Towards a surrogate computational tool to quantify the systematic uncertainties in EDM experiments in storage rings

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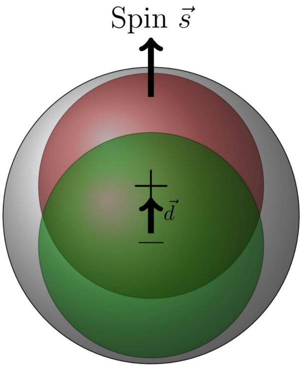
Overview

- Electric Dipole Moment (EDM)
- EDM at COSY and systematic errors
- Need for computational tool
- Brief introduction to proposed method
- Some results
- Summary and outlook

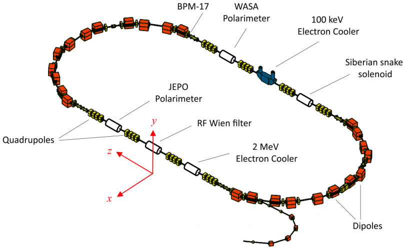
The Electric Dipole Moment (EDM)

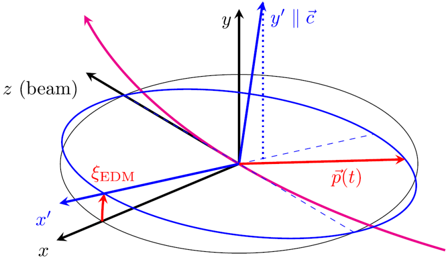
- Permanent separation of positive and negative charges
- Fundamental property of particles
- EDM is only possible via violation of time reversal T and parity P symmetries
- Predominance of matter over antimatter in the Universe



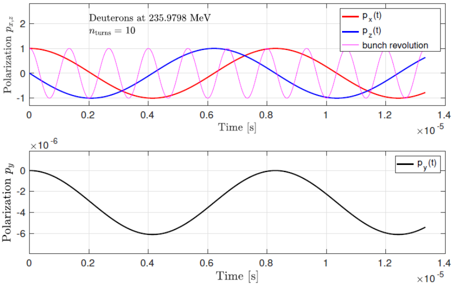
See talks of (J. Pretz and V. Shmakova)

EDM Measurements at COSY (Proof of principle) [1]



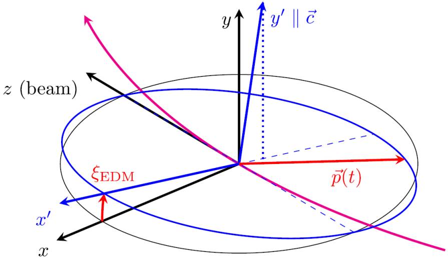


- Spins precess around the $\vec{c}$ axis
- Oscillating vertical polarization component $p_y(t)$ is generated.
- Oscillation amplitude corresponds to EDM tilt angle ξ_{EDM}
- Observed signal dominated by systematic errors



Systematic Errors

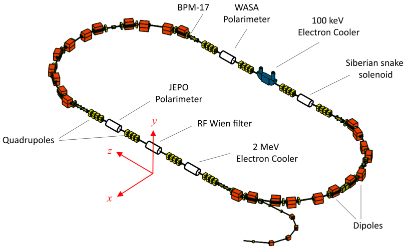
- Real machines are far from ideal conditions
- Systematic errors
  - Mechanical misalignments
  - Electrical tolerances and uncertainties
  - Finite-precision instrumentation (BPMs, orbit)
  - Complex electromagnetic structures (toroidal coils)
  - Unwanted field components (fringe regions and mixed field regions)
- Disentangle desired signal from background: **beam and spin simulations**



The Need for Computational Model

- Computational model of COSY tremendously large:
  - Model considered large if dimension (i.e., number of uncertain parameters) > 10
  - Current model is a **526-dimensional** numerical model (variation of quadrupoles, dipoles and steerers)
- Ready-solution: Monte-Carlo method
  - Curse of dimensionality
  - Very large number of simulations needed for convergence
- Solution: sparse modeling
  - Polynomial Chaos Expansion
- General formulation developed and applied in 2017 [2]
- Non intrusive version freshly implemented in an *in-house* beam and spin tracking code

Demonstration: Simulation of Steerers' Uncertainties



- Use uncertainties of 46 vertical and horizontal steerers (power supplies fluctuations).
- Initial ensemble 46×1000 samples generated
- Single particle simulation considered
- RF Wien filter placed in simulation as resonant transverse spin rotator
- Simulations start at telescope section (injection side)

- Initial spin sector $\vec{S} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$
- $\eta = 0$ (no EDM).
- 1000 turns per simulation.
- Symplectic tracking: Hamiltonian [3] and leap-frogging implementation

Brief Introduction to PCE

- First proposed by Wiener in 1938 as "Homogeneous Chaos"
- Basic idea similar to the Fourier series, where random variables represented as infinite series in terms of orthogonal polynomials
- Let $\mathcal{Y}$ be observable(s), as function of set of uncertain parameters $\boldsymbol{\xi}$ via model $\mathcal{M}$ (spin tracking simulation in this case)

$$\mathcal{Y} = \mathcal{M}\left(\boldsymbol{\xi}\right) = \sum_{\boldsymbol{i} \in \mathcal{I}_{m,p}} \alpha_{\boldsymbol{i}} \Psi_{\boldsymbol{i}}\left(\boldsymbol{\xi}\right) . \qquad (1)$$

- Expansion coefficients validated using Leave-one-out error

$$e_{rr_{\mathrm{LOO}}} = \frac{1}{N} \sum_{k=1}^{N} \left(\mathcal{Y} - \mathcal{Y}^{k}\right)^{2}$$

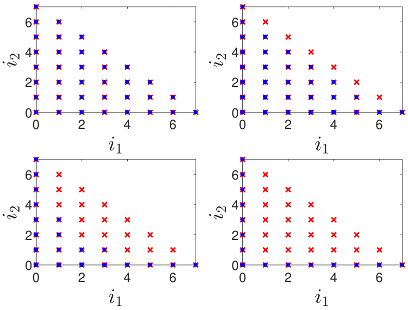
Truncation

- Using only an expansion order of $p = 3$, and $m = 46$

$$P = \binom{m+p}{p} = \frac{(m+p)!}{m!p!}. \quad (2)$$

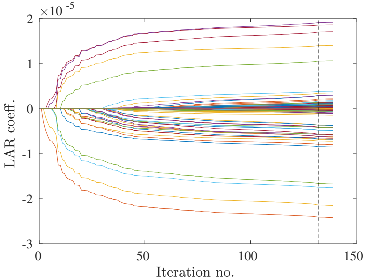
- As a result, 18424 basis functions generated, i.e., 27636 simulations required for convergence
  - Solution: sparse PCE with machine learning
- Hyperbolic truncation

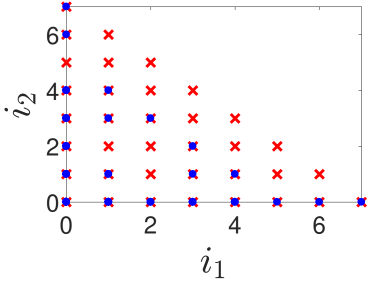
$$\|\cdot\|_q = \left(\sum^m (\cdot)^q\right)^{1/q}. \quad (3)$$



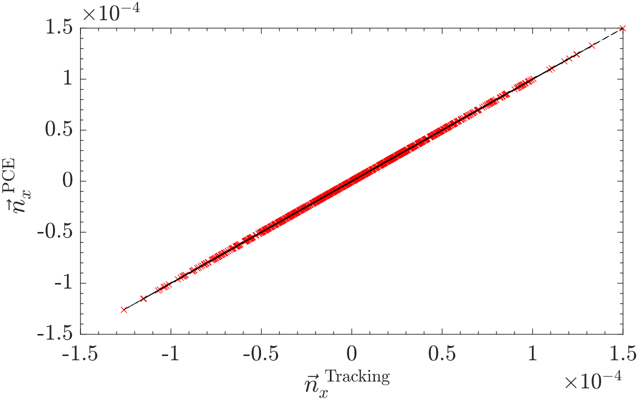
Machine Learning Algorithm Applied

- Using a machine learning algorithm
- Algorithm selects remaining basis functions most correlated to the observable
- Many solutions obtained and selected according to leave-one-out error criteria





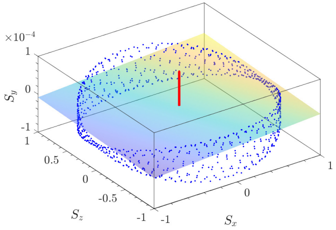
Surrogate Model Validation

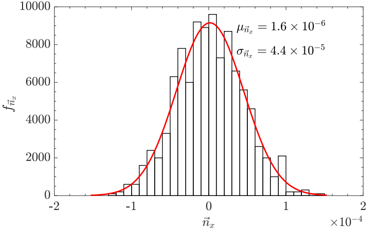


- Expansion order $p = 3$
- Leave-one-out error: 2.1×10^{-8}
- Truncation order q: 0.3
- Basis functions used: only 132 out of 18424

- Power supplies variation
- Orbit variation

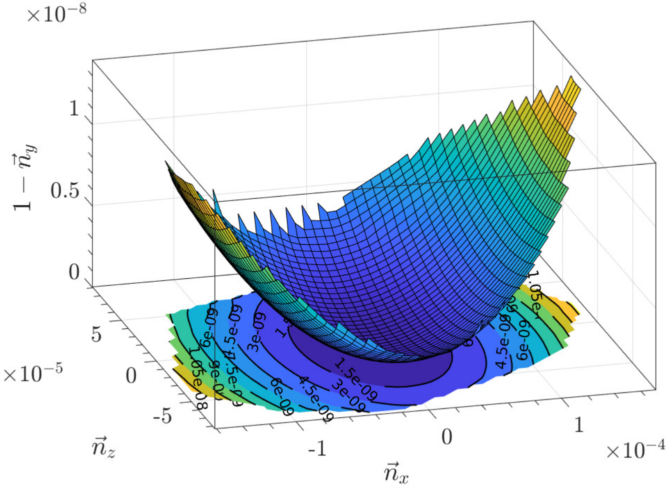
Stable Spin Axis $\vec{n}$ at Wien Filter Location





- $\vec{n} = \begin{pmatrix} 4.98 \times 10^{-5} \\ 0.99999999895 \\ 2.84 \times 10^{-5} \end{pmatrix}$ (induced only by steerers)
- Estimated plane inclination $\xi = 5.74 \times 10^{-5}$

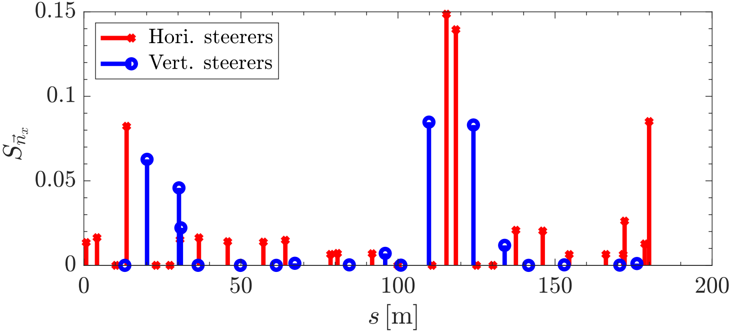
Stable Spin Axis $\vec{n}$



- Quadratic dependence of $\vec{n}$ on the orbit

Sensitivity Analysis

- Quantification of sensitivity of observable on input random parameters (only 1$^{\mathrm{st}}$ order)



- Sensitivity indices computed with zero additional cost

Summary and Outlook

- Precision experiments such as EDM studies require precise and efficient computational tool for quantification of systematic error
- Alternative tool to Monte-Carlo method has been presented: PCE
- Systematic contribution to the EDM limit could be estimated
- Include all misalignments of dipoles and quadrupoles

References

1. Rathmann, F., Nikolaev, N., and Slim, J. (2020). Spin dynamics investigations for the electric dipole moment experiment. Phys. Rev. Accel. Beams, 23, 024601
2. Slim, J., Rathmann, F., and Heberling, D. (2017). Computational framework for particle and spin simulations based on the stochastic Galerkin method. Phys. Rev. E, 96, 063301
3. Wolski, A. (2014). Beam Dynamics in High Energy Particle Accelerators. Imperial College Press

Extra: LAR

1. Set the coefficient to zero and set the residual $\boldsymbol{R} = \mathcal{Y} - \hat{\mathcal{Y}}$
2. Find the vector (basis polynomial) $\boldsymbol{\Psi}_{\boldsymbol{i}_j}$ that is most correlated with the residual $\boldsymbol{R}$
3. Move the corresponding coefficient $a_{\boldsymbol{i}_j}$ from 0 to $\boldsymbol{\Psi}_{\boldsymbol{i}_j}^T \boldsymbol{R}$ until another polynomial $\boldsymbol{\Psi}_{\boldsymbol{i}_k}$ has stronger correlation with the residual
4. Move $a_{\boldsymbol{i}_j}$ and $a_{\boldsymbol{i}_k}$ in the direction defined by their joint least square coefficient on the current residual of $\left(\boldsymbol{\Psi}_{\boldsymbol{i}j}, \boldsymbol{\Psi}_{\boldsymbol{i}_k}\right)$ until some other basis has more correlation with the current residual
5. Continue until P basis (a.k.a. the predictors) have been entered