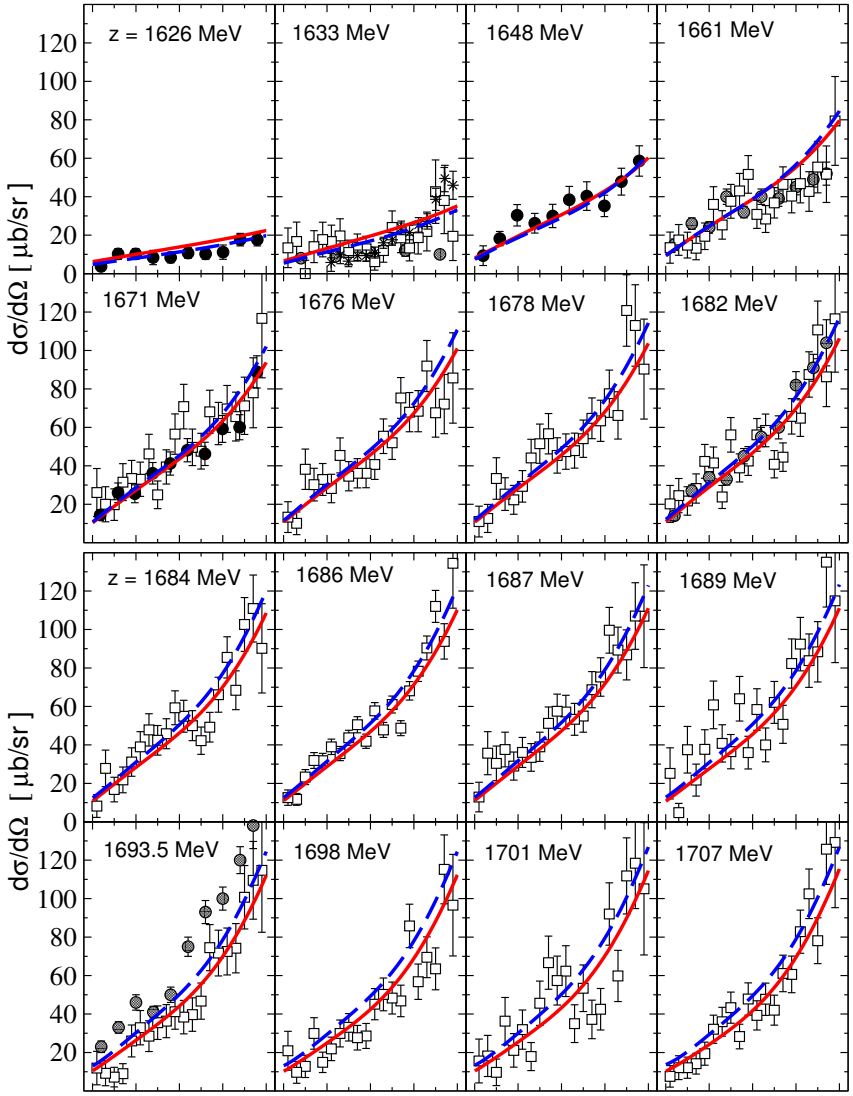
Partial wave content

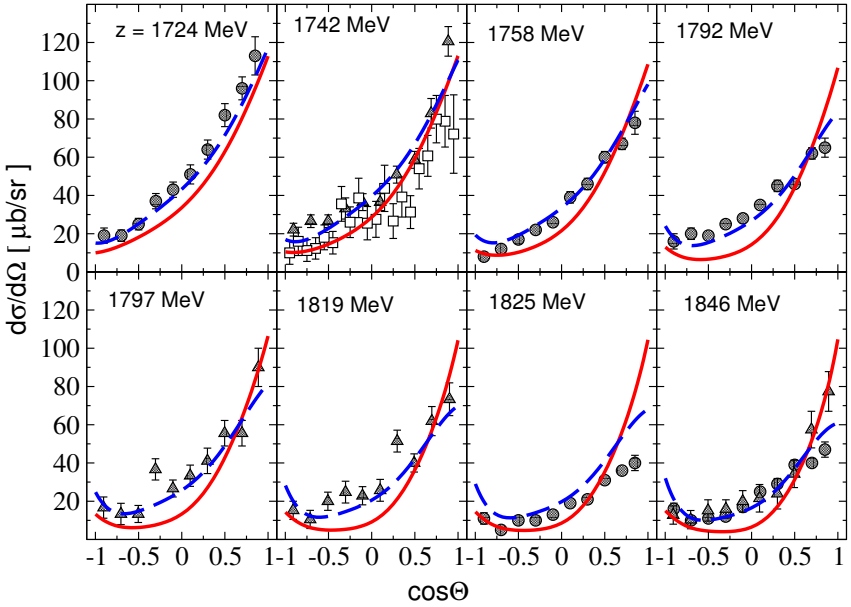
ηN: Partial wave content

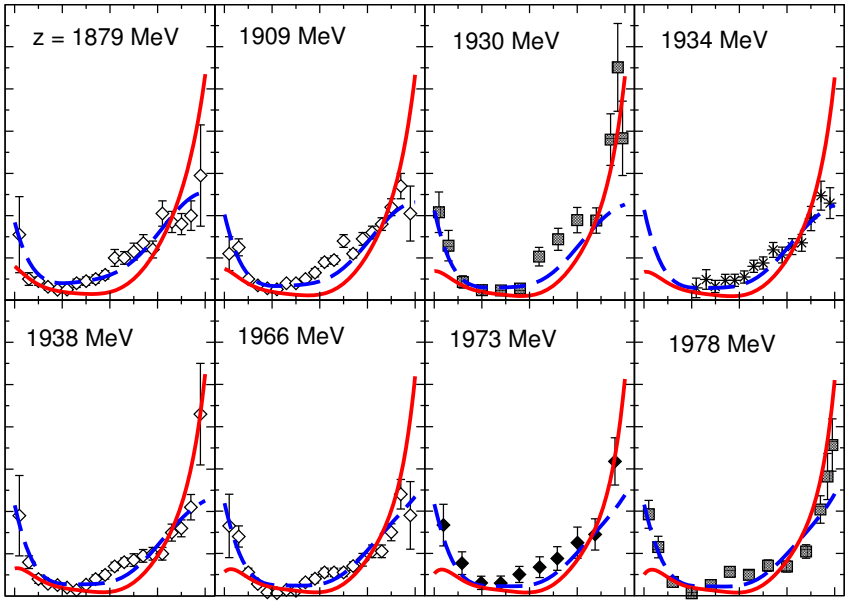


back

Full results $K\Lambda$: $d\sigma/d\Omega$

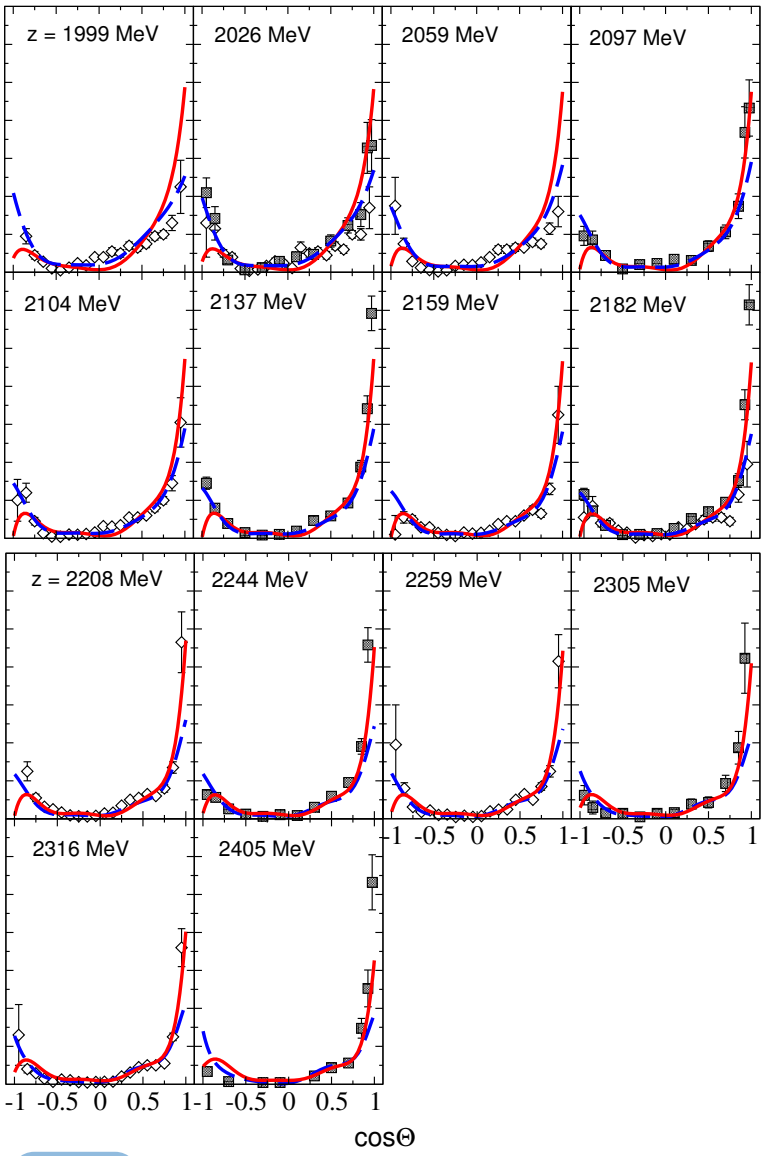






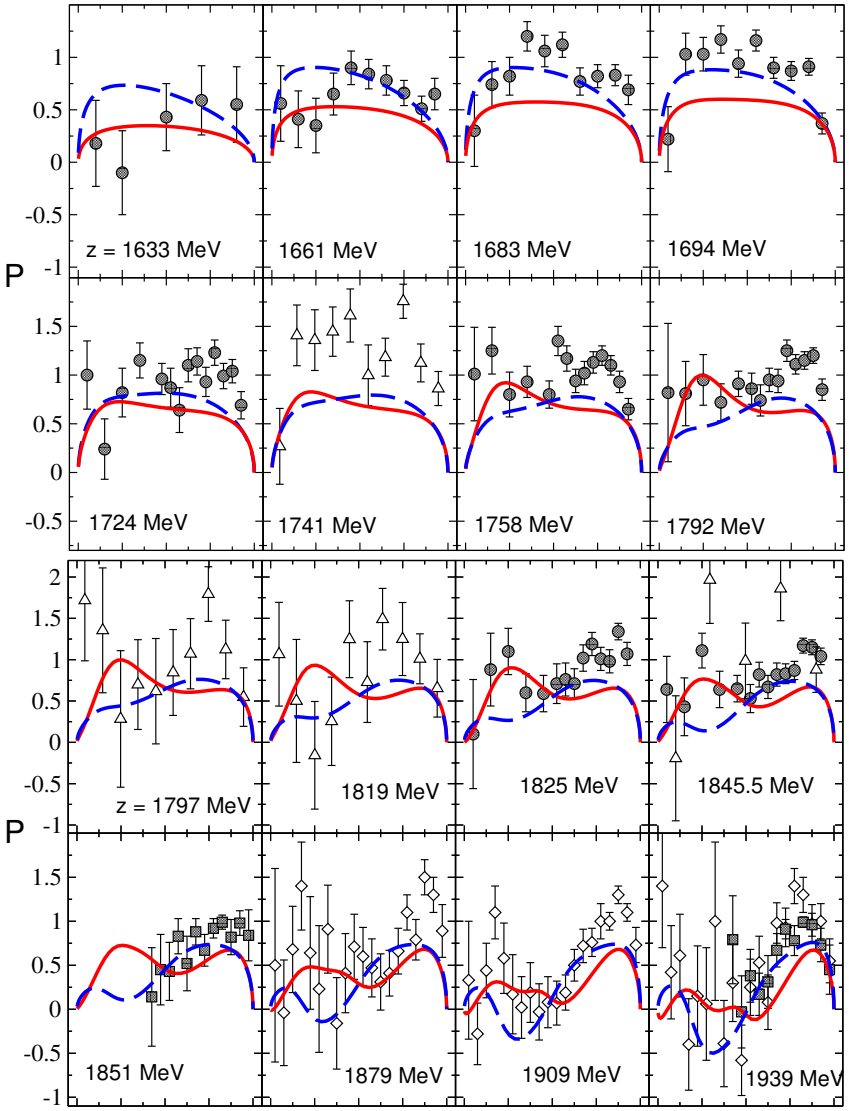
back

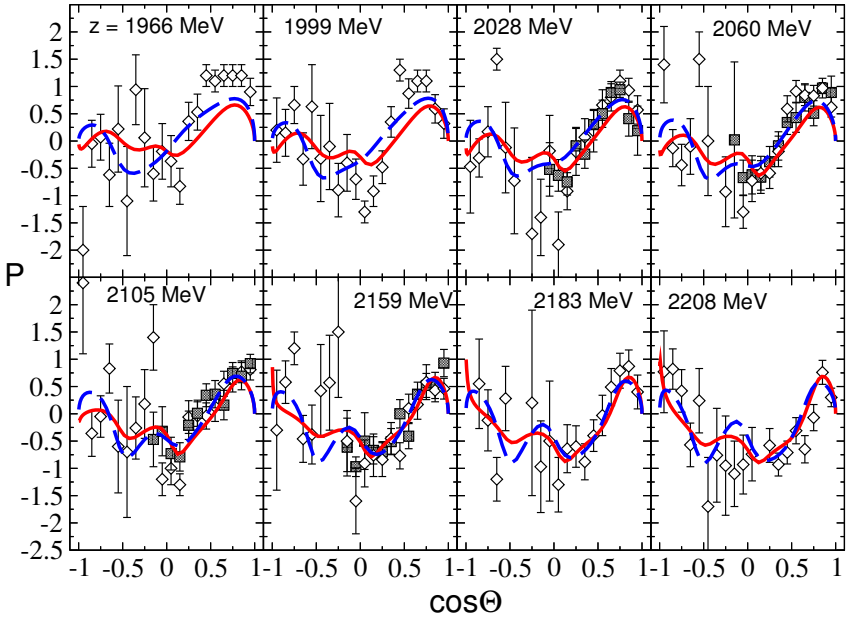
Full results $K\Lambda$: $d\sigma/d\Omega$

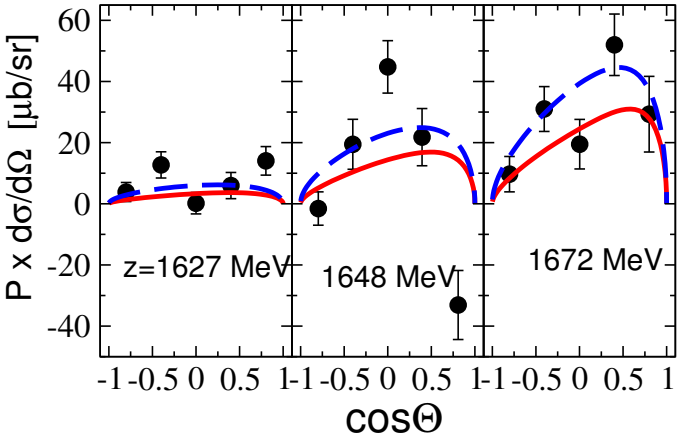


back

Full results $K\Lambda$: Polarization

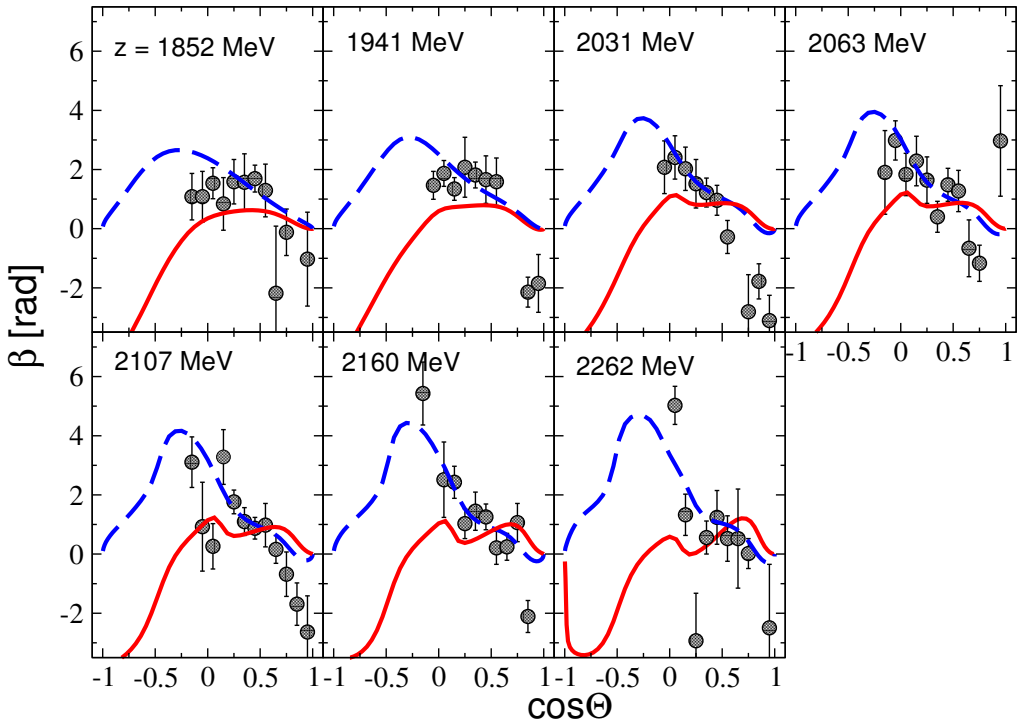






back

Full results $K\Lambda$: Spinrotation parameter β



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$$\beta = \arctan\left(\frac{2Im(h_{fi}^{*}g_{fi})}{|g_{fi}|^{2} - |h_{fi}|^{2}}\right)$$

$\pi^- p \to K^0 \Lambda$: Total cross section

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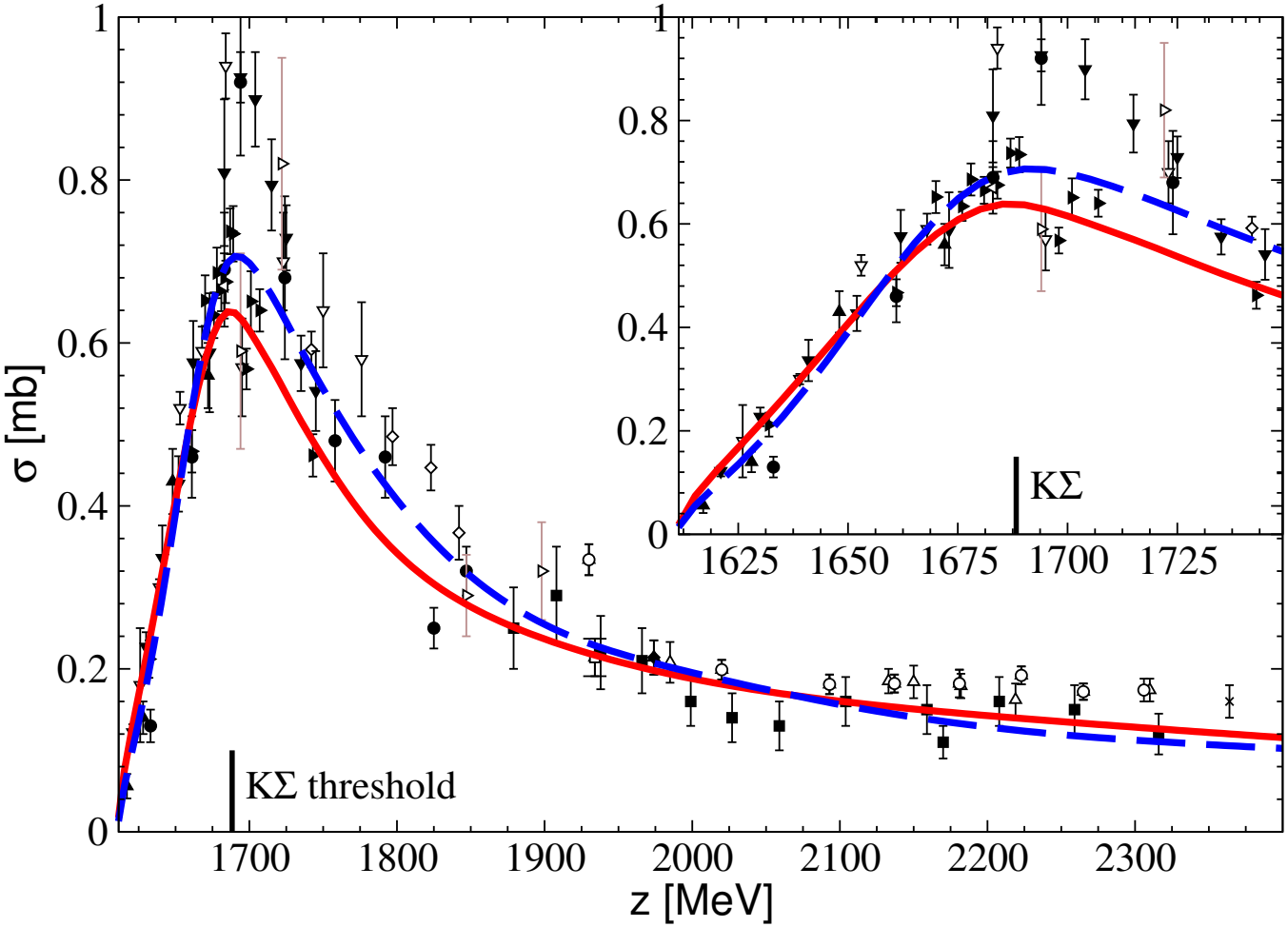

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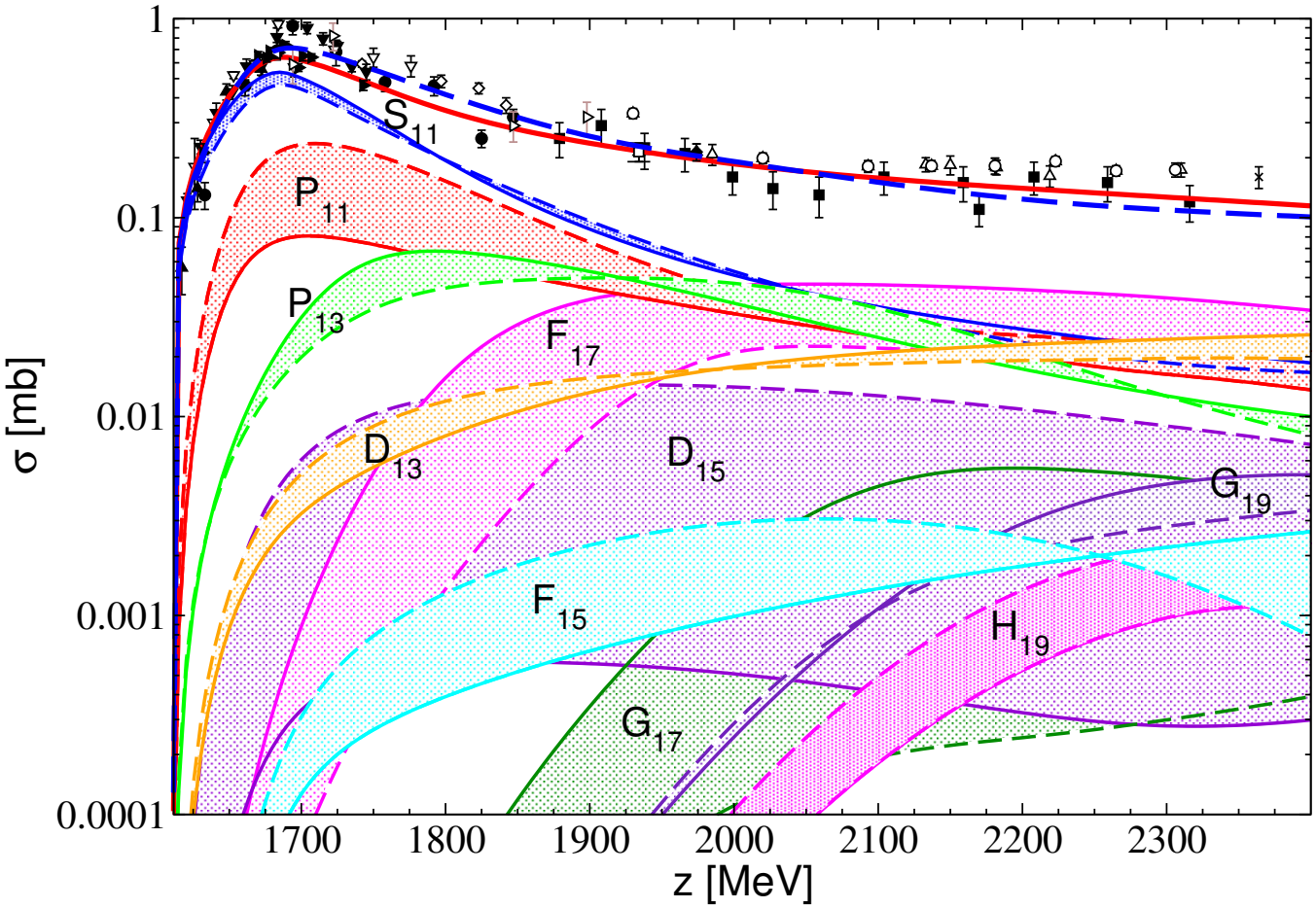
$K\Lambda$: Partial wave content



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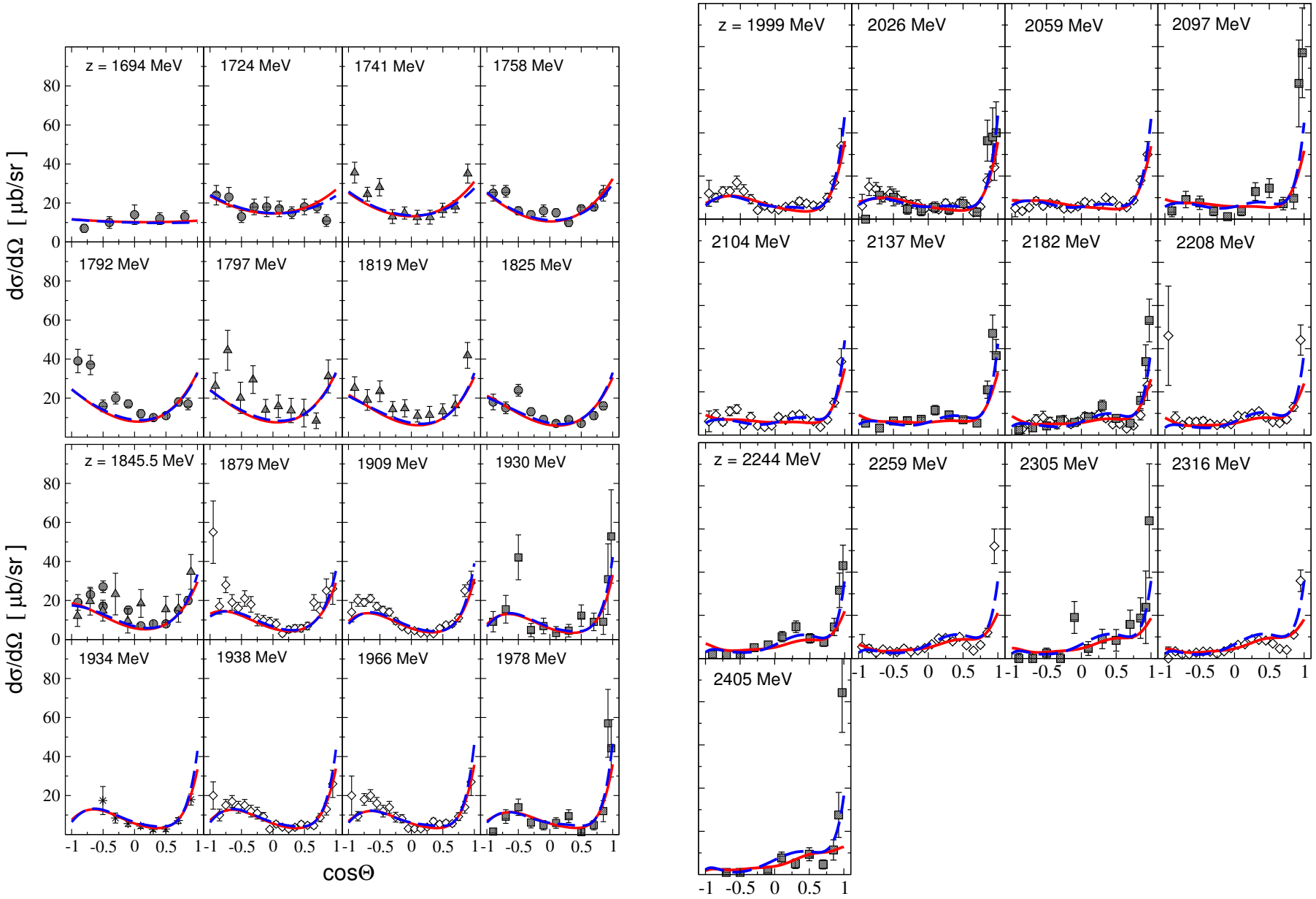
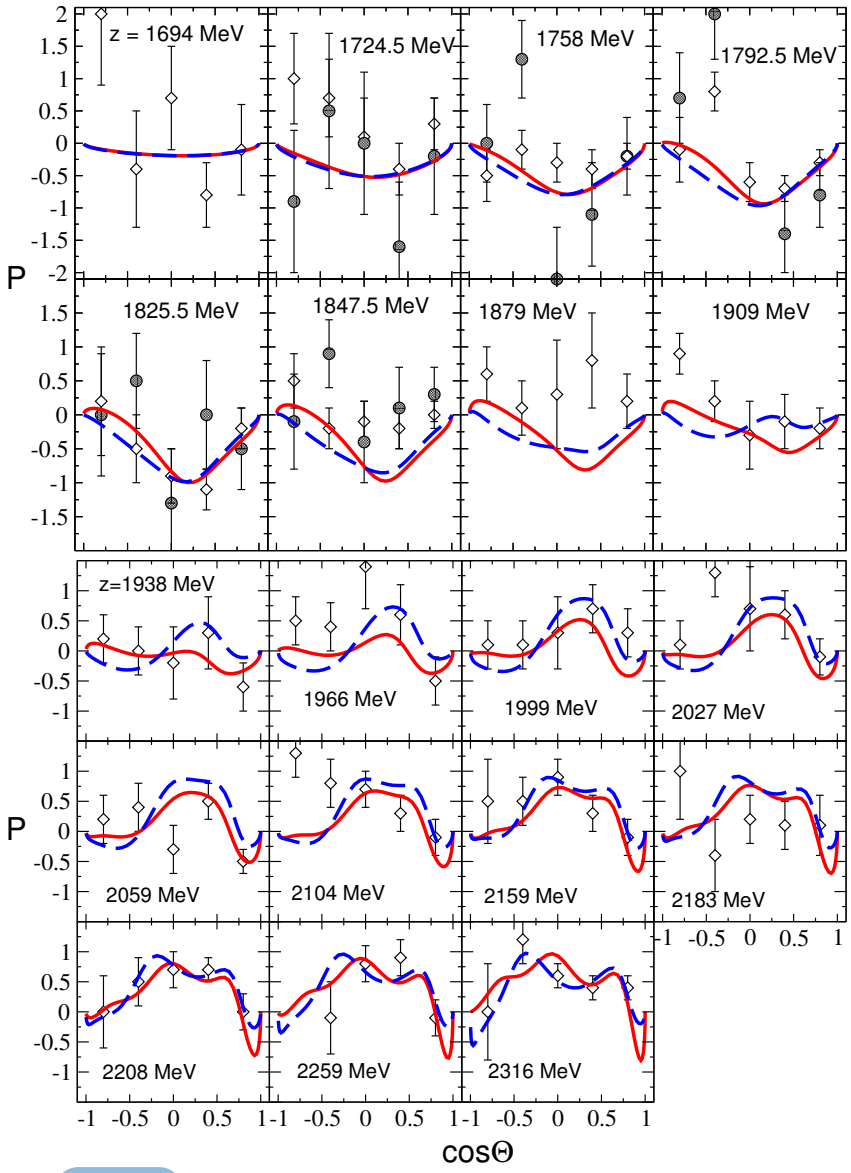
Full results $K^0\Sigma^0$: $d\sigma/d\Omega$



Full results $K^0\Sigma^0$: Polarization



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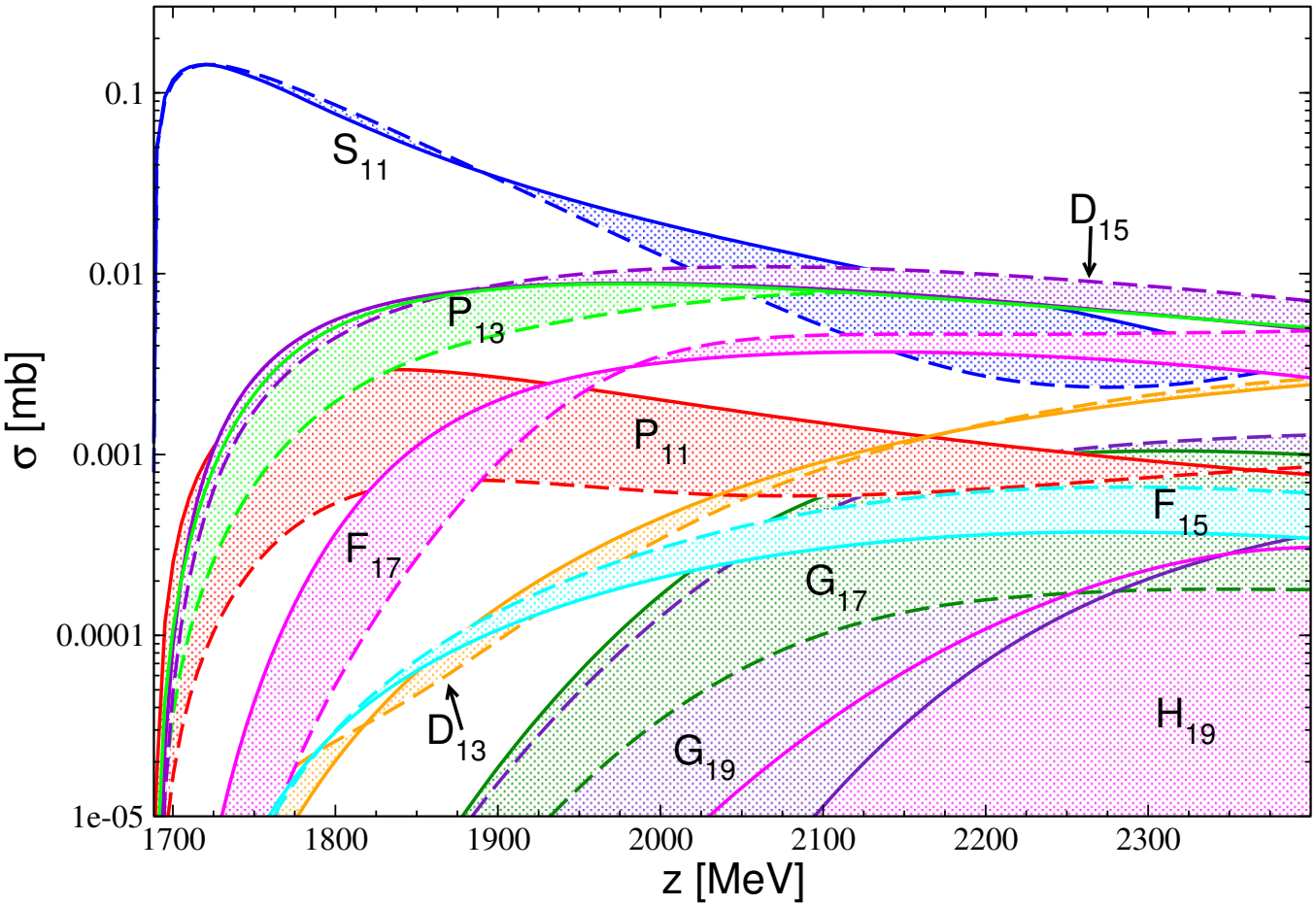
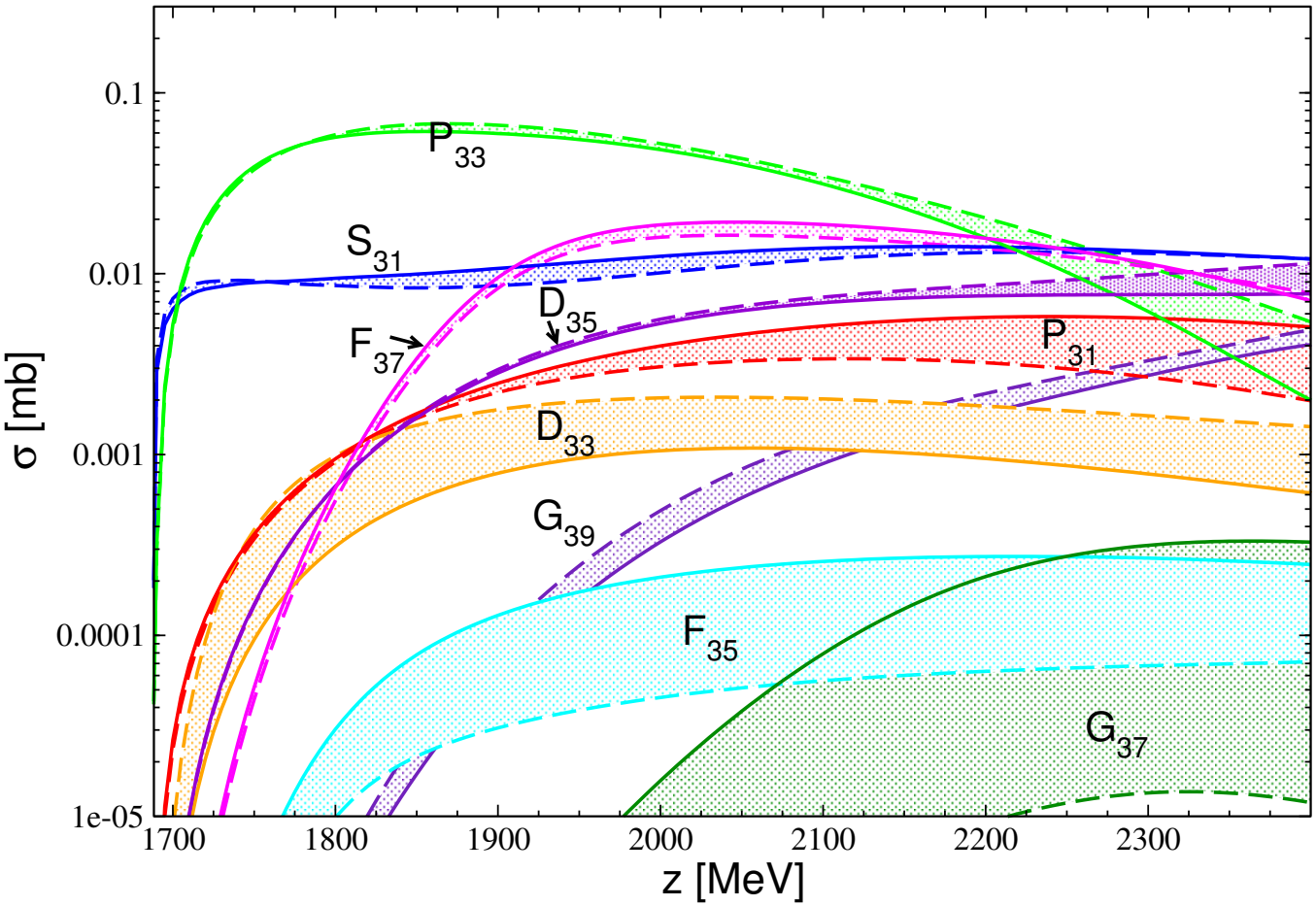
$K^0\Sigma^0$: Partial wave content I=1/2



back

$K^0\Sigma^0$: Partial wave content I=3/2



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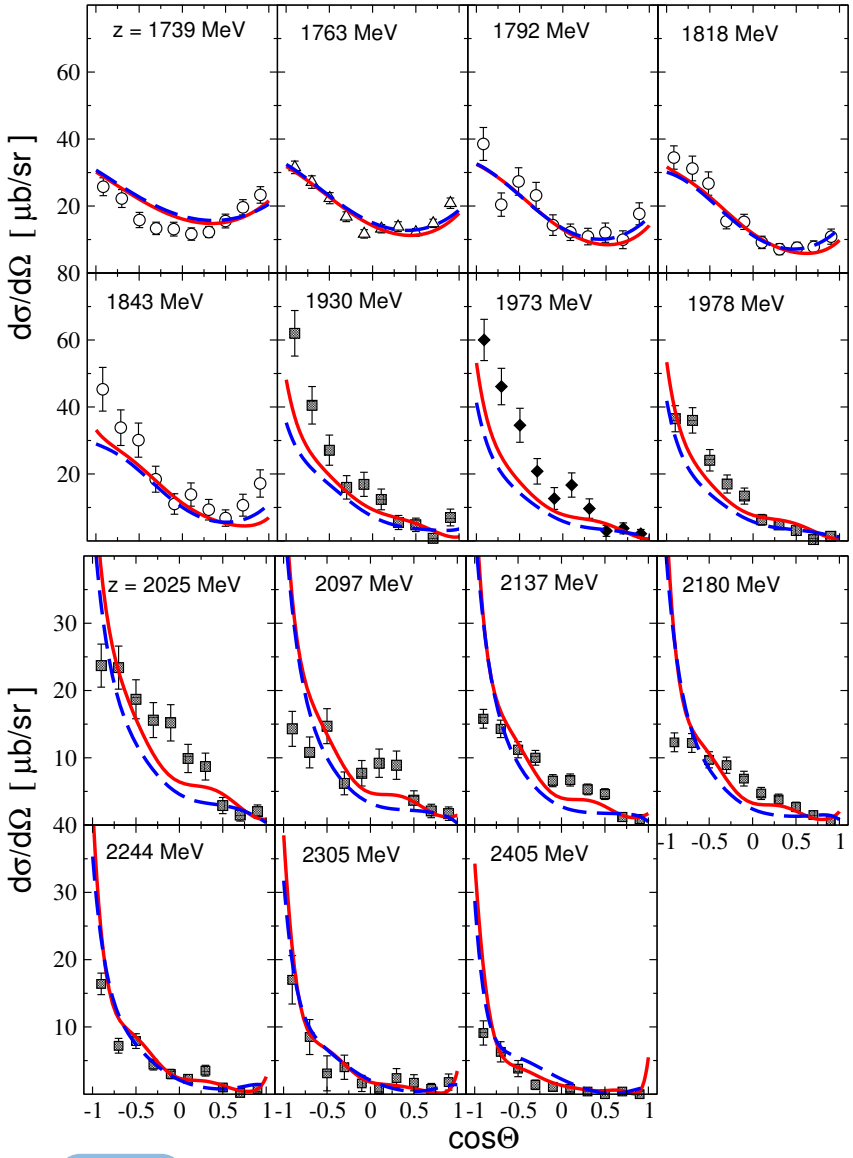
Full results $K^+\Sigma^-$: $d\sigma/d\Omega$



no Polarization data !

back

$\pi^- p \to K^+ \Sigma^-$: Total cross section

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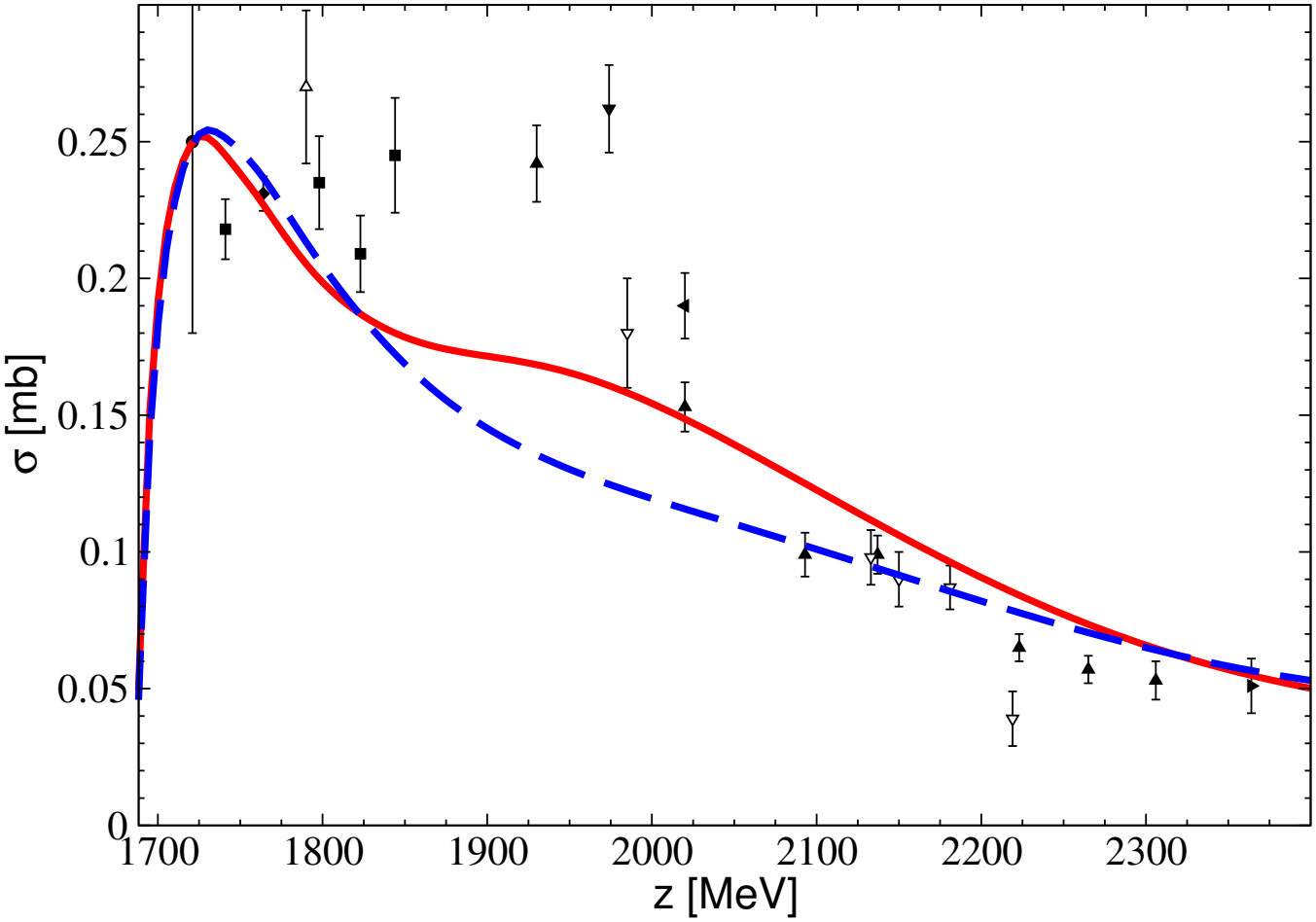
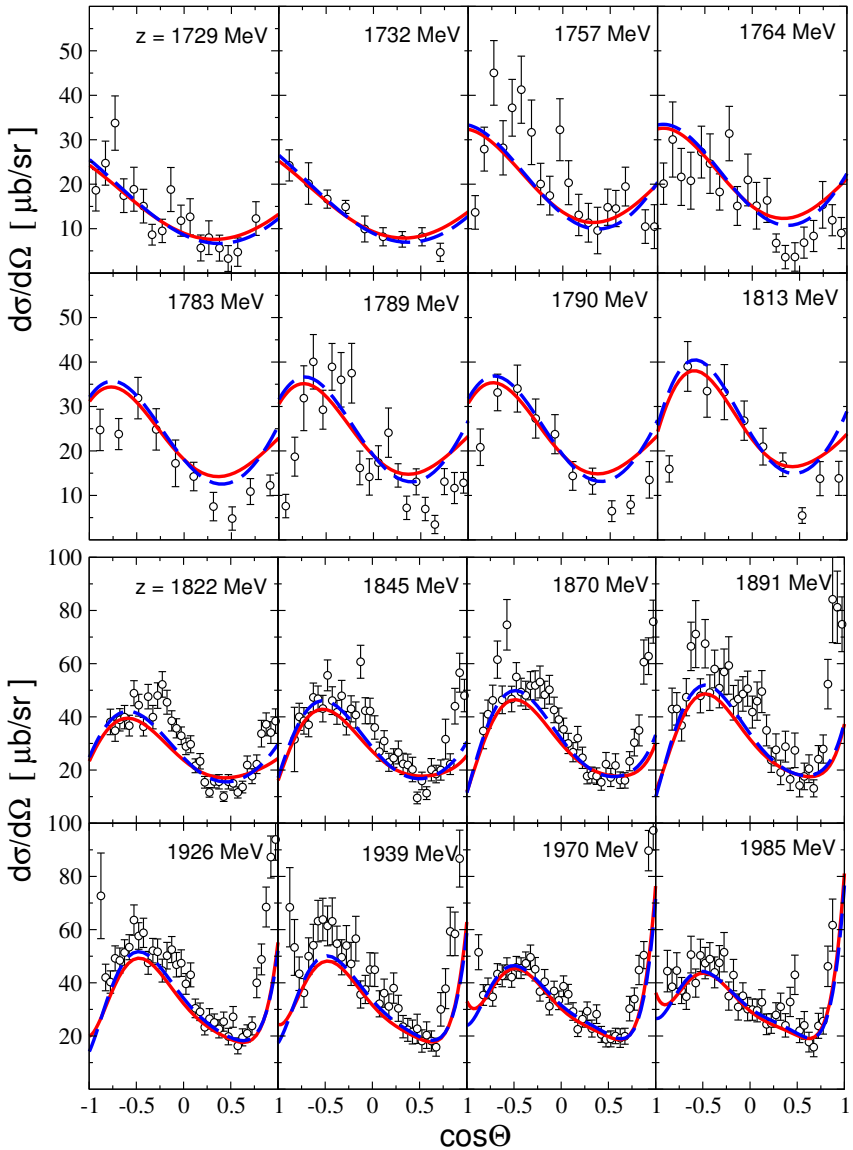
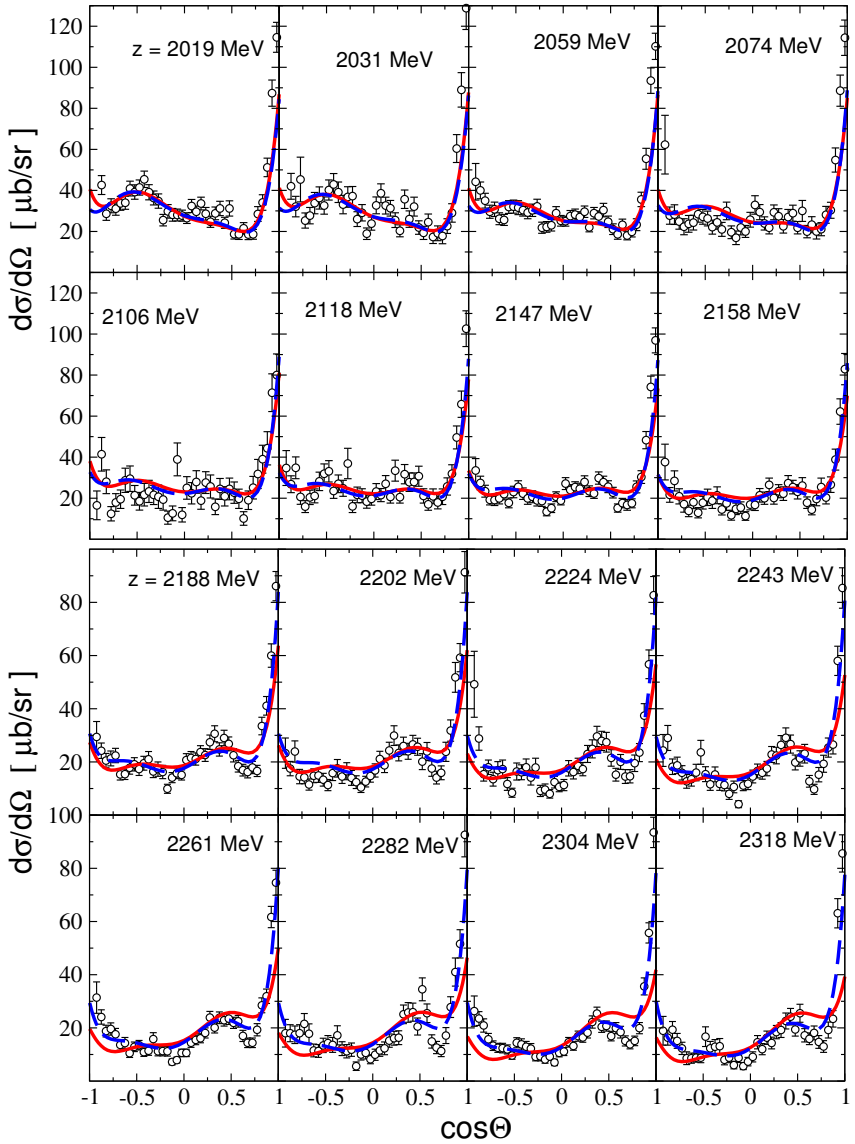


back

Full results $K^+\Sigma^+$: $d\sigma/d\Omega$





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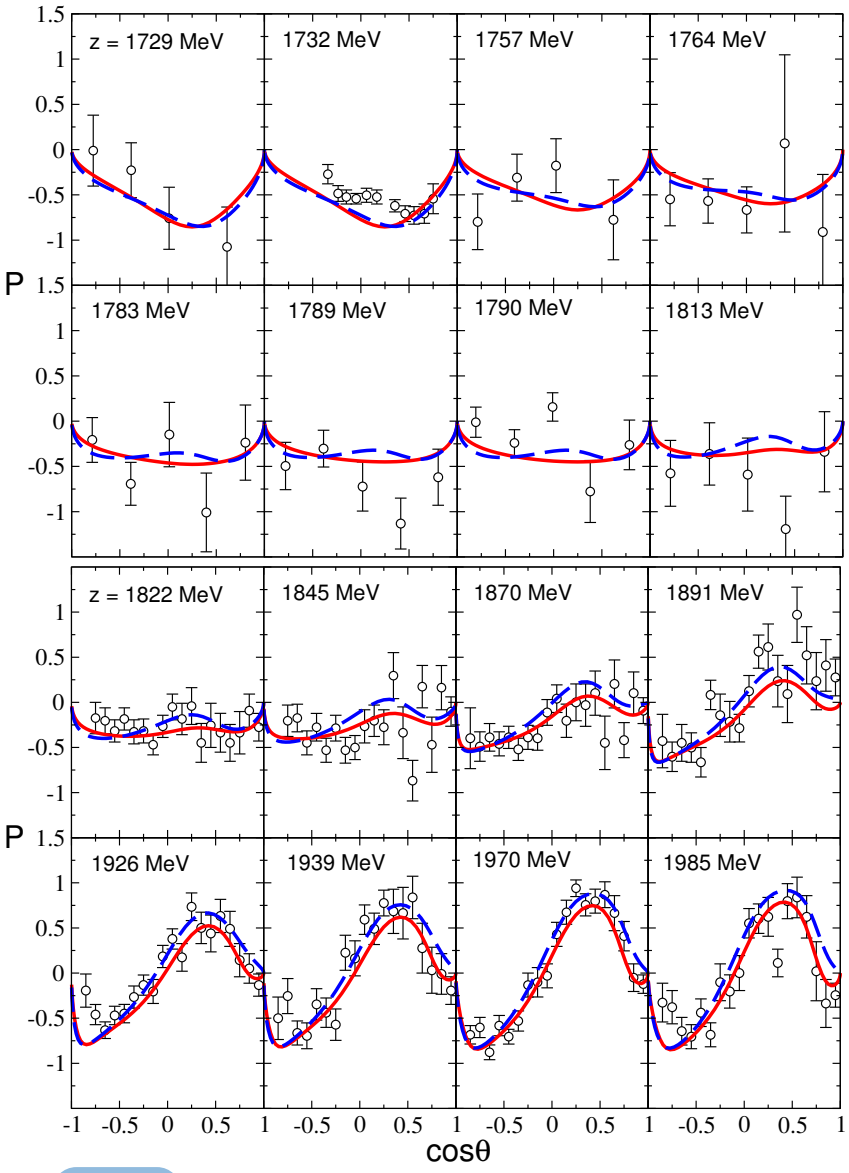
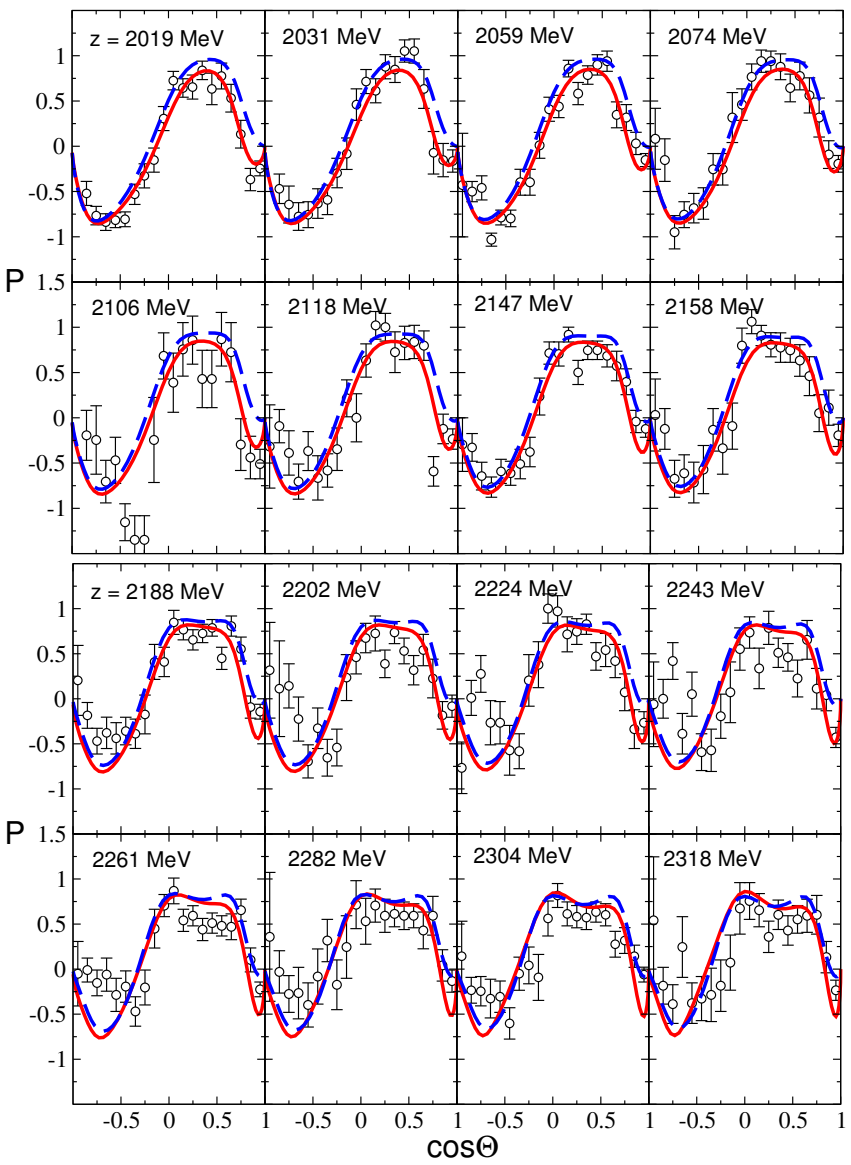
Full results $K^+\Sigma^+$: Polarization





back

$\pi^+ p \to K^+ \Sigma^+$: Total cross section

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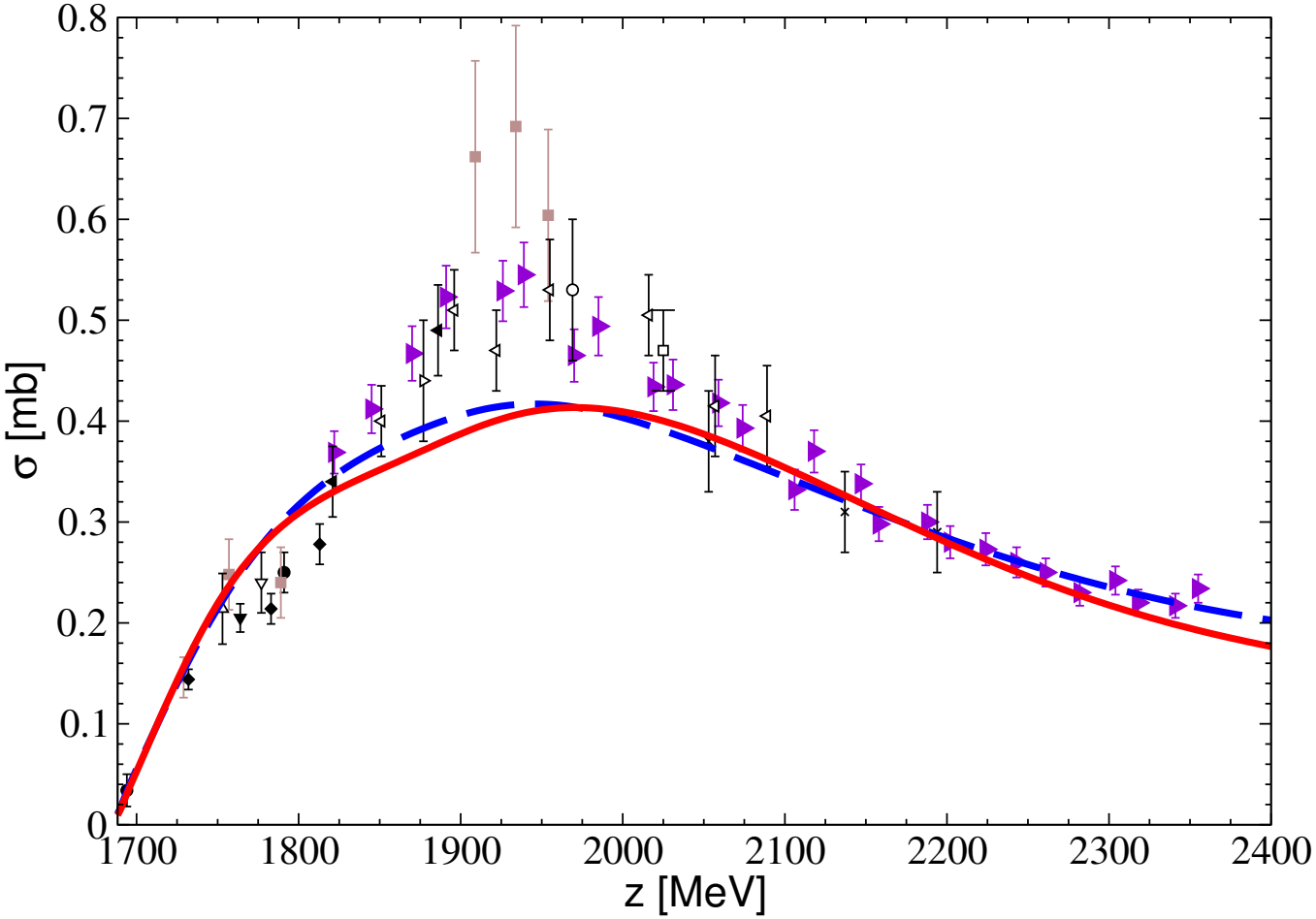

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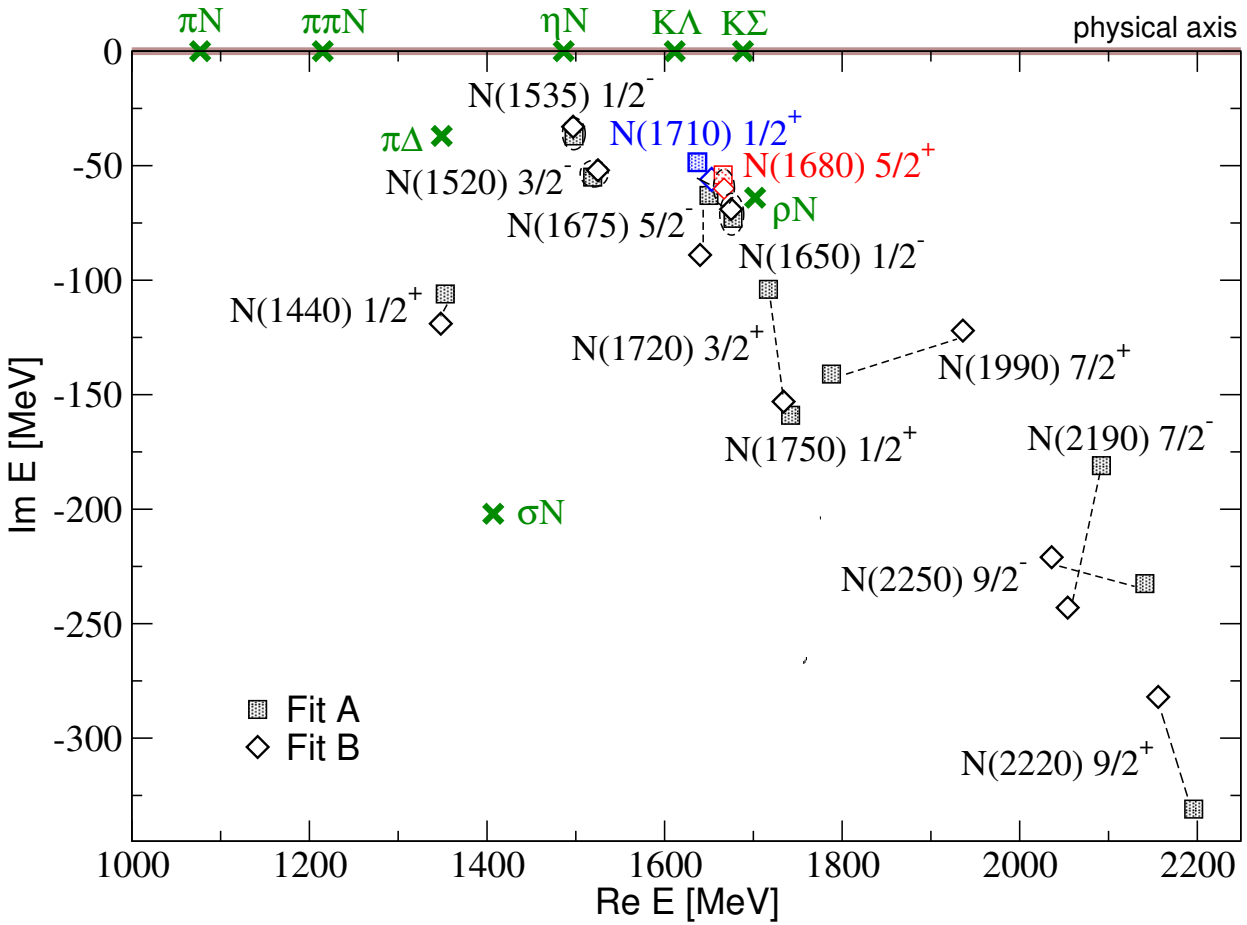
Resonance content: I=1/2

Pole search on the 2nd sheet of the scattering matrix $T_{\mu\nu}$

Resonance parameter:

- "mass" $= Re(E_0)$
- "width"$= -2Im(E_0)$
- Residues $\rightarrow$ branching ratios

E_0: pole position



x: branch points

Notation: $N(\text{"}name\text{"})\,J^{parity}$

Numbers

Resonance content: I = 1/2

	Re z_0 [MeV]		-2Im z_0 [MeV]		$\lvert r_{\pi N}\rvert$ [MeV]		$\theta_{\pi N\to\pi N}$ [deg]		$\Gamma_{\pi N}/\Gamma_{\rm tot}$ [%]	
fit→	A	B	A	B	A	B	A	B	A	B
$N(1535)\ 1/2^-$	1498	1497	74	66	17	13	-37	-43	40	44
$N(1650)\ 1/2^-$	1677	1675	146	131	45	27	-43	-38	61	41
$N(1440)\ 1/2^+_{(a)}$	1353	1348	212	238	59	62	-103	-111	56	52
$N(1710)\ 1/2^+$	1637	1653	97	112	4	8	-30	34	8.2	14
$N(1750)\ 1/2^+_{(*,a)}$	1742	(nf)	318	-	8	-	161	-	5.1	-
$N(1720)\ 3/2^+$	1717	1734	208	306	7	18	-76	-23	6.6	11.6
$N(1520)\ 3/2^-$	1519	1525	110	104	42	36	-16	10	79	69
$N(1675)\ 5/2^-$	1650	1640	126	178	24	34	-19	-32	39	38
$N(1680)\ 5/2^+$	1666	1667	108	120	36	41	-24	-24	67	68
$N(1990)\ 7/2^+$	1788	1936	282	244	4	4	-84	-87	3.2	3.6
$N(2190)\ 7/2^-$	2092	2054	363	486	42	44	-31	-57	23	18
$N(2250)\ 9/2^-$	2141	2036	465	442	17	13	-67	-62	7.5	5.7
$N(2220)\ 9/2^+$	2196	2156	662	565	87	46	-67	-72	26	16

back

Resonance content: I = 1/2 Branching ratios

	$\frac{\Gamma^{1/2}_{\pi N}\Gamma^{1/2}_{\eta N}}{\Gamma_{tot}}$ [%]		$\theta_{\pi N\to\eta N}$ [deg]		$\frac{\Gamma^{1/2}_{\pi N}\Gamma^{1/2}_{K\Lambda}}{\Gamma_{tot}}$ [%]		$\theta_{\pi N\to K\Lambda}$ [deg]		$\frac{\Gamma^{1/2}_{\pi N}\Gamma^{1/2}_{K\Sigma}}{\Gamma_{tot}}$ [%]		$\theta_{\pi N\to K\Sigma}$ [deg]	
	A	B	A	B	A	B	A	B	A	B	A	B
$N(1535)\ 1/2^-$	51	48	120	115	7.7	8.3	68	77	15	34	-74	-83
$N(1650)\ 1/2^-$	15	12	57	46	25	18	-46	-43	26	15	-63	-59
$N(1440)\ 1/2^+_{(a)}$	2	5	-40	-26	2	11	156	152	1	2	67	189
$N(1710)\ 1/2^+$	24	23	130	164	9.4	17	-83	-41	3.9	0.1	-136	-112
$N(1750)\ 1/2^+_{(*,a)}$	0.5	-	-140	-	0.8	-	-170	-	2.2	-	4	-
$N(1720)\ 3/2^+$	1.2	3.1	98	117	3.1	2.9	-89	-63	1.7	2.2	64	90
$N(1520)\ 3/2^-$	3.5	3.0	87	113	5.8	6.3	158	177	0.8	3.6	163	164
$N(1675)\ 5/2^-$	6.0	3.6	-40	-66	0.3	1.7	-93	-122	3.3	3.7	-168	179
$N(1680)\ 5/2^+$	0.4	1.5	-47	-54	0.2	0.3	-99	72	0.1	0.1	141	141
$N(1990)\ 7/2^+$	0.4	0.5	-99	90	1.7	1.5	-123	-99	0.8	1.2	28	70
$N(2190)\ 7/2^-$	0.1	0.4	-28	99	1.9	0.3	-51	-75	1.3	0.5	-63	-105
$N(2250)\ 9/2^-$	0.6	0.1	-92	-96	1.1	0.7	-103	-106	0.3	0.7	-114	62
$N(2220)\ 9/2^+$	0.1	0.3	63	74	0.9	0.8	53	59	0.8	0.1	-138	52

back

Resonance content: I = 3/2

	Re z_0 [MeV]		-2Im z_0 [MeV]		$\lvert r_{\pi N}\rvert$ [MeV]		$\theta_{\pi N\to\pi N}$ [deg]	
fit→	A	B	A	B	A	B	A	B
$\Delta(1620)\ 1/2^-$	1599	1596	71	80	17	18	-107	-107
$\Delta(1910)\ 1/2^+$	1788	1848	575	376	56	20	-140	-143
$\Delta(1232)\ 3/2^+$	1220	1218	86	96	44	50	-35	-38
$\Delta(1600)\ 3/2^+_{(a)}$	1553	1623	352	284	20	27	-158	-124
$[\Delta(1920)\ 3/2^+$	1724	1808	863	887	36	19	163	-70]
$\Delta(1700)\ 3/2^-$	1675	1705	303	185	24	14	-9	-4
$\Delta(1930)\ 5/2^-$	1775	1805	646	580	18	14	-159	3
$\Delta(1905)\ 5/2^+$	1770	1776	259	143	17	9	-59	-40
$\Delta(1950)\ 7/2^+$	1884	1890	234	232	58	58	-25	-19
$\Delta(2200)\ 7/2^-$	2147	2111	477	353	17	20	-52	7
$\Delta(2400)\ 9/2^-$	1969	1938	577	559	25	19	-80	-112

back

Resonance content: I = 3/2 Branching ratios

	$\Gamma_{\pi N}/\Gamma_{\text{tot}}$ [%]		$\frac{\Gamma^{1/2}_{\pi N}\Gamma^{1/2}_{K\Sigma}}{\Gamma_{\text{tot}}}$ [%]		$\theta_{\pi N\to K\Sigma}$ [deg]	
fit→	A	B	A	B	A	B
$\Delta(1620)\ 1/2^-$	48	45	22	24	-107	-107
$\Delta(1910)\ 1/2^+$	20	10	4.7	1.9	-144	-115
$\Delta(1232)\ 3/2^+$	100	100				
$\Delta(1600)\ 3/2^+_{(a)}$	11	19	11	13	-7	41
$[\Delta(1920)\ 3/2^+$	8.3	4.3	16	14	-20	50
$\Delta(1700)\ 3/2^-$	16	16	1.5	1.6	-150	-121
$\Delta(1930)\ 5/2^-$	5.6	4.8	3.1	1.7	-3	135
$\Delta(1905)\ 5/2^+$	13	12	0.5	0.1	-142	-99
$\Delta(1950)\ 7/2^+$	50	50	4.0	3.8	-78	-71
$\Delta(2200)\ 7/2^-$	7.2	11	0.6	0.1	-98	-33
$\Delta(2400)\ 9/2^-$	8.7	5.7	1.3	0.8	40	6

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Data base

simultaneous fit to $\pi N \to \pi N$, ηN, $K\Lambda$, $K\Sigma$

World data base on ηN, $K\Lambda$, $K\Sigma$

	PWA	σ_{tot}	$\frac{d\sigma}{d\Omega}$	P	β
$\pi N \to \pi N$	GWU/SAID 2006 up to J=9/2				
$\pi^- p \to \eta n$		62 data points	38 energy points z=1489 to 2235 MeV	12 energy points 1740 to 2235 MeV	
$\pi^- p \to K^0 \Lambda$		66 data points	46 energy points 1626 to 1405 MeV	27 energy points 1633 to 2208 MeV	7 energy points 1852 to 2262 MeV
$\pi^- p \to K^0 \Sigma^0$		16 data points	29 energy points 1694 to 2405 MeV	19 energy points 1694 to 2316 MeV	
$\pi^- p \to K^+ \Sigma^-$		14 data points	15 energy points 1739 to 2405 MeV		
$\pi^+ p \to K^+ \Sigma^+$		18 data points	32 energy points 1729 to 2318 MeV	32 energy points 1729 to 2318 MeV	2 energy pionts 2021 and 2107 MeV

$\sim$ 6000 data points

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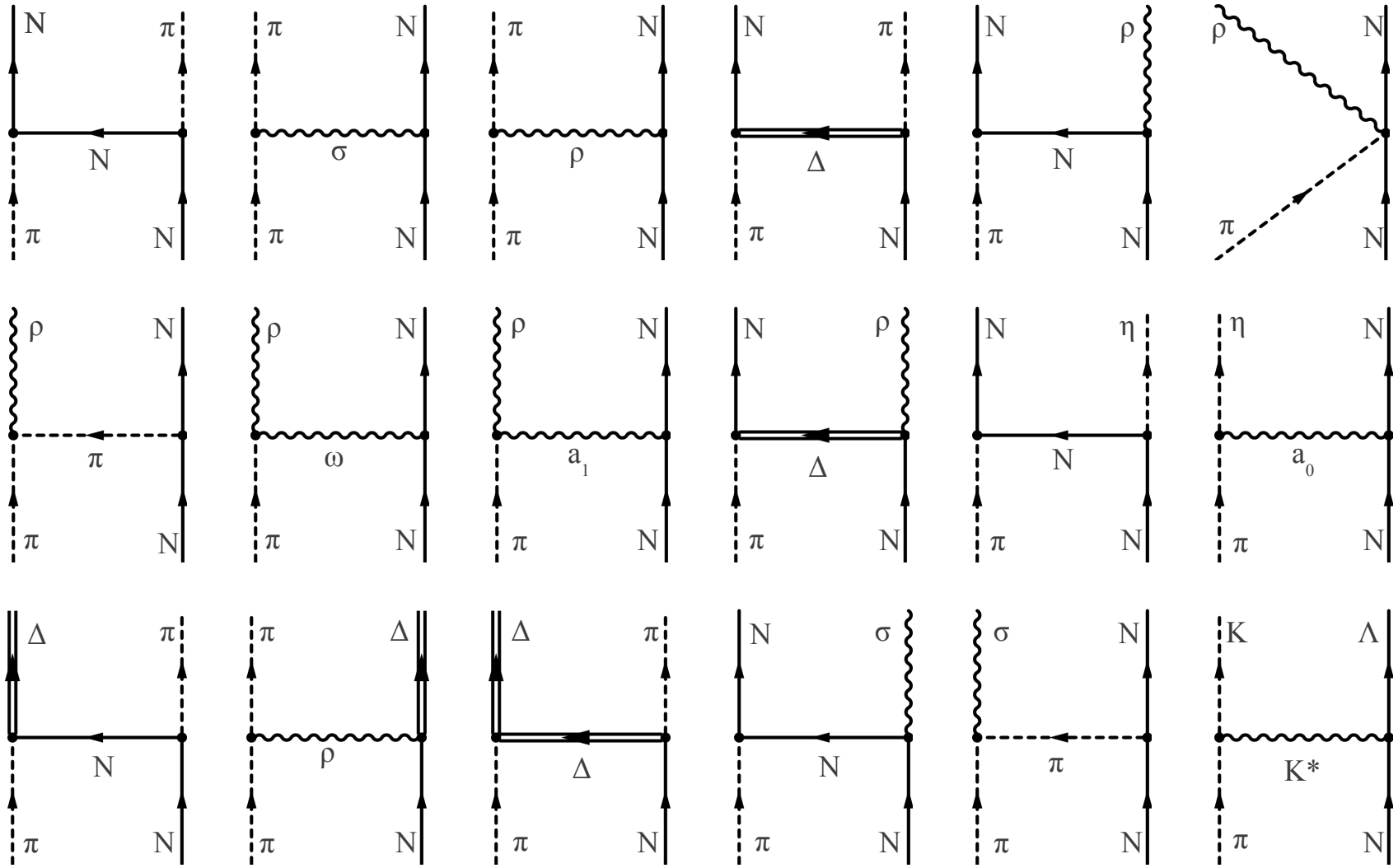
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π N σ π N
π N ρ π N
N π Δ π N
N ρ N π N
ρ N π π N
ρ N π π N
ρ N ω π N
ρ N a1 π N
N ρ Δ π N
N η N π N
η N a0 π N
Δ π N π N
π Δ ρ π N
Δ π Δ π N
N σ N π N
σ N π π N
K Λ K* π N

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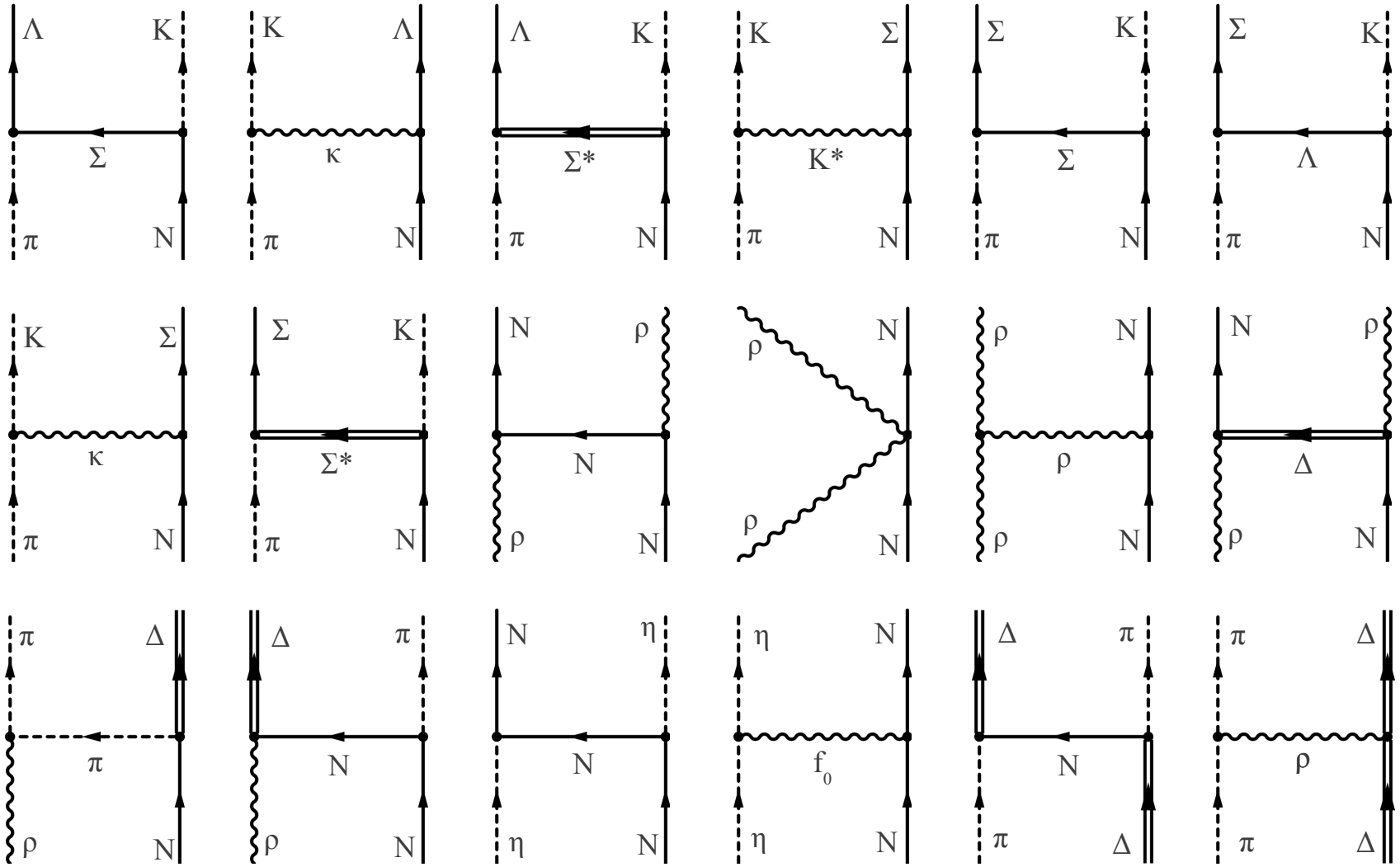
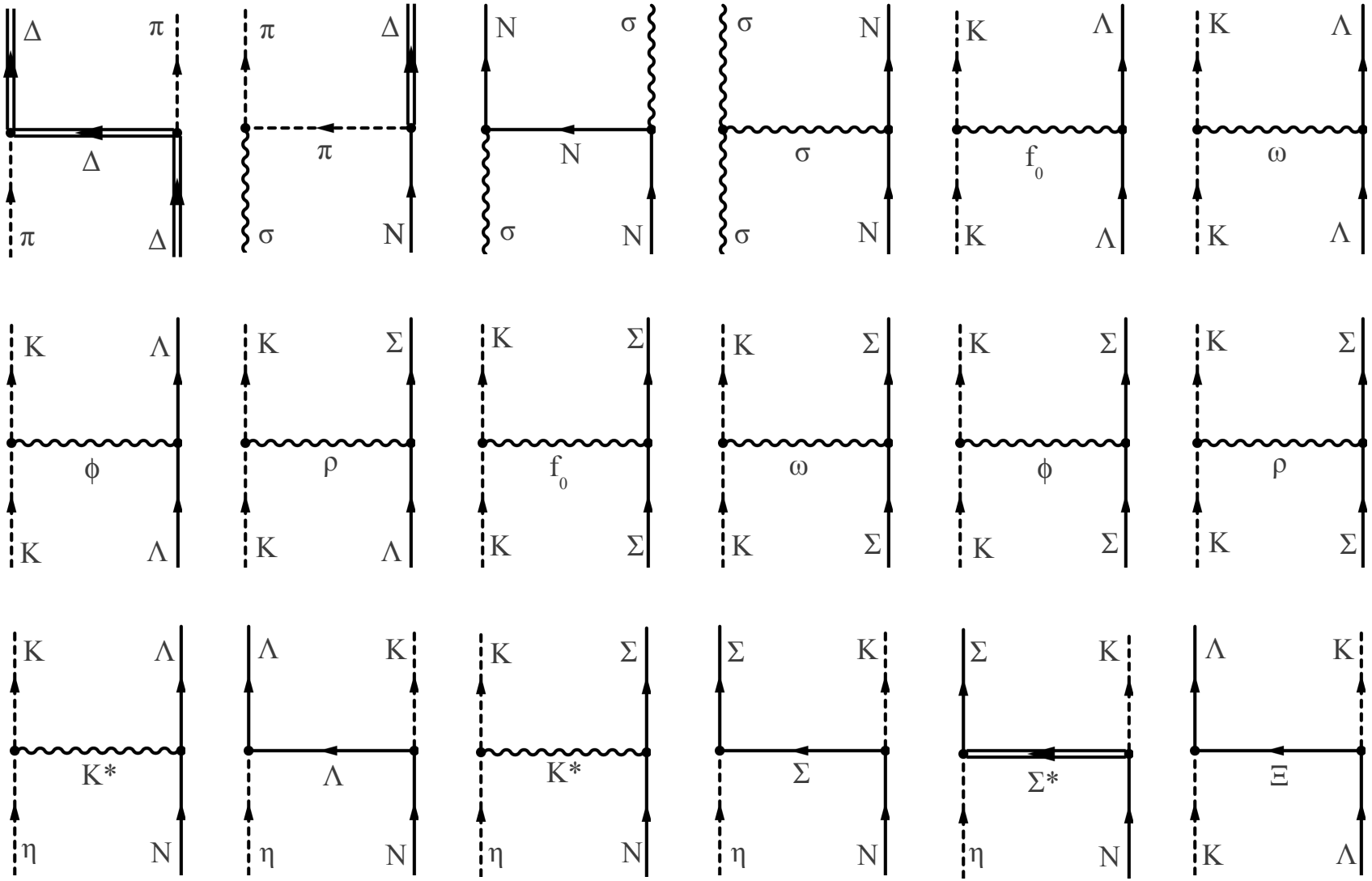

Λ K Σ π N
K Λ κ π N
Λ K Σ* π N
K Σ K* π N
Σ K Σ π N
Σ K Λ π N
K Σ κ π N
Σ K Σ* π N
N ρ N ρ N
ρ N ρ N
ρ N ρ ρ N
N ρ Δ ρ N
π Δ π ρ N
Δ π N ρ N
N η N η N
η N f0 η N
Δ π N π Δ
π Δ ρ π Δ

back


Δ π Δ π
π Δ π σ N
N σ N σ N
σ N σ σ N
K Λ f0 K Λ
K Λ ω K Λ
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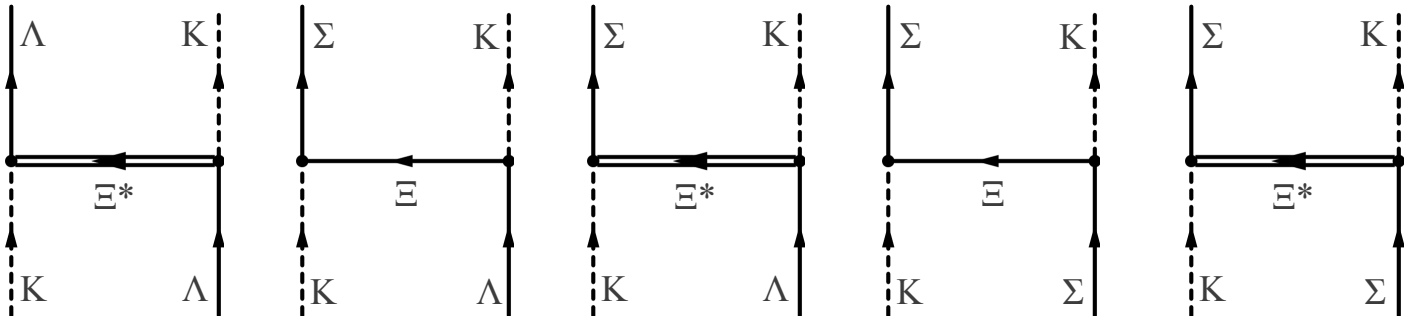
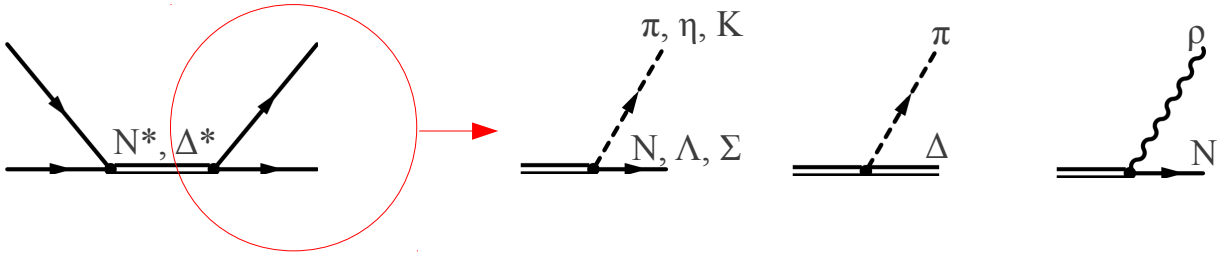
K Σ f0 K Σ
K Σ ω K Σ
K Σ ϕ K Σ
K Σ ρ K Σ
K Λ K* η N
Λ K Λ η N
K Σ K* η N
Σ K Σ η N
Σ K Σ* η N
Λ K Ξ K Λ

t and u, contd.; s-channel exchanges





Potentials for resonances with $J > 3/2$

- Correct dependence on the orbital angular momentum (centrifugal barrier)

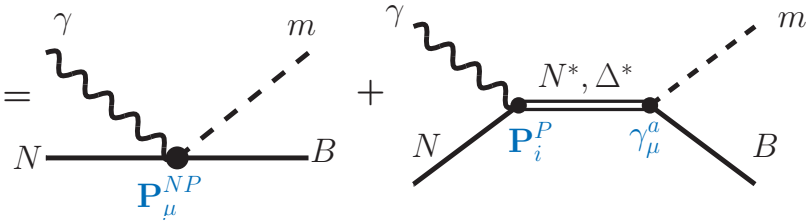
$$v_{\gamma\frac{5}{2}^-} = \frac{k}{M}\, v_{\gamma\frac{3}{2}^+} \qquad v_{\gamma\frac{7}{2}^-} = \frac{k^2}{M^2}\, v_{\gamma\frac{3}{2}^-}$$

$$v_{\gamma\frac{5}{2}^+} = \frac{k}{M}\, v_{\gamma\frac{3}{2}^-} \qquad v_{\gamma\frac{7}{2}^+} = \frac{k^2}{M^2}\, v_{\gamma\frac{3}{2}^+}$$

$\gamma = \pi N$, ρN, ηN, $\pi\Delta$, σN, $K\Lambda$ and $K\Sigma$

M : mass of the baryon in the particular channel

Phenomenological potential:

$\mathbf{V}_{\mu\gamma}(E,q) =$ 

$$= \frac{\tilde{\gamma}^a_{\mu;i}(q)}{m_N} P^{\mathrm{NP}}_\mu(E) + \sum_i \frac{\gamma^a_{\mu;i}(q) P^{\mathrm{P}}_i(E)}{E - m_i^b}$$

P^{P}_i, P^{NP}_μ: energy-dependent polynomials

$\tilde{\gamma}^a_{\mu;i}$, $\gamma^a_{\mu;i}$: hadronic vertices $\rightarrow$ correct threshold behaviour

i: resonance number per multipole

μ: channels πN, ηN, $\pi\Delta$

Polynomials

Helicity multipoles $A^h_{L\pm}$:

- $J = L + 1/2$:

$$A^{1/2}_{L+} = -\frac{1}{2}\left[(L+2)E_{L+} + L\,M_{L+}\right]$$
$$A^{3/2}_{L+} = \frac{1}{2}\sqrt{L(L+2)}\left[E_{L+} - M_{L+}\right]$$

- $J = L - 1/2$:

$$A^{1/2}_{L-} = -\frac{1}{2}\left[(L-1)\,E_{L-} - (L+1)M_{L-}\right]$$
$$A^{3/2}_{L-} = -\frac{1}{2}\sqrt{(L-1)(L+1)}\left[E_{L-} + M_{L-}\right]$$

Matching to lattice

Prediction & analysis of lattice data

[M. Döring et al., EPJ A47, 163 (2011)]

Scattering equation:

$$T(q'',q') \quad = \quad V(q'',q') + \int_0^\infty dq\, q^2\, V(q'',q) \frac{1}{z - E_1(q) - E_2(q) + i\epsilon}\, T(q,q')$$

Discretization of momenta in the scattering equation:

$$\int \frac{\vec{d}\,^3 q}{(2\pi)^3}\, f(|\vec{q}\,|^2) \quad \rightarrow \quad \frac{1}{L^3} \sum_{\vec{n}_i} f(|\vec{q}_i|^2), \quad \vec{q}_i = \frac{2\pi}{L}\, \vec{n}_i, \quad \vec{n}_i \in \mathbb{Z}^3$$

$$T(q'',q') \quad = \quad V(q'',q') + \frac{2\pi^2}{L^3} \sum_{i=0}^{\infty} \vartheta(i) V(q'',q_i) \frac{1}{z - E_1(q_i) - E_2(q_i)}\, T(q_i,q'),$$

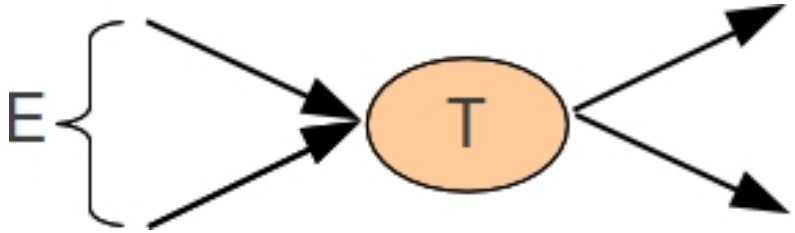
$\vartheta^{(P)}(i)$ series

- Study finite size effects
- Predict lattice spectra

Two-body scattering
in the infinite volume limit

- Unitarity of the scattering matrix S: $SS^\dagger = \mathbb{1}$ $\qquad [S = \mathbb{1} - i\,\frac{p}{4\pi E}\,T]$.

$$\mathrm{Im}\, T^{-1}(E) = \sigma \equiv \frac{p}{8\pi\, E}$$



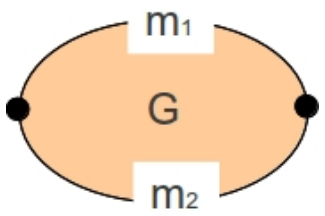
- $\rightarrow$ Generic (Lippman-Schwinger) equation for unitarizing the T-matrix:

$$T = V + V\,G\,T \qquad \mathrm{Im}\, G = -\sigma$$

V: (Pseudo)potential, σ: phase space.

- G: Green's function:

$$\begin{aligned} G &= \int \frac{d^3\vec{q}}{(2\pi)^3}\,\frac{f(|\vec{q}|)}{E^2 - (\omega_1+\omega_2)^2 + i\epsilon}, \\ \omega_{1,2}^2 &= m_{1,2}^2 + \vec{q}^{\,2} \end{aligned}$$



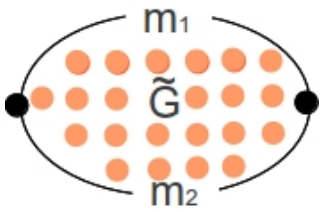
Discretization

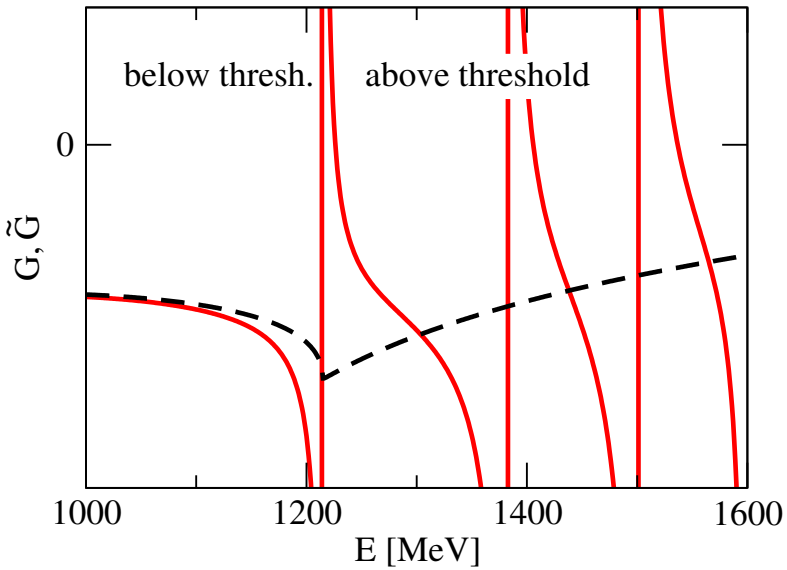
Discretized momenta in the finite volume with periodic boundary conditions

$$\Psi(\vec{x}) \stackrel{!}{=} \Psi(\vec{x} + \hat{\mathbf{e}}_i\, L) = \exp\left(i\, L\, q_i\right) \Psi(\vec{x}) \quad \Longrightarrow \quad q_i = \frac{2\pi}{L}\, n_i, \quad n_i \in \mathbb{Z}, \quad i = 1, 2, 3$$

$$\boxed{\int \frac{d^3\vec{q}}{(2\pi)^3}\, g(|\vec{q}\,|^2) \to \frac{1}{L^3} \sum_{\vec{n}} g(|\vec{q}|^2), \quad \vec{q} = \frac{2\pi}{L}\, \vec{n}, \quad \vec{n} \in \mathbb{Z}^3}$$

$$G \to \tilde{G} = \frac{1}{L^3} \sum_{\vec{q}} \frac{f(|\vec{q}|)}{E^2 - (\omega_1 + \omega_2)^2}$$





- $E > m_1 + m_2$: $\tilde{G}$ has poles at *free energies in the box*, $E = \omega_1 + \omega_2$
- $E < m_1 + m_2$: $\tilde{G} \to G$ exponentially with L (regular summation theorem).

Finite $\to$ infinite volume: the Lüscher equation

Warning: Very crude re-derivation.

- Measured eigenvalues of the Hamiltonian (tower of *lattice levels* $E(L)$) $\to$ Poles of scattering equation $\tilde{T}$ in the finite volume $\to$ determines V:

$$\tilde{T} = (1 - V\tilde{G})^{-1} V \;\to\; V^{-1} - \tilde{G} \stackrel{!}{=} 0 \;\to\; V^{-1} = \tilde{G}$$

- The interaction V determines the T-matrix in the infinite volume limit:

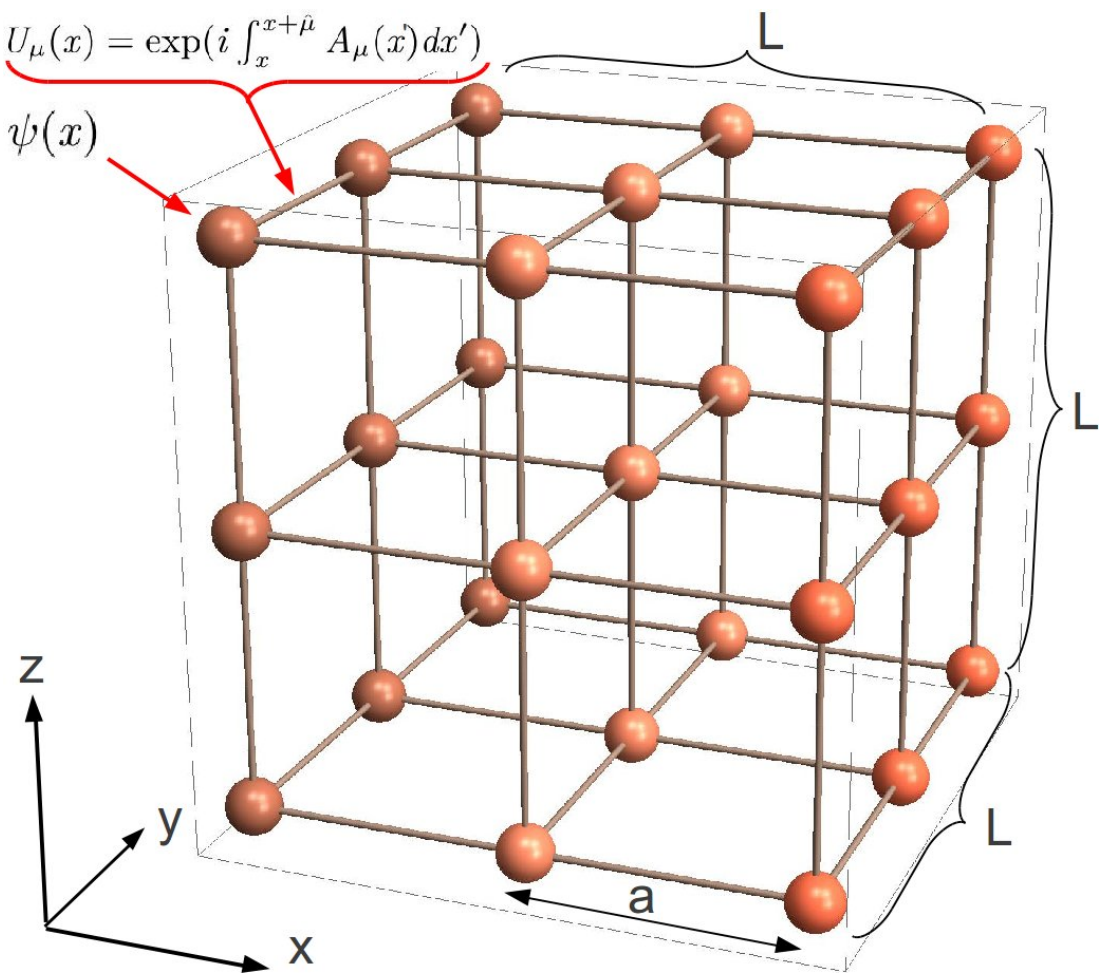
$$T = \left(V^{-1} - G\right)^{-1} = \left(\tilde{G} - G\right)^{-1}$$

- Re-derivation of Lüscher's equation (T determines the phase shift δ):

$$\boxed{p \cot \delta(p) = -8\pi\sqrt{s}\left(\tilde{G}(E) - \mathrm{Re}\, G(E)\right)}$$

  - V and dependence on renormalization have disappeared (!)
  - p: c.m. momentum
  - E: scattering energy
  - $\tilde{G} - \mathrm{Re}G$: known kinematical function ($\simeq \mathcal{Z}_{00}$ up to exponentially suppressed contributions)
  - One phase at one energy.

Connecting phenomenology to Lattice QCD



- Side length L,
  periodic boundary conditions
  $\Psi(\vec{x}) \stackrel{!}{=} \Psi(\vec{x} + \hat{\mathbf{e}}_i L)$
  $\rightarrow$ finite volume effects
  $\rightarrow$ Infinite volume $L \rightarrow \infty$ extrapolation
- Lattice spacing a
  $\rightarrow$ finite size effects
  Modern lattice calculations:
  $a \simeq 0.07$ fm $\rightarrow p \sim 2.8$ GeV
  $\rightarrow$ (much) larger than typical hadronic scales;

  not considered here.
- Unphysically large quark/hadron masses
  $\rightarrow$ (chiral) extrapolation required.