Fringe Field Treatment in COSY INFINITY

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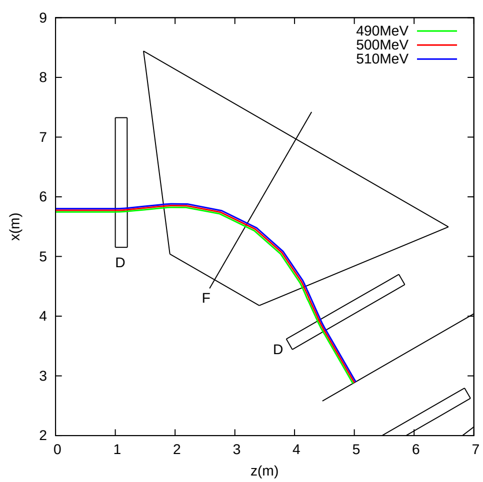
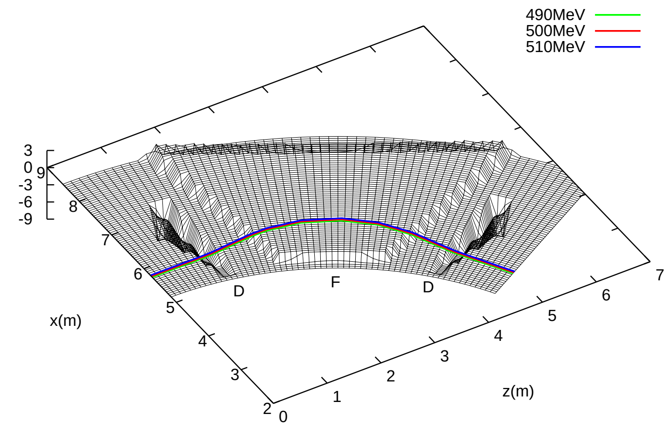
Particle Tracking in a Conventional Style

- Computations of the electromagnetic fields – FEM, BEM,...
  Field values at grid points. → Interpolate anywhere else.
- Numerical integrations of the trajectories through the fields.





A coarse grid (d_R=5cm, d_theta=1deg) is used for the field data, only for the easy demonstration.

Transfer Map Method and Differential Algebras

- The transfer map $\mathcal{M}$ is the flow of the system ODE.

$$\vec{z}_f = \mathcal{M}(\vec{z}_i, \vec{\delta}),$$

  where $\vec{z}_i$ and $\vec{z}_f$ are the initial and the final condition, $\vec{\delta}$ is system parameters.

- For a repetitive system, only one cell transfer map has to be computed. Thus, it is much faster than ray tracing codes (i.e. tracing each individual particle through the system).
- The Differential Algebraic method allows a very efficient computation of high order Taylor transfer maps.
- The Normal Form method can be used for analysis of nonlinear behavior.

Differential Algebras (DA)

- it works to arbitrary order, and can keep system parameters in maps.
- very transparent algorithms; effort independent of computation order.

The code **COSY Infinity** has many tools and algorithms necessary.

Field Description in Differential Algebra

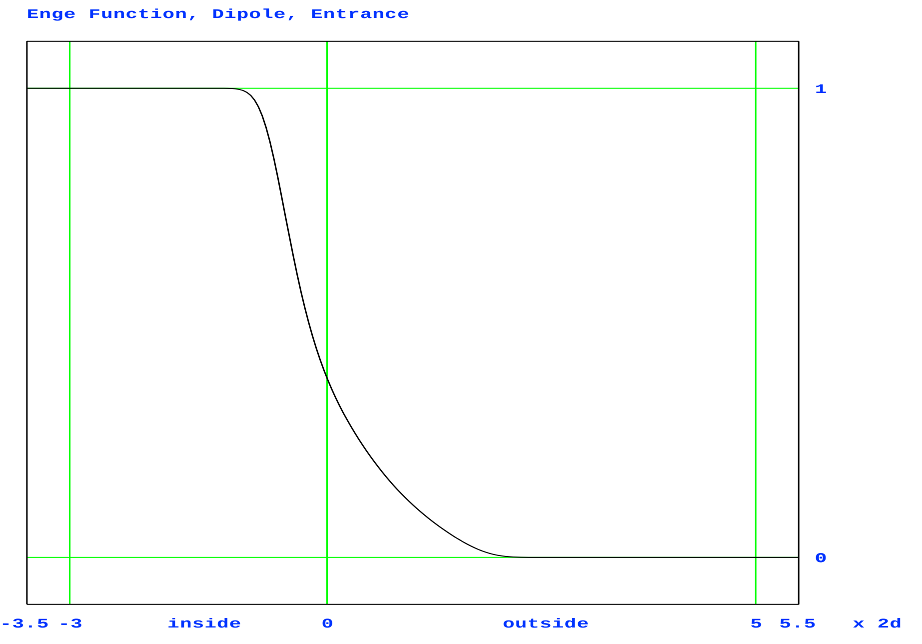
There are various DA algorithms to treat the fields of beam optics efficiently.
For example, **DA PDE Solver**

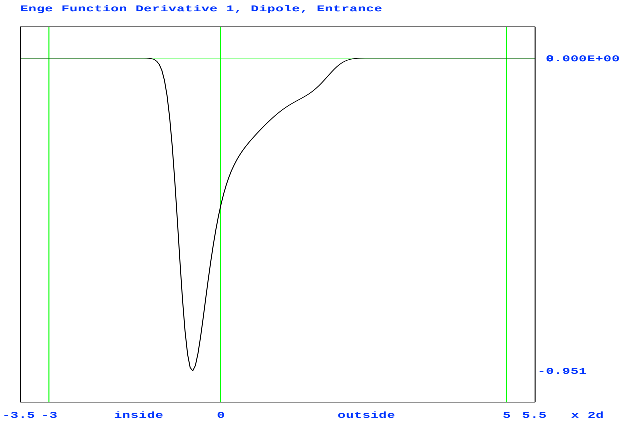
- requires to supply only
  - the midplane field for a midplane symmetric element.
  - the on-axis potential for straight elements like solenoids, quadrupoles, and higher multipoles.
- treats arbitrary fields straightforwardly.
  - Magnet (or, Electrostatic) fringe fields:
    The Enge function fall-off model

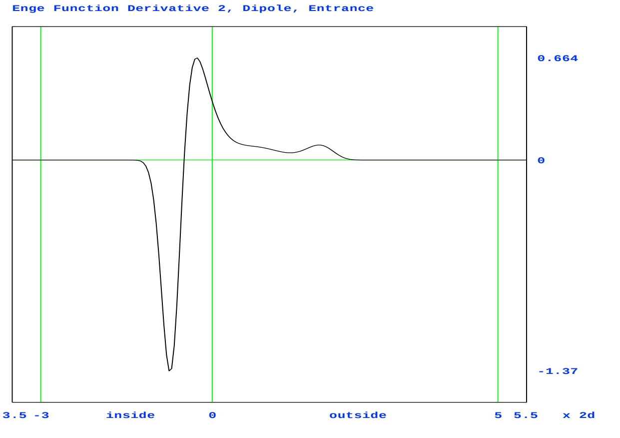
    $$F(s) = \frac{1}{1 + \exp(a_1 + a_2 \cdot (s/D) + ... + a_6 \cdot (s/D)^5)}$$

    where D is the full aperture.
    Or, any arbitrary model including the measured data representation.
  - Solenoid fields including the fringe fields.
  - Measured fields: E.g. Use Gaussian wevelet representation.
  - Etc. etc.

Dipole Enge Function (COSY default)







DA Fixed Point PDE Solvers

The **DA fixed point theorem** allows to **solve PDEs iteratively** in **finitely many steps** by rephrasing them in terms of a fixed point problem.

Consider the rather general PDE

$$a_1 \frac{\partial}{\partial x}\left(a_2 \frac{\partial}{\partial x} V\right) + b_1 \frac{\partial}{\partial y}\left(b_2 \frac{\partial}{\partial y} V\right) + c_1 \frac{\partial}{\partial z}\left(c_2 \frac{\partial}{\partial z} V\right) = 0,$$

where a_i, b_i, c_i are functions of x, y, z.

The PDE is re-written in **fixed point form** as

$$\begin{aligned} V = V\big|_{y=0} &+ \int_0^y \frac{1}{b_2}\left(b_2 \frac{\partial V}{\partial y}\right)\bigg|_{y=0} \\ &- \int_0^y \frac{1}{b_2} \int_0^y \left(\frac{a_1}{b_1}\frac{\partial}{\partial x}\left(a_2 \frac{\partial V}{\partial x}\right) + \frac{c_1}{b_1}\frac{\partial}{\partial z}\left(c_2 \frac{\partial V}{\partial z}\right)\right) dy dy. \end{aligned}$$

Assume the derivatives of V and $\partial V/\partial y$ with respect to x and z are **known in the plane** $y = 0$. Then the right hand side is **contracting** with respect to y (which is necessary for the DA fixed point theorem), and the various orders in y can be **iteratively** calculated by mere iteration.

3.3.2 Bending Elements

COSY INFINITY supports both magnetic and electrostatic elements including so called combined function elements with superimposed multipoles.

...

The following commands let an inhomogeneous combined function bending magnet and a combined function electrostatic deflector act on the map:

MS <radius> <angle> <aperture> $< n_1 >$ $< n_2 >$ $< n_3 >$ $< n_4 >$ $< n_5 >$;

E S <radius> <angle> <aperture> $< n_1 >$ $< n_2 >$ $< n_3 >$ $< n_4 >$ $< n_5 >$;

The radius is measured in meters, the angle in degrees, and the aperture is in meters and corresponds to half of the gap width. The indices n_i describe the midplane radial field dependence which is given by

$$F(x) = F_0 \cdot \left[1 - \sum_{i=1}^{5} n_i \cdot \left(\frac{x}{r_0}\right)^i\right],$$

where r_0 is the bending radius. Note that an electric cylindrical condenser has $n_1 = 1$, $n_2 = -1$, $n_3 = 1$, $n_4 = -1$, $n_5 = 1$, etc, and an electric spherical condenser has $n_1 = 2$, $n_2 = -3$, $n_3 = 4$, $n_4 = -5$, $n_5 = 6$, etc. Homogeneous dipole magnets have $n_i = 0$.

There are various specialized electrostatic deflectors.

...

ECL <radius> <angle> <aperture> ;

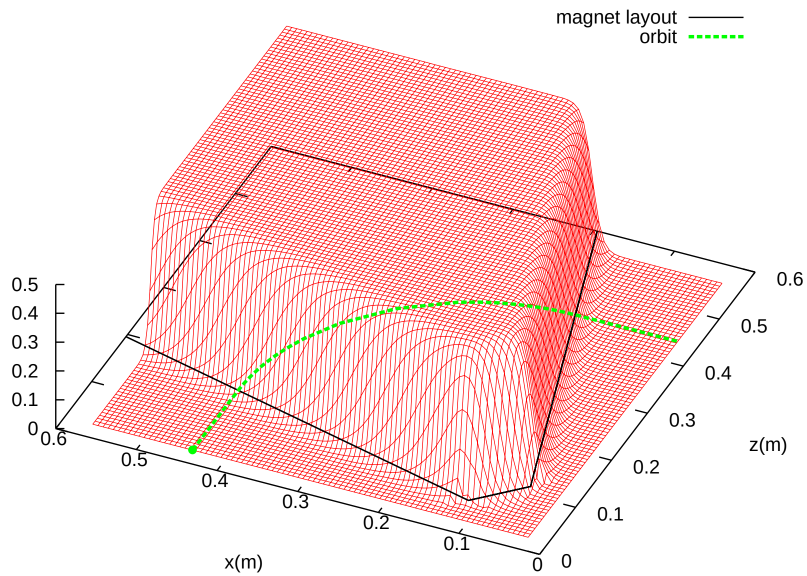
invokes an electrostatic cylindrical deflector, and the element

ESL <radius> <angle> <aperture> ;

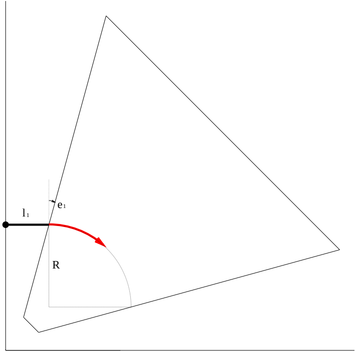
invokes an electrostatic spherical deflector.

A Subtle Problem to Deal with Fringe Fields

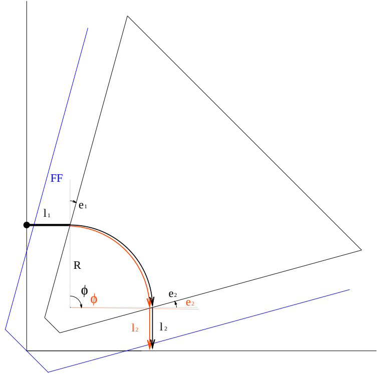
The trajectory differs from that of the idealized one.
If a hard edge model is used at the initial planning stage,
one *would* encounter the necessity to adjust the system.
– The degree of the severity depends on the system parameters.



An illustrating example:
A homogeneous bending magnet with fringe fields.



e1
l1
R


FF
l1
e1
R
ϕ
ϕ
e2
e2
l2
l2

Fringe Fields and Their Nonlinearities

- Fringe fields are often the main source of (non-deliberate) nonlinearities
  - In main fields, one of course attempts very carefully to keep the field constant in direction of reference orbit, and imposes specific axial dependencies
  - In fringe fields, there is natural nonlinearity due to unavoidable curvature of electric or magnetic field lines.
  - These curvatures of fields affect particles at different distances from reference orbit differently, and because of curvature, they do so nonlinearly.
  - All these things are unavoidable; they are a direct consequence of Maxwell's equations.

The Pain with Electrostatic Elements

- Unless one is very careful, there will be various undesirable effects:
  - The motion from before to after the element satisfies energy conservation, but the integrator does not know this
  - Repeated small violations of energy conservation can lead to either oscillations, or big long term effects
  - Particular problem: due to offset of reference orbit, it is very useful to re-align elements. This is normally done after each part:
  - After entrance fringe field, after main field, after exit fringe field
  - Each re-alignment causes small change in geometry, and hence small change in potential!

Electrostatic Elements - Study Case -

A cylindrical element with fringe fields ($22.5°$)

- Fields, especially fringe fields
- Reference orbits
- COSY fringe field computation modes FR 3 and FR 2.5
  - FR 2.5: Computes through the main and the fringe fields
  - FR 3: Further, preserves the mirror symmetry

***** ES cylindrical FR 3 ***** The linear part of the nonlinear transfer maps

0.8764065	-0.2498426E-1	0.000000	0.000000	-0.3431232	100000
9.282307	0.8764065	0.000000	0.000000	-1.697380	010000
0.000000	0.000000	1.000588	0.1214327E-3	0.000000	001000
0.000000	0.000000	9.686087	1.000588	0.000000	000100
0.000000	0.000000	0.000000	0.000000	1.000000	000010
1.697380	0.3431232	0.000000	0.000000	1.712180	000001

Reversed Map

0.8764065	-0.2498426E-1	0.000000	0.000000	-0.3431232	100000
9.282307	0.8764065	0.000000	0.000000	-1.697380	010000
0.000000	0.000000	1.000588	0.1214327E-3	0.000000	001000
0.000000	0.000000	9.686087	1.000588	0.000000	000100
0.000000	0.000000	0.000000	0.000000	1.000000	000010
1.697380	0.3431232	0.000000	0.000000	1.712180	000001

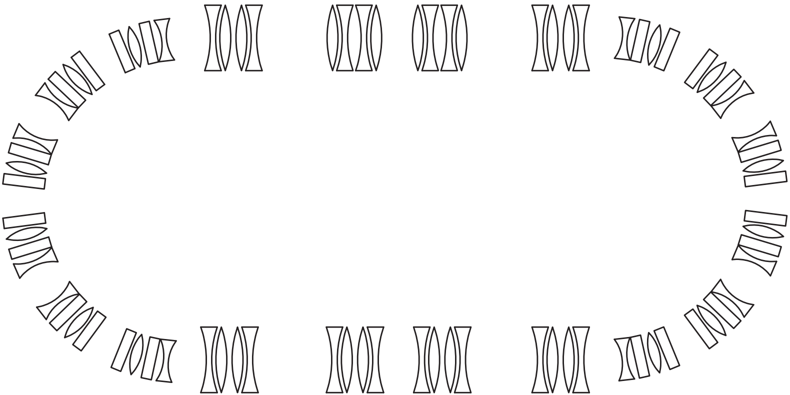
***** ES cylindrical FR 2.5 *****

0.8764077	-0.2498421E-1	0.000000	0.000000	-0.3431202	100000
9.282277	0.8764068	0.000000	0.000000	-1.697352	010000
0.000000	0.000000	1.000588	0.1216104E-3	0.000000	001000
0.000000	0.000000	9.686121	1.000590	0.000000	000100
0.000000	0.000000	0.000000	0.000000	1.000000	000010
1.697364	0.3431199	0.000000	0.000000	1.712164	000001

Reversed Map

0.8764068	-0.2498421E-1	0.000000	0.000000	-0.3431199	100000
9.282277	0.8764077	0.000000	0.000000	-1.697364	010000
0.000000	0.000000	1.000590	0.1216104E-3	0.000000	001000
0.000000	0.000000	9.686121	1.000588	0.000000	000100
0.000000	0.000000	0.000000	0.000000	1.000000	000010
1.697352	0.3431202	0.000000	0.000000	1.712164	000001

COSY Jülich storage ring lattice



COSY INFINITY

-0.9774877	-1.078548	0.00	0.00
0.03521565	-0.9841743	0.00	0.00
0.00	0.00	-0.5176308	-10.90340
0.00	0.00	0.06520659	-0.5583641

MAD8

-0.9774876	-1.0785556	0.00	0.00
0.03521571	-0.984175	0.00	0.00
0.00	0.00	-0.5176308	-10.90340
0.00	0.00	0.06520659	-0.5583641

ZGOUBI

-0.9774910	-1.0783900	0.00	0.00
0.03521423	-0.984180	0.00	0.00
0.00	0.00	-0.5176260	-10.90340
0.00	0.00	.06520679	-0.5583590

Table 1. First order transfer matrices for the three codes without fringe fields. Note that COSY INFINITY and MAD8 use transfer matrices and thus naturally agree to high accuracy, whereas ZGOUBI calculates the transfer map as a result of integration of nearby orbits which is slightly less accurate.

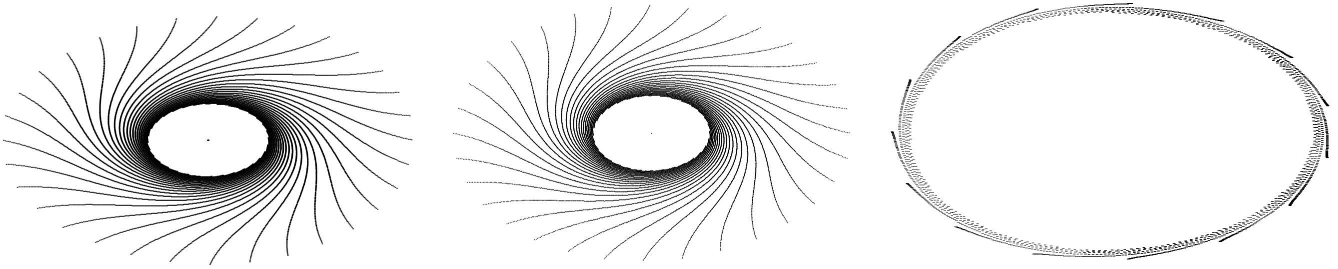


Figure 2. Second order tracking in the horizontal plane created by COSY INFINITY (left) and MAD8 (center), compared with ZGOUBI (right) which does not use transfer maps, under identical initial conditions and lattice parameters. No symplectification is used, showing that second order is insufficient to describe the dynamics.

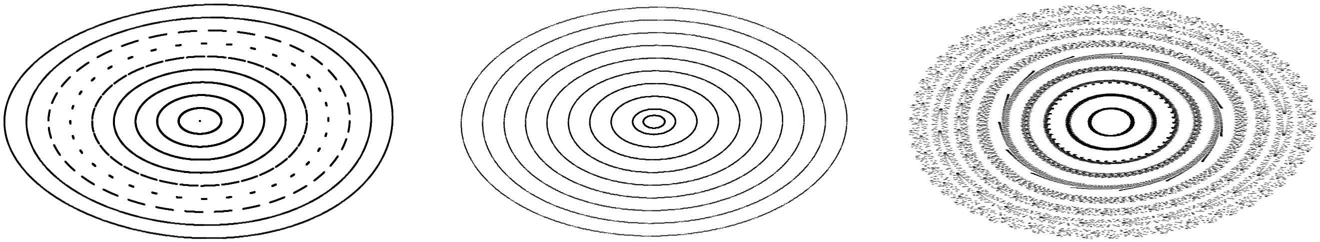


Figure 3. Varying transverse initial conditions from 1 cm to 9 cm in the horizontal plane for second order tracking with COSY INFINITY (left) and MAD8 (center) and compared with ZGOUBI (right). Symplectic tracking is enabled in COSY INFINITY and MAD8. This is not available in ZGOUBI, which shows a widening of the orbit bands indicative of violation of symplecticity.

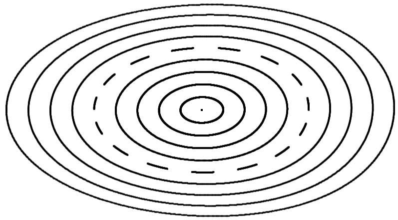


Figure 4. Same tracking as Figure 3, now utilizing COSY INFINITY's 9th order tracking without symplectification, still without fringe fields. The high order of the map and resulting accuracy avoids the violation of symplecticity visible in Figure 2. Even to this high order, there is very little nonlinearity evident in the dynamics.

Fringe Fields

- So far everything is fine, but now we would like to introduce fringe fields.
- COSY Infinity has the convenience of enabling a default set of fringe fields with a single command, "FR 3".

Here is the transfer matrix with fringe fields turned on:

$$\begin{bmatrix} \mathbf{-0.9739104} & 1.954368 & 0.00 & 0.00 \\ 0.01832738 & \mathbf{-1.063567} & 0.00 & 0.00 \\ 0.00 & 0.00 & -0.7993542 & -6.705731 \\ 0.00 & 0.00 & 0.05219644 & -0.8131372 \end{bmatrix}$$

Notice that the motion in the horizontal direction is now *unstable* ($|Trace| > 2$)

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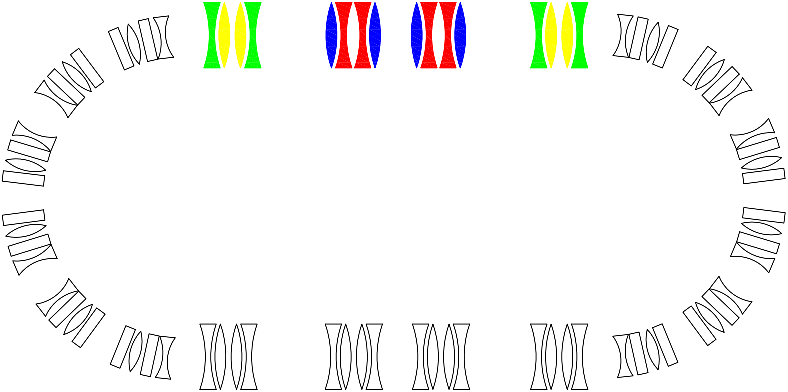
Fringe Fields - Adjusting the lattice



Quadrupole Strength Adjustments

With these adjustments, we are able to recreate the original design transfer matrix

Original

$$\begin{bmatrix} -0.9774877 & -1.078547 & 0 & 0 \\ 0.03521565 & -0.9841743 & 0 & 0 \\ 0 & 0 & -0.5176308 & -10.90340 \\ 0 & 0 & 0.06520659 & -0.5583641 \end{bmatrix}$$

$\nu_x = 0.5312127004376005$ $\nu_y = 0.6595904961761281$

Adjusted

$$\begin{bmatrix} -0.9774877 & -1.078548 & 0.00 & 0.00 \\ 0.03521565 & -0.9841743 & 0.00 & 0.00 \\ 0.00 & 0.00 & -0.5176308 & -10.90340 \\ 0.00 & 0.00 & 0.06520659 & -0.5583641 \end{bmatrix}$$

$\nu_x = 0.5312128411224102$ $\nu_y = 0.6595905356121318$

With these new quad strengths in hand, we can run side by side comparisons with Zgoubi, which also has full fringe field modeling for quads and dipoles.

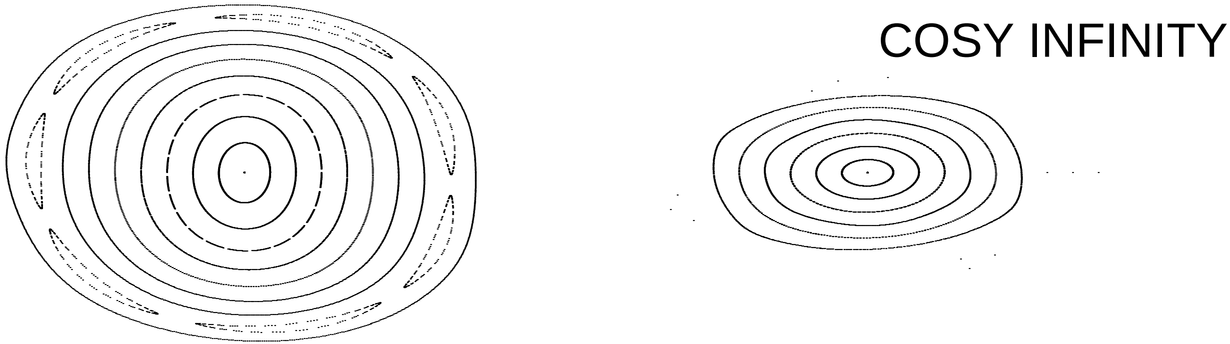


Figure 5. COSY INFINITY 9th order tracking with full fringe field simulation capabilities in vertical (left) and horizontal (right) planes.

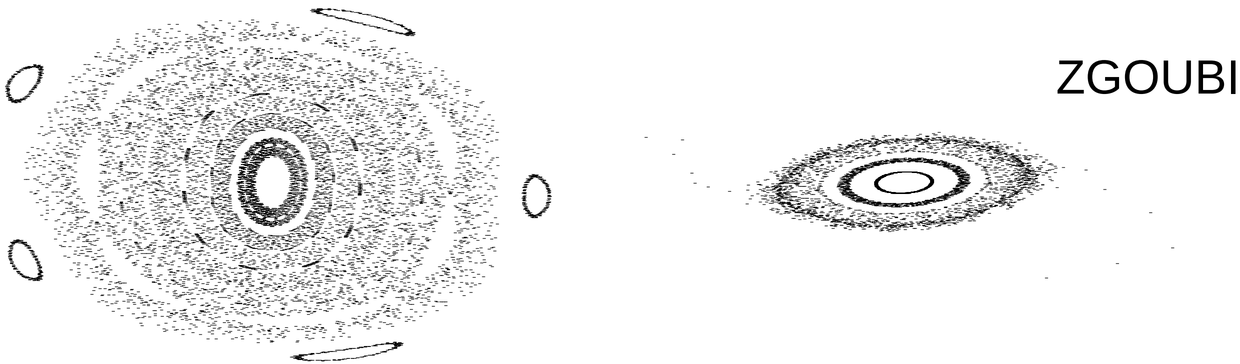
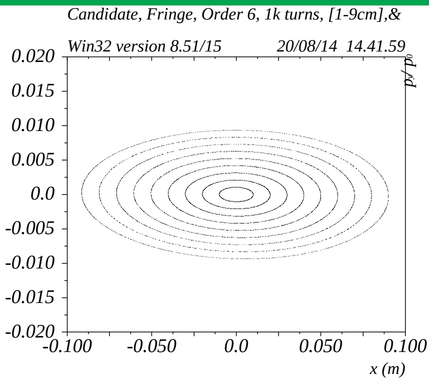
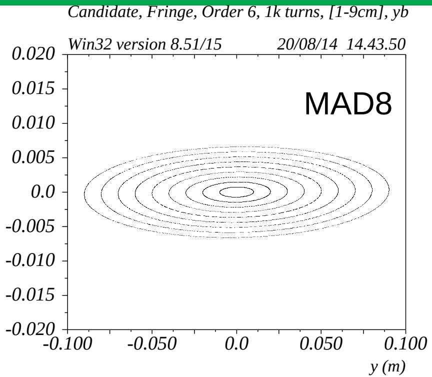
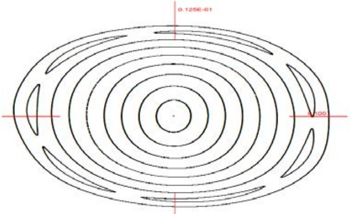


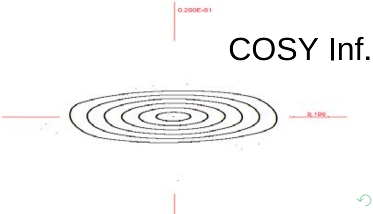
Figure 6. ZGOUBI tracking with full fringe field simulation capabilities in vertical (left) and horizontal (right) planes. The fringe field Enge coefficients are identical to those used in COSY INFINITY. The rough structure and stability boundaries are similar to those of Figure 5, but symplecticity violations are apparent.

MAD8 Fringe Fields -- Nonlinear effects are not reflected !!









Summary

	Tracking Method	Map Order	Out of Plane Order	Symplect
Zgoubi	Integ	NA	3	No
MAD8	Map	$2, 3$	2	Yes
COSY	Map	$\infty(9)$	$\infty(9)$	Yes

Number of Turns	CPU time (seconds)	
	COSY INFINITY	ZGOUBI
10^3	25.082	183.78
10^4	25.297	1831.4
10^5	27.132	18717
10^6	45.343	
10^7	228.26	
10^8	2049.4	
10^9	20193	

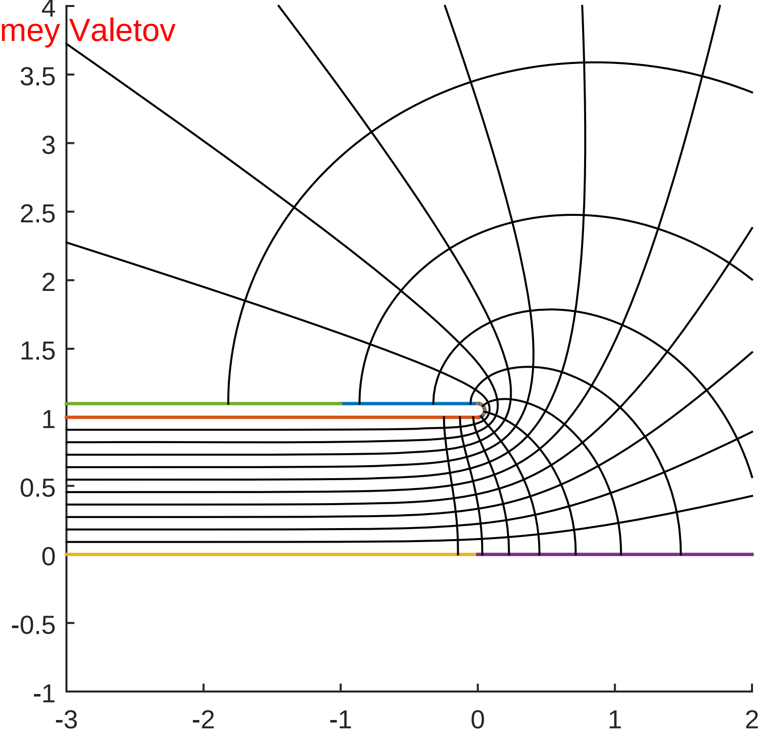
Table 3. Comparison of tracking execution times of COSY INFINITY and ZGOUBI at their respective maximum precisions (order 9 transfer map for COSY INFINITY, 5th order Taylor series integration for ZGOUBI). ZGOUBI execution times are proportional to the number of turns and around 0.187 seconds per turn per particle. COSY INFINITY requires an initial investment in the computation of a transfer map, but for larger turn numbers tracks for 1/50000 of a second per turn.

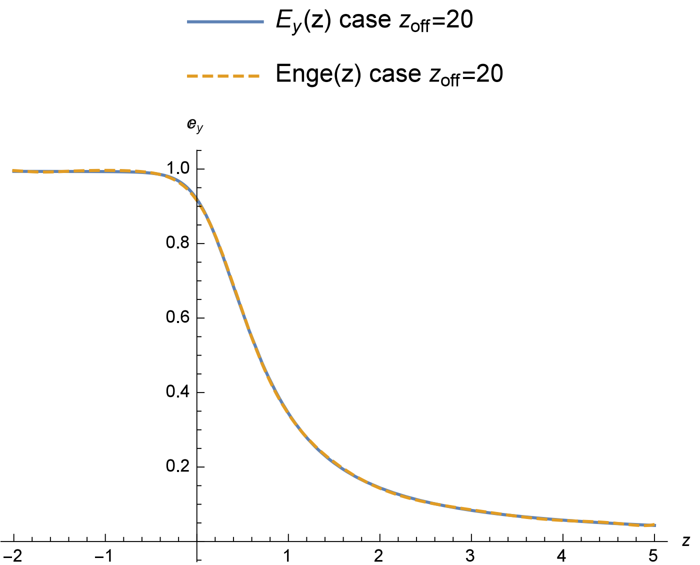
Summary of Comparison Studies with/without Fringe Fields

- COSY Infinity, Zgoubi, MAD8 agree in SCOFF
- High order fringe field tracking is critical to understanding the dynamic aperture of even simple lattices
- COSY Infinity and Zgoubi agree qualitatively when tracking through fringe fields
- MAD8 can handle fringe fields, but only in "thick", nonsymplectic dipoles. Thin elements + fringe a possibility.
- Zgoubi has a powerful graphics output tool - zpop - to assist in understanding the dynamics
- COSY Infinity is a dynamic programming framework; MAD8 is semi-dynamic (variables, element expansions); ZGoubi is static

An E-field calculation program based on Conformal Mapping

by Eremey Valetov




$E_y(z)$ case $z_{\rm off}$=20
Enge(z) case $z_{\rm off}$=20
e_y
1.0
0.8
0.6
0.4
0.2
−2
−1
1
2
3
4
5
z

Summary

- Methods of studying charged particle motions
  - Transfer maps
  - The DA (Differential Algebraic) method
  - The DA PDE solver for obtaining electromagnetic fields
  - Repetitive trackings
- Treatment of fringe fields and special cautions
- Examples
  - Fringe field modes FR 2.5 & 3 in COSY INFINITY
  - Tracking studies for the COSY Jülich storage ring
    Comparisons between COSY INF., ZGOUBI, MAD
  - Modeling the fringe field fall-offs (the Enge function)
    for various E-deflector options at COSY Jülich

Backup pages

Zgoubi fringe fields

- To implement fringe fields in Zgoubi, we need the Enge coefficients used by COSY Infinity.
- It is easy to implement these values in Zgoubi.
- Zgoubi requires you to specify how accurately you integrate through the fringe fields. (Integration zone extent, fringe field extent, step size)

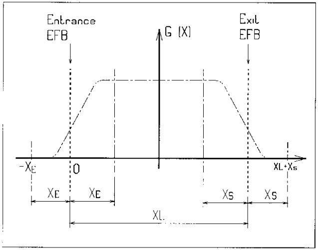


Figure 33: Scheme of the longitudinal field gradient $G(X)$. (OX) is the longitudinal axis of the reference frame $(0, X, Y, Z)$ of **zgoubi** . The length of the element is XL, but trajectories are ray-traced from $-X_E$ to $XL + X_S$, by means of prior and further automatic changes of frame.

MAD8 Fringe Fields

MAD8 also has provisions for fringe fields, but only in the thick bending elements. We can add approximate fringe fields to the aforementioned thin-element model using the fringe integral techniques "Fringe Integral (FINT)" techniques developed by Wollnik, Matsuda, Brown, etc. We must provide the value of FINT as a parameter to the software.

$$\begin{bmatrix} x \\ a \\ y \\ b \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{\rho}\tan\beta & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{\rho}\tan\beta_v & 1 \end{bmatrix} \begin{bmatrix} x_0 \\ a_0 \\ y_0 \\ b_0 \end{bmatrix} + \begin{bmatrix} \frac{g^2}{\rho\cos^2\beta} I_1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{ where } I_1 = \int\int (B_y^0 - B_y)\, d\sigma\, d\sigma$$

and

$$\tan\beta_v = \tan\left(\beta - \frac{g}{\rho}\frac{1+\sin^2\beta}{\cos^3\beta} I_2\right) \text{ with } I_2 = \int_{-\infty}^{\infty} \frac{B_y(B_0 - B_y)}{B_0^2}\, d\sigma$$

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