Theory and Application of UAL/ETEAPOT for EDM Storage Rings

Richard Talman and John Talman

Laboratory for Elementary-Particle Physics
Cornell University

IPAC-2015, Richmond, VA
Satellite Meeting on Spin Tracking for Precision Measurements
May 5, 2015

2 Outline

The ETEAPOT Code

- Code Status
- Relativistic kinematics in electric potential $V(r)$
- "Integrable", exact, betatron motion
- Lumped correction for field index deviation
- "Exact" solution of the BMT equation

Figures from ETEAPOT-Simulated AGS Electron Analogue Ring

- The AGS Electron Analogue Ring
- Reconstruction of Ernest Courant tune plane resonance machine studies using AGS Analogue
- Long term spin tracking in a proton EDM ring

Extra Slides

- "Conceptual Design Report" 1953 letter from BNL to AEC
- Two detailed AGS Analogue figures from Plotkin

3 Code Status

- This presentation is extracted from the following two papers, both of which have been submitted for publication to PRST-AB.
- arXiv:1503.08468v1 [physics.acc-ph] 29 Mar 2015 , *ETEAPOT: symplectic orbit/spin tracking code for all-electric storage rings*, Richard Talman and John Talman
- arXiv:1503.08494v1 [physics.acc-ph] 29 Mar 2015, *EDM planning using ETEAPOT with a resurrected AGS Electron Analogue ring* , Richard Talman and John Talman

ETEAPOT PUBLICATION SECTION HEADINGS

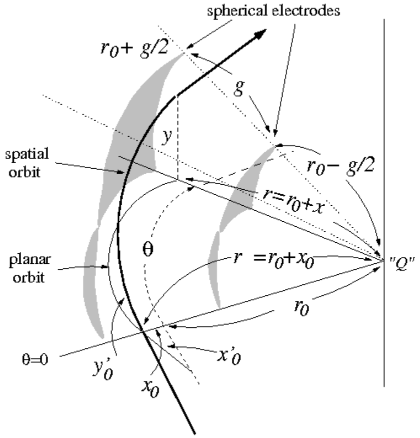


Figure: The bold curve shows a particle orbit passing through a *spherical*, $m = 1$, electrostatic bending element. The shaded surfaces are spherical electrodes. The "Q" shown at the origin is the "effective point charge" that would give the same electric field as the electrodes.

6 Relativistic kinematics in electric potential $V(r)$

- Radial electric field with index m power law dependence on radius r;

$$\mathbf{E} = -E_0 \frac{r_0^{1+m}}{r^{1+m}} \hat{\mathbf{r}}, \tag{1}$$

- The Lorentz force equation in the m=1 spherical case is

$$\frac{d\mathbf{p}}{dt} = -k \frac{\hat{\mathbf{r}}}{r^2}, \tag{2}$$

- Both the total energy $\mathcal{E} = \gamma mc^2 + V(\mathbf{r})$ and angular momentum $\mathbf{L} = \mathbf{r} \times \mathbf{p}$ are conserved in $m = 1$ bend elements.
- The orbit equation for radial coordinate $r(\theta)$ is

$$r(\theta) = \frac{\lambda}{1 + \epsilon \cos \kappa(\theta - \theta_0)}; \tag{3}$$

- This differs from the Newton/Kepler ellipse formula only because the relativistic effect is to make κ (the "tune" in accelerator jargon) deviate from 1. Eccentricity ϵ is close to zero.
- The orbit is not closed; rather, the "perihelion advances".

7 “Integrable”’ Betatron Motion

- Muñoz and Pavic (relativistic astrophysicists) show that the “generalized”-Hamilton vector

$$\mathbf{h} = h_r\,\hat{\mathbf{r}} + h_\theta\,\hat{\theta}, \tag{4}$$

is ideal for describing 2D relativistic Kepler orbits.
- (Roughly speaking) h_r and h_θ are betatron coordinates x and x'.
- But h_r and h_θ evolve sinusoidally for all betatron amplitudes.
- In spite of being nonlinear, the motion is “integrable”.
- This is what permits ETEAPOT tracking to be exact, and exactly symplectic (with no artificial symplectification).
- With conventional accelerator formulation (with thick multipole elements allowed) this is impossible.

8 Lumped correction for field index deviation

- Normally $m \neq 1$. This is handled by inserting zero thickness "effective quadrupoles" of appropriate strength.
- The transfer matrices for the thin effective quadrupole for electric bend angle $\Delta\theta$ at bend radius r_0 are

$$\mathbf{K_x^{(m)}}(\Delta\theta) = \begin{pmatrix} 1 & 0 \\ (m-1)\Delta\theta/r_0 & 1 \end{pmatrix},$$
$$\mathbf{K_y^{(m)}}(\Delta\theta) = \begin{pmatrix} 1 & 0 \\ (-m+1)\Delta\theta/r_0 & 1 \end{pmatrix}. \tag{5}$$

- This approximation becomes arbitrarily accurate with sufficiently fine element slicing.

9 “Exact” solution of the BMT equation

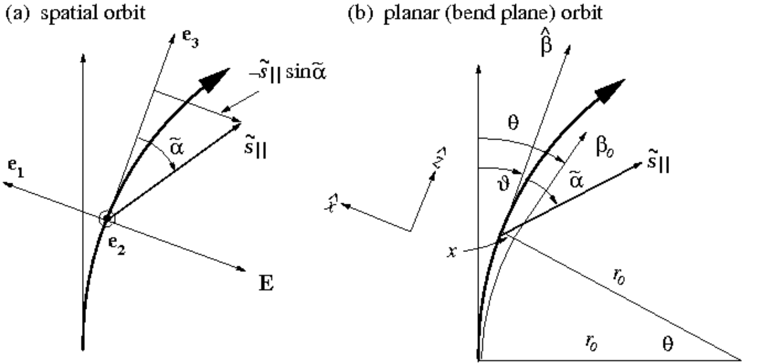


Figure: (a) In the bend plane the spin vector **s** has precessed through angle $\tilde{\alpha}$ away from its nominal direction along the proton’s velocity. (b) Projection of figure (a) onto the laboratory horizontal plane. x is the deviation of the (bold face) particle orbit from the (pale face) design orbit. θ is the reference particle deviation angle from longitudinal and ϑ is the tracked particle deviation angle from longitudinal. Betatron oscillations cause them to differ on a turn by turn basis, and also cause the instantaneous bend plane to wobble away from horizontal.

- Bend plane axes are $\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}}$.
- Spin component $\mathbf{s}_\perp = \tilde{s}_y \hat{\mathbf{y}}$ normal to bend plane is conserved.
- Spin vector parallel to the bend plane is

$$\tilde{\mathbf{s}} = -\tilde{s}_{\|} \sin\tilde{\alpha}\,\hat{\mathbf{x}} + \tilde{s}_y \hat{\mathbf{y}} + \tilde{s}_{\|} \cos\tilde{\alpha}\,\hat{\mathbf{z}}. \tag{6}$$

  $\tilde{s}_{\|}$ is the (conserved) magnitude of the in-plane projection of $\tilde{\mathbf{s}}$, and $\tilde{\alpha}$ is the angle between the projection of $\tilde{\mathbf{s}}$ onto the plane and the tangent vector to the orbit.
- Jackson gives the rate of change of the longitudinal spin component in an electric field $\mathbf{E}$ as

$$\frac{d}{dt}\,(\hat{\beta}\cdot\mathbf{s}) = -\frac{e}{m_p c}\,(\mathbf{s}_\perp \cdot \mathbf{E})\left(\frac{g\beta}{2} - \frac{1}{\beta}\right). \tag{7}$$

Substituting from Eq. (6) the BMT equation becomes

$$\frac{d}{dt}\left(\tilde{s}_{\|}\cos\tilde{\alpha}\right) = -\frac{e}{m_p c}\left(\tilde{s}_{\|}\sin\tilde{\alpha}\,E\right)\left(\frac{g\beta}{2} - \frac{1}{\beta}\right), \tag{8}$$

where $\tilde{s}_{\|}$ is constant. Then Eq. (8) reduces to

$$\frac{d\tilde{\alpha}}{dt} = \frac{eE}{m_p c}\left(\frac{g\beta}{2} - \frac{1}{\beta}\right). \tag{9}$$

Because the curvature is $1/r = eE/(vp)$, the advance of particle angle ϑ is governed by the equation

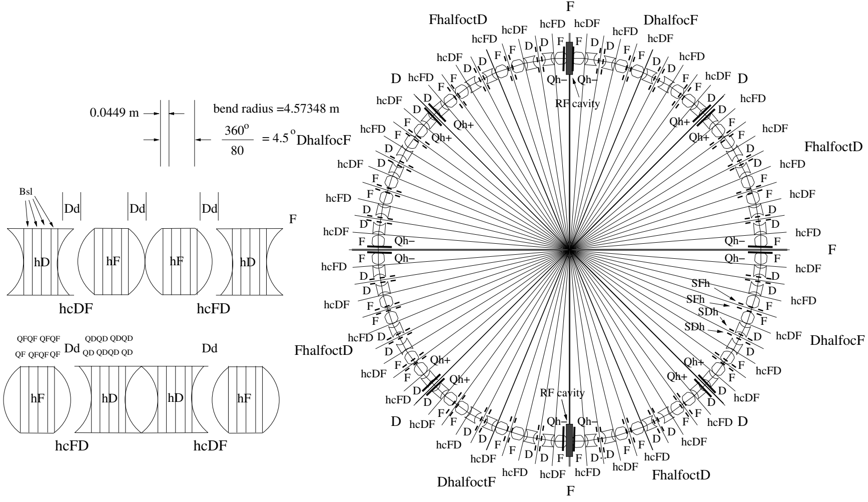
$$\frac{d\vartheta}{dt} = \frac{d}{dt}\left(\frac{s}{r}\right) = \frac{eE}{p}. \tag{10}$$

- Combine Eqs. (9) and (10)
- Angles θ and ϑ, though not quite identical, differ only in higher order. Furthermore they advance at exactly the same rate on the average.
- Setting $\theta = \vartheta$,

$$\boxed{\frac{d\tilde{\alpha}}{d\theta} = \left(\frac{g}{2} - 1\right)\gamma - \frac{g/2}{\gamma}.} \tag{11}$$

- Even allowing for the θ dependence of $\gamma(\theta)$ (which is small) this equation can be integrated in closed form.
- Fringe field precession is handled similarly, assuming the fringe field bend plane is identical to the bend element bend plane (which is very nearly, but not exactly true).

13 The AGS Electron Analogue Ring



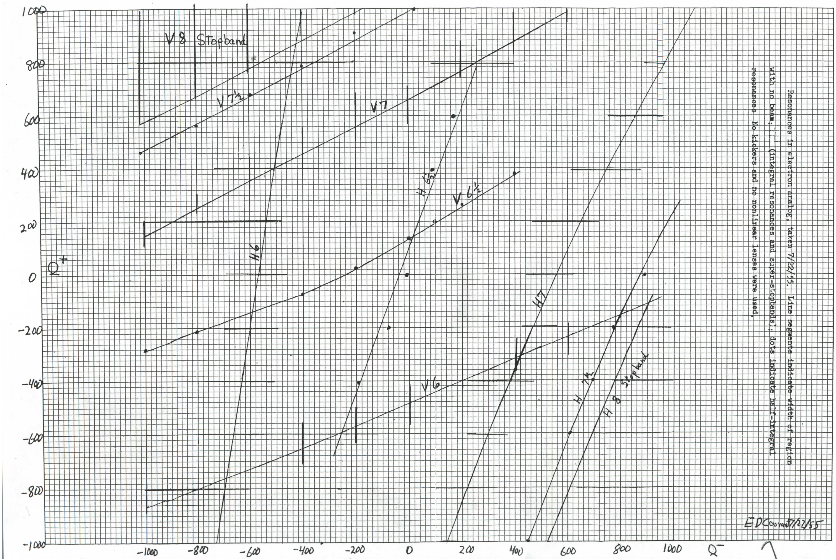


Figure: Ernest Courant 1954 machine studies tune plane resonance plot. Short lines indicate *no beam* integer resonance. Dots indicate *perturbed beam* half-integer resonance. The axes are quad-family strengths.

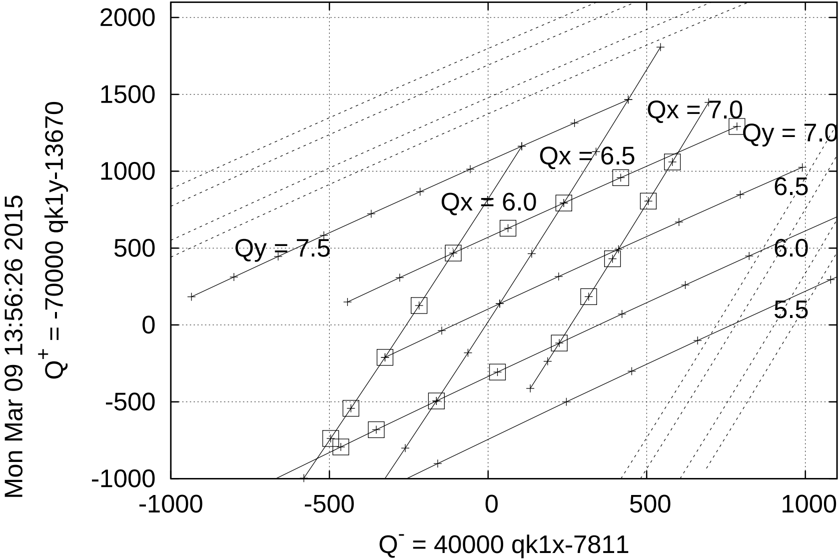


Figure: Reconstruction of Ernest Courant 1954 machine studies tune plane resonance plot using TEAPOT. Boxes indicate integer tune, crosses indicate half-integer tune. The axes are quad-family strengths.

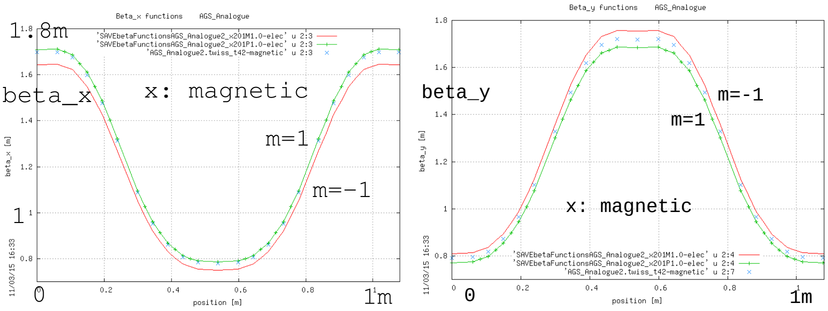


Figure: β_x and β_y lattice function plots for the AGS-Analogue Ring, modeled as all-electric ($m = \pm 1$) or all-magnetic. Because of the strong focusing, $(Q_x, Q_y) = (6.5, 6.5)$, there is little difference between electric and magnetic—comparable to the change in electric field index from $m = -1$ to $m = 1$.

- This will not be true for proton EDM lattices where the tunes will be much smaller; e.g. $(Q_x, Q_y) = (2.5, 0.2)$, where quadrupole and electric bend focusing strengths are comparable.

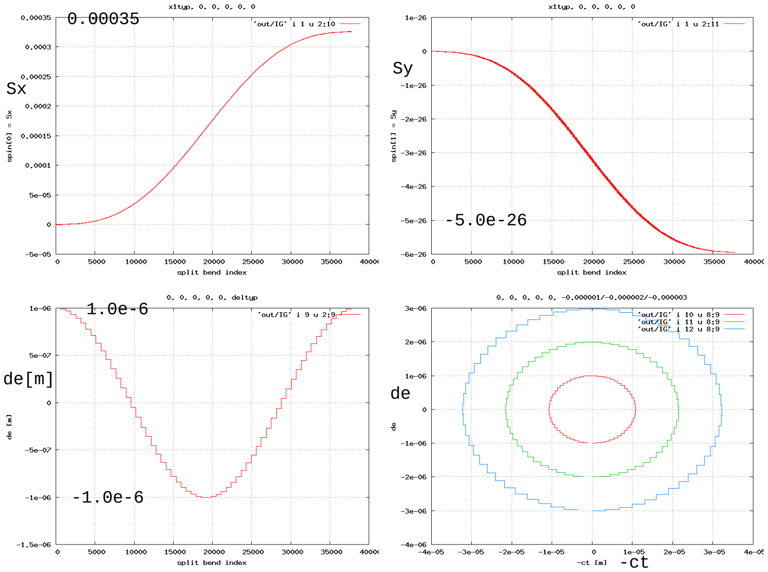


Figure: Spin and longitudinal phase space evolution during one synchrotron oscillation period of the AGS Analogue all-electric ring.

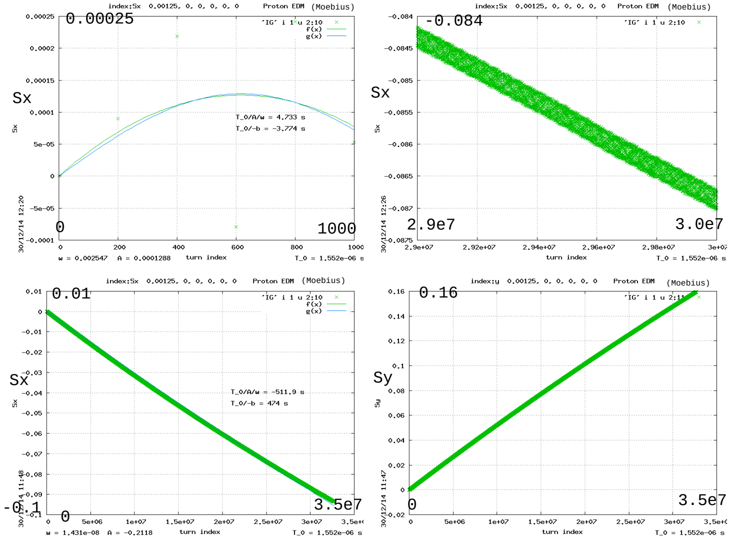


Figure: 32 million turn (50 s real time) spin tracking in Möbius-converted (to produce long spin coherence time (SCT)) proton EDM lattice. The upper graphs show brief intervals near the beginning and end of the run.

BNL Director Haworth Electric Analogue Proposal Letter to AEC

August 21, 1953

Dr. T.H. Johnson, Director
Division of Research
U.S. Atomic Energy Commission
Washington 25, D.C.

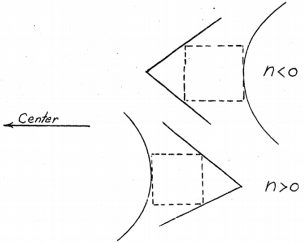
Dear Tom:

This letter concerns certain aspects of our accelerator development program, particularly the proposed electron model.

As you know, the general development of a very high energy alternating gradient synchrotron is proceeding actively at Brookhaven, utilizing operating funds allocated to Basic Physics Research. As I explained in my letter of August 12, however, these funds are insufficient to carry forward the development as rapidly as desirable. Also, there are certain steps which should be taken for which the expenditure of operating funds is not appropriate. The first and most important of these is the construction of an electron model intended to provide final assurance of the technical feasibility of the chosen machine and, more importantly, to provide information enabling us to design in the most effective and economical manner. (We have no doubt of the general feasibility of accelerators of this type.)

We have given considerable thought to the requirements for such a model and to the philosophy which should guide us in designing and building it. In the alternating gradient synchrotron, two problems require especially careful exploration by extensive calculation and experimental modelling. These are the close-spaced resonances in the betatron oscillations and the shift of phase stability at intermediate energies. It seems best to study these problems with an electron accelerator which would be essentially an analogue rather than an exact model. This device should, in our opinion, be designed to yield the maximum of orbital data with a minimum of engineering complications, especially those not applicable to a final machine. After considerable thought we have arrived at a tentative description and list of parameters which follow.

The device would consist of an accelerator having an orbital radius of 15 feet and an overall diameter including the straight sections, of approximately 45 feet; the guide and focussing fields would be electrostatic, with electrode shapes as indicated in the sketch (full scale).



Electrons of about 1 MeV energy would be injected from a small horizontal Van de Graaff generator (of the 2 MeV type manufactured by the High Voltage Engineering Corporation) so that 5% to 6% frequency modulation would be required.

Use of a reasonably large radius helps the radio frequency and observing equipment in frequency range where good techniques exist, and permits high n-values which are necessary for strong alternating-gradient focussing. (This, and phase transition, will not be modeled in the Cornell machine.) A moderate rise rate, consistent with attainable vacuum requirements, still permits the use of small, air-cooled amplifier tubes and a heavily loaded low-Q rf cavity.

A tentative list of parameters is:

Radius of curvature	15 ft
Over-all diameter	45 ft
n	200
No. of periods	37
No. of straight sections	74
No. of lenses per period	4
Length of lens	7.6 in.
Length of straight section	7.6 in.

Field strength (magnetic type)	
at injection	10.5 gauss
at 10 MeV	74 gauss
Field strength (electrostatic type)	
at injection	3 kV/cm
at 10 MeV	22 kV/cm
Rise time	.01 sec
Phase transition energy	2.8 MeV
Frequency (final)	7 mc
Frequency change	5½ %
Volts/turn	150 V
RF power	about 1 kw
No. of betatron wavelengths	about 6.2
aperture	1 X 1 in.
Betatron amplitude for 10^{-3} rad. error	0.07 in.
Maximum stable amplitude, synchrotron osc.	-0.16 in.
Radial spacing of betatron resonances	about 0.4 in.
Vacuum requirement	about 10^{-6} mm Hg

Total power requirements will be small and available with existing installations. The test shack seems to be a suitable location since the ring will be erected inside a thin magnetic shield which can be thermally insulated and heated economically.

We estimate the cost to be approximately $600,000, distributed as shown in the following table:

Model	Direct	Overhead	Total
Staff S. & W.	$135,000	$ 65,000	$200,000
Van de Graaff	70,000	-	70,000
Other E. & S.	130,000	-	130,000
Shops	135,000	65,000	200,000
	$470,000	$130,000	$600,000

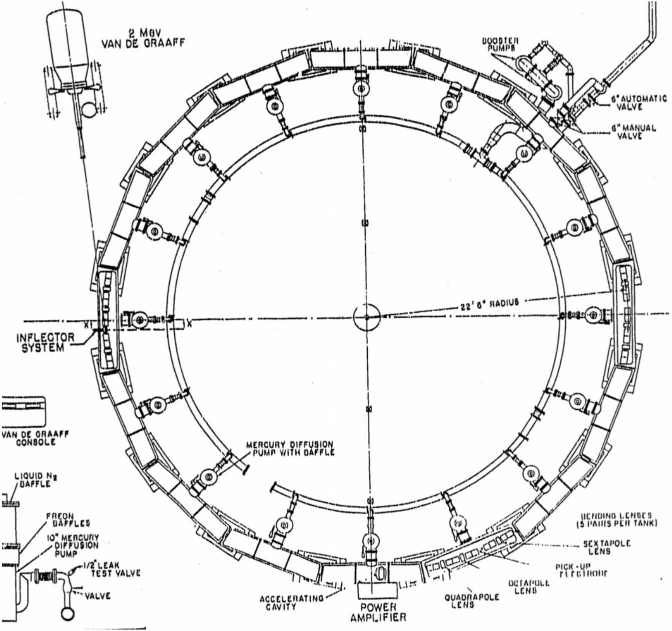
It is seen that total direct expense would be approximately $470,000 and the overhead assigned by our methods of accounting would be approximately $130,000. Since the total Laboratory overhead cost would not actually be appreciably affected by the project, the assignment of this overhead would result in relieving Laboratory operations of cost in approximately the same amount. A small but finite savings to operations would also be affected by way of shop charges since approximately one-third of the shop costs represent fixed expenses for foremen, tool crib attendants and other helpers.

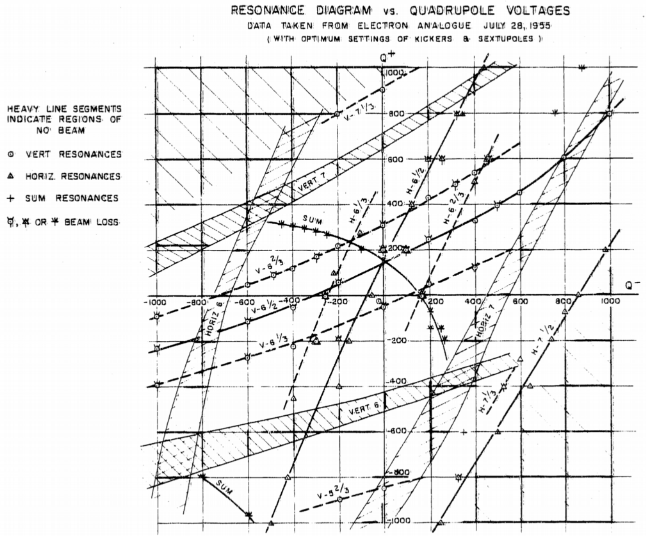
The above facts would, of course, reduce the budgetary difficulties in which we find ourselves as a result of the present operational ceilings. Furthermore, the construction of the model would result is some transfers of scientific, technical and shop personnel from the operating program, resulting in further savings. We would naturally like to minimize this by adding new personnel either in operations or for the construction of the model. Allowing for a reasonable compromise in this connection, I believe that our Physical Research Program could be carried forward without too serious sacrifices by the provision of the $600,000 required for the electron model, the additional $100,000 promised for Applied Chemistry and $350,000 additional operating funds for Applied Physics. Were all of this forthcoming it would not, in my opinion, be necessary to reduce the power level of the reactor but we would endeavor to make all possible economies in equipment and supplies and in overhead costs and would hold physics personnel to present levels except for a few additions in the accelerator development work.

Sincerely yours,

Leland J. Haworth
Director

LJH:DS
cc: E.L. VanHorn


2 MeV
VAN DE GRAAFF
BOOSTER PUMPS
6" AUTOMATIC VALVE
6" MANUAL VALVE
22' 6" RADIUS
INFLECTOR SYSTEM
VAN DE GRAAFF CONSOLE
MERCURY DIFFUSION PUMP WITH BAFFLE
LIQUID N_2 BAFFLE
FREON BAFFLES
10" MERCURY DIFFUSION PUMP
1/2" LEAK TEST VALVE
VALVE
BENDING LENSES (5 PAIRS PER TANK)
SEXTAPOLE LENS
PICK-UP ELECTRODE
OCTAPOLE LENS
QUADRAPOLE LENS
ACCELERATING CAVITY
POWER AMPLIFIER


RESONANCE DIAGRAM vs. QUADRUPOLE VOLTAGES
DATA TAKEN FROM ELECTRON ANALOGUE JULY 28, 1955
(WITH OPTIMUM SETTINGS OF KICKERS & SEXTUPOLES)
HEAVY LINE SEGMENTS INDICATE REGIONS OF NO BEAM
VERT RESONANCES
HORIZ. RESONANCES
SUM RESONANCES
OR BEAM LOSS
Q+
Q−
VERT 7
VERT 6
HORIZ 6
HORIZ 7
SUM
SUM
V-7 1/3
V-6 2/3
V-6 1/2
V-6 1/3
V-5 2/3
H-6 1/2
H-6 1/3
H-6 2/3
H-7 1/2
H-7 1/3