Tracking results of all-electric ring with Runge-Kutta integration

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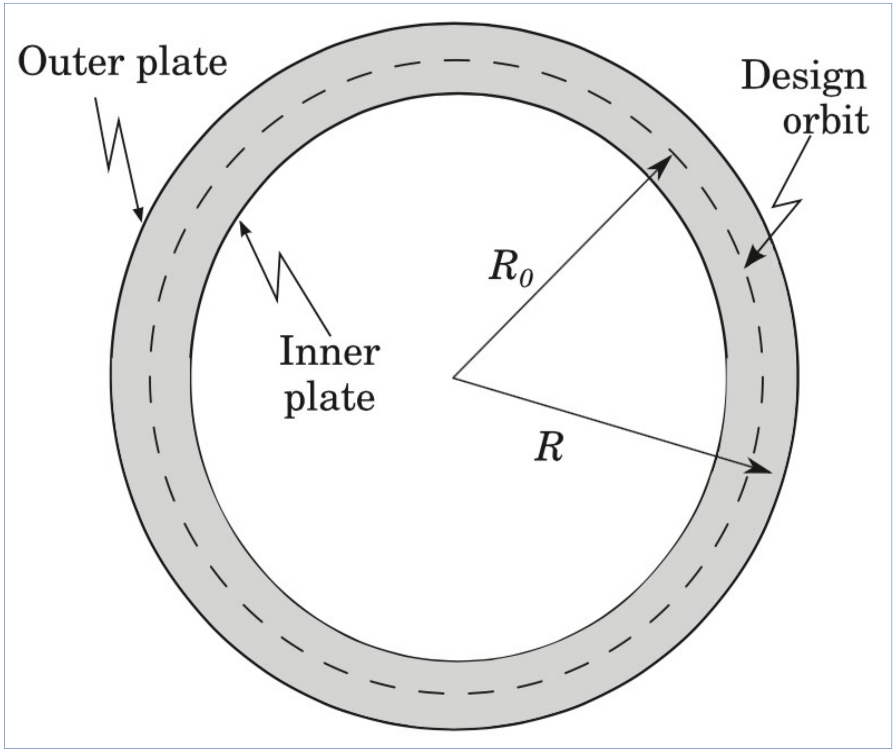
CAPP
Center for Axion and Precision Physics Research

Beam and spin tracking – Ring elements (Dipole)

Electric ring



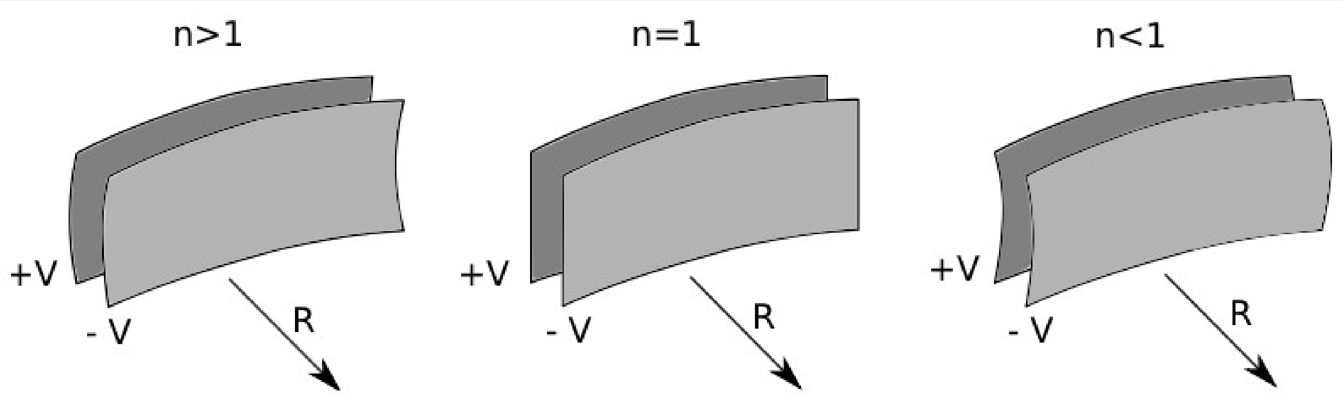
the E-field can be obtained using
- Maxwell's eq. and
- the symmetries of the ring

$$E_z = -E_0 \left(\frac{R_0}{R}\right)^n \left[(n-1)\frac{z}{R} + \frac{1}{6}(n^2-1)(n+1)\frac{z^3}{R^3} + O(z^5) \right]$$

$$E_r = -E_0 \left(\frac{R_0}{R}\right)^n \left[1 - \frac{n^2-1}{2}\frac{z^2}{R^2} + \frac{1}{24}(n^2-1)(n+1)(n+3)\frac{z^4}{R^4} + O(z^6) \right]$$

n: field index

determines the focusing



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Beam and spin tracking – Diff. Equations

$$\frac{d\vec{\beta}}{dt}=\frac{e}{\gamma m}\left[\vec{\beta}\times\vec{B}+\frac{\vec{E}}{c}-\vec{\beta}\frac{(\vec{\beta}\cdot\vec{E})}{c}\right]$$

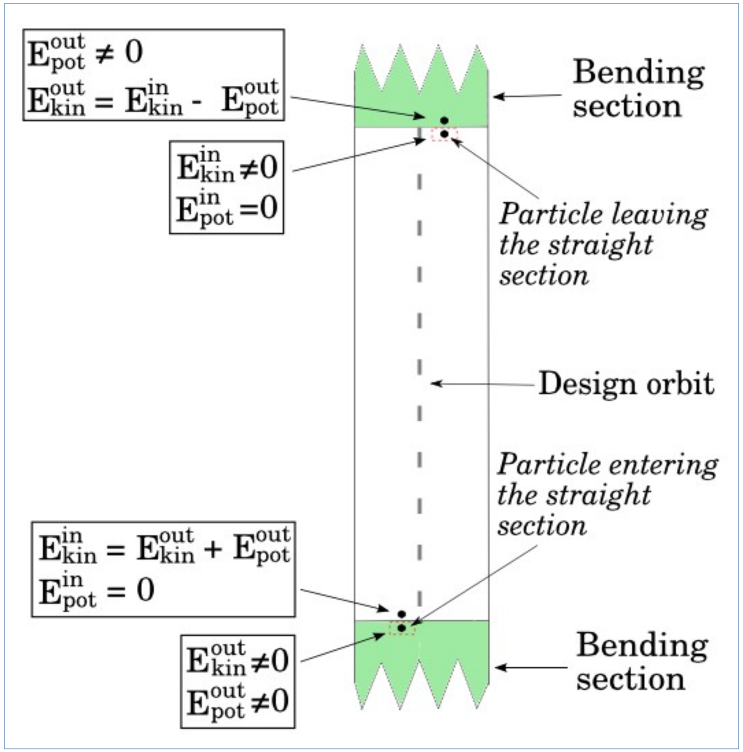
$$\frac{d\vec{s}}{dt}=\frac{e}{m}\vec{s}\times\left[\left(\frac{g}{2}-\frac{\gamma-1}{\gamma}\right)\vec{B}-\left(\frac{g}{2}-1\right)\frac{\gamma}{\gamma+1}(\vec{\beta}\cdot\vec{B})\vec{\beta}-\left(\frac{g}{2}-\frac{\gamma}{\gamma+1}\right)\frac{\vec{\beta}\times\vec{E}}{c}\right]$$

BMT-equation

- Velocity and spin equations are solved with stepwise integrators:
  - Runge-Kutta
  - Predictor-Corrector
  - Bulirsch-Stoer
  - etc.
- E and B-fields are determined using position of the particle
- Ring elements like dipoles, quadrupoles, sextupoles, RF cavities etc. are defined by their electromagnetic and geometrical properties.

Beam and spin tracking – Ring elements (Straight section)

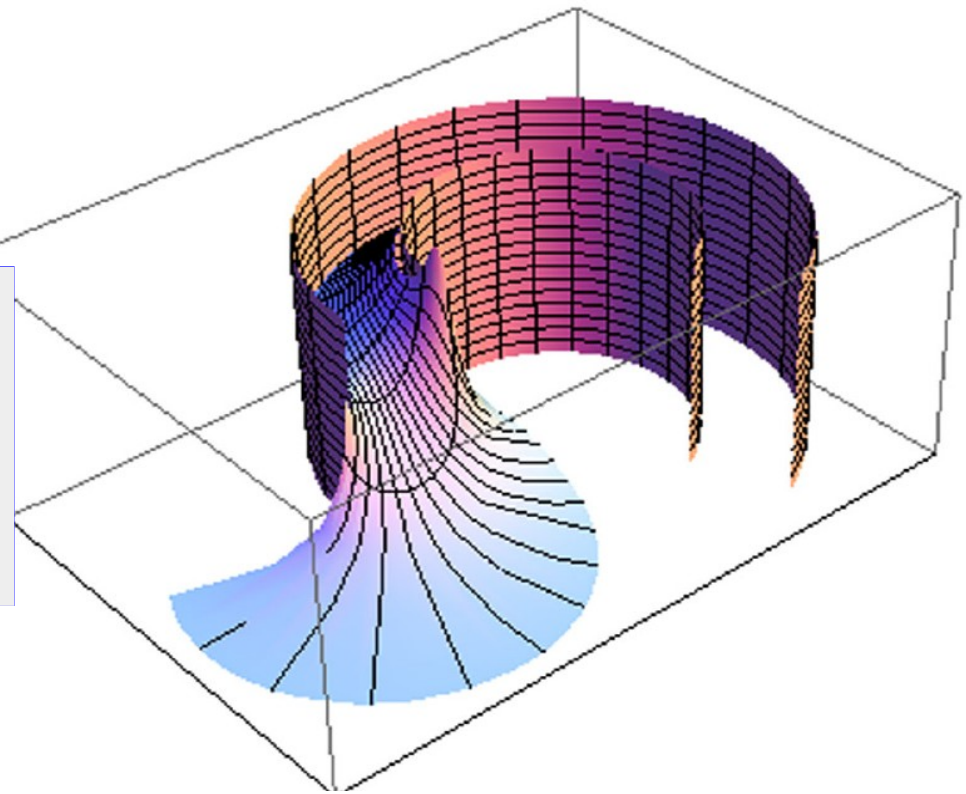
Hard edge approximation



- the particle has
  - some potential energy in bending section
  - no potential energy in straight section
- the total energy is conserved
- the longitudinal velocity changes at the edges
- moves with constant velocity in straight section

Fringe fields

- Eric Metodiev studied fringe fields using conformal mapping.
- Their study confirmed that fringe field and hard-edge approximation give similar spin coherence time



PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS **17,** 074002 (2014)

Fringe electric fields of flat and cylindrical deflectors in electrostatic charged particle storage rings

E. M. Metodiev,[1,2,3,4] K. L. Huang,[1,2] Y. K. Semertzidis,[1,3,4] and W. M. Morse[1]

Pros and cons

- Requires almost no estimation besides the definitions of ring geometry and the position and size of ring elements
- Tunes, twiss parameters, acceptance etc. are estimated after simulation
- Slow for long term tracking (1ms: 2 hours with 1ps step size for RK)
- Good for
  - Studying systematic errors like offsets in ring elements, geometric phase etc.
  - Benchmarking independent studies
  - Estimating spin coherence time

Benchmarking

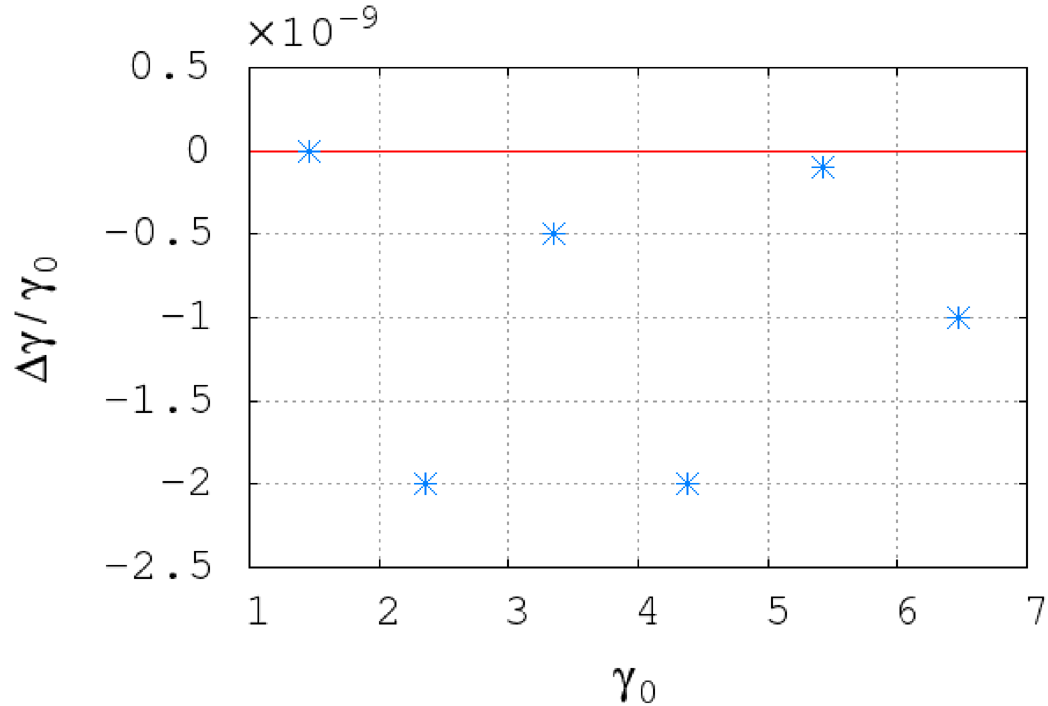
- Benchmarking in electric and magnetic rings
  - Y. Orlov recently estimated average radial position and kinetic energy for electric and magnetic rings with RF cavity
  - RF cavity averages the revolution period of the particle to the ideal one
  - It averages out the energy too, in a electric ring with weak focusing (in magnetic ring, it does not average out)
  - γ averages to some constant term proportional to vertical pitch when no vertical focusing
- The tracking results show agreement to high accuracy

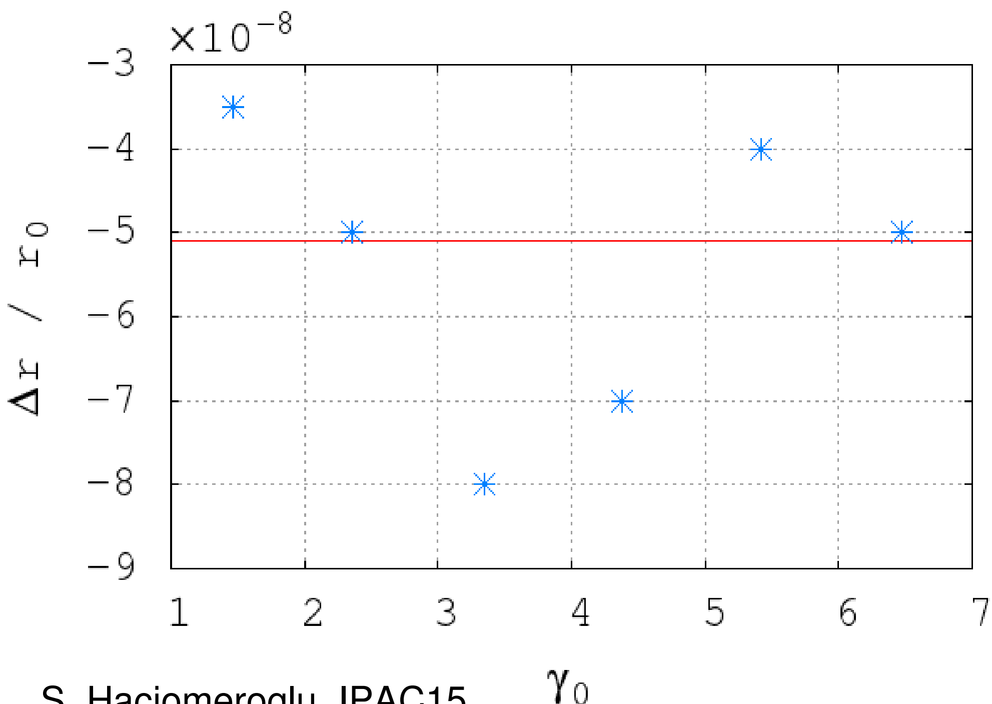
Benchmarking – weak electric focusing

$$\langle \frac{\Delta \gamma}{\gamma_0} \rangle = 0$$

$$\langle \frac{\Delta r}{r_0} \rangle = -\frac{1}{2} \langle \theta_y^2 \rangle$$

http://arxiv.org/abs/1504.07304





γ : Lorentz factor
r : Radial offset
Q_y: Vertical pitch

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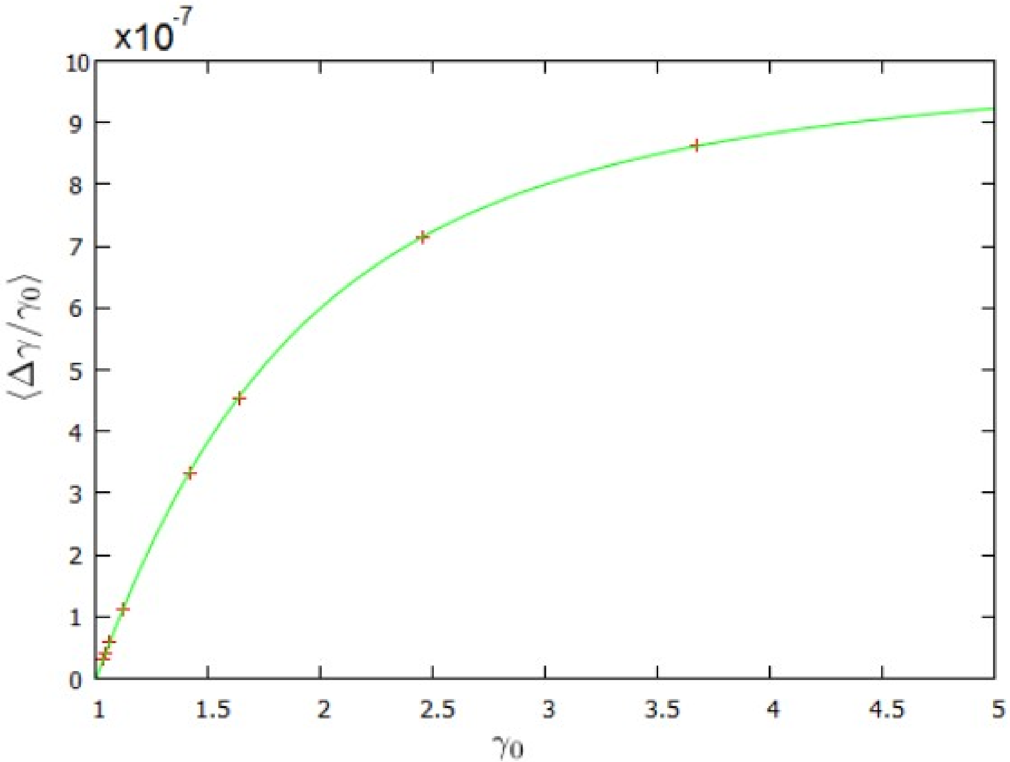
Benchmarking – no focusing

$$\left\langle \frac{\Delta\gamma}{\gamma_0} \right\rangle = \langle \theta_y^2 \rangle \frac{\gamma^2 - 1}{\gamma^2 + 1}$$

$$\left\langle \frac{\Delta r}{r_0} \right\rangle = -\frac{1}{2} \langle \theta_y^2 \rangle \frac{\gamma^2 - 1}{\gamma^2 + 1}$$



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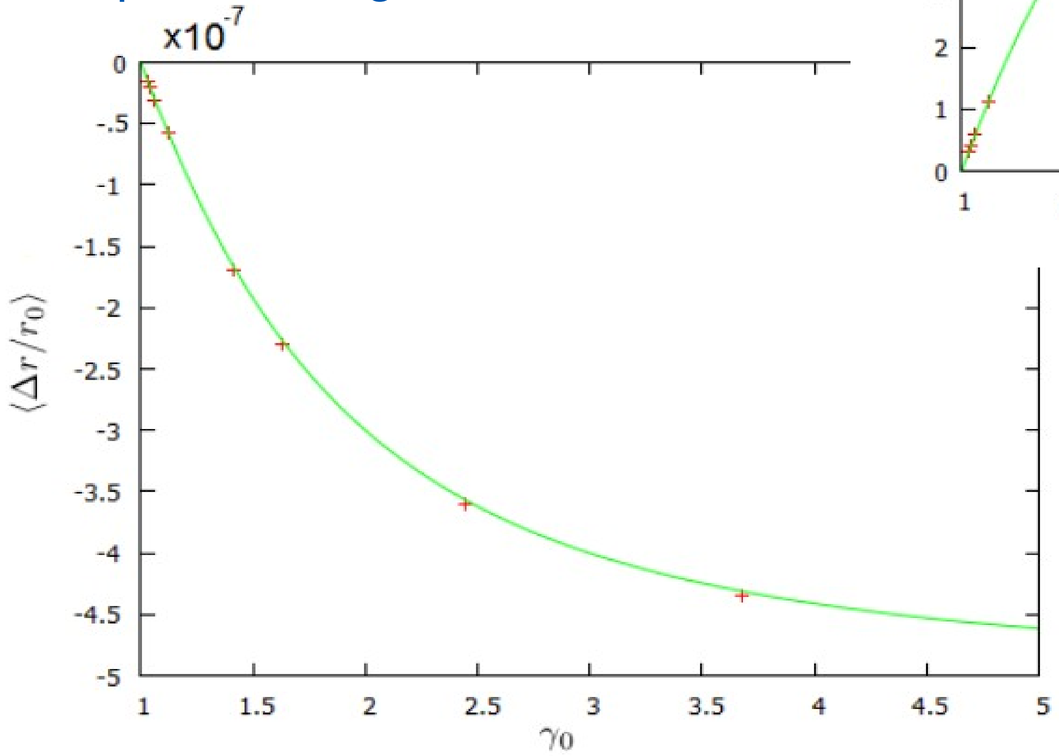

γ : Lorentz factor
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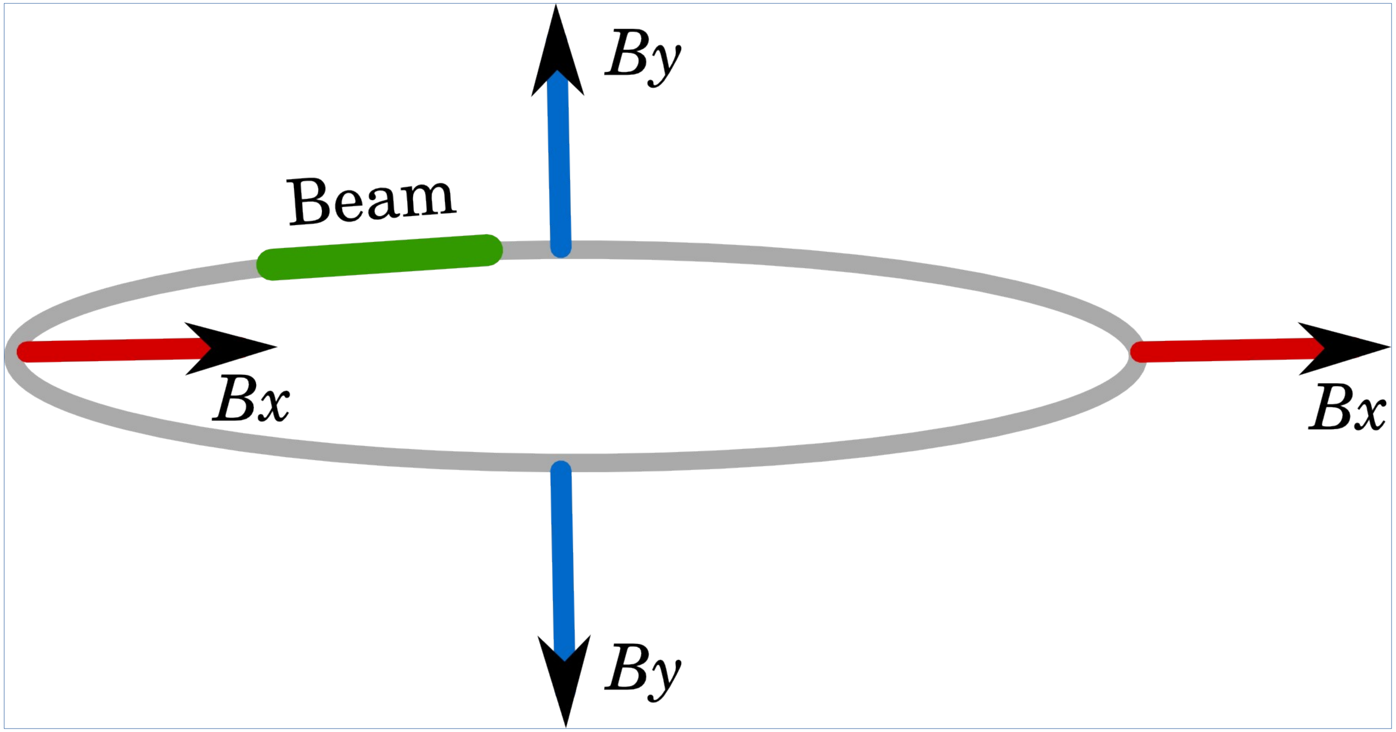
Benchmarking

Several benchmarking results related to both electric and magnetic rings are submitted to NIM.

Geometric phase

- Some configurations of electric and magnetic field in perpendicular directions accumulate spin precession that mimics EDM



Geometric phase – B-field

Longitudinal and radial B-fields oscillating around the ring with a phase difference gives vertical component to the spin precession.

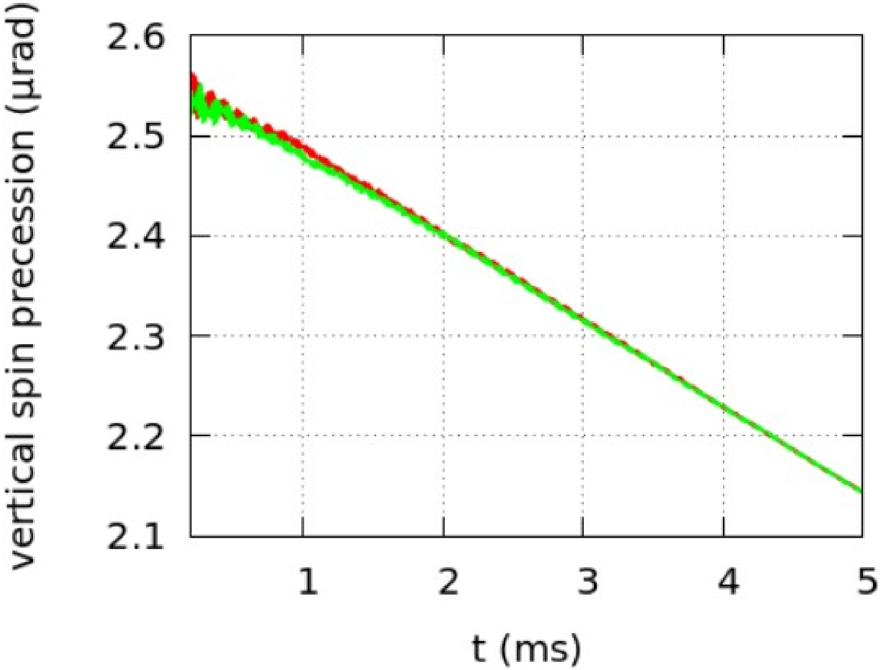
1 nT longitudinal and vertical B-field

# of oscillation	Analytical	Simulation
N=1	16.3 nrad/s	23.83 nrad/s
N=2	8 mrad/s	8 nrad/s
N=3	5.5 nrad/s	5.5 nrad/s

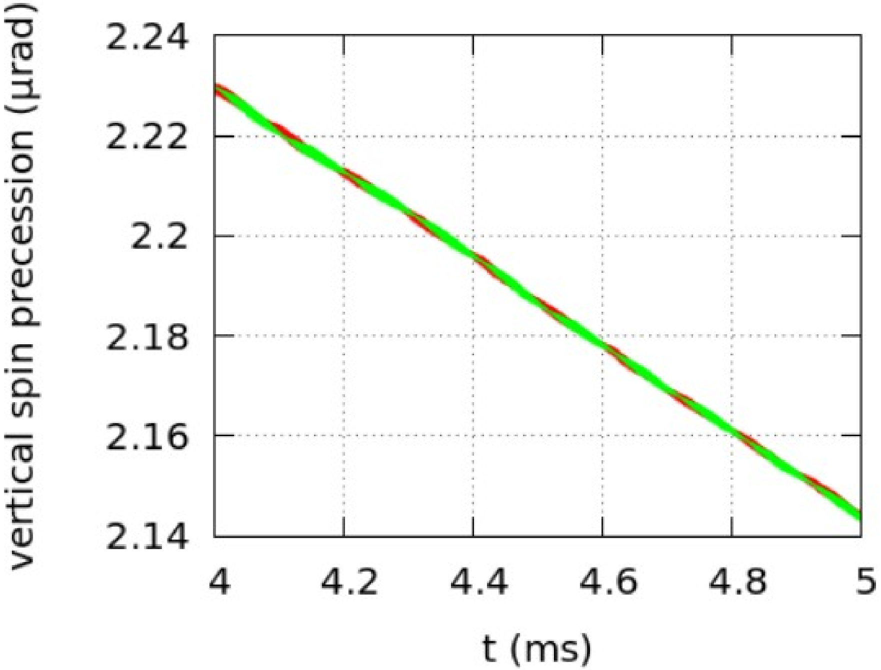
CW and CCW cancel out completely

Deflector misalignment

- Two plates in the ring are given offset: one vertical, the other horizontal by 1cm.
- The vertical spin component of CW and CCW beams cancel out to 4 μrad/s.
- Scales down to 4 nrad/s for 10 μm precision of plate positioning.

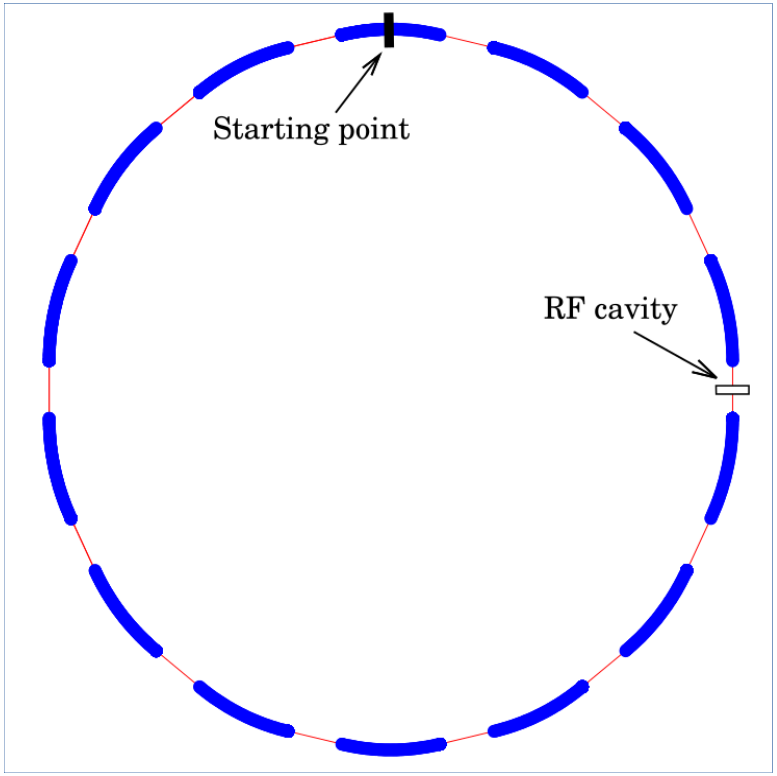


Zoomed in from the left graph



pEDM ring

- we need to achieve ≈ 1000s of SCT
- restrictions:
  - Qy ≈ 0.3- 0.4
    error in initial momentum Δp/p ≈ 10^{-4}
  - reasonable acceptance
- we try different designs:
  - various field index (*n*) values
  - length of straight sections
  - initial offset of the partice w.r.t design orbit
  - quadrupole strengths
  - etc.
- work on systematics like misalignments
- on plates, quadrupoles, RF cavity etc.



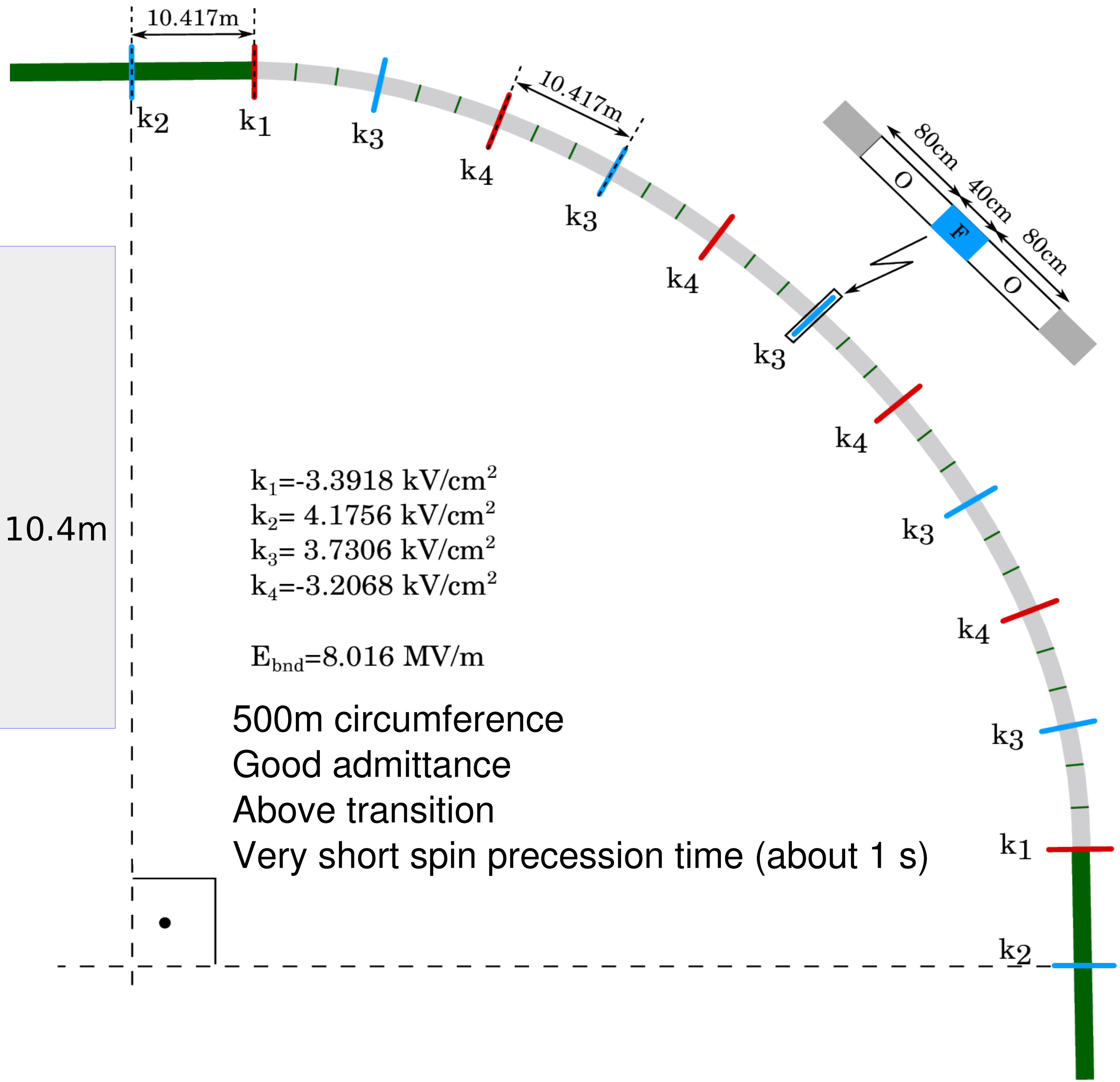
PEDM ring

- The simulations showed so far:
  - Effect of the straight section length is negligible
  - Spin precession is dominated by momentum spread rather than positional oscillations
  - Spin precession shows strong dependence on field index
  - Alternating gradient found to have long spin precession time

Beam and spin tracking – pEDM ring - 2

V. Lebedev's lattice

- E_0=8 MV/m
- R_0=52m
- L_{defl}=40 x 3 x 2.75m
- L_{str}=20 x 0.8m + 8 x 10.4m
- L_{quad}=40cm



500m circumference
Good admittance
Above transition
Very short spin precession time (about 1 s)

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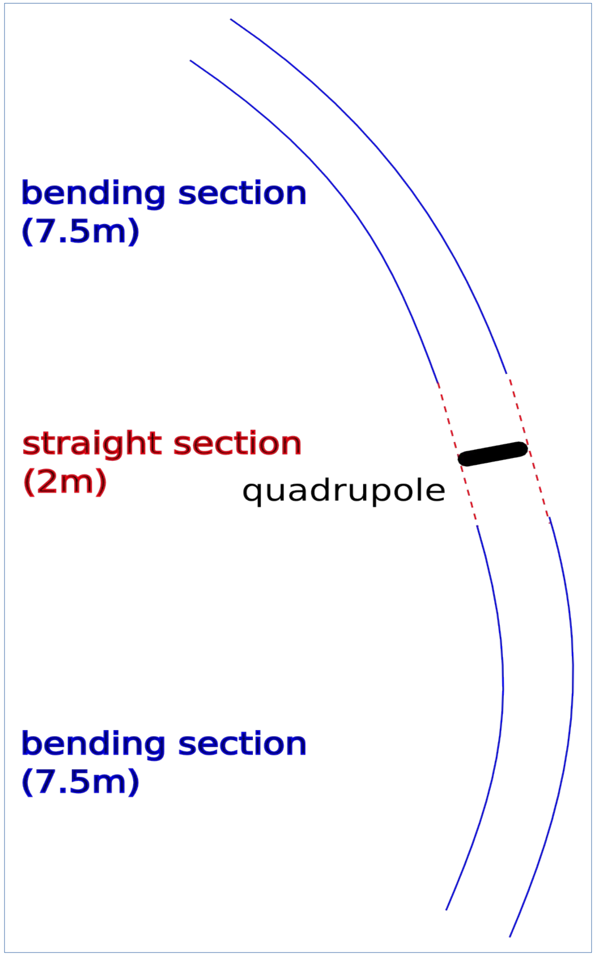
Beam and spin tracking – pEDM ring - 1

Richard Talman's lattice

- E_0=3.5 MV/m
- R_0=96m
- L_{defl}=80 x 7.5m
- L_{str}=76 x 2m + 4 x 10m
- L_{quad}=15cm
- 1 RF cavity
- field index n=1
- even numbered straight sections include focusing quadrupole (K=50kV/m^2)
- odd numbered straight sections include defocusing quadrupole (K=-100 kV/m^2)



- Reasonable admittance for AGS of BNL
- Very long spin precession time (about 10^4 s)
- Below transition

Conclusion/Future plan

- Runge-Kutta tracking is efficient for
  - spin tracking
  - studying systematic effects
  - benchmarking independent studies.
- Slow for long term tracking. Practically we simulate a few ms.
- Simulation results show that CW-CCW design is useful in eliminating geometric phase effect
- We have a working lattice (Talman's lattice). Slip factor and long SCT needs to be together.
- Soohyung Lee from IBS/Korea is working on parallelization
- Martin Gaisser from IBS/Korea is working on fast algorithms