Contributions of CP Violating Operators to the Neutron/Proton EDM from Lattice QCD: Status and Future Prospects

Rajan Gupta

Los Alamos
NATIONAL LABORATORY
EST.1943

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LANL EDM collaboration

Tanmoy Bhattacharya

Vincenzo Cirigliano

Rajan Gupta

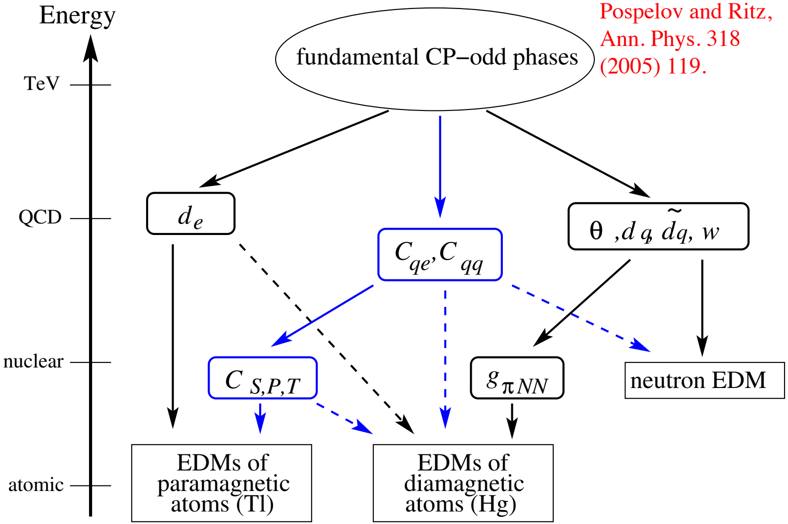
Emanuelle Mereghetti

Boram Yoon

LANL Publications

- Bhattacharya et al, “Dimension-5 CP-odd operators: QCD mixing and renormalization”, PhysRevD.92.114026
- Bhattacharya et al, “Neutron Electric Dipole Moment and Tensor Charges from Lattice QCD”, PhysRevLett.115.212002
- Bhattacharya et al, “Isovector and isoscalar tensor charges of the nucleon from lattice QCD”, PhysRevD.92.094511
- Gupta et al, “Flavor diagonal tensor charges of the nucleon from $(2 + 1 + 1)$-flavor lattice QCD” PhysRevD.98.091501
- Bhattacharya et al, “Neutron Electric Dipole Moment from Beyond the Standard Model”, arXiv:1812.06233
- Bhattacharya et al, “Contribution of the QCD Θ-term to nucleon electric dipole moment”, arXiv:2101.07230

Hierarchy of EDM Scales



Pospelov and Ritz, Ann. Phys. 318 (2005) 119.

$$\mathcal{L}_{\mathrm{CPV}} = \mathcal{L}_{\mathrm{CKM}} + \mathcal{L}_{\bar{\theta}} + \mathcal{L}_{\mathrm{BSM}} \quad \longrightarrow \quad \mathcal{L}_{\mathrm{CPV}}^{\mathrm{eff}}$$

Effective CPV Lagrangian at Hadronic Scale

$$\begin{aligned}
\mathcal{L}_{\rm CPV}^{d\leq 6} = & -\frac{g_s^2}{32\pi^2}\overline{\theta}G\tilde{G} && \text{dim=4 QCD } \theta\text{-term} \\
& -\frac{i}{2}\sum_{q=u,d,s,c} d_q\overline{q}(\sigma\cdot F)\gamma_5 q && \text{dim=5 Quark EDM (qEDM)} \\
& -\frac{i}{2}\sum_{q=u,d,s,c} \tilde{d}_q g_s\overline{q}(\sigma\cdot G)\gamma_5 q && \text{dim=5 Quark Chromo EDM (CEDM)} \\
& + d_w\frac{g_s}{6}G\tilde{G}G && \text{dim=6 Weinberg's 3g operator} \\
& + \sum_i C_i^{(4q)}O_i^{(4q)} && \text{dim=6 Four-quark operators}
\end{aligned}$$

- $\overline{\theta} \leq \mathcal{O}(10^{-8} - 10^{-11})$: Strong CP problem
- Dim=5 terms suppressed by $d_q \approx \langle \mathrm{v}\rangle/\Lambda_{BSM}^2$; effectively dim=6
- All terms up to $d = 6$ are leading order

Contributions to the Neutron EDM d_n

$$d_n = \overline{\theta} \cdot C_\theta + d_q \cdot C_{\mathrm{qEDM}} + \tilde{d}_q \cdot C_{\mathrm{CEDM}} + \cdots$$

- SM and BSM theories
  $\longrightarrow$ Coefficients of the effective CPV Lagrangian $(\overline{\theta}, d_q, \tilde{d}_q, \ldots)$
- Lattice QCD
  $\longrightarrow$ Nucleon matrix elements in presence of CPV interactions

  $C_\theta = \langle N | J^{\mathrm{EM}} | N \rangle |_\theta$

  $C_{\mathrm{qEDM}} = \langle N | J^{\mathrm{EM}} | N \rangle |_{\mathrm{qEDM}}$

  $C_{\mathrm{CEDM}} = \langle N | J^{\mathrm{EM}} | N \rangle |_{\mathrm{CEDM}}$

  $\ldots$

Lattice QCD $\Longrightarrow$ Physical Results

- **Removing Excited state contamination**
  – Lattice meson and nucleon interpolating operators also couple to excited states
- **Renormalization: Lattice scheme $\longrightarrow$ continuum $\overline{\mathrm{MS}}$**
  – involves complicated/divergent mixing
- **Heavier $\rightarrow$ Physical Pion Mass. Almost there!**
  – As $M_\pi \rightarrow 135$ MeV $\Longrightarrow$ larger errors as computational cost increases
- **Finite Lattice Spacing**
  – Extrapolate from finite lattice spacings $0.045 < a < 0.15$ fm
- **Finite Volume**
  – Finite lattice volume effects small in most EDM calculations for $M_\pi L > 4$

Extrapolate data at $\{a, M_\pi, M_\pi L\}$ to $a = 0$, $M_\pi = 135$ MeV, $M_\pi L \rightarrow \infty$

Neutron EDM from Quark EDM term

$$\mathcal{L}_{\mathrm{CPV}}^{d\leq 6} = -\frac{g_s^2}{32\pi^2}\bar{\theta}G\tilde{G}$$ dim=4 QCD θ-term

$$-\frac{i}{2}\sum_{q=u,d,s} d_q\bar{q}(\sigma\cdot F)\gamma_5 q$$ dim=5 Quark EDM (qEDM)

$$-\frac{i}{2}\sum_{q=u,d,s} \tilde{d}_q g_s\bar{q}(\sigma\cdot G)\gamma_5 q$$ dim=5 Quark Chromo EDM (CEDM)

$$+\,d_w\frac{g_s}{6}G\tilde{G}G$$ dim=6 Weinberg's 3g operator

$$+\sum_i C_i^{(4q)}O_i^{(4q)}$$ dim=6 Four-quark operators

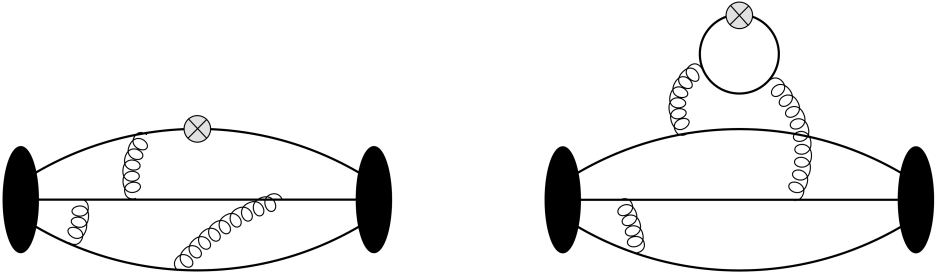
C_{qEDM} are given by the Tensor Charges

- leading contributions of Quark EDMs are given by the tensor charges g_T

$$-\frac{i}{2}\sum_{q=u,d,s,c} d_q\bar{q}(\sigma\cdot F)\gamma_5 q \quad\longrightarrow\quad d_N = d_u g_T^u + d_d g_T^d + d_s g_T^s + d_c g_T^c$$

$$\langle N|\bar{q}\sigma_{\mu\nu}q|N\rangle = g_T^q\bar{u}_N\sigma_{\mu\nu}u_N$$

- $d_q \propto m_q$ in many models $\Rightarrow$ Precision determination of $g_T^{\{s,c\}}$ is important



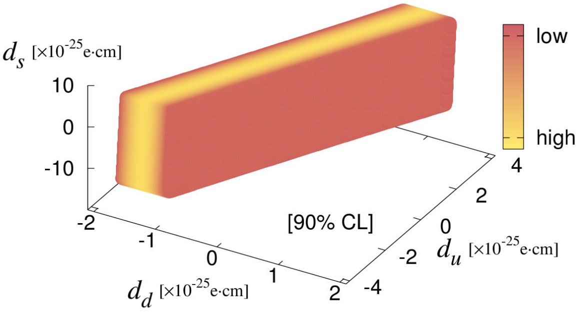
"Disconnected" diagram is noisy (expensive), small, but only contribution for $g_T^{\{s,c\}}$

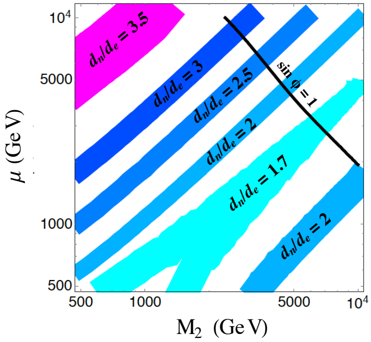
qEDM: FLAG2019 and Current Status

Collaboration	N_f	a	m_π	FV	Z	ESC	g_T^u	g_T^d
PNDME 20	2+1+1	★‡	★	★	★	○	0.783(27)(10)	−0.205(10)(10)
ETM 19	2+1+1	■	○	★	★	○	0.729(22)	−0.2075(75)
PNDME 18B	2+1+1	★‡	★	★	★	○	0.784(28)(10)#	−0.204(11)(10)#
PNDME 16	2+1+1	○‡	★	★	★	○	0.792(42)#&	−0.194(14)#&
Mainz 19	2+1	★	○	★	★	○	0.77(4)(6)	−0.19(4)(6)
JLQCD 18	2+1	■	○	○	★	○	0.85(3)(2)(7)	−0.24(2)(0)(2)
ETM 17	2	■	○	○	★	○	0.782(16)(2)(13)	−0.219(10)(2)(13)

Collaboration	N_f	a	m_π	FV	Z	ESC	g_T^s
PNDME 20	2+1+1	★‡	★	★	★	○	−0.0022(12)
ETM 19	2+1+1	■	○	★	★	○	−0.00268(58)
PNDME 18B	2+1+1	★‡	★	★	★	○	−0.0027(16)#
Mainz 19	2+1	★	○	★	★	○	−0.0026(73)(42)
JLQCD 18	2+1	■	○	○	★	○	−0.012(16)(8)
ETM 17	2	■	○	○	★	○	−0.00319(69)(2)(22)

Constraints on BSM from qEDM and Future Prospects





[Bhattacharya, *et al.* (2015), Gupta, *et al.* (2018)]

Status:

- $g_T^{u,d,s}$ results from multiple collaborations with control over $a \to 0$ extrapolation
- Single result from ETM 19 $g_T^c = -0.00024(16)$

Neutron EDM from QCD θ-term

$$\mathcal{L}_{\text{CPV}}^{d\leq 6} = -\frac{g_s^2}{32\pi^2}\bar{\theta}G\tilde{G} \quad \text{dim=4 QCD } \theta\text{-term}$$

$$-\frac{i}{2}\sum_{q=u,d,s} d_q\bar{q}(\sigma\cdot F)\gamma_5 q \quad \text{dim=5 Quark EDM (qEDM)}$$

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$$+\, d_w\frac{g_s}{6}G\tilde{G}G \quad \text{dim=6 Weinberg's 3g operator}$$

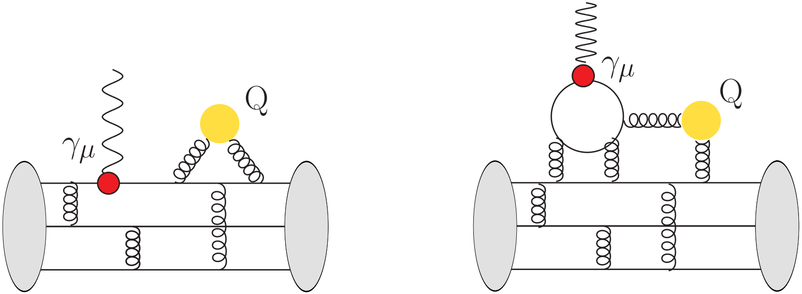
$$+\sum_i C_i^{(4q)}O_i^{(4q)} \quad \text{dim=6 Four-quark operators}$$

QCD θ-term

$$S = S_{QCD} + i\theta Q, \qquad Q = \int d^4x \frac{G\tilde{G}}{32\pi^2}$$

At the leading order, the correlation functions calculated are

$$\langle N \mid J_\mu^{\mathrm{EM}} \mid N\rangle\Big|^{\overline{\Theta}} \approx \langle N \mid J_\mu^{\mathrm{EM}} \mid N\rangle\Big|^{\overline{\Theta}=0} - i\overline{\Theta}\left\langle N \middle| J_\mu^{\mathrm{EM}} \int d^4x \, \frac{G_{\mu\nu}^a \widetilde{G}_{\mu\nu}^a}{32\pi^2} \middle| N \right\rangle ,$$



Three different approaches for the QCD θ-term

- External electric field method: $\langle N\overline{N}\rangle_\theta(\vec{\mathcal{E}},t) = \langle N(t)\overline{N}(0)e^{i\theta Q}\rangle_{\vec{\mathcal{E}}}$

  Aoki and Gocksch (1989), Aoki, Gocksch, Manohar, and Sharpe (1990), CP-PACS Collaboration (2006), Abramczyk, *et al.* (2017)

- Simulation with imaginary θ: $\theta = i\tilde{\theta}$, $S^q_\theta = \tilde{\theta}\frac{m_l m_s}{2m_s+m_l}\sum_x \overline{q}\gamma_5 q$

  Horsley, *et al.*, (2008), Guo, *et al.* (2015)

- Expansion in small θ:

$$\langle O(x)\rangle_\theta = \frac{1}{Z_\theta}\int d[U,q,\overline{q}]O(x)e^{-S_{QCD}-i\theta Q}$$
$$= \langle O(x)\rangle_{\theta=0} - i\theta\langle O(x)Q\rangle_{\theta=0} + \mathcal{O}(\theta^2)$$

  Shintani, *et al.*, (2005); Berruto, *et al.*, (2006); Shindler *et al.* (2015); Shintani, *et al.* (2016); Alexandrou *et al.*, (2016)

  Abramczyk, *et al.* (2017)

  Dragos, *et al.* (2019); Alexandrou, *et al.* (2020); Bhattacharya, *et al.* (2021)

The form factor $eF_3(0) = 2M_N d_\Theta$

In simulations with *'imaginary θ'* and *'expansion in θ'* we extract $F_3(0)$ from the most general decomposition:

$$\begin{aligned}\langle N(p',s') \mid J_\mu^{\mathrm{EM}} \mid N(p,s)\rangle_{\not{C}\mathrm{P}}^{\overline{\Theta}} &= \overline{u}_N(p',s')\Bigg[\gamma_\mu F_1(q^2) \\ &+ \frac{1}{2M_N}\sigma_{\mu\nu}q_\nu\Big(F_2(q^2) - iF_3(q^2)\gamma_5\Big) \\ &+ \frac{F_A(q^2)}{M_N^2}(\not{q}q_\mu - q^2\gamma_\mu)\gamma_5\Bigg]u_N(p,s)\,,\end{aligned}$$

F_3 in the naive decomposition is not the correct CP-odd form factor

The neutron state aquires a phase α which mixes F_2 and F_3 in "standard" approach

Phase α with P and CP violation and impact on F_3

The most general spectral decomposition of the 2-point nucleon correlator is

$$\langle\Omega|\mathcal{T}N(\boldsymbol{p},\tau)\overline{N}(\boldsymbol{p},0)|\Omega\rangle = \sum_{i,\boldsymbol{s}} e^{-E_i\tau}\ \mathcal{A}^*{}_i\mathcal{A}_i\ \mathcal{M}_i^{\boldsymbol{s}}\ ,$$

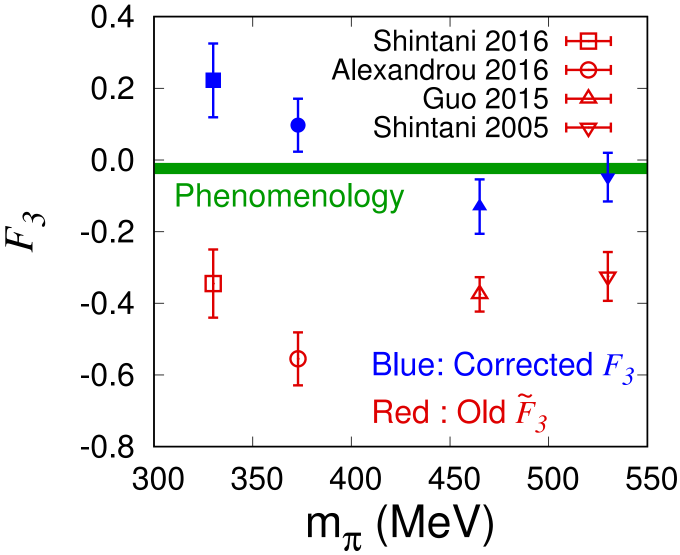
$$\sum_{\boldsymbol{s}} \mathcal{M}_i^{\boldsymbol{s}} = e^{i\alpha_i\gamma_5}\frac{(-i\not{p}_i + M_i)}{2E_i^p}e^{i\alpha_i^*\gamma_5} = e^{i\alpha_i\gamma_5}\sum_{s} u_N^i(\boldsymbol{p},\boldsymbol{s})\overline{u}_N^i(\boldsymbol{p},\boldsymbol{s})e^{i\alpha_i^*\gamma_5}$$

With CPV

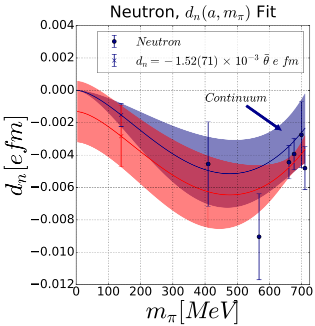
- γ_4 is no longer the parity operator for the neutron state
- There is a unique α for each state and each CPV interaction

Calculations of the Θ-term pre 2017

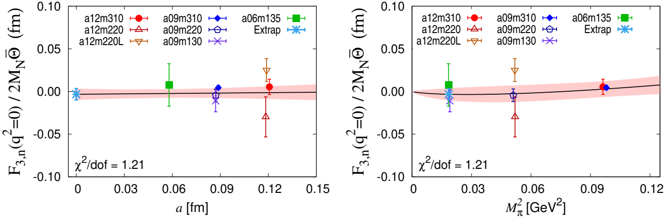
Abramczyk, *et al.* clarified the issue of $\alpha \Longrightarrow$ previous lattice give $d_n \approx 0$



Recent calculations with the Θ-term



Dragos, *et al.* (2019)



Bhattacharya, *et al.*, (2020)

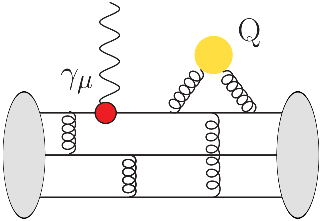
[Dragos, *et al.* (2019)]:

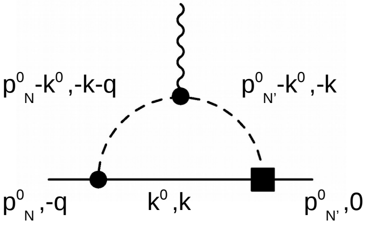
- multiple a but large pion mass $m_\pi > 400\text{MeV}$
- $d_n = -1.52(71) \times 10^{-3}\ \overline{\theta}\ e \cdot fm$
- Inflection point occurs near smallest M_π to satisfy $d_n = 0$ at $M_\pi = 0$

Does the $N\pi$ excited state contribute

Bhattacharya *et al.* (2021) perform a χPT analysis:

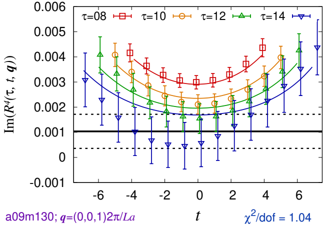
$\Longrightarrow$ Contribution of low energy $N\pi$ excited-state should grow as $M_\pi \to 135$ MeV



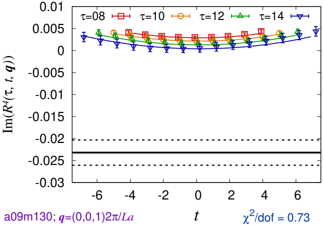


Including the $N\pi$ state gives a very different value for ground-state matrix element

$M_2 \sim 2\,\mathrm{GeV}$



$M_2 \sim 1\,\mathrm{GeV} + M_\pi$



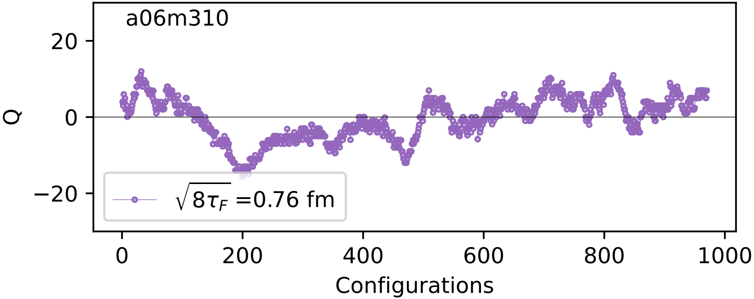
Status from Bhattacharya *et al.* (2021)

	Neutron $\overline{\Theta}$ e · fm	Proton $\overline{\Theta}$ e · fm
Bhattacharya 2021	$d_n = -0.003(7)(20)$	$d_p = 0.024(10)(30)$
Bhattacharya 2021 with $N\pi$	$d_n = -0.028(18)(54)$	$d_p = 0.068(25)(120)$
ETMC 2020	$\lvert d_n \rvert = 0.0009(24)$	–
Dragos 2019	$d_n = -0.00152(71)$	$d_p = 0.0011(10)$
Syritsyn 2019	$d_n \approx 0.001$	–

Table: Summary of lattice results for the contribution of the Θ-term to the neutron and proton electric dipole moment.

- No reliable estimate of the contribution of the Θ-term to nEDM
- Including the contribution of the lowest energy $N\pi$ excited state gives a much larger result

QCD θ-term future: All lattice systematics need better control



- Simulations at $M_\pi = 135$ MeV
- Check for long autocorrelations in Q, which increase as $a \to 0$
- High statistics needed
- Resolving the contribution of $N\pi$ excited state
- Simulations on small a lattices required to reduce discretization artifact
- Chiral-continuum fits

New algorithms needed for lattice generation at $a \lesssim 0.6$ fm to get high statistics

Neutron EDM from quark Chromo-EDM (CEDM)

$$\mathcal{L}_{\mathrm{CPV}}^{d\leq 6} = -\frac{g_s^2}{32\pi^2}\overline{\theta} G\tilde{G}$$ dim=4 QCD θ-term

$$-\frac{i}{2}\sum_{q=u,d,s} d_q \overline{q}(\sigma\cdot F)\gamma_5 q$$ dim=5 Quark EDM (qEDM)

$$-\frac{i}{2}\sum_{q=u,d,s} \tilde{d}_q g_s \overline{q}(\sigma\cdot G)\gamma_5 q$$ dim=5 Quark Chromo EDM (CEDM)

$$+ d_w \frac{g_s}{6} G\tilde{G}G$$ dim=6 Weinberg's 3g operator

$$+ \sum_i C_i^{(4q)} O_i^{(4q)}$$ dim=6 Four-quark operators

Lattice QCD approaches for CEDM

$$S = S_{QCD} + S_{CEDM}; \qquad S_{CEDM} = \frac{g_s}{2} \sum_{q=u,d,s} \tilde{d}_q \int d^4x \overline{q}(\sigma \cdot G)\gamma_5 q$$

- Three different approaches developed
  - Schwinger source method [Bhattacharya, *et al.* (2016)]:

$$D_{clov} \to D_{clov} + \frac{i}{2}\varepsilon\sigma^{\mu\nu}\gamma_5 G_{\mu\nu}$$

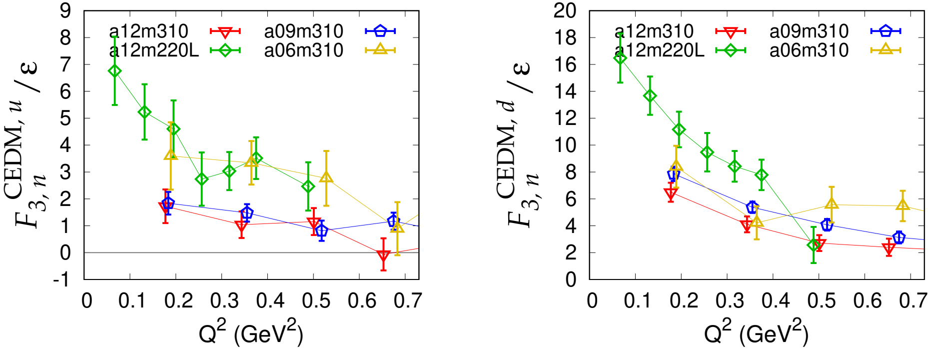
  - Direct 4-point method with expansion in $\sum_q O_{CEDM}$ [Abramczyk, *et al.* (2017)]:

$$\langle N V_\mu \overline{N} \rangle_{CEDM} = \langle N V_\mu \overline{N} \rangle + \tilde{d}_q \langle N V_\mu \overline{N} \sum_q O_{CEDM} \rangle + \mathcal{O}(\tilde{d}_q^2)$$

  - External electric field method [Abramczyk, *et al.* (2017)]:

$$\langle N \overline{N} \rangle_{CEDM}(\vec{\mathcal{E}}, t) = \langle N(t) \overline{N}(0) O_{CEDM} \rangle_{\vec{\mathcal{E}}}$$

Signal in F_3

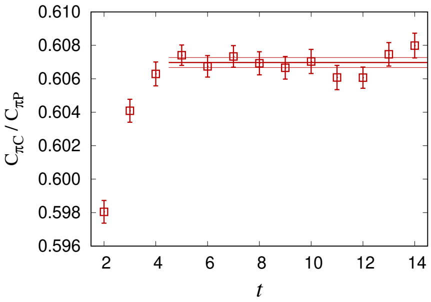


- These data are without renormalization
  - RI-MOM scheme result for CEDM with 1-loop conversion factors to $\overline{\mathrm{MS}}$ available
  - Divergent mixing with pseudoscalar operator: $O^{sub}_{\mathrm{CEDM}} = O_{\mathrm{CEDM}} - Aa^{-2}P$

[Bhattacharya, *et al.* (2015), Constantinou, *et al.*(2015)]

- Working on understanding the behavior versus Q^2 and M_π^2

Operator Mixing $O_{\mathrm{CEDM}}^{sub} = O_{\mathrm{CEDM}} - Aa^{-2}P$



- Determining the mixing coefficient A to define the subtracted operator

Renormalization using Gradient Flow

Gradient flow [Lüscher and Weisz (2011)]:

$$\partial_t B_\mu(t) = D_\nu G_{\nu\mu}, \qquad B_\mu(x, t=0) = A_\mu(x),$$
$$\partial_t \chi(t) = \Delta^2 \chi, \qquad \chi(x, t=0) = \psi(x)$$

- Smear (flow) gluon and quark fields along the gradient of an action to a fixed physical size (sets ultraviolet cutoff of the theory)
- The flowed operators have finite matrix elements except for an universal Z_ψ $\longrightarrow$ Allow us to take continuum limit without power-divergent subtractions
- Mixing and connection to $\overline{\mathrm{MS}}$: simpler perturbative calculation in continuum
- Calculations for CPV ops underway [Rizik, Monahan, and Shindler (2020)]

CEDM: Future Prospects

- Working on renormalization and operator mixing using the gradient flow scheme
- Need algorithm developments for large scale simulations at physical pion mass and lattice spacing $a < 0.09$ fm
- Investigating machine learning methods to reduce computational cost
  [Yoon, Bhattacharya, and Gupta (2019)]

Neutron EDM from Weinberg's ggg and Various Four-quark Ops

$$\mathcal{L}_{\rm CPV}^{d\le 6} = -\frac{g_s^2}{32\pi^2}\overline{\theta} G\tilde{G}$$ dim=4 QCD θ-term

$$-\frac{i}{2}\sum_{q=u,d,s} d_q \overline{q}(\sigma\cdot F)\gamma_5 q$$ dim=5 Quark EDM (qEDM)

$$-\frac{i}{2}\sum_{q=u,d,s} \tilde{d}_q g_s \overline{q}(\sigma\cdot G)\gamma_5 q$$ dim=5 Quark Chromo EDM (CEDM)

$$+ d_w \frac{g_s}{6} G\tilde{G}G$$ dim=6 Weinberg's 3g operator

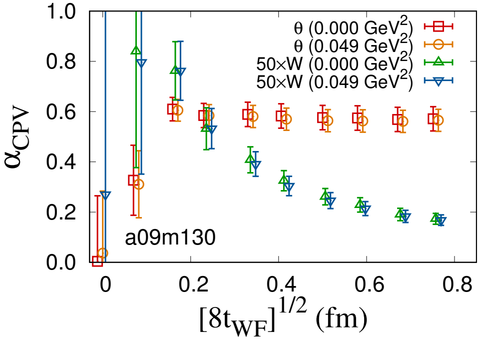
$$+\sum_i C_i^{(4q)} O_i^{(4q)}$$ dim=6 Four-quark operators

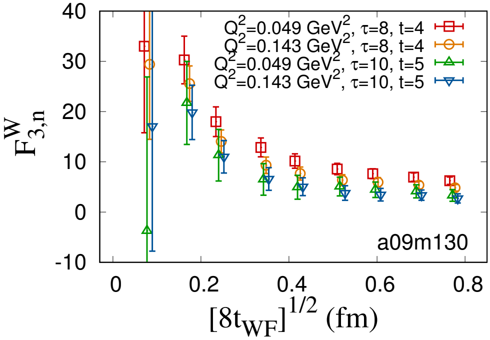
Weinberg's $G\widetilde{G}$ Operator: Status and Future Prospects

$$\mathcal{L}_{W_{ggg}} = \frac{1}{6} d_w g_s G\tilde{G}G$$

- Calculation is almost the same as for the QCD θ-term
- No publications yet, only a few preliminary studies
  [Yoon, Bhattacharya, Cirigliano, and Gupta (2019)]
- Signal is noisier than QCD θ-term
- Suffers from the long autocorrelations on $a \lesssim 0.06$ fm lattices
- Requires solving operator renormalization and mixing
  - RI-MOM scheme and its perturbative conversion to $\overline{\mathrm{MS}}$ is available
    [Cirigliano, Mereghetti, and Stoffer (2020)]
  - Gradient flow scheme is being investigated to address divergent mixing structure
    [Rizik, Monahan, and Shindler (2020)]

Weinberg's $G\widetilde{G}G$ Operator: Mixing with the Θ-term

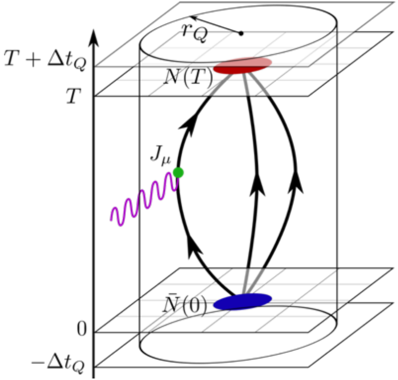


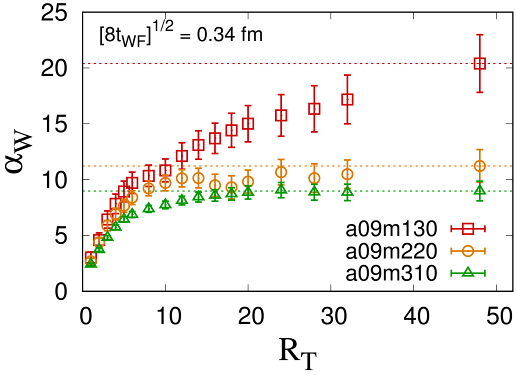


$1/t_{\rm WF}$ mixing with the Θ-term

Variance reduction by integrating $G\widetilde{G}G$ over a local volume

Motivation: correlation between $G\widetilde{G}$ or $G\widetilde{G}G$ and $\langle N J^{\mathrm{EM}} N \rangle$ expected to be short range. Region outside contributes only noise (Shintani, Liu, ...)





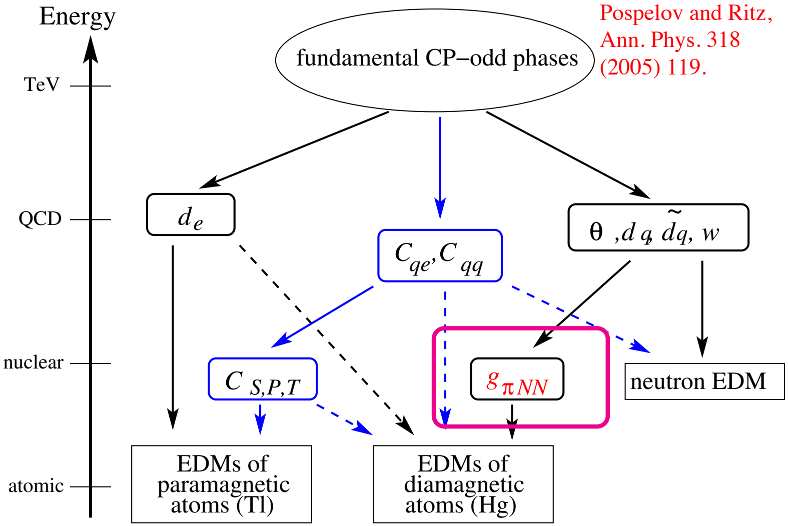
Ensuring that the "safe" volume is large enough to not give rise to a bias.
On current lattices, safe region approaches the full volume as $M_\pi \to 135$ MeV

Four-quark operators: Current Status and Future Prospects

$$\mathcal{L}_{4q} = \sum_i C_{ij}^{(4q)} (\bar{\psi}_i \psi_i)(\bar{\psi}_j i\gamma_5 \psi_j) + \cdots$$

- No lattice QCD calculations yet!
- Calculation expected to be statistically noisy and computationally expensive
- Hopefully we can include this calculation in a long range (5–10) year plan

Lattice Calculations for $g_{\pi NN}$



Pospelov and Ritz, Ann. Phys. 318 (2005) 119.

$g_{\pi NN}$: Current Status and Future Prospects

$$\mathcal{L}_{\pi NN}^{CPV} = -\frac{\bar{g}_0}{2F_\pi}\bar{N}\boldsymbol{\tau}\cdot\boldsymbol{\pi}N - \frac{\bar{g}_1}{2F_\pi}\pi_0\bar{N}N - \frac{\bar{g}_2}{2F_\pi}\pi_0\bar{N}\tau^3N + \cdots$$

- Chiral symmetry relations + nucleon σ-term & mass splittings $\longrightarrow g_{\pi NN}$
  [Vries, Mereghetti, Seng, and Walker-Loud (2017)]
- No direct lattice calculation of $g_{\pi NN}$ published yet

Can be calculated from $\langle N|A_\mu(q)|N\rangle_{\mathrm{CPV}}$ following the same methodology used for neutron EDM via $\langle N|V_\mu(q)|N\rangle_{\mathrm{CPV}}$

Conclusion

- Significant progress, issues of signal, statistics and renormalization remain
- Gradient flow scheme is, so far, best for renormalization
- quark-EDM: Lattice QCD has provided results with $\lesssim 5\%$ uncertainty
- Θ-term: Significant Progress. No reliable estimates yet
  - Statistics
  - Does $N\pi$ provide leading excited-state contamination?
- quark chromo-EDM: Signal in both methods
  - Renormalization and mixing (Working on gradient flow scheme)
  - Does $N\pi$ provide leading excited-state contamination?
- Weinberg $G\widetilde{G}G$ Operator: Signal
  - Address the mixing with Θ-term in gradient flow scheme
- Four-quark operators: Yet to be initiated

Could use 10x Larger Computational Resources