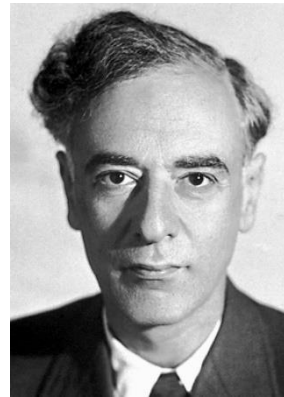


# General Relativity Effects in Storage Ring Searches for EDM

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Towards Storage Ring Electric Dipole Moment Measurements

744. WE-Heraeus-Seminar, 29 Mar - 31 Mar 2021

## Gravity: the weakest known force in nature

Planck mass is the best known measure of the strength of gravity

$$M_p = 1.2 \times 10^{19} \text{ GeV}$$

Newton constant

$$G = M_p^{-2} = 7 \times 10^{-39} \text{ GeV}^{-2}$$

## Interaction of nonrelativistic protons

Gravity : Electrostatics =  $6 \times 10^{-39} : \alpha_{em}$

In the everyday life free fall acceleration is quite significant... still no one has ever cared about the free fall of stored beams

# Why: EDM as a high-precision window at physics Beyond Standard Model

- Sakharov (1967): CP violation is imperative for baryogenesis in the Big Bang Cosmology

	<i>observed</i>	<i>SM prediction</i>
$\frac{n_B - n_{\bar{B}}}{n_\gamma}$	$(6.1 \pm 0.3) \times 10^{-10}$	$10^{-18}$
PSI neutron EDM limit ( $e \cdot cm$ )	$< 1.8 \times 10^{-26}$	$10^{-31}$

C. Abel et al., Phys. Rev. Lett. 124 no. 8, 081803 (2020)

- nEDM: plans to increase sensitivity by still another order in magnitude
- pEDM: statistical accuracy of  $10^{-29} e \cdot cm$  is aimed at **dedicated all-electric magic energy storage rings:**  
 $10^{-15}$  of the magnetic moment
- dEDM and pEDM in precursor experiment at COSY: dEDM  $\sim 10^{-20}$  is within reach
- Sequel to JEDI:** CPEDM & prototype 30 MeV pure electric proton ring PTR (at CERN? at COSY?...) --- big international effort, positive response from community. Paving road to ultimate all electric frozen spin proton EDM ring.

## Small parameters of Earth's curved space-time :

We are after EDM  $\sim 10^{-15}$  MDM

Earth's rotation  $\omega = 7 \times 10^{-5} \text{ s}^{-1} \sim 3 \times 10^{-11} \Omega_p$

Ring radius  $\rho \sim 80 \text{ m}$ :  $\omega\rho \sim 2 \times 10^{-11} c$

Equatorial velocity  $\omega R \sim 1.6 \times 10^{-6} c$

Earth's gravitational radius  $r_g = 2G_N M / c^2 = 0.9 \text{ cm} = 1.3 \times 10^{-9} R$

Free fall acceleration  $g_0 = (r_g / 2R^2) c^2$

# Layout:

- All electric magic energy proton storage ring at rest on the rotating Earth
- Earth's gravity pull :
  - beam displacement taken care of, and closed orbit is ensured, by focusing
  - a source of false EDM
  - Standard Candle for the EDM ring performance
- Maxwell equations in curved space-time :
  - are pure electrostatic laboratories bound to the rotating Earth free of (geometric) magnetic fields?
  - geometric magnetic field in the neutron EDM experiments
  - impact on proton EDM in storage ring expts?
- Fleeting comments on detection of single gravitons and storage rings as gravitational wave antennae
- Apologies for gory details of the General Relativity :
  - unavoidable but reduced to minimum minimorum

To be on safe side: full fledged GR formalism with the Kerr metrics for rotating Earth (perturbative expansion)

$$g_{00} = 1 - \frac{r_g}{R} - \frac{[\boldsymbol{\omega} \times \mathbf{R}]^2}{c^2},$$

$$g_{0i} = - \left\{ 1 + \frac{r_g}{R} \left( 1 - I \cdot \frac{R_{\oplus}^2}{R^2} \right) \right\} \frac{[\boldsymbol{\omega} \times \mathbf{R}]^i}{c},$$

$$g_{ij} = - \left( 1 + \frac{r_g}{R} \right) \delta^{ij}.$$

**Tetrad aka vierbine** (square root from metric tensor) = a must for the Dirac equation in curved space-time:

-- Landau-Lifshitz convention is convenient to define beam closed orbit

$$ds^2 = g_{00} \left( dx^0 + \frac{g_{0i} dx^i}{g_{00}} \right)^2 -$$

$$- \left( -g_{ij} + \frac{g_{0i} g_{0j}}{g_{00}} \right) dx^i dx^j =$$

$$= \eta_{ab} (e_{\mu}^a dx^{\mu}) (e_{\nu}^b dx^{\nu})$$

$\eta_{ab}$  = **Minkowski metrics**,  $I \sim 1/3$  is the reduced moment of inertia

## Tetrads

$$e_0^0 = \sqrt{g_{00}} = 1 - \frac{r_g}{2R} - \frac{[\boldsymbol{\omega} \times \mathbf{R}]^2}{2c^2}, \quad e_0^\alpha = 0,$$

$$e_i^0 = \frac{g_{0i}}{\sqrt{g_{00}}} = - \left\{ 1 + \frac{r_g}{R} \left( \frac{3}{2} - I \cdot \frac{R_\oplus^2}{R^2} \right) \right\} \frac{[\boldsymbol{\omega} \times \mathbf{R}]^i}{c},$$

$$e_i^\alpha = \left( 1 + \frac{r_g}{2R} \right) \delta_i^\alpha + \frac{[\boldsymbol{\omega} \times \mathbf{R}]^\alpha [\boldsymbol{\omega} \times \mathbf{R}]^i}{2c^2},$$

## Connections (curved space-time witnesses)

$$\nabla_\mu X^a \equiv e_\nu^a \nabla_\mu X^\nu = \partial_\mu X^a + \gamma_{b\mu}^a X^b$$

$$\gamma_{\alpha 00} = \frac{1}{c^2} (\mathbf{g}_0 - [\boldsymbol{\omega} \times [\boldsymbol{\omega} \times \mathbf{R}]])^\alpha,$$

$$\gamma_{\alpha 0\beta} = \gamma_{\alpha\beta 0} = \varepsilon_{\alpha\beta\sigma} \left\{ \left( 1 - \frac{r_g}{2R_\oplus} (1 - I) \right) \frac{\boldsymbol{\omega}^\sigma}{c} + \frac{r_g}{R_\oplus} \left( 1 - \frac{3}{2} I \right) \frac{(\boldsymbol{\omega} \cdot \mathbf{n}_\oplus) \mathbf{n}_\oplus^\sigma}{c} \right\},$$

$$\gamma_{\alpha\beta\rho} = \varepsilon_{\alpha\beta\sigma} \left\{ -\varepsilon_{\sigma\rho\delta} \frac{\mathbf{g}_0^\delta}{c^2} + \frac{3}{2c^2} \boldsymbol{\omega}^\sigma [\boldsymbol{\omega} \times \mathbf{R}]^\rho \right\},$$

## Connections enter the equations of motion

$$\frac{d\gamma}{dt} = (\gamma_{\alpha 0 c} u^c) \mathbf{v}^\alpha + \frac{q}{mc} \mathbf{E} \cdot \boldsymbol{\beta}, \quad \boldsymbol{\beta} \equiv \frac{\mathbf{v}}{c},$$

$$\frac{du^\alpha}{dt} = c \left( (\gamma_{\alpha 0 c} u^c) + (\gamma_{\alpha \beta c} u^c) \boldsymbol{\beta}^\beta \right) +$$

$$+ \frac{q}{mc} (\mathbf{E}^\alpha + [\boldsymbol{\beta} \times \mathbf{H}]^\alpha),$$

The gravity term in the beam angular velocity:

$$\boldsymbol{\Omega}_c^{Gr} = \frac{1}{\gamma \mathbf{v}^2} \left[ \mathbf{v} \times \left\{ \left( \frac{2\gamma^2 - 1}{\gamma} \mathbf{g}_0 + \gamma [\boldsymbol{\omega} \times [\mathbf{R} \times \boldsymbol{\omega}]] \right) + \right. \right.$$

$$+ 2\gamma \left( \left( 1 - \frac{r_g}{2R_\oplus} (1 - I) \right) + \frac{3(\mathbf{v} \cdot [\boldsymbol{\omega} \times \mathbf{R}])}{4c^2} \right) [\mathbf{v} \times \boldsymbol{\omega}] +$$

$$\left. \left. + \frac{2\gamma r_g}{R_\oplus} \left( 1 - \frac{3}{2} I \right) (\boldsymbol{\omega} \cdot \mathbf{n}_\oplus) [\mathbf{v} \times \mathbf{n}_\oplus] \right\} \right].$$



# Safe approach to the spin motion, especially for higher spin particles: Foldy-Woutheyesen transformation of the Dirac Hamiltonian in curved space-time

Strongly advocated by Obukhov, Silenko, Teryaev

$$\mathbf{P} = \mathbf{S} + \frac{\gamma^2}{c^2(\gamma + 1)} (\mathbf{S}\mathbf{v})\mathbf{v}, \quad P^0 = \frac{\gamma}{c} (\mathbf{S}\mathbf{v}), \quad u_a P^a = 0.$$

$$\frac{dP^a}{ds} + (\gamma_{bc}^a u^c) P^b = \frac{2}{\hbar c} \left\{ \mu F_b^a P^b - \mu' u^a F_c^b u_b P^c - d \cdot \tilde{F}_b^a P^b + d \cdot u^a \tilde{F}_c^b u_b P^c \right\}$$

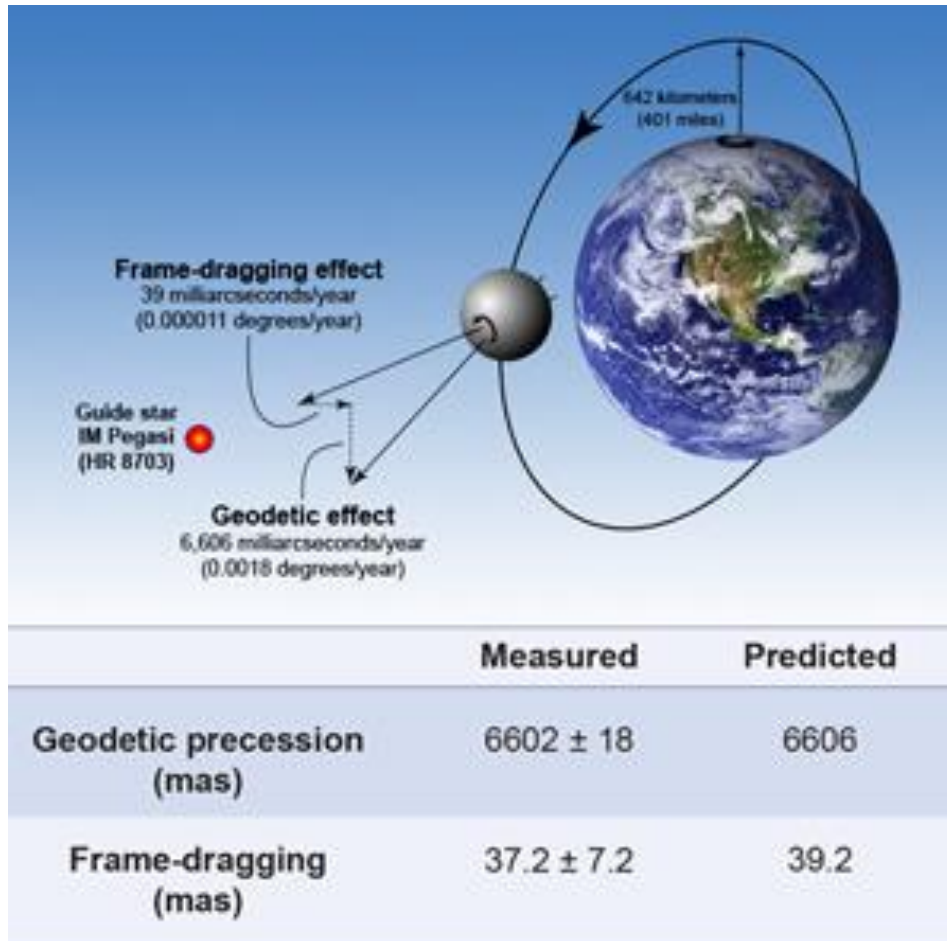
parallel transfer in GR

familiar FT-BMT equation

# Freely falling gyroscope on Earth's polar orbit: de Sitter geodetic precession and Lense-Thirring frame dragging effect

L.I. Schiff, Phys. Rev. Lett. 4(1960) 215

**Gravity Probe B:** C.W.F. Everitt, Phys. Rev. Lett. 106 (2011) 221101



mas/yr = milli-second of arc per year

De Sitter => **spin-orbit** interaction

Lense-Thirring => **spin-spin** interaction

# Impose the storage ring constraint of the beam closed orbit: cancellation of the GR effects by focusing fields and the feedback of focusing on spin

First discussion of focusing effect for the magnetic ring:

A.Silenko and O.Teryaev., Phys. Rev. D71 : 064016, 2005; Phys. Rev., D76:061101, 2007.

GR treatment of the electrostatic frozen spin ring

Y. Orlov, E. Flanagan, and Y K. Semertzidis. Phys. Lett., A376:2822-2829, 2012.

## Electrostatic focusing

$$\frac{q}{mc} \mathbf{E}_f = \gamma \left\{ -\frac{(2\gamma^2 - 1)\mathbf{g}_0}{\gamma^2 c} + \frac{(\boldsymbol{\beta} \cdot \mathbf{g}_0)\boldsymbol{\beta}}{c} + \frac{[\boldsymbol{\omega} \times [\boldsymbol{\omega} \times \mathbf{R}]]}{c} + \left( 2 - \frac{r_g}{R_\oplus} (1 - I) + \frac{3}{2c} (\boldsymbol{\beta} \cdot [\boldsymbol{\omega} \times \mathbf{R}]) \right) [\boldsymbol{\omega} \times \boldsymbol{\beta}] - \frac{r_g}{R_\oplus} (2 - 3I) (\boldsymbol{\omega} \cdot \mathbf{n}_\oplus) [\boldsymbol{\beta} \times \mathbf{n}_\oplus] \right\}.$$

Vertical E-field

→ radial motional B-field

→ false EDM from MDM

Earth's rotation:

S.N. Vergeles, N.N. Nikolaev, [JETP, 129\(4\), 541-552 \(2019\)](#); [JHEP., 2004, 191 \(2020\)](#)

$$\boldsymbol{\beta} \cdot \mathbf{g}_0 = 0 \rightarrow \text{ring loop integral } \oint \mathbf{E}_f d\mathbf{r} = 0$$

## GR correction to spin precession

$$\begin{aligned}
 \Omega_f^{GR} &= \left( G + \frac{1}{\gamma + 1} \right) \frac{q}{mc} [\boldsymbol{\beta} \times \mathbf{E}_f] + \Omega_s^{Gr} = \Omega_g^{GR} + \Omega_\omega^{GR} \\
 &= \frac{1 - (2\gamma^2 - 1)G}{\gamma} \frac{|\mathbf{g}_0|}{c} [\mathbf{n}_\oplus \times \boldsymbol{\beta}] \\
 &\quad - \gamma \frac{G\gamma}{c} (\boldsymbol{\omega}\boldsymbol{\beta}) \cdot [\boldsymbol{\omega}_t \times \mathbf{R}] \\
 &+ \left[ -\frac{1 - 2(\gamma^2 - 1)G}{\gamma} \left( 1 - \frac{r_g}{2R_\oplus} (1 - I) \right) \right. \\
 &\quad \left. + \frac{1}{2\gamma} \left( (5\gamma^2 - 3)G - 3 \right) \frac{1}{c} (\boldsymbol{\omega}_t [\mathbf{R} \times \boldsymbol{\beta}]) \right] \boldsymbol{\omega} \\
 &+ \left( \frac{2(\gamma^2 - 1)G}{\gamma} - \frac{1}{\gamma} \right) \frac{r_g}{R_\oplus} \left( 1 - \frac{3}{2}I \right) (\boldsymbol{\omega}\mathbf{n}_\oplus) \mathbf{n}_\oplus \\
 &\quad - \left[ \left( 2\gamma G + \frac{\gamma}{\gamma + 1} \right) \left( 1 - \frac{r_g}{2R_\oplus} (1 - I) \right) \right. \\
 &\quad \left. + \left( \gamma G + \frac{\gamma}{\gamma + 1} \right) \frac{3}{2c} (\boldsymbol{\omega}_t [\mathbf{R} \times \boldsymbol{\beta}]) \right] (\boldsymbol{\omega}_t \boldsymbol{\beta}) \boldsymbol{\beta}.
 \end{aligned}$$

$\mathbf{n}_\oplus$  - normal to the ring plane

$\boldsymbol{\omega}_t$  - in-plane component

The gravity pull + geodetic precession --- the dominant effect

Earth's rotation:

S.N. Vergeles, N.N. Nikolaev, [JETP, 129\(4\), 541-552 \(2019\)](#); [JHEP, 2004, 191 \(2020\)](#)

Earth's rotation  $\boldsymbol{\omega}$ : can be neglected at  $\sim 1$  hour spin coherence time. Can be subjected to active compensation by Wien filter?

The radial & longitudinal in-plane contributions from Earth's rotation do vanish upon BKM averaging (wait a while)

## Numerics for false EDM in all electric frozen spin ring

F. Abusaif et al., arXiv: 1912.07881 [nucl.ex] (CYR)

$$\Omega_g^{GR} = \frac{1 - G(2\gamma^2 - 1)}{\gamma} \cdot \frac{|\mathbf{g}_0|}{c} [\mathbf{n}_\oplus \times \boldsymbol{\beta}] = \frac{1 - G(2\gamma^2 - 1)}{\gamma} \cdot \frac{|\mathbf{g}_0|}{c} \beta \mathbf{e}_r$$

Obukhov, Silenko, Teryaev (2016)

Opposite sign for the CW and ACW beams!

Special case of frozen spin at magic  $\beta^2 = \frac{1}{1+G} \rightarrow \Omega_g^{GR} = \sqrt{G} \frac{|\mathbf{g}|_0}{c} \mathbf{e}_r$

GR derivation by Orlov, Flanagan, Semertzidis (2012); Nikolaev et al., PoS SPIN2018 (2019) 089

$$\text{CYR: } d_p^{\text{GR}} \approx 2.88 \cdot 10^{-28} \text{ e cm (i.e. } \eta_p^{\text{GR}} \approx 2.74 \cdot 10^{-14})$$

Identical orbits of CW and ACW beams in all electric ring:

CW + ACW = EDM signal

CW-ACW = Standard Candle for controlling systematics

What will be the Standard Candle for Hybrid Ring with magnetic focusing: identity of spin and momentum rotations on CW and ACW orbits in question?

## Beam vertical displacement by gravity pull (nonmeasurable)

Vertical betatron oscillations: **spring constant** is an effective quadrupole strength

$$q \langle k \rangle = q \langle dE_y/dy \rangle$$

Relate spring constant to betatron frequency (**vertical tune  $\sim 0.45$** )

V. Anastassopoulos et al., Rev. Sc. Inst. 87, 11156 (2016) & **CYR**

$$\langle k \rangle = \gamma m \omega_y^2 / q \sim 1.3 \times 10^{-8} \text{ V cm}^{-2}$$

$$\omega_y \sim 1 \text{ M rad/s}$$

$$\langle y \rangle = \langle E_f \rangle / \langle k \rangle \sim 13 \text{ pm}$$

**RF Wien filter commissioning by JEDI @ COSY**: collective oscillation amplitude of a rarefied gas beam

$A_y < 1 \text{ } \mu\text{m} \sim 10$  **Heisenberg quantum limit** vs. beam size  $\sim 1 \text{ mm}$

J.Slim et al., arXiv: 2101.07582 [nucl-ex]

**Quantum mechanics does not preclude  $\sim 1 \text{ pm}$  for a beam centroid**

## Pure electrostatic lab bound to the rotating Earth: can it be free of the intralab magnetic fields?

- I abstract from Earth's magnetic field --- that's an entirely separate story
- Observer from distant stars: moving charges do obviously generate magnetic fields with configurations subject to the charge distributions
- Will there be any ghost (**geometric**) magnetic field seen by an observer at rest in the Earth bound laboratory?
- Background to EDM from interaction of MDM with geometric magnetic field?

# Pure electrostatics in the Earth bound lab (all derivations in the lab frame)

S.N. Vergeles, N.N. Nikolaev,  
[JETP, 129\(4\), 541-552 \(2019\)](#);  
[JHEP, 2004, 191 \(2020\)](#)

$$\frac{1}{\sqrt{-g}} \partial_\nu (\sqrt{-g} F^{\nu\mu}) = 4\pi J^\mu$$

$$(\partial/\partial x^0) J^\mu(x) = 0, \quad J^0 \neq 0, \quad J^i = 0.$$

$$\partial_i (\sqrt{-g} F^{ij}) = 0 \quad \rightarrow \quad \sqrt{-g} F^{ij} = \varepsilon_{ijk} \partial_k \psi.$$

$$\varepsilon_{\mu\nu\lambda\rho} \partial_\nu F_{\lambda\rho} = 0$$

$$\partial_i \left( \sqrt{-g} (e_0^0)^{-2} g^{ij} \partial_j \psi \right) = -\varepsilon_{ijk} e_0^0 e_i^\alpha \mathbf{E}^\alpha \partial_k (e_j^0 / e_0^0)$$

$$e_j^0 = g_{0j} / \sqrt{g_{00}} \neq 0$$



$$e_i^0 = \frac{g_{0i}}{\sqrt{g_{00}}} = - \left\{ 1 + \frac{r_g}{R} \left( \frac{3}{2} - I \cdot \frac{R_{\oplus}^2}{R^2} \right) \right\} \frac{[\boldsymbol{\omega} \times \mathbf{R}]^i}{c}$$

To the leading approximation

$$\mathbf{H}_{\boldsymbol{\omega}} = -\nabla\psi, \quad \Delta\psi = \frac{2}{c}(\boldsymbol{\omega}\mathbf{E})$$

Nonvanishing geometric magnetic field

Has the correct parity

## Charged spherical shell of radius $a$ :

$$\mathbf{E}(\mathbf{r}) = \begin{cases} \frac{Q\mathbf{r}}{r^3}, & \text{for } r > a, \\ 0, & \text{for } r < a. \end{cases}$$

$$\mathbf{H}_\omega(\mathbf{r}) = \begin{cases} \frac{1}{r^3} \left\{ 3(\boldsymbol{\mu}\mathbf{n})\mathbf{n} - \boldsymbol{\mu} \right\} + \frac{1}{c} [\mathbf{E}(\mathbf{r}) \times [\boldsymbol{\omega} \times \mathbf{r}]], & \text{for } r > a, \\ \frac{2Q\boldsymbol{\omega}}{3ca}, & \text{for } r < a, \end{cases}$$

$\mathbf{n} = \mathbf{r}/r$

Magnetic moment induced by Earth's rotation

$$\boldsymbol{\mu} = \frac{Qa^2}{3c} \boldsymbol{\omega}$$

## Plane capacitor in the nEDM expts

$$f_n = \frac{1}{\pi\hbar} |\mu_n \mathbf{B} + d_n \mathbf{E}|$$

Flip the E-field  $\rightarrow$  
$$d_n = \frac{\pi\hbar\Delta f}{2|\mathbf{E}|}$$

Geometric H-field flips as the E-field 
$$\mathbf{H}_\omega = -\frac{2\omega_z z}{c} \mathbf{E}$$

Offset of the center of gravity of neutrons with respect to the uniformly distributed comagnetometer mercury atoms

$\rightarrow$  average geometric magnetic field acting on the neutron MDM

$$\begin{aligned} \mathbf{H}_\omega^{(n)} &= -\frac{2\langle z \rangle \omega_z}{c} \mathbf{E}_0 & \rightarrow & d_{false} = -\frac{2\langle z \rangle \omega_z}{c} \mu_n \\ \langle z \rangle &\simeq 2.8\text{mm} & \rightarrow & d_{false} \approx 2.5 \times 10^{-28} \text{ e}\cdot\text{cm} \end{aligned}$$

Spread of false EDM signal in a storage cell of height 12 cm

$$\Delta d_{false} = \pm \frac{h\omega_z}{c} \mu_n \simeq \pm 5 \times 10^{-27} \text{ e}\cdot\text{cm}$$

# All-electric magic-energy proton ring

Gap of the electrostatic deflector ( $\rho$  - midgap radius,  $\mathbf{n} = \mathbf{r}/r$ )

$$\mathbf{E}_0 = -\mathcal{E}_0 \frac{\rho \mathbf{r}}{r^2} = -\nabla A_0(r), \quad A_0(r) = \mathcal{E}_0 \rho \ln \frac{r}{\rho}$$

Ansatz for the magnetic potential

$$\psi = f(r) \cdot (\boldsymbol{\omega}_t \mathbf{r})$$

Solution has a quadrupole behavior along the orbit

$$\mathbf{H}_\omega^i = \frac{\mathcal{E}_0 \rho}{c} \left\{ \ln \left( \frac{r}{\rho} \right) \cdot \delta_{ij} + \left( \frac{1}{2} \delta_{ij} - \mathbf{n}_i \mathbf{n}_j \right) \right\} \omega_t^j \simeq \frac{\mathcal{E}_0 \rho}{2c} (\delta_{ij} - 2\mathbf{n}_i \mathbf{n}_j) \omega_t^j$$

$$\mathbf{H}_\omega = H_\omega (\sin 2\theta, \cos 2\theta)$$

$$\omega \rho / c \sim 2 \times 10^{-11}$$

# Impact of in-plane geometric field from Earth's rotation on spin precession

Most dangerous radial magnetic field

$$H_{\omega}^{(r)} = (\mathbf{n} \cdot \mathbf{H}_{\omega}) = H_{\omega} \sin \theta.$$

Bogoliubov-Krylov-Mitropolsky averaging

$$\langle H_{\omega}^{(r)} \rangle = \frac{1}{2\pi} \oint d\theta H_{\omega}^{(r)} = 0$$

Similar averaging out of the in-plane component of Earth's rotation effects from focusing fields compensating for the gravity pull

Non-vanishing Berry geometric phase?

# Gravitational waves and storage rings



Any new idea for detection is interesting, even the most “ambitious” ones

# Storage rings as gravitational wave antennae

Impressions of the

**ARIES WP6 Workshop: Storage Rings and Gravitational Waves "SRGW2021"**

Sessions held on : **3x5 talks on 2, 18 February, 4,11, 18 March 2021**

Typical attendance ~ 40-50

Better consult original talks which can be downloaded from

<https://indico.cern.ch/event/982987/contributions/4201176/>

Very superficial comments upon piracy from the above:

- Detection of single gravitons (Gertsenstein effect)
- Excitation of betatron & synchrotron oscillations by gravitational waves



**IPhT**  
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DRF-INP UMR 3681

# Revisiting Gravitational Wave Detection in an SRF Cavity

---

Sebastian A. R. Ellis

IPhT, CEA Saclay

Based on:

210x.xxxxx

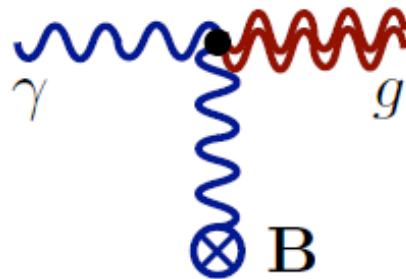
A. Berlin, R. T. D'Agnolo, SARE



# GW interaction w/ EM

Gertsenshtein effect, 1962

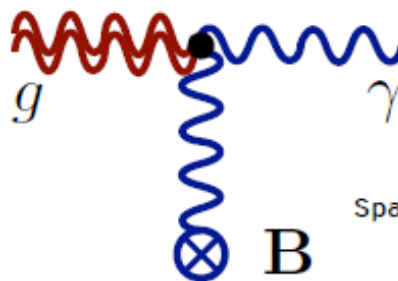
Also Zeldovich 1973



$$\square h_{\{+, \times\}} = 16\pi G_N B \partial_i A_{\{+, \times\}}$$

$h = \text{strain}$  (a dimensionless departure from the Minkowski metrics)

Naturally, inverse process also allowed



Inverse Primakoff effect  
Deja vue: **axions!**

Spatial and temporal variations of graviton contribute

See e.g. Domcke & Garcia-Cely PRL126 (2021)

# GW interaction w/ EM strategy: venerable history

Braginskii & Menskii, 1971

JETP LETTERS

VOLUME 13, NUMBER 11

5 JUNE 1971

HIGH-FREQUENCY DETECTION OF GRAVITATIONAL WAVES

V. B. Braginskii and M. B. Menskii

Physics Department, Moscow State University

Submitted 18 March 1971

ZhETF Pis. Red. 13, No. 11, 585 - 587 (5 June 1971)

Pegoraro, Picasso & Radicati, 1978

J. Phys. A: Math. Gen., Vol. 11, No. 10, 1978. Printed in Great Britain

**On the operation of a tunable electromagnetic detector for gravitational waves**

F Pegoraro<sup>†</sup>, E Picasso<sup>‡</sup> and L A Radicati<sup>†§</sup>

<sup>†</sup>Scuola Normale Superiore, Pisa, Italy

<sup>‡</sup>CERN, Geneva, Switzerland

Received 6 December 1977, in final form 20 April 1978

Pegoraro, Radicati, Bernard & Picasso, 1978

Led to MAGO collaboration @ CERN

early 2000's

See also Caves 1979, Reece, Reiner & Melissinos 1982, 1984

**ELECTROMAGNETIC DETECTOR FOR GRAVITATIONAL WAVES**

F. PEGORARO, L.A. RADICATI

*Scuola Normale Superiore, Pisa, Italy*

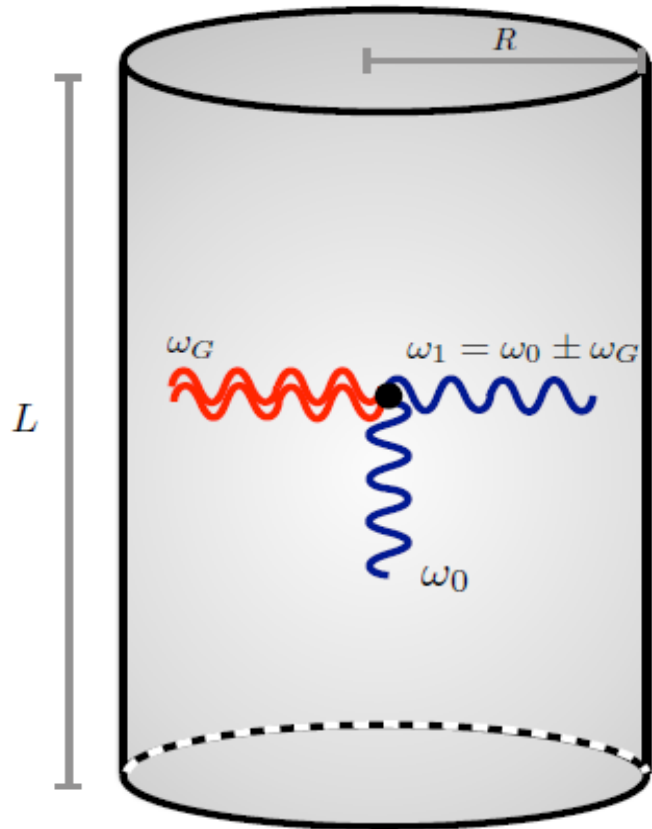
and

Ph. BERNARD and E. PICASSO

*CERN, Geneva, Switzerland*

Received 29 June 1978

# Gravity Wave Resonant Frequency Conversion



*Cylinder for illustrative purposes only!*

Superconducting RF Cavity

$$\omega_i \sim \text{GHz}$$

$$Q_{\text{int}} \sim 10^9 \div 10^{13}$$

Tunability:

$$\delta\omega \lesssim \text{MHz} \quad \text{piezos}$$

$$\delta\omega \gtrsim \text{MHz} \quad \text{fins}$$

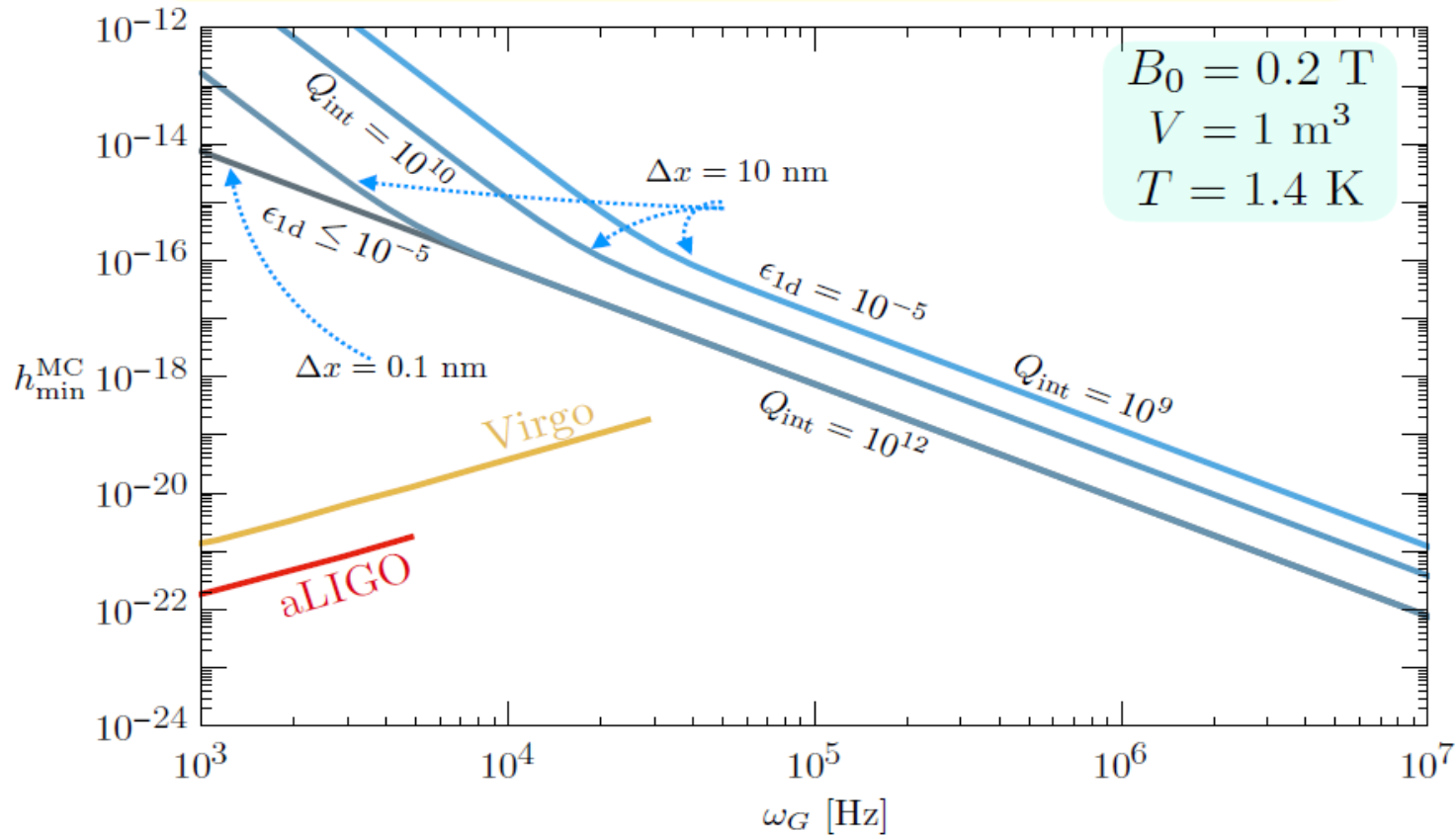
Fields must have quadrupole moment

**N.B. :  $J^P=2^+ \rightarrow 2\gamma$  is allowed !**

# Reach – Monochromatic source\*\*

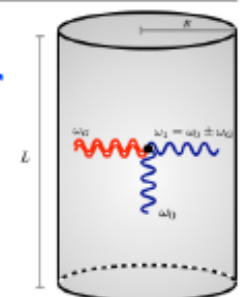
\*\*ultra-preliminary

$$h_{\min}^{\text{MC}} \sim \frac{1}{\omega_G^2} \left( \frac{T}{\omega_1 Q_1} \right)^{1/2} \left( \frac{\delta\omega}{t_{\text{int}}} \right)^{1/4} \frac{1}{E_0 V^{7/6}} \sim 10^{-22} \left( \frac{10^7 \text{ Hz}}{\omega_G} \right)^2$$



# Conclusion

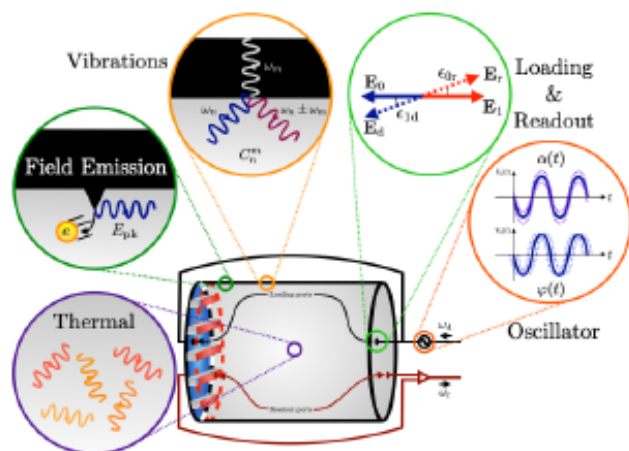
Direct signal – cavity as a Gertsenshtein converter



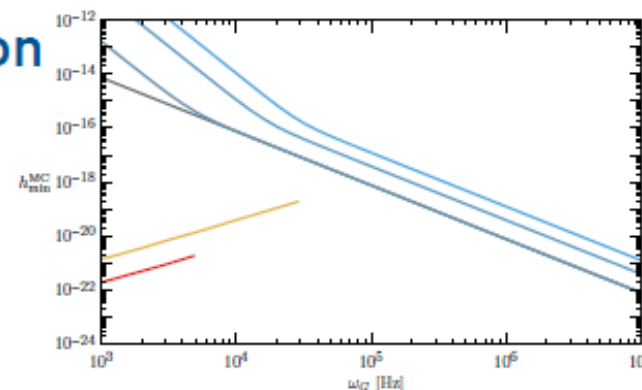
Cylinder for illustrative purposes only!

Noise in SRF Cavity requires precise Cavity control:

- Careful loading
- vibration control
- mode isolation

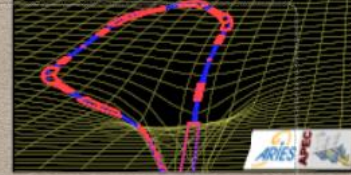


Strain sensitivity up to  $h \sim 10^{-22} - 10^{-20}$



*Bonus! Technology useful for axion DM direct detection*



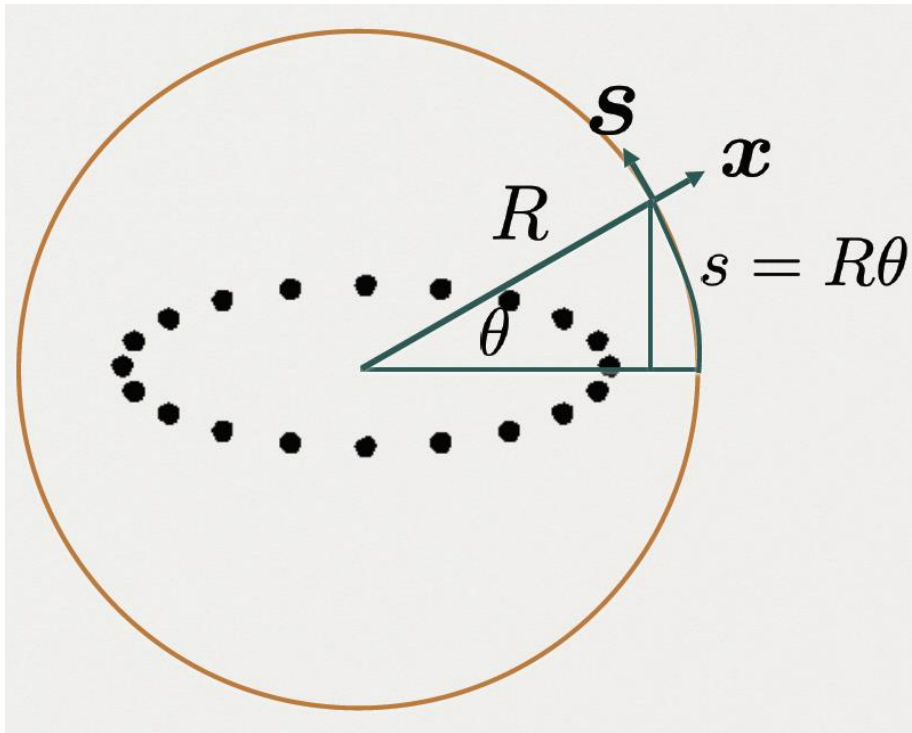


# Response of a storage-ring beam to a gravitational wave

Feb. 3, 2021 @SRGW2021

K. Oide (KEK/CERN)

Many thanks to G. Franchetti and F. Zimmermann for discussions.



$$\frac{d^2 X_\mu}{dt^2} = \frac{1}{2} \ddot{h}_{\mu\nu} X^\nu$$

$$\omega_{\text{GR}} = 2\pi c/\lambda = ck$$

Betatron resonance condition

$$kR \pm 2 = \nu_x$$

Betatron amplitude gain per turn

$$\Delta x \approx \pi k^2 R^2 \beta_x h \approx \pi \nu_x R h$$

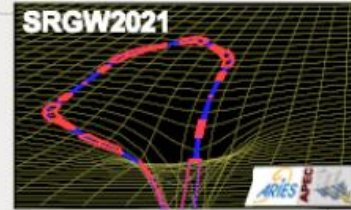
$$h = 10^{-22}, R = 5 \text{ km and } \nu_x = 130 \rightarrow \Delta x \sim 0.2 \text{ fm}$$

JEDI with deuterons in COSY :  $A_y < 1 \mu\text{m}$

J.Slim et al., arXiv: 2101.07582 [nucl-ex]

## 4 Summary

- The beam in a storage ring can respond to a wave of gravitational radiation, somewhat similar to the Weber bar.
- The betatron motion of the beam in a ring can resonate to the GR.
- A special beam optics “beam antenna” may enhance the sensitivity,
- There will be many noise sources to overcome: thermal motion of quadrupoles, beam fluctuation due to emittance, synchrotron radiation and the acceleration, etc.



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Feb. 3, 2021 K. Oide



# GRAVITATIONAL WAVES AT PARTICLE STORAGE RINGS



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Everything interacts with the gravitational wave: magnets, tunnel protons, EM fields, us observers, ...



We can choose coordinates that move together with the wave  
(TT frame)



A bit counterintuitive: freely falling objects don't move, but  
proper distances change



We can use this symmetry to our advantage: choose a frame where our Newtonian intuition applies (**proper detector frame**) at least for small distances compared to the wavelength

$$\frac{L}{\lambda_g} \ll 1$$

This is the frame of an observer standing still next to the LHC and watching the protons fly by, i.e. exactly what we want







We can use this symmetry to our advantage: choose a frame where our Newtonian intuition applies (**proper detector frame**)

- The wave acts as a Newtonian force
- Coordinate distances = proper distances (for non-relativistic objects)
- Rigid objects don't move (much)
- At zero frequency we are in flat space

# Synchrotron resonance: $w_g = w_l$ ( $l = \text{longitudinal}$ )

Staggering of the revolution time

$$\delta_t = \frac{\delta_l}{c} \simeq (hT)(\omega_l \tau_l)$$

$$\left(\frac{\Delta T}{T}\right)_{\text{exp}} \simeq 10^{-7}$$

$$\omega_l \simeq 10 \text{ Hz}$$

$$\tau_l \simeq 1 \text{ hour}$$

$$h \gtrsim 10^{-11}$$

AT LEAST 10 ORDERS OF MAGNITUDE ABOVE  
KNOWN SOURCES

Heated discussion of the **destructive role of the RF cavity** and of  
synchrotron radiation in LHC → suggestions of a **non-relativistic single  
ion coasting beams**.



## New Testament after D'Agnolo:

- MEASURING GRAVITATIONAL WAVES USING STORAGE RINGS MIGHT BE HOPELESS
- HOWEVER MEASURING GRAVITATIONAL WAVES WAS CONSIDERED ALTOGETHER HOPELESS FOR DECADES AND WE WERE WRONG
- THE PRESENCE OF A POTENTIAL PATH TOWARDS GW-LEVEL SENSITIVITY FOR LONGITUDINAL DEFORMATIONS IS INTERESTING AND MIGHT DESERVE FURTHER STUDY

# Epilogue:

- Unexpectedly large manifestations of the feeblest force in nature in ultrahigh precision particle physics expts
- Spin precession as a gravimeter: gravity as a unique Standard Candle in all-electric EDM ring
- What about CW vs ACW comparison in hybrid rings ?
- Impact of Earth's rotation on the EDM experiments deserves more scrutiny
- Splash of curiosity about accelerators as gravitational wave antennae

Stay tuned !