Rogowski beam position monitors

Falastine Abusaif

Forschungszentrum Jülich, Nuclear Physics Institute IKP 2
RWTH Aachen University, III. Physikalisches Institut B
For the JEDI collaboration

f.abusaif@fz-juelich.de

744. WE-Heraeus-Seminar
Towards Storage Ring Electric Dipole Moment Measurements

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Overview

Principle of Rogowski coil

- Toroidal helical wire (principle of magnetic induction)
- Segmented into four parts (link to beam coordinates):
  - $\frac{\Delta_x}{\Sigma} = \frac{right-left}{right+left}$
  - $\frac{\Delta_y}{\Sigma} = \frac{up-down}{up+down}$

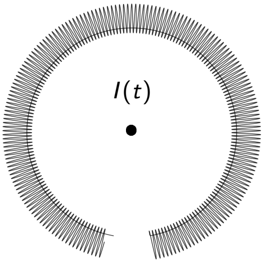


Figure 1: Rogowski winding

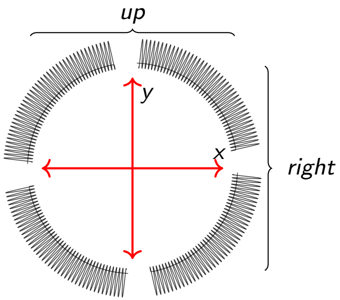


Figure 2: Rogowski BPM winding

Assembly

Figure 3: Core-winding combination

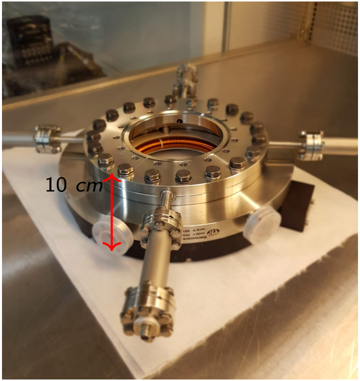
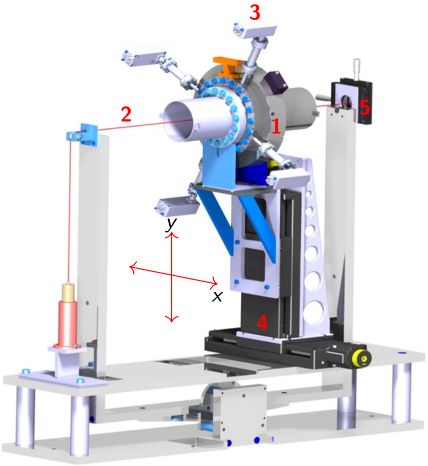


Figure 4: Rogowski BPM

Calibration measurement



- **1**: Rogowski BPM
- **2**: Copper wire (beam) $\vec{z}$
- **3**: Pre-amplifier
- **4**: Stepping drives
- **5**: Manual tables

- Apply AC signal (mimic beam)
- Scan both x and y planes
- Amplify output signal
- Measure integrated signal
- Construct calibration ratios

Theoretical investigations

Response for fully wound coils

For fully wound coils, measured voltage difference is related to beam positions through:

$$
\begin{aligned}
U_{out1}(x, y, \omega) = \ & \omega\gamma_1(\omega_0, \omega_1, \omega) n\mu_0 c_0 I \Bigg[\bigg(1 + c_1(x + y) + c_2(xy) \\
& + c_3(-x^3 - y^3 + 3yx^2 + 3xy^2) \\
& + c_5(x^5 + y^5 - 10x^3y^2 - 10y^3x^2 + 5y^4x + 5x^4y) + \ldots\bigg) + \ldots\Bigg]
\end{aligned}
\tag{1}
$$

x,y: beam transverse positions.
c_i: geometrical factors, units of mm^{-n}.

Theoretical investigations

Not fully wound coils

- Incomplete angular coverage
- Each coil has $\Delta\psi < \frac{\pi}{2}$

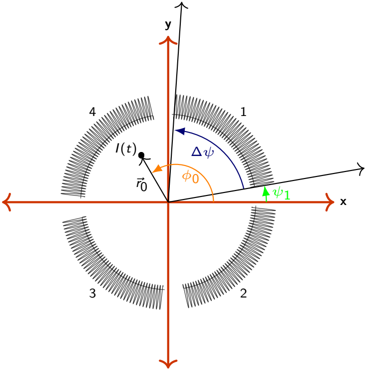


Figure 5: Rogowski BPM winding viewed in the xy plane with $\Delta\psi < \frac{\pi}{2}$.

Following the derivations:

- same polynomials (in terms of x,y)
- non-vanishing fourth degree
- different geometrical factors

Theoretical investigations

$$U_{out1}(x,y,\omega) = \omega\gamma_1(\omega_0,\omega_1,\omega)n\mu_0 c_0 I\Big[\Big(1 + c_1(x+y) + c_2(xy) + c_3(-x^3 - y^3 + 3yx^2 + 3xy^2) + c_5(x^5 + y^5 - 10x^3y^2 - 10y^3x^2 + 5y^4x + 5x^4y) + \dots\Big) + \dots\Big] \quad (2)$$

Not fully wound coils

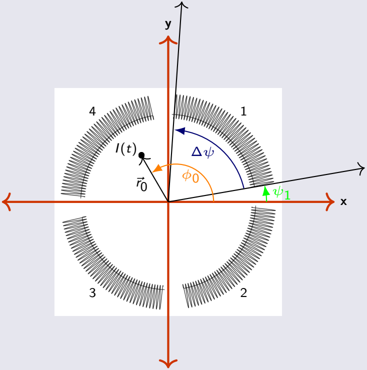
Voltage response:

$$U_{out1}(x,y,\omega) = \omega\gamma_1(\omega_0,\omega_1,\omega)n\mu_0 c_0^\dagger I\Big[\Big(1 + c_1^\dagger(x+y) + c_2^\dagger(xy) + c_3^\dagger(-x^3 - y^3 + 3yx^2 + 3xy^2) + c_4^\dagger(-x^4 - y^4 + 6x^2y^2) + c_5^\dagger(x^5 + y^5 - 10x^3y^2 - 10y^3x^2 + 5y^4x + 5x^4y) + \dots\Big) + \dots\Big] \quad (3)$$

Theoretical investigations

Not fully wound coils

Gemetrical parameters in terms of fully wound coils case:



- $c_0^\dagger = c_0$
- $c_1^\dagger = \frac{\pi(\cos(\psi_1) - \sin(\psi_1))}{2\Delta\psi} c_1$
- $c_2^\dagger = \frac{\pi(\cos^2(\psi_1) - \sin^2(\psi_1))}{2\Delta\psi} c_2$
- $c_3^\dagger = \frac{\pi(\cos(3\psi_1) + \sin(3\psi_1))}{2\Delta\psi} c_3$
- $c_4^\dagger = \frac{2\pi \sin(4\psi_1)}{2\Delta\psi} c_4$
- $c_5^\dagger = \frac{\pi(\cos(5\psi_1) - \sin(5\psi_1))}{2\Delta\psi} c_5$

Calibration example

Calibration example ($\Delta\psi < \frac{\pi}{2}$)

- To validate the results in previous three slides
- Rogowski BPM with winding range $< \frac{\pi}{2}$ was calibrated
- Sine wave with $f = 750$ kHz, and a square map of range(-10,10,1) mm

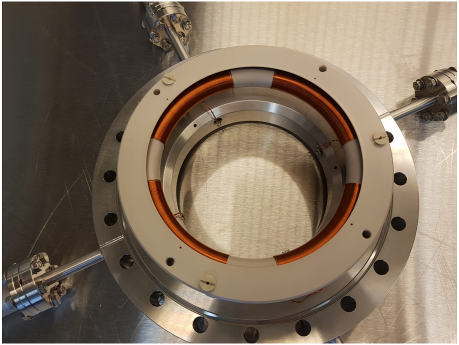


Figure 6: Rogowski BPM winding with incomplete angular range.

Calibration example

Calibration example

Two models were applied on data from calibration measurement:

- ⋆ the first with typical equations (no 4th order, see eq. 1)
- ⋆ the second with modified set of equations (with fourth order, see eq. 3)

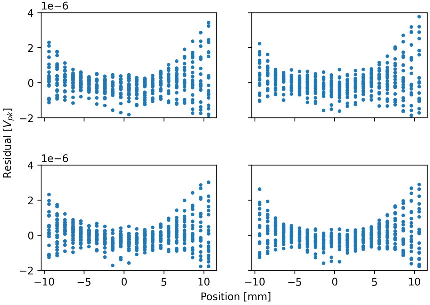


Figure 7: Model 1 (based on fully wound coils).

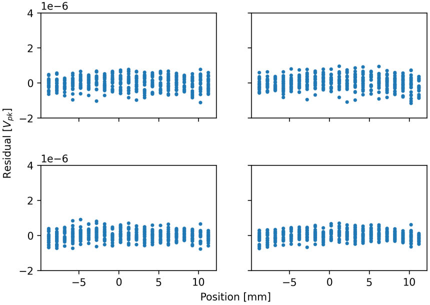


Figure 8: Model 2 (based on realistic used coils).

Calibration example

Values of geometrical parameters for the modified set of equations (model 2) from this calibration are listed below in table 1.

Parameter	Value
$c_1^{\dagger}$	$(1.8 \pm 0.000005) \times 10^{-2}$
$c_2^{\dagger}$	$(6.12 \pm 0.0001) \times 10^{-4}$
$c_3^{\dagger}$	$(2.44 \pm 0.0004) \times 10^{-6}$
$(2.32 \pm 0.004) \times 10^{-8}$	
$c_5^{\dagger}$	$(-9.38 \pm 0.26) \times 10^{-11}$

Table 1: Values of geometrical parameters.

Parameter	Value
c_1	$(1.63519 \pm 0.0000001) \times 10^{-2}$
c_2	$(4.62688 \pm 0.0001) \times 10^{-4}$
$c_3 \dagger$	$(1.27195 \pm 0.0006) \times 10^{-6}$
c_4 c_5	$(1.888 \pm 0.023) \times 10^{-10}$

Table 2: Values of geometrical parameters from another calibration of a Rogowski BPM with fully wound coils.

Clearly, these values as shown in table 1 if compared to results from calibration with fully wound coils are making an agreement with the results obtained from theoretical investigations considering $\psi_1 \approx 14 - 15\ ^{\circ}$ and $\Delta\psi \approx \frac{\pi}{3}$.

Coils orientation vs. sensitivity

- How orienting the winding can affect the sensitivity to BEAM positions
- Responses were derived for the case in figure 10
- Calibration functions were then calculated
- Comparison to default (figure 9)

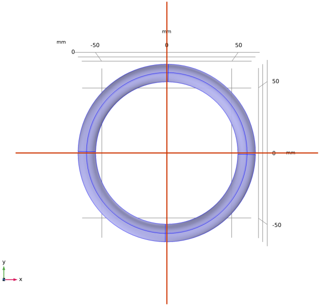


Figure 9: Winding orientation 1.

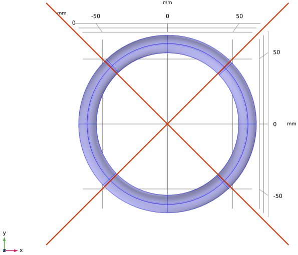


Figure 10: Winding orientation 2.

Theoretical investigations

Coils orientation vs. sensitivity

- How orienting the winding can affect the sensitivity to positions
- Responses were derived for the case in figure 10
- Calibration functions were then calculated
- Comparison to default (figure 9)

Horizontal ratio becomes:

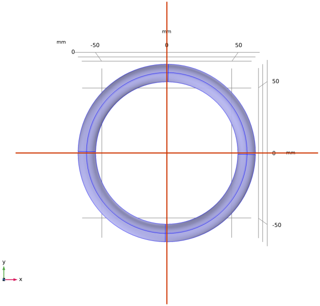
$$\frac{\Delta_x}{\Sigma} = \frac{1}{2}\left[c_1^{'}(x) + c_3^{'}(x^3 - 3xy^2) + c_5^{'}(-x^5 + 10x^3y^2 - 5y^4x)\right]. \tag{4}$$

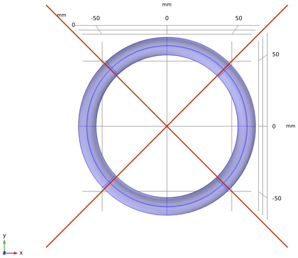
By comparing the parameters in both cases, and considering the linear approximation for the delta over sigma, the difference in sensitivity for both orientations is approximated to:

$$\frac{\frac{1}{2}c_1^{'}}{c_1} \approx 0.7\,. \tag{5}$$

The two orientations were investigated within COMSOL Multiphysics:

- 3d study (for each configuration)
- a central coil was used to mimic beam (harmonic excitation)
- frequency domain analysis (AC DC module/magnetic field interface)
- parametric sweep study step (change beam position)
- $y = 10$ mm, $x =$ range$(-10, 10, 1)$ mm





COMSOL simulations

Results from COMSOL Multiphysics study:

- Sensitivity is better in the case of orientation 1
- Agreement with theoretical expectations (ratio of slopes is ≈ 0.7)

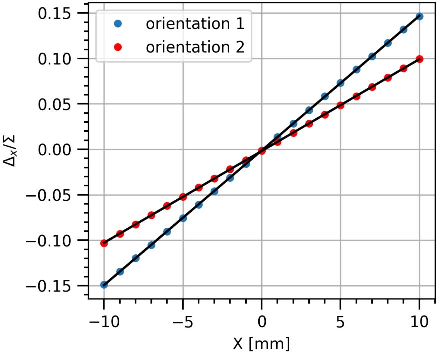


Figure 11: $\frac{\Delta_x}{\Sigma}$ against the central coil (beam) x positions for the two winding orientations shown in figures 9 and 10.

SNR

The signal to noise ratio from Rogowski BPM can vary depending on:

- operational frequency
- beam current amplitude
- electronics (filter bandwidth and other relevant settings)

But in general it can reach a value of few thousands.

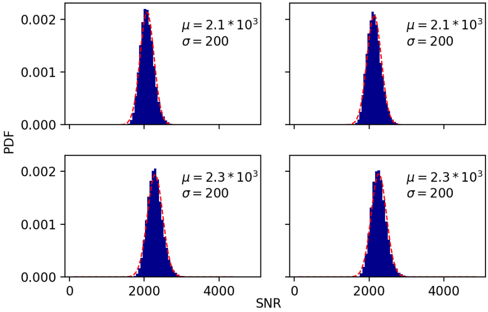


Figure 12: An example for the SNR from the four quadrants of Rogowski BPM.

Evaluations

Resolution

An estimate for the spatial resolution, neglecting the differences between individual coils (for simplicity) and assuming that beam position is some where around the center where linear position dependence is just sufficient:

$$\delta_x = \frac{1}{2c_1 \times SNR} . \tag{6}$$

A resolution of few µm for one single measured beam position (averaged over a second) is reachable.

Accuracy

An estimate for the accuracy considering errors from the stepping motors and errors introduced by temporal changes of electrical signal is about 20 µm.

Installation and use in COSY

Installation

Two Rogowski BPMs were successfully installed in COSY.

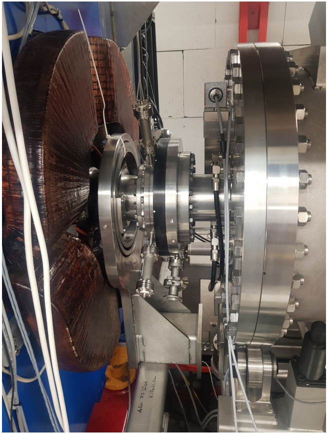


Figure 13: A photo for one Rogowski BPM installed in COSY.

Installation and use in COSY

Local orbit bump

- Fixed horizontal orbit
- apply vertical bumps (range$(5, -5, 1)$ mm)

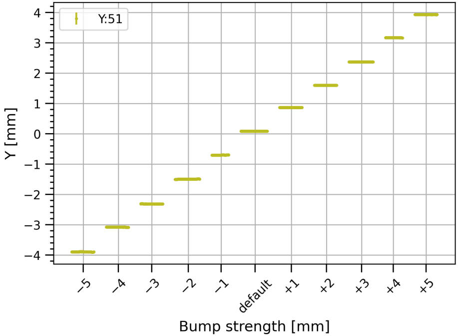


Figure 14: Vertical orbit measured by Rogowski BPM after applying a local orbit bump.

Outlook

Further improvements and developments need to be done in order to fulfill the requirements of EDM measurement:

- operation of Rogowski BPM for Clockwise-Counterclockwise Beams (CCB) has to be deeply investigated
- bunch size and longitudinal bunch shape vs. positions
- bunch-to-bunch positions (upgrade in existing electronics)

Summary

- New types of compact, non-destructive BPMs based on Rogowski coil have been built
- Agreement between some theoretical investigations and both measurement and COMSOL simulation
- Successful installation and operation in COSY
- Further improvements are needed to fulfill the requirements of EDM measurement

The End

Spare

For one single quadrant covering an angular range of $\Delta\psi$ that starts and ends at the angles ϕ_1 and $\phi_1 + \Delta\psi$, respectively, the induced magnetic flux is:

$$\begin{aligned}\Phi =& \mu_0 I(t)(R - \sqrt{R^2 - a^2}) \Big[1 + 2c_1 r_0 [\sin(\phi_1 - \phi_0 + \Delta\psi) - \sin(\phi_1 - \phi_0)] \\ &+ \frac{c_2 r_0^2}{2} [\sin(2(\phi_1 - \phi_0 + \Delta\psi)) - \sin(2(\phi_1 - \phi_0))] \\ &+ \frac{c_3 r_0^3}{3} [\sin(3(\phi_1 - \phi_0 + \Delta\psi)) - \sin(3(\phi_1 - \phi_0))] \\ &+ \frac{c_4 r_0^4}{4} [\sin(4(\phi_1 - \phi_0 + \Delta\psi)) - \sin(4(\phi_1 - \phi_0))] \\ &+ \frac{c_5 r_0^5}{5} [\sin(5(\phi_1 - \phi_0 + \Delta\psi)) - \sin(5(\phi_1 - \phi_0))] \\ &+ \mathcal{O}(r_0^6) + ... \Big]\end{aligned} \tag{7}$$

Coils orientation vs. sensitivity

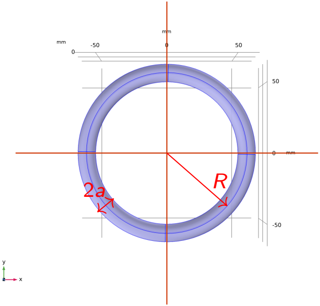


Table 3: The geometrical parameters for complete winding angular coverage

Parameter	Expression
c_0	$R - u$[a]
c_1	$\frac{2}{\pi u}$
c_2	$\frac{a^2}{2\pi u^3(R-u)}$
c_3	$\frac{Ra^2}{3\pi u^5(R-u)}$
c_4	$\frac{a(a^2+4R^2)}{4\pi u^7(R-u)}$
c_5	$\frac{a^2R(3a^2+4R^2)}{20\pi u^9(R-u)}$

[a] $u = \sqrt{R^2 - a^2}$.

Spare

Calibration example

For example, values of geometrical parameters for a Rogowski BPM with fully wound coils are listed below in table.

Parameter	Value
c_1	$(1.63519 \pm 0.0000001) \times 10^-2$
c_2	$(4.62688 \pm 0.0001) \times 10^-4$
c_3	$(1.27195 \pm 0.0006) \times 10^-6$
c_5	$(1.888 \pm 0.023) \times 10^-10$

Table 4: Geometrical parameters for modified set of equations

Clearly, these values if compared with those shown in table 1 are making an agreement with the results obtained from theoretical investigations for the case of incomplete winding range (see slide 9) considering $\psi_1 \approx 14 - 15\ ^\circ$ and $\Delta\psi \approx \frac{\pi}{3}$.

Spare

Coils orientation vs. sensitivity

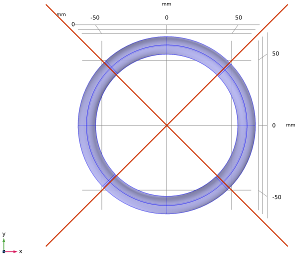


Figure 15: Core-winding combination

Table 5: The geometrical parameters for complete winding angular coverage in the second geometrical orientation.

Parameter	Expression
c_0'	$R - u$
c_1'	$\frac{2\sqrt{2}}{\pi u}$
c_2'	$\frac{a^2}{\pi u^3(R-u)}$
c_3'	$\frac{\sqrt{2}Ra^2}{3\pi u^5(R-u)}$
c_5'	$\frac{\sqrt{2}a^2R(3a^2+4R^2)}{20\pi u^9(R-u)}$

Spare

Calibration example

The values for the remaining model parameters obtained from calibration measurement are listed in the table below.

Parameter	Value	Parameter	Value
x_{off}	$(-0.62 \pm 0.0002)\ mm$	k_{32}	$(2.16 \pm 0.0014) \times 10^-2$
y_{off}	$(0.736 \pm 0.001)\ mm$	k_{34}	$(2.34 \pm 0.0012) \times 10^-2$
s_1	0.976 ± 0.00002	k_{41}	$(2.65 \pm 0.0012) \times 10^-2$
s_2	1.001 ± 0.00002	k_{42}	$(1.6 \pm 0.0007) \times 10^-2$
s_3	1.001 ± 0.00002	k_{43}	$(2.69 \pm 0.0011) \times 10^-2$
s_4	0.972 ± 0.00002	k_{31}	$(1.08 \pm 0.0006) \times 10^-2$
k_{12}	$(2.29 \pm 0.0012) \times 10^-2$	k_{13}	$(1.42 \pm 0.0008) \times 10^-2$
k_{14}	$(2.02 \pm 0.0015) \times 10^-2$	k_{21}	$(2.08 \pm 0.001) \times 10^-2$
k_{23}	$(2.32 \pm 0.0015) \times 10^-2$	k_{24}	$(8.77 \pm 0.0076) \times 10^-3$
θ	(-3.6 ± 0.001) mrad		

Table 6: Rest of model parameters for modified set of equations

Spare

For the default coil configurations as in figure 9, and neglecting coils differences, horizontal ratio becomes:

$$\frac{\Delta_x}{\Sigma} = \left[c_1(x) - c_3(x^3 - 3xy^2) + c_5(x^5 - 10x^3y^2 + 5y^4x)\right]. \tag{8}$$

While the vertical ratio becomes:

$$\frac{\Delta_y}{\Sigma} = \left[c_1(y) - c_3(y^3 - 3yx^2) + c_5(y^5 - 10y^3x^2 + 5x^4y)\right]. \tag{9}$$

Spare

Coils orientation vs. sensitivity

- How orienting the winding can affect the sensitivity to positions
- Responses were derived for the case in figure 10
- Calibration functions were then calculated
- Comparison to default (figure 9)

Horizontal ratio becomes:

$$\frac{\Delta_x}{\Sigma} = \frac{1}{2}\left[c_1^{'}(x) + c_3^{'}(x^3 - 3xy^2) + c_5^{'}(-x^5 + 10x^3y^2 - 5y^4x)\right]. \tag{10}$$

While the vertical ratio becomes:

$$\frac{\Delta_y}{\Sigma} = \frac{1}{2}\left[c_1^{'}(y) + c_3^{'}(y^3 - 3yx^2) + c_5^{'}(-y^5 + 10y^3x^2 - 5x^4y)\right]. \tag{11}$$

Spare

Resolution

The horizontal and vertical ratios (delta-over-sigma) are defined as:

$$\frac{\Delta_x}{\Sigma} = \frac{U_1 + U_2 - U_3 - U_4}{U_1 + U_2 + U_3 + U_4}\,, \tag{12}$$

$$\frac{\Delta_y}{\Sigma} = \frac{U_1 + U_4 - U_2 - U_3}{U_1 + U_2 + U_3 + U_4}\,. \tag{13}$$

Applying Gaussian error on the delta-over-sigma:

$$\delta_{\frac{\Delta_x}{\Sigma}} = \frac{2\sqrt{({\delta_{U_1}}^2 + {\delta_{U_2}}^2)(U_3 + U_4)^2 + ({\delta_{U_3}}^2 + {\delta_{U_4}}^2)(U_1 + U_2)^2}}{\Sigma^2}\,, \tag{14}$$

$$\delta_{\frac{\Delta_y}{\Sigma}} = \frac{2\sqrt{({\delta_{U_1}}^2 + {\delta_{U_4}}^2)(U_2 + U_3)^2 + ({\delta_{U_2}}^2 + {\delta_{U_3}}^2)(U_1 + U_4)^2}}{\Sigma^2}\,. \tag{15}$$

Spare

Resolution

From the previous relations, we arrive at:

$$\delta x = \frac{\delta_{\frac{\Delta_x}{\Sigma}}}{c_1}, \\ = \frac{1}{2c_1 \times SNR}. \tag{16}$$