Accelerator Physics Limitations on an EDM ring Design

Valeri Lebedev
Fermilab

R. Talman
annotated
for the Juelich
January 2018
EDM Task Force
meeting.

November 10-11, 2014
EDM collaboration meeting
=>
March 13, 2015
Julich, Forschungszentrum

Contents

- Bending and Focusing with Pure Electric Field
  Optics code: http://www-ap.fnal.gov/~ostiguy/OptiM/
- Linear and non-linear optics
- IBS and Coulomb tune shift
- Spin precession and its suppression by a feedback system
- Suppression of vertical magnetic field
- Limitations and requirements for radial magnetic field

This presentation is aimed to discuss issues related accelerator physics

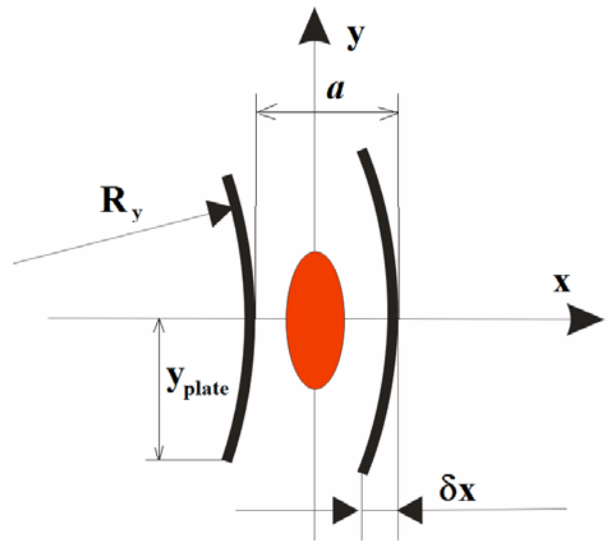
It is not aimed to make a credible suggestion for a ring

R Talman agrees with transverse dynamics, but not longitudinal dynamics.
This is because the formalism is valid only OUTSIDE electric bends. But particles spend most of their time INSIDE electric bends.

Bending and Focusing with Pure Electric Field

- Electric field in electrostatic bend
  - Non linearities are present at fundamental level

$$\varphi(r,z) = A_1 \ln\left(\frac{r}{R}\right) + \sum_{n=2}^{\infty} A_n \left(y^n - \frac{n(n-1)}{4} y^{n-2} r^2 \right)$$



  - If required E_y can be made linear:

$$\Phi(x,y) = E_0 R_0 \left(\left(1+\frac{m}{2}\right) \ln\left(1+\frac{x}{R_0}\right) - \frac{m}{4}\left(\left(1+\frac{x}{R_0}\right)^2 - 1\right) + \frac{my^2}{2R_0{}^2} \right)$$

$$\xrightarrow{x \ll R_0} E_0 R_0 \left(\frac{x}{R_0} - (1+m)\frac{x^2}{2R_0{}^2} + m\frac{y^2}{2R_0{}^2} + (2+m)\frac{x^3}{6R_0{}^3} + O\left(x^4\right) \right)$$

  - Non-linear contribution to E_x is $\Delta E/E_0 \approx (x/R_0)^2 \approx 10^{-7}$ ($\Delta G/G \approx (x/R_0) \approx 3\cdot 10^{-4}$)

    It is well below expected manufacturing accuracy agreed

- Electric field in electrostatic quadruple

$$\varphi(\rho,\theta) = C_2 r^2 \cos(2\theta) + C_6 r^6 \cos(6\theta) + \ldots \quad \Rightarrow \quad \varphi(x,y) = G_0 \frac{x^2 - y^2}{2} + \ldots$$

  - Motion non-linearity comes from kin. en. change ($\Delta F/F \approx 2\cdot 100$ keV/230 MeV$\approx 10^{-3}$)

$$\frac{d\mathbf{p}}{ds} = -\frac{e}{\mathrm{v}}\nabla\varphi \approx -\frac{e}{\mathrm{v}_0}\left(1 - \frac{\delta\mathrm{v}}{\mathrm{v}_0}\right)\nabla\varphi \approx -\frac{e}{\mathrm{v}_0}\left(1 + \frac{e\varphi}{mc^2\beta^2\gamma^3}\right)\nabla\varphi = -\frac{e}{\mathrm{v}_0}\left(\nabla\varphi + \frac{e}{2mc^2\beta^2\gamma^3}\nabla\left(\varphi^2\right)\right)$$

    - It cannot be compensated by electrode geometry adjustments: $\Delta(\phi^2) \neq 0$

same order as
energy spread

Linear Optics

- Methods of optics analyses developed for magnet-based beam optics are directly applicable to the optics based on electrostatics
  - Difference comes from kinetic energy change in electric field

$$K = E_0 - e\phi$$

  - Analysis is based on transfer matrix
    - kinetic energy is the same in drifts => K changes are irrelevant
- Transfer matrix of electric bend (see H. Wollnik, "Optics of charged particles") This is the basis for most electrostatic simulations Page 124

$$M = \begin{bmatrix} c_x & s_x & 0 & 0 & 0 & d_x N_t \\ -k_x^2 s_x & c_x & 0 & 0 & 0 & s_x N_t / R_0 \\ 0 & 0 & c_y & s_y & 0 & 0 \\ 0 & 0 & -k_y^2 s_y & c_y & 0 & 0 \\ -s_x N_t / R_0 & -d_x N_t & 0 & 0 & 1 & -N_t^2 t_d \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad V = \begin{bmatrix} x \\ \theta_x \\ y \\ \theta_y \\ s \\ \Delta p / p \end{bmatrix}$$

$$k_x = \frac{1}{R_0}\sqrt{1 - m + \frac{1}{\gamma^2}} \qquad k_y = \frac{1}{R_0}\sqrt{m} \qquad N_t = 1 + \frac{1}{\gamma^2}$$

$$c_x = \cos(k_x L) \qquad s_x = \sin(k_x L) \qquad d_x = \frac{1 - c_x}{R_0 k_x^2}$$

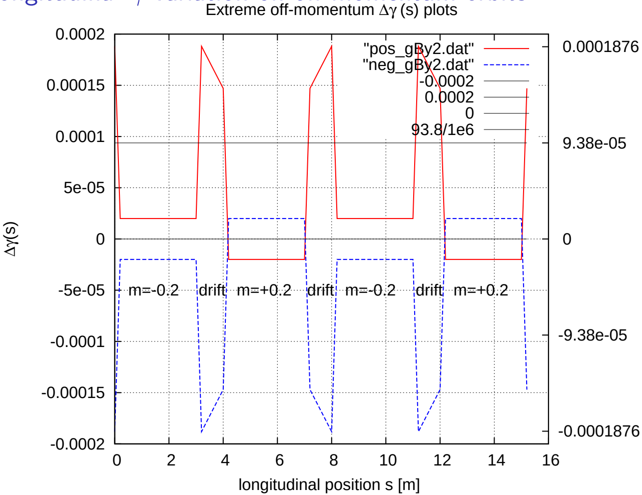
$$c_y = \cos(k_y L) \qquad s_y = \sin(k_y L) \qquad t_d = \frac{k_x L - s_x}{R_0^2 k_x^3}$$

!!! In this definition M_{56} accounts for the effective orbit lengthening

- includes both orbit lengthening and velocity change due to radial displacement in the bend
- Does not include the effect of velocity change due to momentum change outside bend - standard acc. phys. definition.

This is NOT VALID for near-cylindrical electrodes |m|<<1 longitudinal dynamics

22 Longitudinal γ variation on off-momentum orbits



Behavior of strong focsing lattices will be similar

This alters longitudinal dynamics

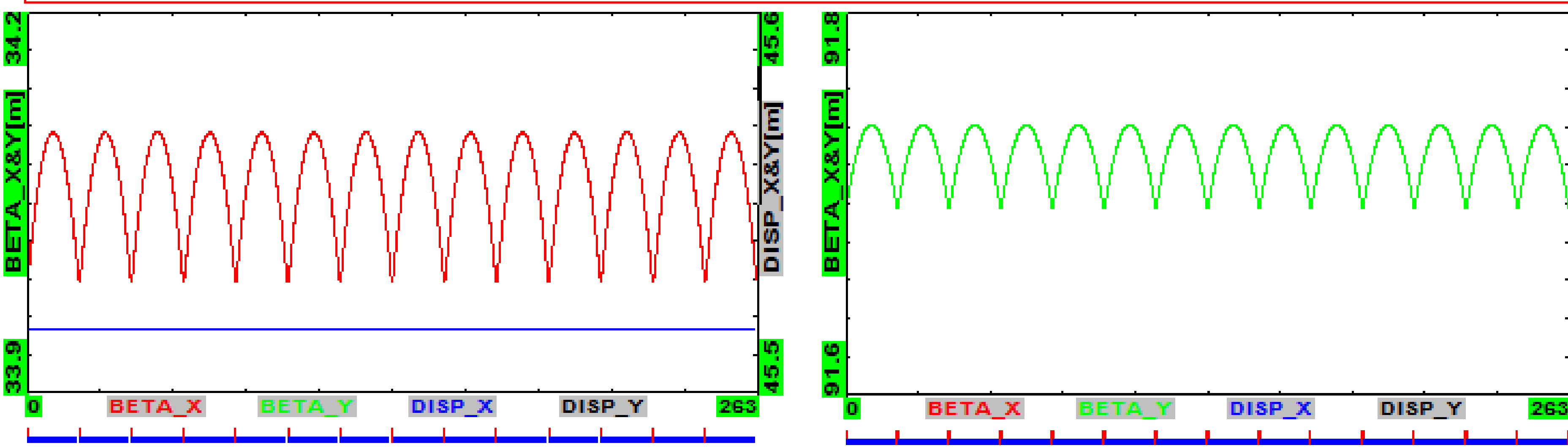
because particles are mainly "inside" not "ouside"

Figure: Dependence of deviation from "magic" $\Delta\gamma(s) = \gamma(s) - \gamma_0$ on longitudinal position s, for off-momentum closed orbits (circular arcs within bends) just touching inner or outer electrodes at $x = \pm 0.015$ m. *Notice the anomalous cross-overs in $m > 0$ bends.*

Weak Focusing Ring

- Major part of focusing comes from bends, m≈0.2
- Ring structure
  - 14 periods:
    - each include electric bending with R_0=40 m and 0.834 m drift
      - o Gap between plates 3 cm, Voltage ±157 kV
- Small variations of β-functions excited by drifts, constant dispersion
  - Large beta-functions => Ring is extremely sensitive to focusing errors
  - Trim quads are required for final tuning and optics correction
    - GdL≈2 kV/cm^2 for tune correction

agreed

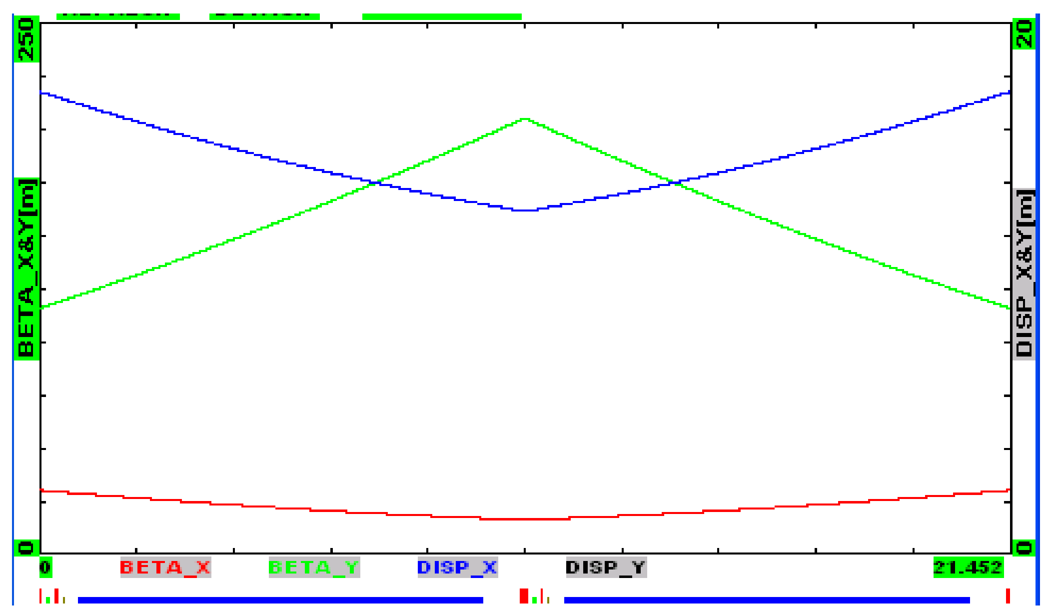


”Strong” Focusing Ring

Focusing dominated by quads

- defeats self-magnetometry
- limits CW/CCW sensitivity

- Ring structure
  - Same 14 periods. Each includes:
    - 2 electric bends with R_0=40 m and L=8.97 m
      - o Gap between plates 3 cm, V= ±157 kV,
      - o m = 0 (no vert. focusing)
    - 2 electric quads, one F and one D
      - o L=15 cm, G_F = 17.2 kV/cm^2, G_D = -13.8 kV/cm^2
      - o Each quad can be independently adjusted for optics correct.
  - One of two 80 cm gaps between quads and bends are filled with
    - H or V corrector, skew-quad corrector, F or D sextupole, and BPM
    - Other can be used by experiment, + RF cavity



- Circumference – 300 m
- Kinetic energy - 232.79 MeV

Weak Focusing versus "Strong" Focusing

	Weak foc.	Strong foc.
Kinetic energy [MeV]	232.79	
Number of periods	14	14
Circumference [m]	263	300
Focusing parameter in bends, m	0.199	0
Tunes, Q_x / Q_y	1.229 / 0.456	2.32 / 0.31
Maximum beta-function, β_x/β_y [m]	34 / 91.7	29.1 / 204
Dispersion	45.5	17.35
Maximum momentum deviation: $\Delta p/p\|_{max}$	$\pm 3.3\cdot 10^{-4}$ ♦	$\pm 8.6\cdot 10^{-4}$ ♦
Rms momentum spread	$1.1\cdot 10^{-4}$ ♥	$2.9\cdot 10^{-4}$ ♥
Hor. norm. acceptance [mm mrad]	5 ♦	5.8 ♦
Hor. /vert. norm. emittance [mm mrad]	0.56♥/1.52	0.31♥/2.2♥
Revolution frequency [kHz]	682.1	597.3
Momentum compaction, α	1.785	0.51
Slip-factor: $\eta = \alpha - 1/\gamma^2$	1.144	-0.132
Transition energy ($\gamma_{tr} = 1/\sqrt{\alpha}$), [MeV]	N/A *	376

♦ Limited by distance between bending plates ($2a=3$ cm)

* Operation above transition because α>1

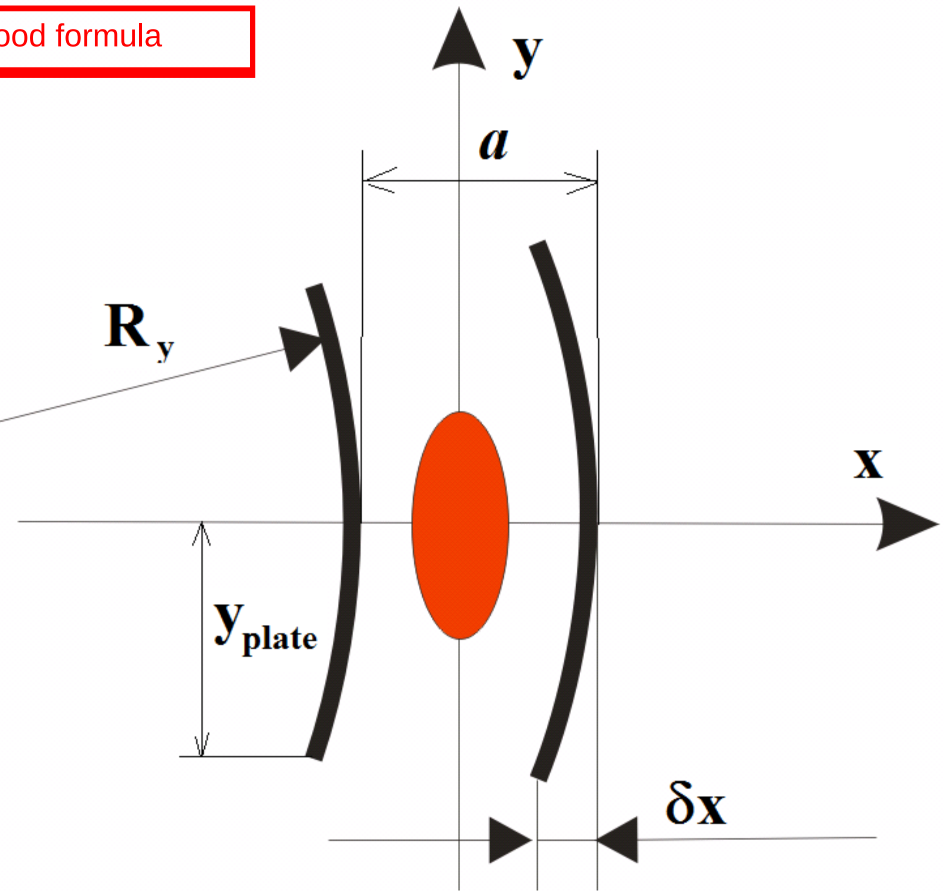
♥ Set by IBS

above transition

I believe this is incorrect

Requirements to Manufacturing and Installation of Bending Plates

- The plate bending radius in the vertical plane is $R_y = R_0 / m = 201$ m
  good formula
  - For vertical displacement y_{plate}=10 cm it yields δx of 25 μm
    - Looks close to impossible to manufacture and install with required accuracy ??‘



- Non-parallel plates (non-concentric plate surfaces) create skew-quad field with gradient $G_s = \theta E_0 / a$, where θ is the angle between plates
  - Requirement to have this skew gradient much smaller than the gradient of vertical focusing field, $\theta E_0 / a \ll m E_0 / R_0$, yields: $\theta \ll 1.5 \cdot 10^{-4}$
    - with margin of 100 (skew quads are still required) one obtains very tight requirement: $\theta < 1.5 \cdot 10^{-6}$
- Looks like that the required mechanical and installation accuracies are too tight => "Soft-focusing -> Normal quad focusing machine!!!"

Space Charge Tune Shifts

REALLY HANDY FORMULAS

- Space charge tune shift is weakly affected by ring optics

$$\Delta Q \approx \frac{r_p}{(2\pi)^{3/2}\beta^2\gamma^3}\left(\frac{C}{\sigma_s}\right)\frac{N_p}{2\varepsilon}$$

	Weak foc.	Strong foc.
Protons per bunch: N_p	$1.5\cdot 10^8$	$7\cdot 10^8$
Beam current, [mA]	1.1	4.7
$\Delta Q_x / \Delta Q_y$, $[10^{-3}]$	4.7/6.6	15/27

  - ♦ Exact formulas were used

$$\Delta Q_x = \frac{r_p N_p C}{(2\pi)^{3/2}\beta^2\gamma^3\sigma_s}\left\langle \frac{\beta_x}{(\sigma_x+\sigma_y)\sigma_x}\right\rangle_s$$

$$\Delta Q_y = \frac{r_p N_p C}{(2\pi)^{3/2}\beta^2\gamma^3\sigma_s}\left\langle \frac{\beta_y}{(\sigma_x+\sigma_y)\sigma_y}\right\rangle_s$$

Not good

- Beam emittances and momentum spreads are set by aperture (gap)
- Requirement to have sufficiently small IBS sets the beam current for soft-focusing ring
  - ♦ Tune shifts due to space charge are the main beam current limitation for strong focusing ring

and also weak and weak/weaker

- The tune shift due to counter rotating beam, $\sqrt{2\pi}C / N_b\sigma_s \approx 5.5$, is smaller and does not represent a problem

agreed

IBS

This is another justification for buildin prototype low energy ring

- IBS is the major source of emittance growth and, consequently, the major source of particle loss

agreed

- Dependence of potential energy on radial position in the electrostatic bend yields an additional term to the dependence of average particle momentum on radius
  - However it does not change local velocity spreads => "standard" IBS theory is applicable

I believe this is incorrect "inside"

- For the case when derivatives of dispersions and beta-functions can be neglected the growth rates for Gaussian beam are expressed by comparatively simple formula

$$\frac{d}{dt}\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \sigma_p{}^2\end{bmatrix}=\frac{r_p c N_p L_c}{4\sqrt{2}\beta^3\gamma^5\sigma_x\sigma_y\sigma_s\sqrt{\theta_x{}^2+\theta_y{}^2+\theta_p{}^2}}\begin{bmatrix}\left\langle \beta_x\psi\left(\theta_x,\theta_y,\theta_p\right)+\gamma^2\frac{D_x{}^2}{\beta_x}\psi\left(\theta_p,\theta_x,\theta_y\right)\right\rangle_s\\ \left\langle \beta_y\psi\left(\theta_y,\theta_p,\theta_x\right)\right\rangle_s\\ \left\langle \gamma^2\psi\left(\theta_p,\theta_x,\theta_y\right)\right\rangle_s\end{bmatrix},\quad \begin{array}{l}\sigma_x=\sqrt{\varepsilon_x\beta_x+\left(\sigma_p D_x\right)^2},\quad \sigma_x=\sqrt{\varepsilon_y\beta_y},\\ \theta_x=\sqrt{\varepsilon_x/\beta_x},\quad \theta_x=\sqrt{\varepsilon_y/\beta_y},\quad \theta_p=\sigma_p\sqrt{\varepsilon_x\beta_x}/\left(\gamma\sigma_x\right),\\ L_c=\ln\left(\sqrt{\sigma_x\sigma_y}/r_{\min}\right),\quad r_{\min}=2r_p/\left(\beta^2\gamma^2\left(\theta_x{}^2+\theta_y{}^2+\theta_p{}^2\right)\right),\end{array}$$

$$\Psi\left(x,y,z\right)=\frac{\sqrt{2}\sqrt{x^2+y^2+z^2}}{3\pi}\left(y^2\,\mathrm{R_D}\left(z^2,x^2,y^2\right)+z^2\,\mathrm{R_D}\left(x^2,y^2,z^2\right)-2x^2\,\mathrm{R_D}\left(y^2,z^2,x^2\right)\right),\mathrm{R_D}\left(x,y,z\right)=\frac{3}{2}\int_0^\infty\frac{dt}{\sqrt{\left(t+x\right)\left(t+y\right)\left(t+z\right)^3}}$$

- Growth rates look significantly more complicated in the general case
  - Exact formulas are used in below estimates

The simplified formulas are especially valid in the weak/weaker case

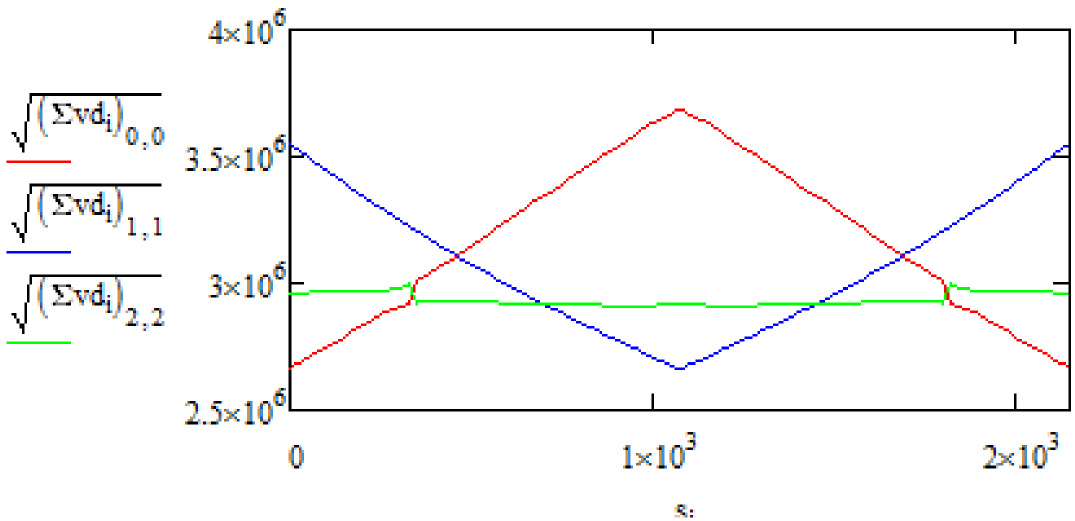
IBS for the Weak and Strong Focusing Rings

I believe that both rings are actually above transition

- Weak focusing ring
  - Operates above transition
    - It is impossible to get to a quasi-equilibrium between temperatures of planes
      $\Rightarrow$ Fast emit. growth
  - Emittance growth leads to particle loss which determines the experiment time scale

	Weak foc.	Strong foc.
Protons per bunch	$1.5 \cdot 10^8$	$7 \cdot 10^8$
$\Delta Q_x / \Delta Q_y$, $[10^{-3}]$	4.7/6.6	15/27
$\tau_x = \varepsilon_x / (d\varepsilon_x/dt)$ [s]	305	7500
$\tau_y = \varepsilon_y / (d\varepsilon_y/dt)$ [s]	-1400	7500
$\tau_s = \varepsilon_s / (d\varepsilon_s/dt)$ [s]	250	7500

  - 1000 s measurement
    => IBS growth rates to be greater or about 300 s (see below)
    => Protons per bunch



- Strong focusing ring
  - IBS is suppressed in a quasi-equilibrium state
  - Beam space charge is the major limitation

IBS Growth Rates and Beam Lifetime (continue)

- Taking into account that particle scraping happens due to both betatron and synchrotron motion we obtain the boundary condition

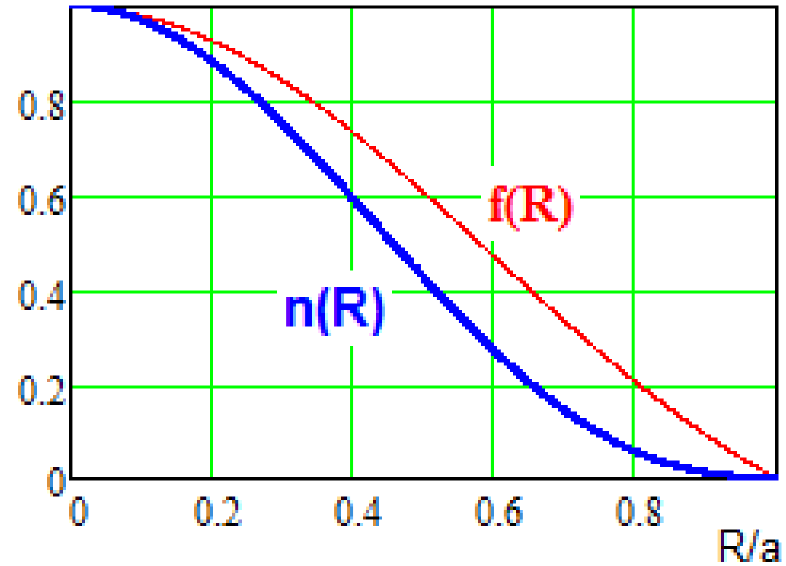
Nice description

$$f\left(t,R\right)_{R=a}=0,\quad R=\sqrt{x^2+\Theta_x{}^2+s^2+\Theta_s{}^2}$$

- Looking for spherically symmetric solution with exponential decay in time we obtain ordinary differential equation for the distribution function

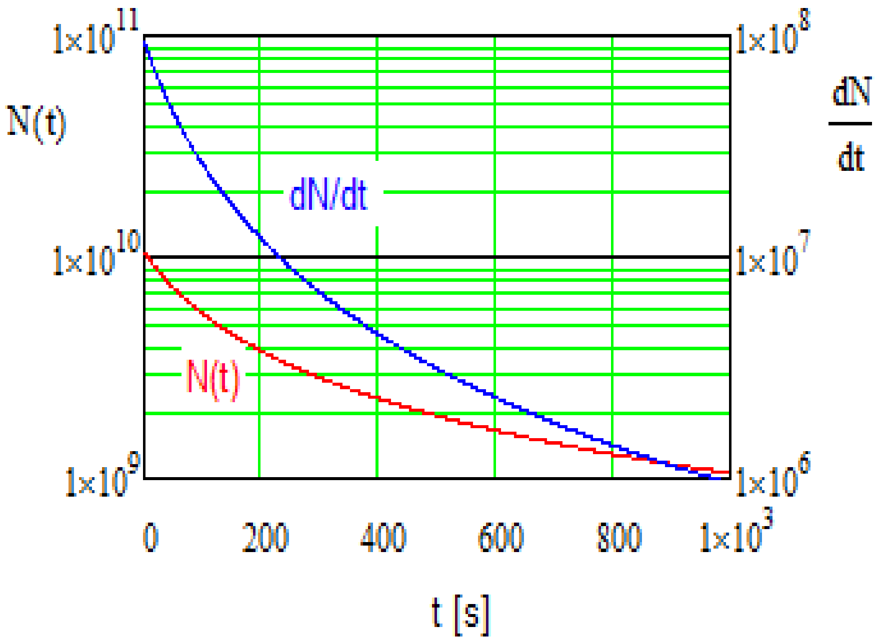
$$-\lambda f=D\frac{1}{R^3}\frac{d}{dR}\left(R^3\frac{df}{dR}\right),\quad f\left(\mathrm{a}\right)=0$$

Solution is $f(R)=2J_1\left(\mu_{11}R/a\right)/\left(R/a\right)$



- Integrating it over s, Θ_x, Θ_s we obtain particle distribution, n(x)
- Taking into account that diffusion depends on N_p we obtain a dependence of particle population on time

$$N(t)=\frac{N_p}{1+\lambda_D t},\quad \lambda_D\approx\frac{\mu_0{}^2}{n_\sigma\left(\tau_x+\tau_s\right)},\quad \begin{array}{c}\mu_{11}\approx 3.832\\ n_\sigma\approx 2.96,\quad \tau_x\approx\tau_s\end{array}$$



Vertical Magnetic Field Compensation

- Magnetic field does not change as fast as electric field and a suppression its fluctuations with feedback system is easier ??‘
- Vertical component of magnetic field
  - To nullify $\langle B_y \rangle$ we can use the second spin feedback which measures the horizontal spin precession of the counter rotating beam and corrects it with vertical magnetic field excited by coils
    - only average magnetic field needs to be compensated
      $\Rightarrow$ 0.1 rad after 1000 s corresponds ΔB_y=0.058 μG
  - Keeping the spin horizontal components equal to zero for both beams keeps us at the magic energy if $dB_y/dx = 0$.
    - Otherwise the beam separation introduced by magnetic field makes average magnetic field different for both beams => slightly different energies

Two single loops above and below the beam carrying mA currents would be sufficient

Effect of Radial Magnetic Field

- First assume that B_x does not change along circumference. Then, β has only longitudinal component, and the spin precession into vertical plane is:
$$\frac{d\mathbf{s}_y}{dt} = \frac{e}{m_p c}\left(\left(\frac{g_p}{2} - \frac{\gamma - 1}{\gamma}\right) B_x s_z + \left(\frac{g_p}{2} - \frac{\gamma}{\gamma + 1}\right) \beta E_y s_z\right)$$
- Equation of motion bounds electrical and magnetic fields
  - That yields
$$\frac{dp_y}{dt} = e\left(E_y + \beta B_x\right) = 0 \quad \Rightarrow \quad E_y = -\beta B_x$$
$$\frac{d\mathbf{s}_y}{dt} = \frac{e}{m_p c}\left(\left(\frac{g_p}{2} - \frac{\gamma - 1}{\gamma}\right) - \left(\frac{g_p}{2} - \frac{\gamma}{\gamma + 1}\right)\beta^2\right) B_x s_z \xrightarrow{\text{At magic energy}} \approx \frac{e}{m_p c}\left(2.59 - 0.8\right) B_x s_z$$
    - Thus, electric field which comes from the vertical focusing reduces the spin precession by about 30%
    - Effect has the same sign for both counter-rotating beams. It mimics EDM
- The half-separation of beams comes from: $mE_0 \Delta y / R_0 = \beta B_x$
  - It is directly related to the spin precession to the vertical plan
$$\frac{\Omega_{sy}}{\Omega_0} \equiv \frac{1}{\Omega_0}\frac{ds_y}{s_z dt} = m\gamma\left(\left(\frac{g_p}{2} - \frac{\gamma - 1}{\gamma}\right) - \left(\frac{g_p}{2} - \frac{\gamma}{\gamma + 1}\right)\beta^2\right)\frac{\Delta y}{R_0} \xrightarrow{\text{At magic energy}} \approx 2.24 m \frac{\Delta y}{R_0}$$

⇨ $5 \cdot 10^{-6}$ rad after 1000 s => $B_x \approx 0.29$ pG and $\Delta y \approx 0.1$ pm

See also analysis by Christian Carli

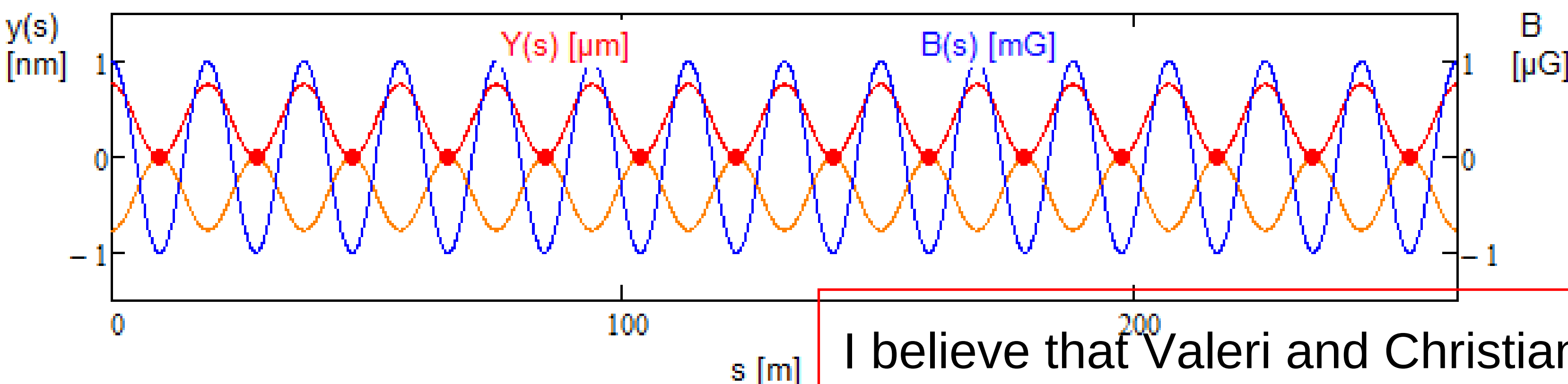
Limitations on Radial Magnetic Field Cancellation

- Typical values of residual magnetic fields
  - in SC cavities ~10 mG
  - in high precision experiments with active suppression in small volume ~ 1 μG
- In our estimate we assume that
  - the magnetic field is suppressed to 1 μG
  - we have 14 infinite accuracy BPMs
  - after correction of differential orbit for counter rotating beams we assume the magnetic field being distributed in worst possible scenario

$$B_x(s) = \frac{m_p c^2 \gamma}{e}\left(\left(\kappa^2 - k^2\right)\cos(\mathrm{ks}) + \kappa^2\right)A_y \,, \quad k = \frac{2\pi}{C} N_{BPM} \,, \quad \kappa = \frac{2\pi}{C} Q_y \,, \quad N_{BPM} = 14$$

which corresponds to the beam displacement of $y(s) = \left(1 + \cos(\mathrm{ks})\right) A_y$

- B_{max}=1 μG yields A_y=380 pm and the average $\overline{B_x} = \Delta B_{\max}\left(Q_y / N_{BPM}\right)^2$ =1.06 nG



I believe that Valeri and Christian (and me) are in agreement

Limitations on Radial Magnetic Field Cancellation

- The above estimate results in that it does not make much sense to have
  - BPM accuracy better than ~50 pm and that
  - the best expectations for average magnetic field cancellation is about 1 nG if 1 μG is achieved in non-beam measurements

I disagree

- This is 4 orders of magnitude worse than the desired values
  - Note that 1 μG field used in the estimate looks as extremely optimistic requirement
  - Note that this residual field is not determined by random fluctuations and therefore cannot be averaged out with more measurements

THIS IS WHY WEAK/WEAKER FOCUSING and ULTRA-SENSITIVE SELF-MAGNETOMETRY WIL EVENTUALLY BE REQUIRED

Sources of Magnetic Field Gradient

- Eddy magnetic field in the cavity is the major source of magnetic field gradient and, consequently, magnetic field
  - Beam crosses the cavity at zero voltage

⇨ $\int GdL\Big|_{cavity} \approx \omega_{RF} V_{RF} / 2c \approx 0.22\ G$ for V_{RF}=13 kV

  - For the differential precession rate <5 nrad/s we obtain the beam offset from the cavity center < **0.35 nm**
    - Typical microseism > 1 μm @ ~1 s

      ⇒ we need 3000 times suppression
- Main limitations on beam position measurements
  - Shot noise after 1 sec averaging ~ 60 pm @ full intensity
  - Thermal noise with 5 kΩ coupling impedance and room temperature amplifier ~ 50 pm in 10 Hz band @ full intensity
- These accuracy limitations correspond to expectations from the previous slide
- Can we achieve such accuracy?
  - Systematic errors are expected to be the main problem

Conclusions

- Overall concept of proton EDM electrostatic machine is not limited by the considered beam-physics issues
- Judged on pure acceleration physics grounds the strong focusing ring looks better than the soft focusing ring
  - Larger momentum acceptance and particle number
  - Suppressed IBS rates

	Soft focusing	Strong focusing
Circumference, m	263	300
Qx/Qy	1.229/0.456	2.32/0.31
Particle per bunch	$1.5\cdot10^8$	$7\cdot10^8$
Coulomb tune shifts, $\Delta Q_x/\Delta Q_y$	0.0046/0.0066	0.0146/0.0265
Rms emittances, x/y, norm, μm	0.56/1.52	0.31/2.16
Rms momentum spread	$1.1\cdot10^{-4}$	$2.9\cdot10^{-4}$
IBS growth times, x/y/s, s	300/(-1400)/250	7500
RF voltage	13	10.3
Synchrotron tune	0.02	0.006

- Analysis of spin decoherence for both rings is required to see their potential for EDM
  - In particular, the sensitivity of spin decoherence to sextupoles

I doubt the need for sextupoles

- Small vertical tune requires exceptionally high mechanical accuracies of bending plates manufacturing
  - Corrections are required for any ring
  - At minimum a standard set: dipole correctors, trim quads, skew-quads and sextupoles.
  - Note that all soft-focusing machines which were built had much larger ratio of the gap to radius
    - It greatly alleviates problems
- 2 feedback systems are required to cancel the vertical magnetic field and keep the spin aligned along velocity
- It is not feasible how the average radial magnetic field can be suppressed below 1 nG
  - It is already unprecedented level of magnetic field suppression for such large vacuum chamber
  - Looks like we are above the desired value by about 4 order of magnitude

Possibly true but probably not necessay

500 m Electric Ring: IBS and Beam Parameters

Valeri Lebedev
Fermilab

I BELIEVE THIS WAS THE SOURCI
OF THE LATTICE USED BY YANNI
IN THE RSI 87, 115116(2016)
PUBLICATION

December 24, 2014

NICE DISTILLATION OF IMPORTANT ISSUE
BUT I DISAGREE WITH SOME ASPEC

Optics

Main parameters

Beam energy	232.792 MeV
Circumference	500 m
Qx/Qy	2.42/0.44
Number of super-periods	4
FODO sells per super period	6
FODO sell length	20.83333 m
Number of arcs	4
Sells per arc	5
Number of straights	4
Sells per straight	1
Bends per half cell	3
Bending radius	52.3089
Gap	3 cm
Bending voltage	±120 kV
Slip-factor, $\eta=\alpha-1/\gamma^2$	-0.192

Can the vertical tune be coaxed down close to zero. I don't thinkk so.

I believe this is incorrect

Structure of FODO half-cell in arc (other half is mirror symmetric)

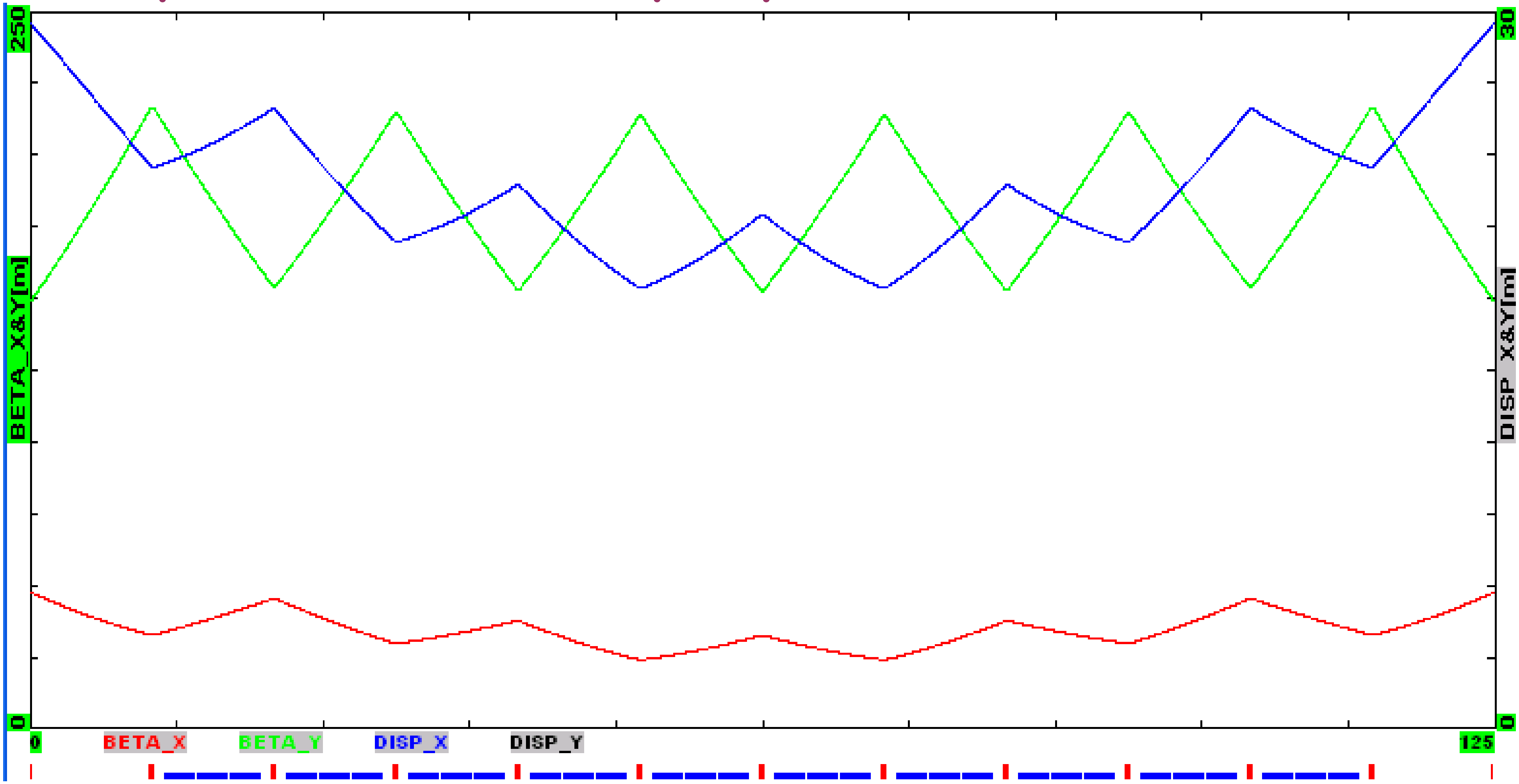
N	Name	S[cm]	L[cm]		
1	LqDh1	1061.67	20		Ge[kV/cm**2]=-3.3918
2	oQ	1141.67	80		
3	Rbend	1415.56	273.889	E[kV/cm]=80.16	Ge[kV/cm**2]=0
4	ob	1425.56	10		
5	Rbend	1699.44	273.889	E[kV/cm]=80.16	Ge[kV/cm**2]=0
6	ob	1709.44	10		
7	Rbend	1983.33	273.889	E[kV/cm]=80.16	Ge[kV/cm**2]=0
8	oQ	2063.33	80		
9	LqF	2083.33	40		Ge[kV/cm**2]=3.7306
10	oQ	2183.33	80		
11	Rbend	2457.22	273.889	E[kV/cm]=80.16	Ge[kV/cm**2]=0
12	ob	2467.22	10		
13	Rbend	2741.11	273.889	E[kV/cm]=80.16	Ge[kV/cm**2]=0
14	ob	2751.11	10		
15	Rbend	3025	273.889	E[kV/cm]=80.16	Ge[kV/cm**2]=0
16	oQ	3105	80		
17	LqDh	3125	20		Ge[kV/cm**2]=-3.2068

Optical structure of in straight lines

1	Rbend	273.889	273.889	E[kV/cm]=80.16	Ge[kV/cm**2]=0
2	oQ	353.889	80		
3	LqD1	393.889	40		Ge[kV/cm**2]=-3.3918
4	oLong	1395.56	1001.67		
5	LqF1	1435.56	40		Ge[kV/cm**2]=4.1756
8	oLong	2437.22	1001.67		
6	LqD1	2477.22	40		Ge[kV/cm**2]=-3.3918
7	oQ	2557.22	80		
8	Rbend	2831.11	273.889	E[kV/cm]=80.16	Ge[kV/cm**2]=0

Optics (3)

Twiss parameters for 1 super-period (out of four)



β_{xmax}=47 m, β_{ymax}=216 m, D_{xmax}=29.5 m

Optics (4)

- Ring structure
  - 4 super periods. Each includes:
    - 5 FODO cells with 3 bends per half cell
      - o electric bends with R_0=52.3 m and L=2.7389 m
        - Gap between plates 3 cm, V= ±120 kV,
        - m = 0 (no vert. focusing)
    - 1 FODO cell of the same length but without bends (~22 m straight line)
  - One of two 80 cm gaps between quads and bends are filled with
    - H or V corrector, skew-quad corrector, F or D sextupole, and BPM
    - Other can be used by experiment
  - RF cavity, injection kicker and septum are located in the straights
- Large flexibility in adjustments of beam optics
- For chosen optics its major parameters are:
  - β_{xmax}=47 m, β_{ymax}=216 m, D_{xmax}=29.5 m
- Weak vertical focusing was chosen for control of radial magnetic field
  - It results in high sensitivity to focusing errors

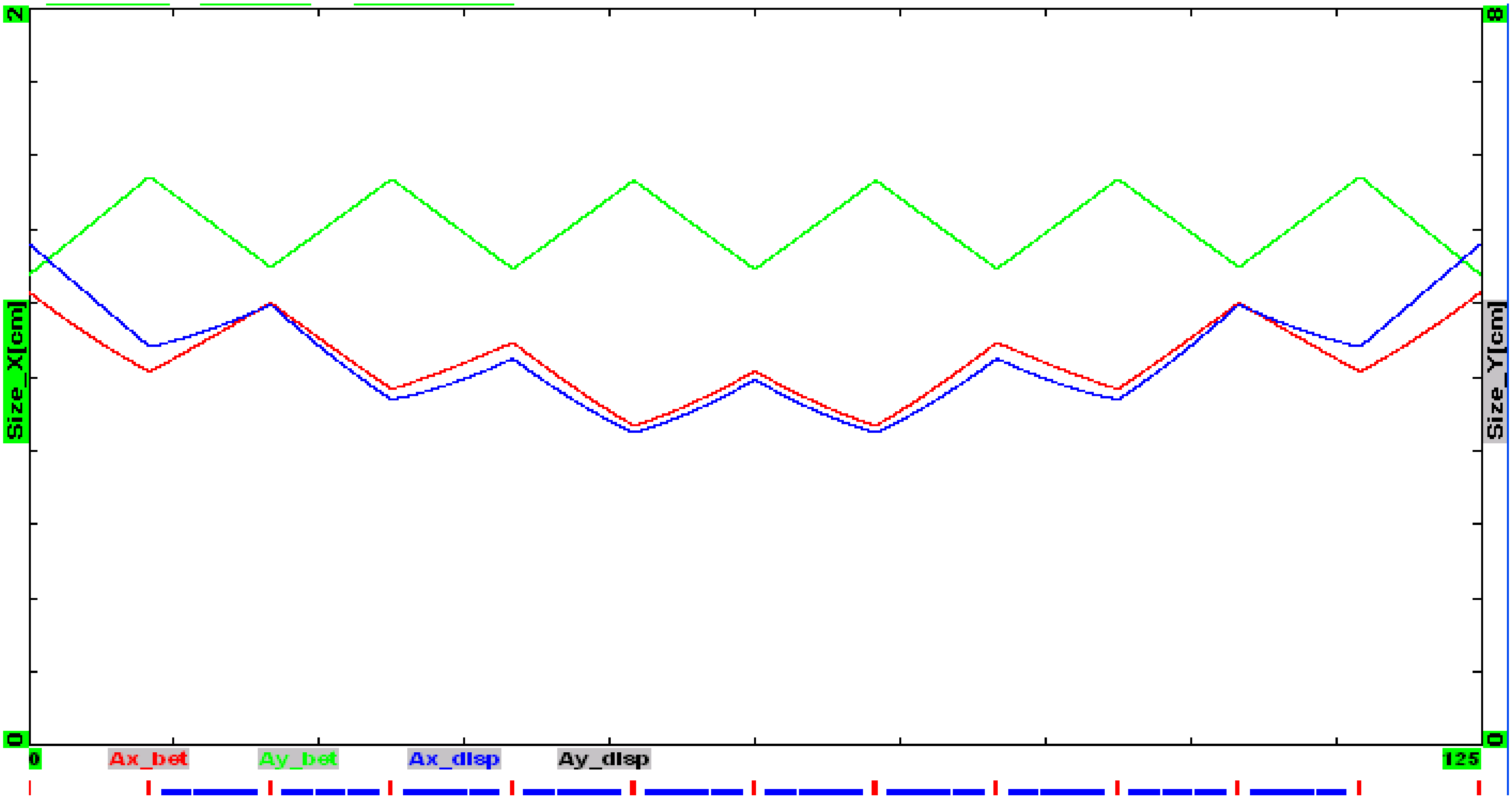
Acceptances

- Horizontal acceptance is set by the gap between plates of 3 cm and maximum of beta-function in the bends of 44 m
  - Assuming 3 mm orbit error one obtains ε_{Hmax}=3.2 mm mrad
- The maximum dispersion in the bends is 27 m
  - Similarly, one obtains the maximum momentum spread: $(\Delta p/p)_{max}=4.6\cdot10^{-4}$
- We determine the vertical acceptance to be large enough so that the vertical and horizontal degrees of freedom would be in thermal equilibrium, i.e. vertical and transverse velocity spreads would be equal
  - That minimizes the IBS

This assumes the slip factor has been calculated correctly, which may not be true

  - It results in: ε_{Vmax}=17 mm mrad and the maximum beam size (at acceptance boundary) of 6.2 cm which is about 5 times larger than the horizontal beam size

Acceptances (2)



Beam boundary at acceptances:

ε_{Hmax}=3.2 mm mrad, ε_{Vmax}=17.5 mm mrad, $(\Delta p/p)_{max}=4.6\cdot10^{-4}$

RF and Related Parameters

- Synchrotron frequency has to be large enough to minimize spin decoherence within one synchrotron period but small relative to the distance to strong resonances, Q_s=0.0066 was chosen (ΔQ_{SC}~0.04)
- Sum of bunch lengths, $n_b \sigma_s$, has to be as large as possible to reduce space charge tune shifts and IBS
  - Bucket height, $\Delta p/p|_{\text{bucket}}$, has to be only slightly larger than the longitudinal acceptance, $\Delta p/p|_{\max}$, but linearity is still desirable
    => $\Delta p/p|_{\text{bucket}}$ / $\Delta p/p|_{\max}$=1.5
- Main parameters
  - RF voltage: V_0=6 kV
  - Harmonic number: h=100
  - RF frequency: f_{RF}=35.878 MHz
  - Synchrotron tune: Q_s=0.0066
  - Bucket height: $\Delta p/p|_{\text{bucket}}$=6.9·10^{-4}
  - Bucket length: 5.0 m
  - Bunch length: σ_s = 32 cm

$$Q_s = \sqrt{\frac{heV_0\eta}{2\pi mc^2\gamma\beta^2}}$$

$$\left.\frac{\Delta p}{p}\right|_{\text{bucket}} = \frac{2Q_s}{h\eta}$$

$$\sigma_s = \frac{C\eta\sigma_p}{2\pi Q_s}$$

Space Charge Tune Shifts

- Beam emittances and momentum spreads are set by aperture (gap)
- Tune shifts due to space charge are the main beam current limitation for strong focusing ring
- The tune shifts due to counter rotating beam, $\sqrt{2\pi}C / N_b\sigma_s \approx 40$, are smaller and do not represent a problem

$$\Delta Q_x = \frac{r_p N_p C}{(2\pi)^{3/2} \beta^2 \gamma^3 \sigma_s} \left\langle \frac{\beta_x}{(\sigma_x + \sigma_y)\sigma_x} \right\rangle_s$$

$$\Delta Q_y = \frac{r_p N_p C}{(2\pi)^{3/2} \beta^2 \gamma^3 \sigma_s} \left\langle \frac{\beta_y}{(\sigma_x + \sigma_y)\sigma_y} \right\rangle_s$$

Protons per bunch: N_p	$2.5 \cdot 10^8$
Beam current, [mA]	1.4
Rms bunch length [cm]	32
Rms norm. emittances, $\varepsilon_x/\varepsilon_y$ [μm]	0.12/0.61
$\Delta Q_x / \Delta Q_y$, [10^{-2}]	2.9/5.0

IBS Growth Rates

- Operation below transition greatly reduces IBS growth rates for operation at the thermal equilibrium: $\tau_{x,y,s} \approx 1600$ s (no scraping)
- Temperature exchange between different degrees of freedom proceeds faster by more than an order of magnitude for a states close to the equilibrium
- Collimation stops emittance growth when the rms beam size achieves approximately 1/3 of the aperture
- For chosen optics the major aperture limitation is in the longitudinal plane. That results in a quasi-equilibrium state when the growth rates of horizontal and vertical emittances due to IBS are equal to zero. The IBS driven emittance growth rate for longitudinal plane of 1/830 s^{-1} is stopped by collimation which results in the intensity loss:

$$N(t) = \frac{N_p}{1+\lambda_D t}, \quad \lambda_D \approx \frac{{\mu_{01}}^2}{n_\sigma \tau_s}, \qquad \begin{array}{c} \mu_{01} \approx 2.405 \\ n_\sigma \approx 2.96, \quad \tau_x = \tau_y = 0 \end{array}$$

- The parameters of the quasi-equilibrium state are: ε_x=0.21 μm, ε_y=1.0 μm, σ_p=1.4·10^{-4} => τ_s=830 s, 1/λ_D=350 s