

Electric Dipole Moment Measurements at Storage Rings

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Outline

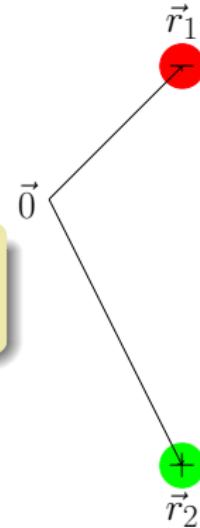
- **Introduction: Electric Dipole Moments (EDMs):**
What is it?
- **Motivation:**
Why is it interesting?
- **Experimental Method:**
How to measure charged particle EDMs?
- **Results of first test measurements:**
Spin tune and Spin Coherence time
- **Polarisation Measurements:**
Topics for working group

What is it?

Electric Dipoles

Classical definition:

$$\vec{d} = \sum_i q_i \vec{r}_i$$



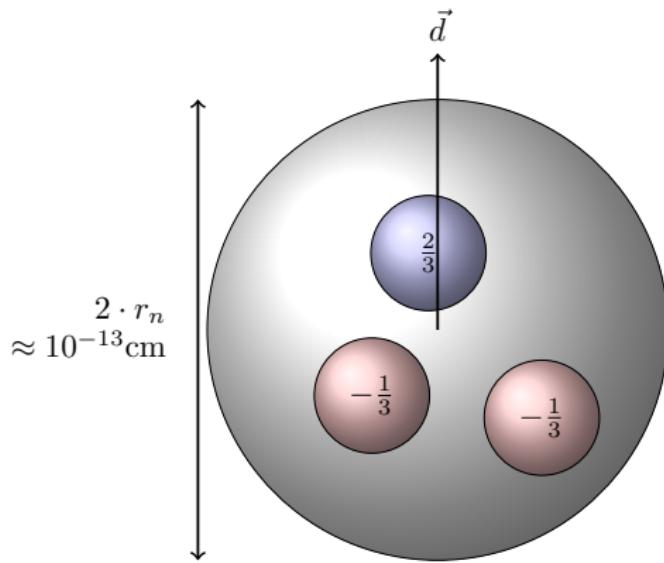
Order of magnitude

	atomic physics	hadron physics
charges	e	
$ \vec{r}_1 - \vec{r}_2 $	$1 \text{ \AA} = 10^{-8} \text{ cm}$	
EDM		
naive expectation	$10^{-8} e \cdot \text{cm}$	
observed	water molecule	
	$2 \cdot 10^{-8} e \cdot \text{cm}$	

Order of magnitude

	atomic physics	hadron physics
charges	e	e
$ \vec{r}_1 - \vec{r}_2 $	$1 \text{ \AA} = 10^{-8} \text{ cm}$	$1 \text{ fm} = 10^{-13} \text{ cm}$
EDM		
naive expectation	$10^{-8} e \cdot \text{cm}$	$10^{-13} e \cdot \text{cm}$
observed	water molecule $2 \cdot 10^{-8} e \cdot \text{cm}$	neutron $< 3 \cdot 10^{-26} e \cdot \text{cm}$

Order of magnitude



neutron EDM of $d_n = 3 \cdot 10^{-26} \text{ e}\cdot\text{cm}$ corresponds to separation
of $u-$ from $d-$ quarks of $\approx 5 \cdot 10^{-26} \text{ cm}$

$$\text{Operator } \vec{d} = q\vec{r}$$

is odd under parity transformation ($\vec{r} \rightarrow -\vec{r}$):

$$\mathcal{P}^{-1}\vec{d}\mathcal{P} = -\vec{d}$$

Consequences:

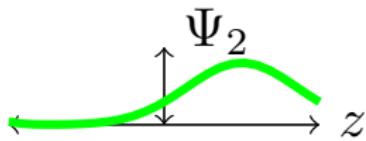
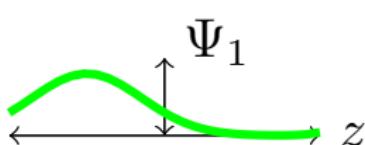
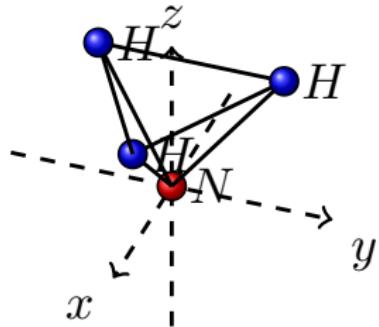
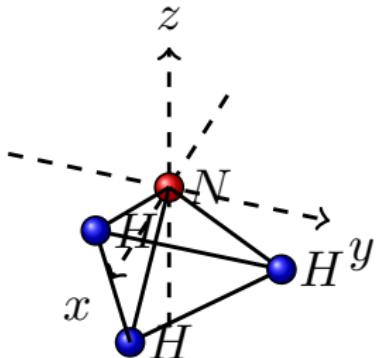
In a state $|a\rangle$ of given parity the expectation value is 0:

$$\langle a|\vec{d}|a\rangle = -\langle a|\vec{d}|a\rangle$$

but if $|a\rangle = \alpha|P=+\rangle + \beta|P=-\rangle$

in general $\langle a|\vec{d}|a\rangle \neq 0 \Rightarrow$ i.e. molecules

EDM of molecules



ground state: mixture of $\Psi_s = \frac{1}{\sqrt{2}} (\Psi_1 + \Psi_2)$ $P = +$

$$\Psi_a = \frac{1}{\sqrt{2}} (\Psi_1 - \Psi_2) \quad P = -$$

(Cohen-Tannoudji, B. Diu, F. Laloë, Mécanique quantique)

Order of magnitude

Molecules can have large EDM because of degenerated ground states with different parity

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Elementary particles (including hadrons) have a definite parity and cannot posses an EDM

$$P|\text{had}\rangle = \pm 1 |\text{had}\rangle$$

Order of magnitude

Molecules can have large EDM because of degenerated ground states with different parity

Elementary particles (including hadrons) have a definite parity and cannot posses an EDM

$$P|\text{had}\rangle = \pm 1 |\text{had}\rangle$$

unless

\mathcal{P} and time reversal \mathcal{T} invariance are violated!

\mathcal{T} and \mathcal{P} violation of EDM

\vec{d} : EDM

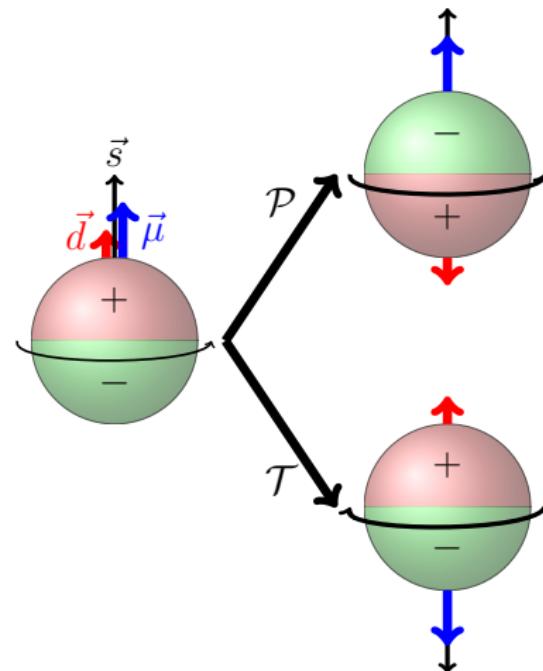
$\vec{\mu}$: magnetic moment

both \parallel to spin

$$H = -\mu \vec{\sigma} \cdot \vec{B} - d \vec{\sigma} \cdot \vec{E}$$

$$\mathcal{T}: H = -\mu \vec{\sigma} \cdot \vec{B} + d \vec{\sigma} \cdot \vec{E}$$

$$\mathcal{P}: H = -\mu \vec{\sigma} \cdot \vec{B} + d \vec{\sigma} \cdot \vec{E}$$



⇒ EDM measurement tests violation of fundamental symmetries \mathcal{P} and \mathcal{T} ($\stackrel{\mathcal{CPT}}{=} \mathcal{CP}$)

Symmetries in Standard Model

	electro-mag.	weak	strong
\mathcal{C}	✓	✗	✓
\mathcal{P}	✓	✗	✓
$T \xrightarrow{CPT} \mathcal{CP}$	✓	(✗)	(✓)

- \mathcal{C} and \mathcal{P} are maximally violated in weak interactions (Lee, Yang, Wu)
- \mathcal{CP} violation discovered in kaon decays (Cronin,Fitch) described by CKM-matrix in Standard Model
- \mathcal{CP} violation allowed in strong interaction but corresponding parameter $\theta_{QCD} \lesssim 10^{-10}$ (strong \mathcal{CP} -problem)

Sources of \mathcal{CP} -Violation

Standard Model	
Weak interaction	
CKM matrix	→ unobservably small EDMs
Strong interaction	
θ_{QCD}	→ best limit from neutron EDM
beyond Standard Model	
e.g. SUSY	→ accessible by EDM measurements

Why is it interesting?

Matter-Antimatter Asymmetry

Excess of matter in the universe:

	observed	SM prediction
$\eta = \frac{n_B - n_{\bar{B}}}{n_\gamma}$	6×10^{-10}	10^{-18}

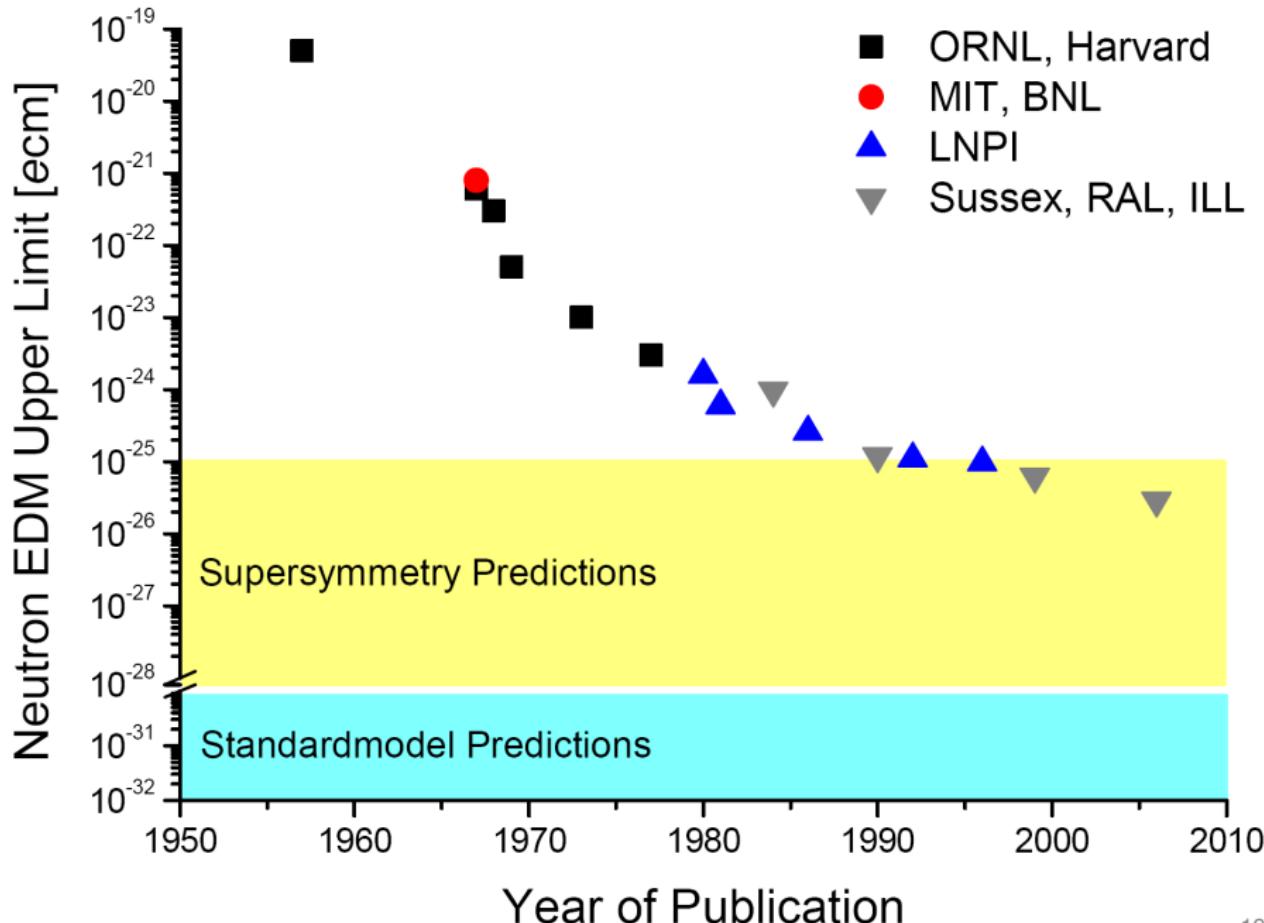
Sakharov (1967): \mathcal{CP} violation needed for baryogenesis

⇒ New \mathcal{CP} violating sources beyond SM needed to explain this discrepancy

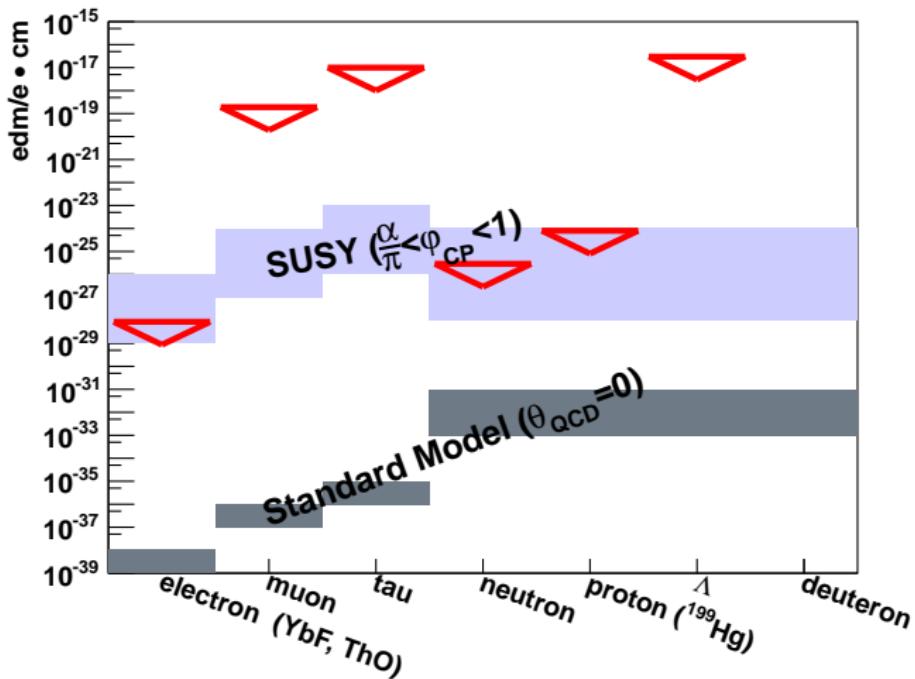
They could manifest in EDMs of elementary particles

What do we know about
EDMs?

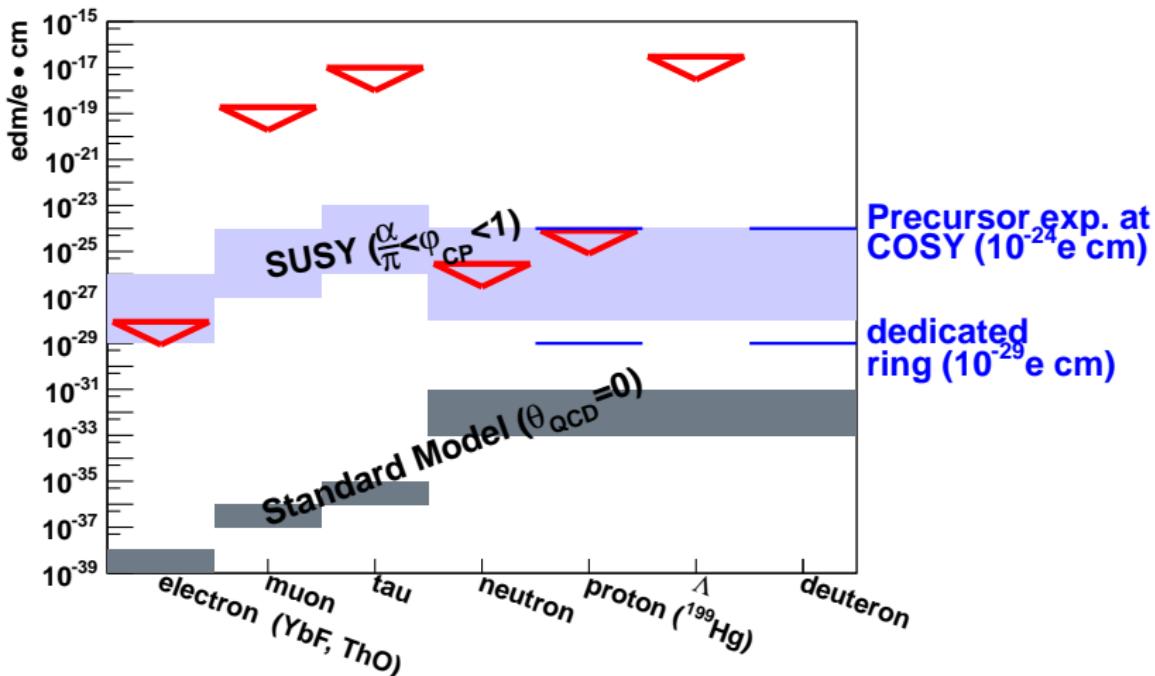
History of Neutron EDM



EDM: Current Upper Limits



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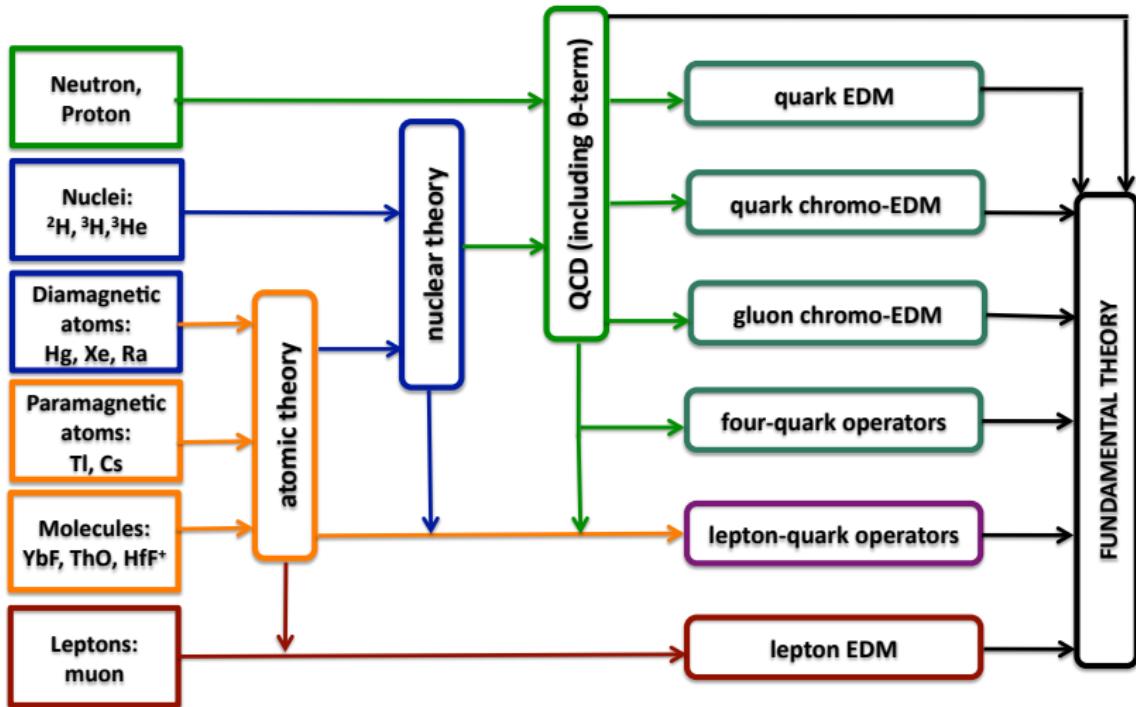


FZ Jülich: EDMs of **charged** hadrons: $p, d, {}^3\text{He}$

Why Charged Particle EDMs?

- no direct measurements for charged hadrons exist
- potentially higher sensitivity (compared to neutrons):
 - longer life time,
 - more stored protons/deuterons
- complementary to neutron EDM:
 $d_d \stackrel{?}{=} d_p + d_n \Rightarrow$ access to θ_{QCD}
- EDM of one particle alone not sufficient to identify \mathcal{CP} -violating source

Sources of \mathcal{CP} Violation



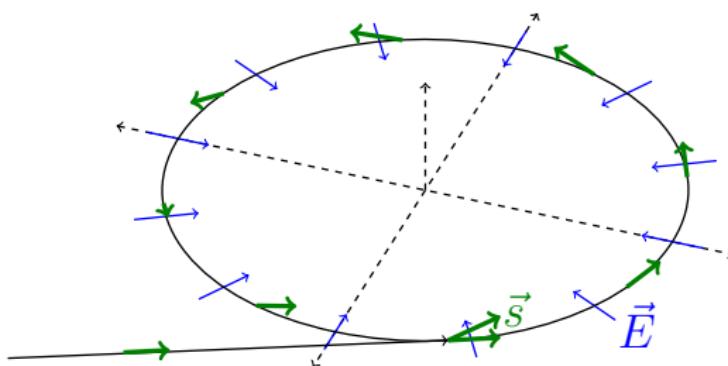
How to measure charged
particle EDMs?

Experimental Method: Generic Idea

For **all** EDM experiments (neutron, proton, atoms, ...):

Interaction of \vec{d} with electric field \vec{E}

For charged particles: apply electric field in a storage ring:



$$\frac{d\vec{s}}{dt} \propto \vec{d} \vec{E} \times \vec{s}$$

In general:

$$\frac{d\vec{s}}{dt} = \vec{\Omega} \times \vec{s}$$

build-up of vertical polarization $s_{\perp} \propto |\vec{d}|$

Experimental Requirements

- high precision storage ring
(alignment, stability, field homogeneity)
- high intensity beams ($N = 4 \cdot 10^{10}$ per fill)
- polarized hadron beams ($P = 0.8$)
- large electric fields ($E = 10$ MV/m)
- long spin coherence time ($\tau = 1000$ s),
- polarimetry (analyzing power $A = 0.6$, acc. $f = 0.005$)

$$\sigma_{\text{stat}} \approx \frac{1}{\sqrt{Nf_\tau PAE}} \Rightarrow \sigma_{\text{stat}}(1\text{year}) = 10^{-29} \text{ e}\cdot\text{cm}$$

challenge: get σ_{sys} to the same level

Systematics

Major source:

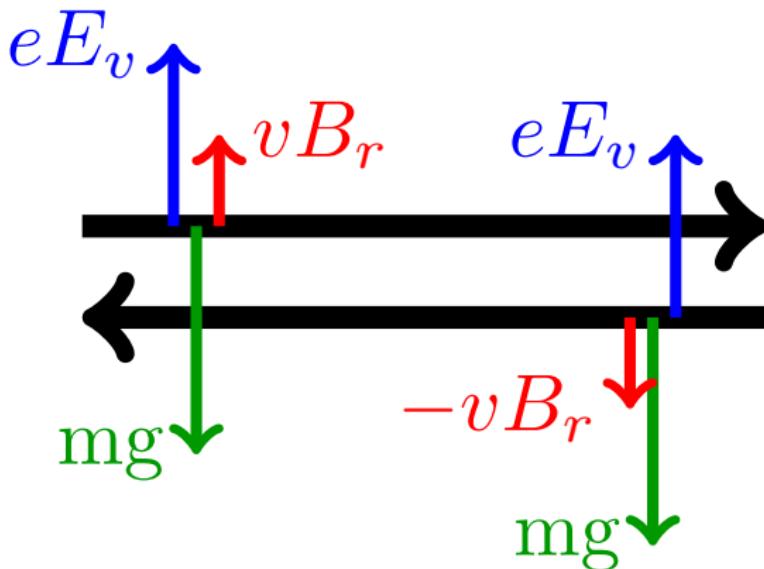
Radial B field mimics an EDM effect:

- Difficulty: even small radial magnetic field, B_r can mimic EDM effect if $\mu B_r \approx dE_r$
- Suppose $d = 10^{-29} \text{ e}\cdot\text{cm}$ in a field of $E_r = 10 \text{ MV/m}$
- This corresponds to a magnetic field:

$$B_r = \frac{dE_r}{\mu_N} = \frac{10^{-22} \text{ eV}}{3.1 \cdot 10^{-8} \text{ eV/T}} \approx 3 \cdot 10^{-17} \text{ T}$$

Solution: Use two beams running clockwise and counter clockwise, separation of the two beams is sensitive to B_r

Systematics



Sensitivity needed: $1.25 \text{ fT}/\sqrt{\text{Hz}}$ for $d = 10^{-29} \text{ e cm}$
(possible with SQUID technology)

Spin Precession: Thomas-BMT Equation

$$\frac{d\vec{s}}{dt} = \vec{\Omega} \times \vec{s} = \frac{e}{m} [\textcolor{red}{G}\vec{B} + \left(\textcolor{red}{G} - \frac{1}{\gamma^2 - 1} \right) \vec{v} \times \vec{E} + \frac{m}{es} \textcolor{blue}{d}(\vec{E} + \vec{v} \times \vec{B})] \times \vec{s}$$

Ω : angular precession frequency

d : electric dipole moment

G : anomalous magnetic moment

γ : Lorentz factor

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dedicated ring: pure electric field,

freeze horizontal spin motion $\left(G - \frac{1}{\gamma^2 - 1} \right) = 0$

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COSY: pure magnetic ring

access to EDM via motional electric field $\vec{v} \times \vec{B}$,

requires additional radio-frequency E and B fields

to suppress $\textcolor{red}{G}\vec{B}$ contribution

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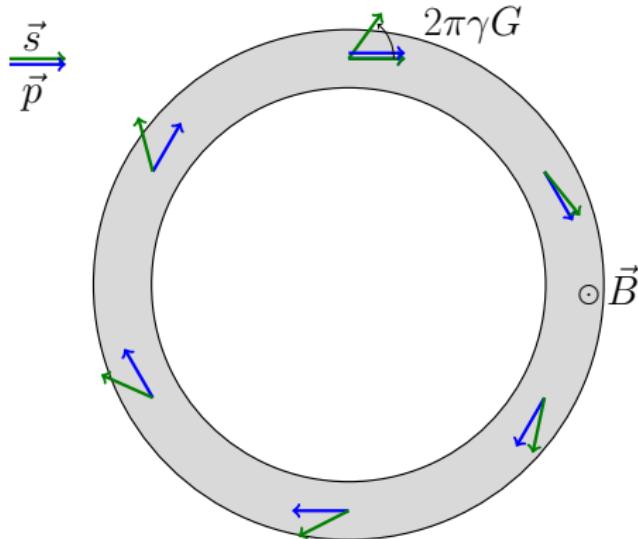
to suppress $\textcolor{red}{G}\vec{B}$ contribution

neglecting EDM term

spin tune: $\nu_s \approx \frac{|\vec{\Omega}|}{|\omega_{cyc}|} = \gamma \textcolor{red}{G}$, $(\vec{\omega}_{cyc} = \frac{e}{\gamma m} \vec{B})$

Spin Tune ν_s

Spin tune: $\nu_s = \gamma G = \frac{\text{nb. of spin rotations}}{\text{nb. of particle revolutions}}$



deuterons: $p_d = 1 \text{ GeV}/c$ ($\gamma = 1.13$), $G = -0.14256177(72)$

$$\Rightarrow \nu_s = \gamma G \approx -0.161$$

Results of first test measurements

Cooler Synchrotron COSY

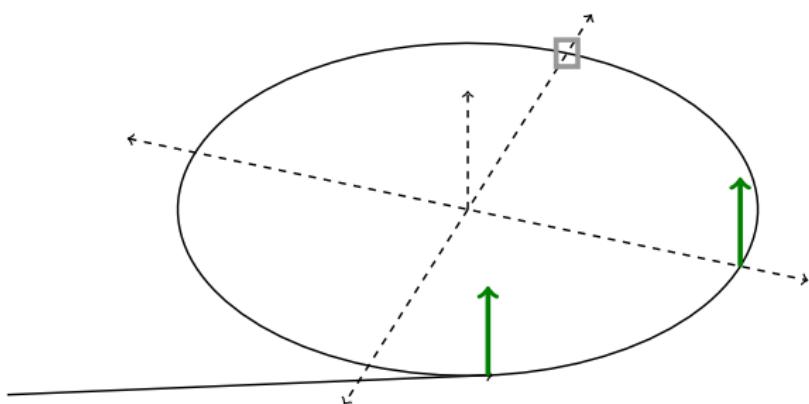


COSY provides (polarized) protons and deuterons with
 $p = 0.3 - 3.7 \text{ GeV}/c$

⇒ **Ideal starting point for charged particle EDM searches**

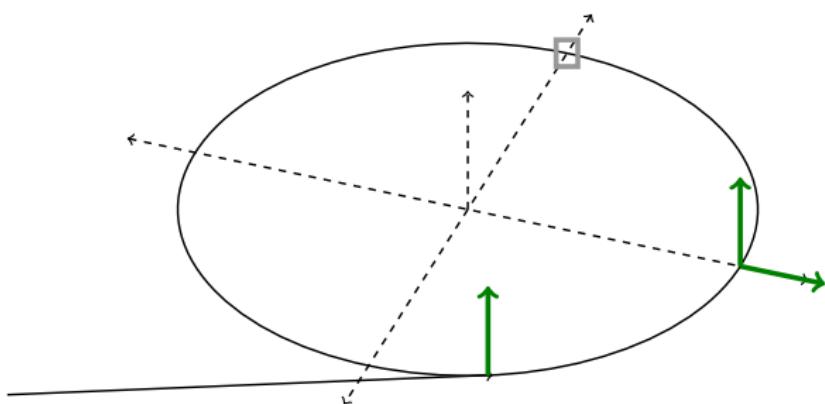
Experimental Setup

- Inject and accelerate vertically polarized deuterons to $p \approx 1 \text{ GeV}/c$



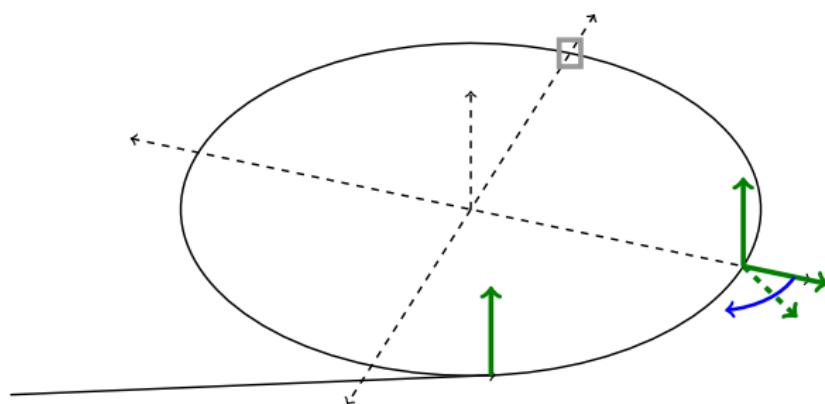
Experimental Setup

- Inject and accelerate vertically polarized deuterons to $p \approx 1 \text{ GeV}/c$
- flip spin with help of solenoid into horizontal plane



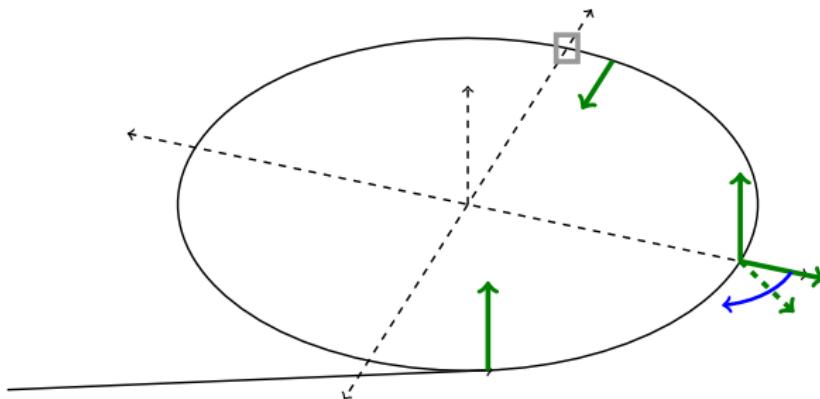
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Experimental Setup

- Inject and accelerate vertically polarized deuterons to $p \approx 1 \text{ GeV}/c$
- flip spin with help of solenoid into horizontal plane
- Spins start to precess
- Extract beam slowly (in 100 s) on target
- Measure asymmetry and determine spin precession



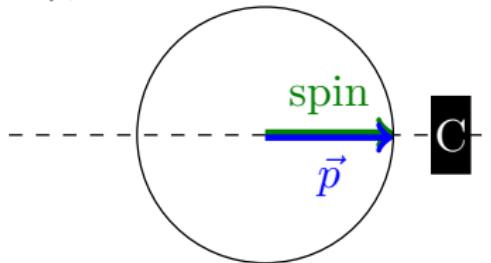
Asymmetry Measurements

- Detector signal $N^{up,dn} \propto (1 \pm PA \sin(\gamma G f_{rev} t))$

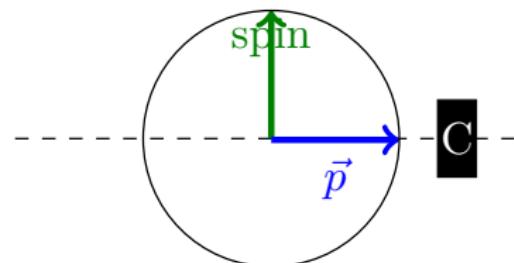
$$A_{up,dn} = \frac{N^{up} - N^{dn}}{N^{up} + N^{dn}} = PA \sin(\gamma G f_{rev} t)$$

A : analyzing power, P : polarization

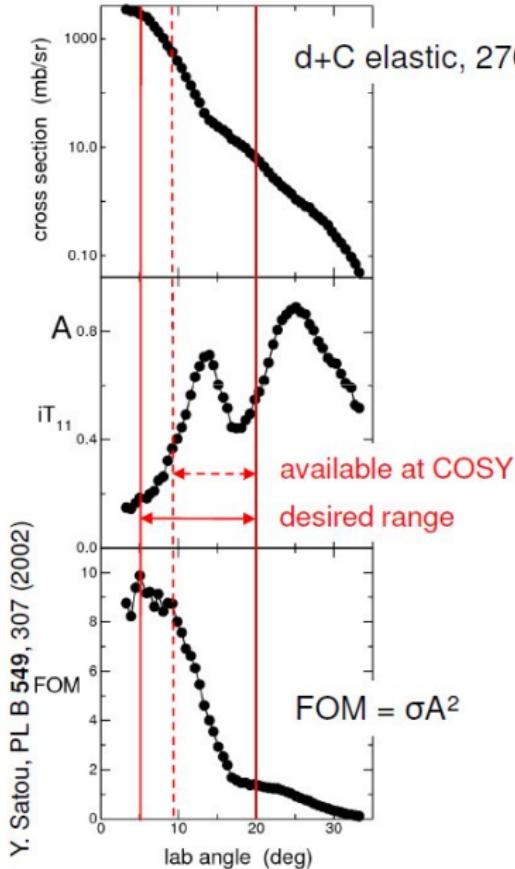
$$A_{up,dn} = 0$$



$$A_{up,dn} = PA$$



Polarimetry



Cross Section &
Analyzing Power
for deuterons

$$N_{up,dn} \propto (1 \pm P A \sin(\nu_s f_{rev} t))$$

$$\begin{aligned} A_{up,dn} &= \frac{N^{up} - N^{dn}}{N^{up} + N^{dn}} \\ &= P A \sin(\nu_s f_{rev} t) \end{aligned}$$

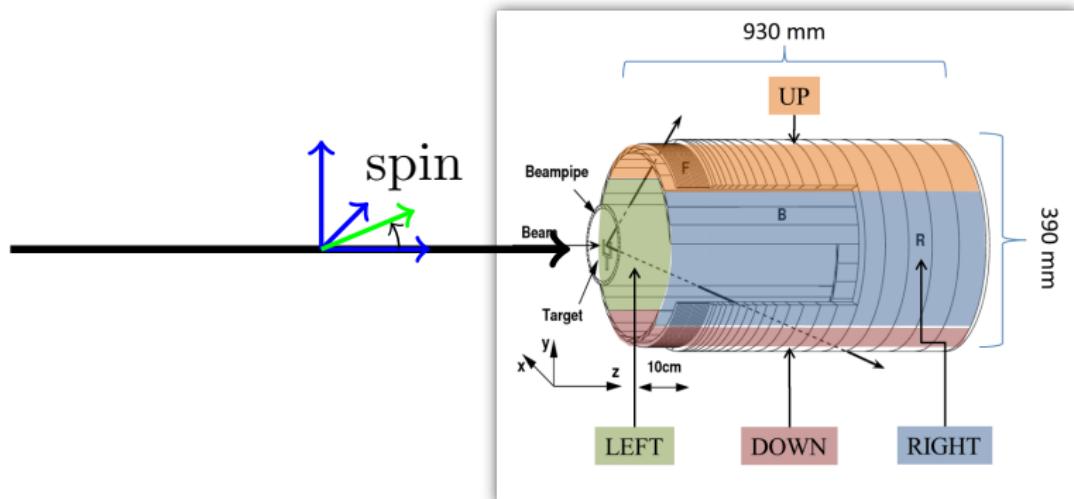
A : analyzing power
P : beam polarization

Polarimeter

elastic deuteron-carbon scattering

Up/Down asymmetry \propto horizontal polarization $\rightarrow \nu_s = \gamma G$

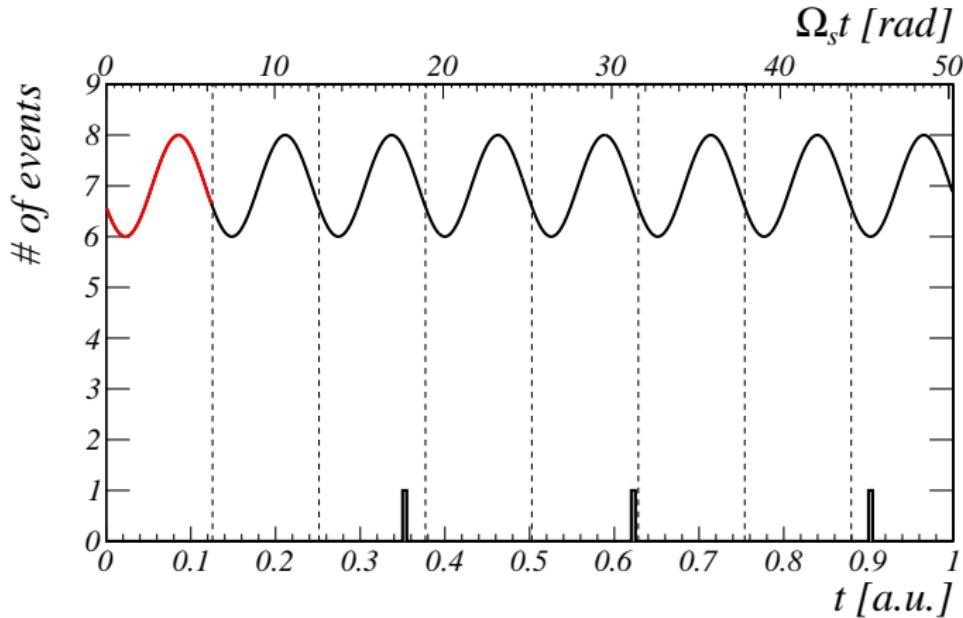
Left/Right asymmetry \propto vertical polarization $\rightarrow d$



$$N_{up,dn} \propto 1 \pm PA \sin(\nu_s f_{rev} t), \quad f_{rev} \approx 781 \text{ kHz}$$

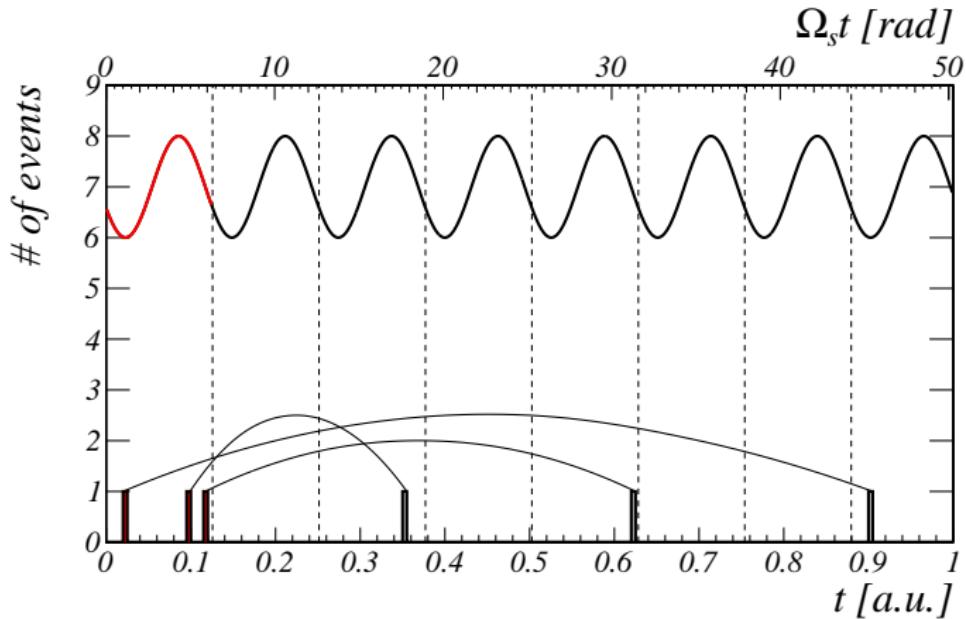
Spin Tune ν_s measurement

- Problem: detector rate ≈ 5 kHz, $f_{rev} = 781$ kHz
 \Rightarrow only 1 hit every 25th period
- not possible to use usual χ^2 -fit
- use unbinned Maximum Likelihood (under investigation)

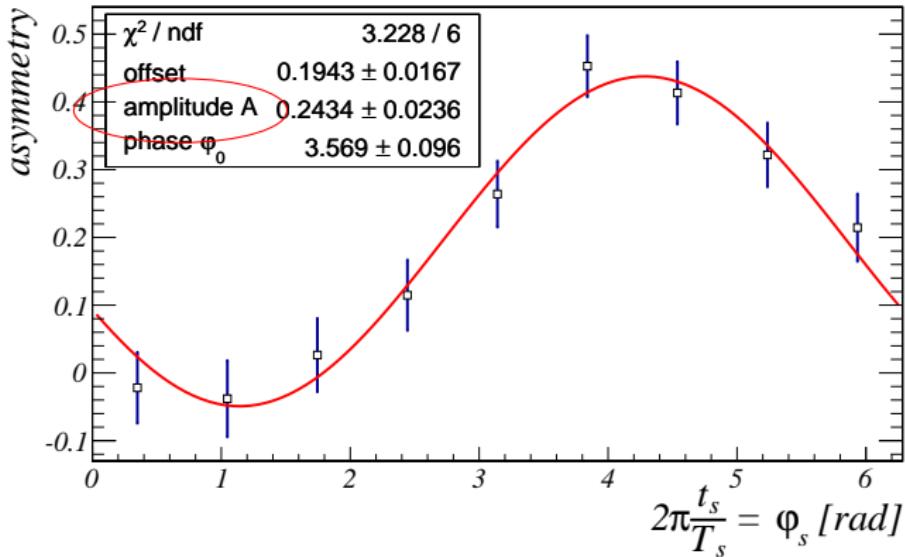


Spin Tune ν_s measurement

- map all events into first period ($T = 1/(\nu_s f_{rev}) \approx 8\mu s$) and perform χ^2 -fit
(requires knowledge of $\nu_s f_{rev}$)
- Analysis is done in macroscopic time bins of $\approx 2s$

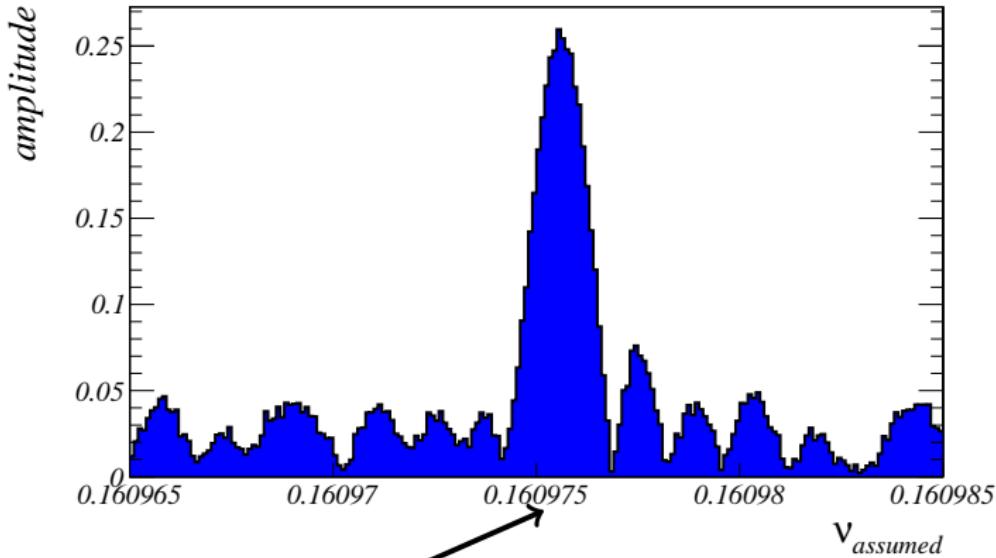


Asymmetry in 1st period



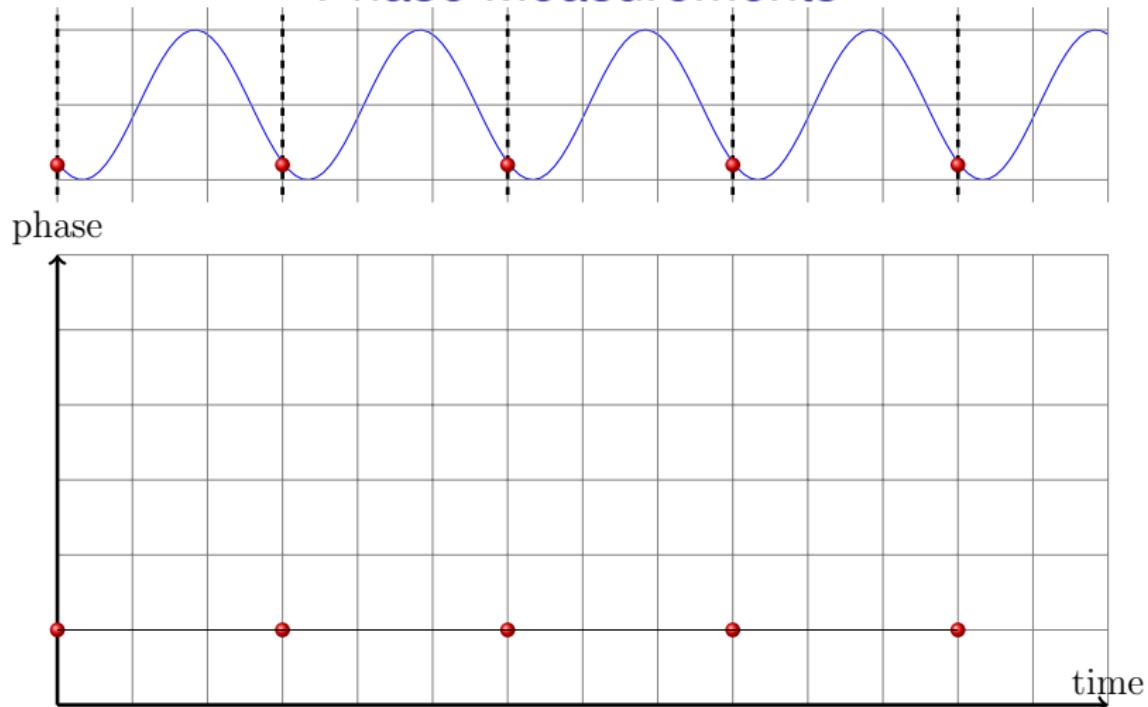
- only works if $T_s = \frac{1}{\nu_s f_{rev}}$ is correct.

Scan of ν_s

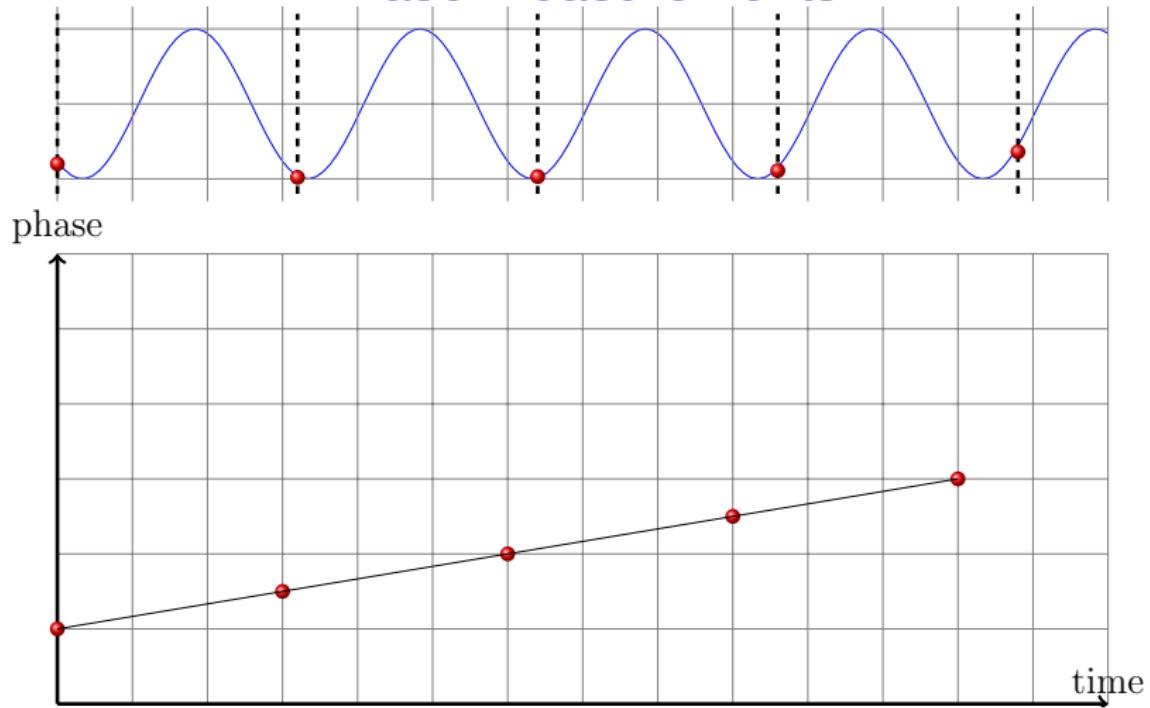


- set $\nu_s = \nu_{max}$ and determine phase in macroscopic time bins of $\approx 2\text{s}$
- allows for $\sigma_{\nu_s} \approx 10^{-6}$

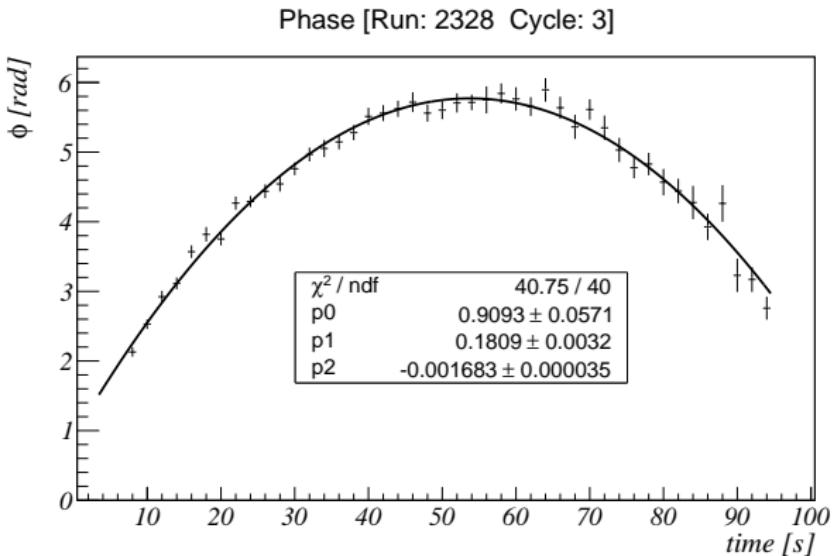
Phase Measurements



Phase Measurements

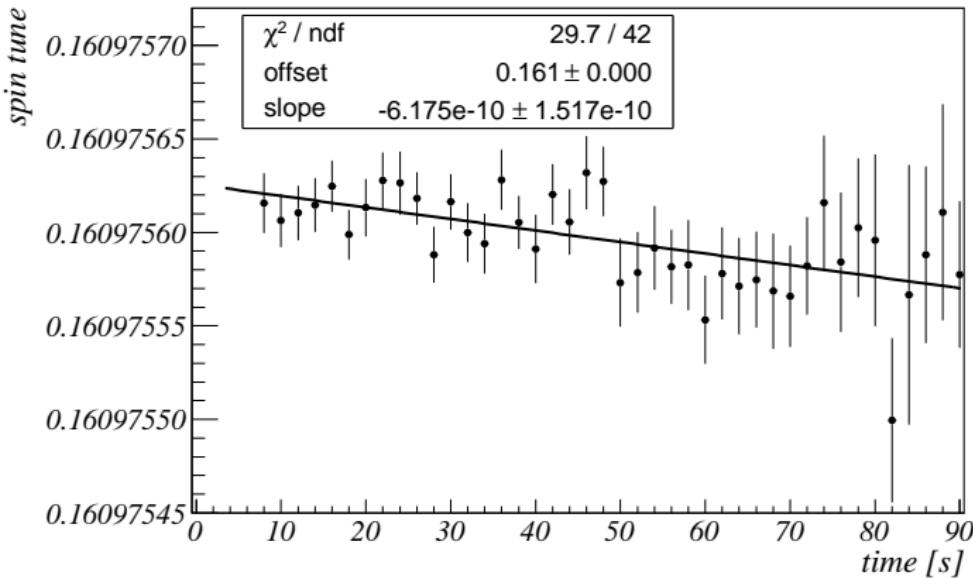


Phase Measurements



- 1st derivative gives deviation from assumed spin tune

Results: Spin Tune ν_s



- Spin tune ν_s can be determined to $\approx 10^{-8}$ in 2 s
- Average $\overline{\nu_s}$ in cycle (≈ 100 s) determined to 10^{-10}
(for $G = 0$, $d = 10^{-24} \text{ e}\cdot\text{cm} \Rightarrow$ spin tune = $5 \cdot 10^{-11}$)

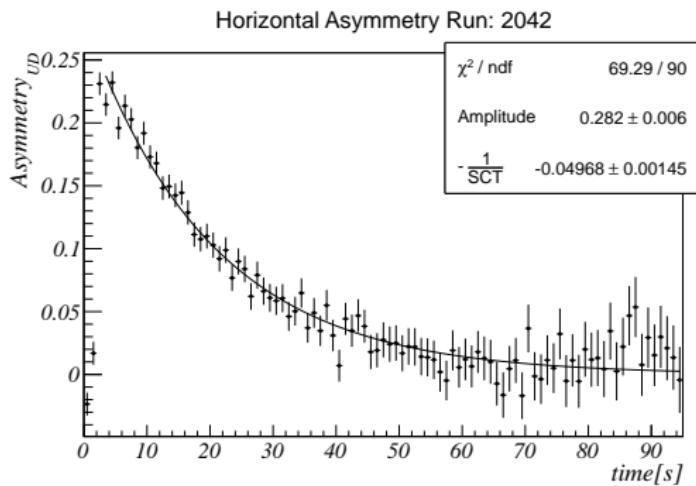
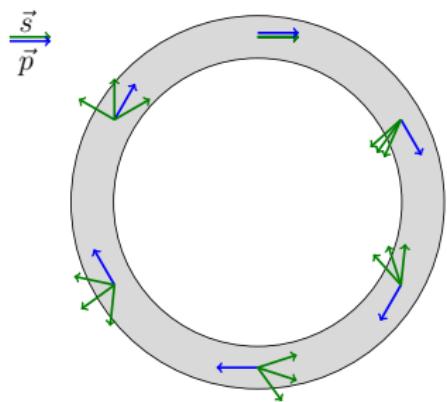
Spin Tune ν_s

Experiment	Gedankenexperiment
$G \approx -0.14, d \approx 0$	$G = 0, d = 10^{-24} \text{ e cm}$
$\nu_s = \gamma G = -0.16$	$\nu_s = \frac{\nu m \gamma d}{e s} = 5 \cdot 10^{-11}$

compare to $\sigma(\nu_s) = 10^{-10}$ in 100 s measurement

Results: Spin Coherence Time (SCT)

Short Spin Coherence Time



unbunched beam

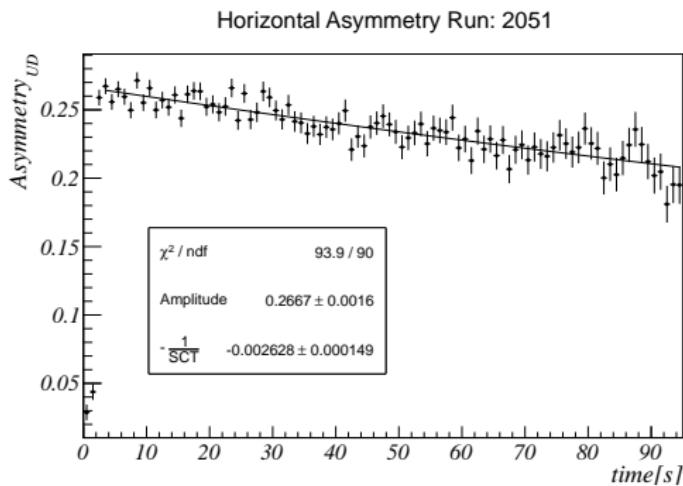
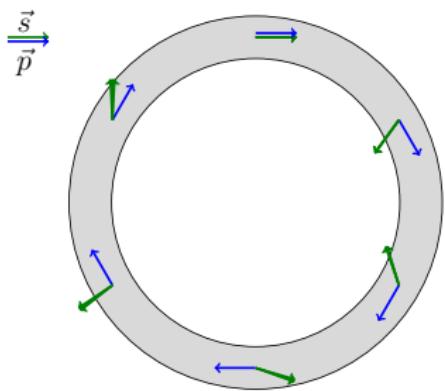
$$\Delta p/p = 10^{-5} \Rightarrow \Delta \gamma/\gamma = 3 \cdot 10^{-6}, T_{rev} \approx 10^{-6} \text{ s}$$

\Rightarrow decoherence after $< 1 \text{ s}$

cooled bunched beam \Rightarrow SCT $\tau = 20 \text{ s}$

Results: Spin Coherence Time (SCT)

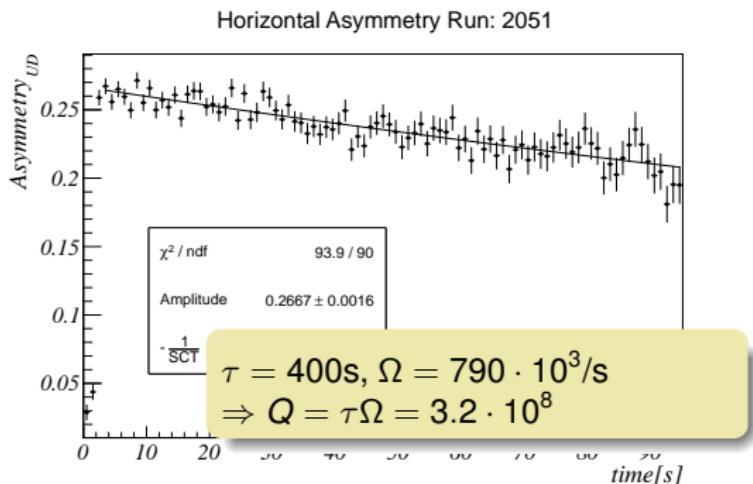
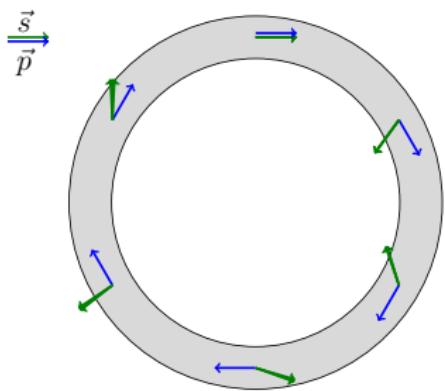
Long Spin Coherence Time



using correction sextupole to correct for higher order effects
leads to SCT of $\tau = 400$ s

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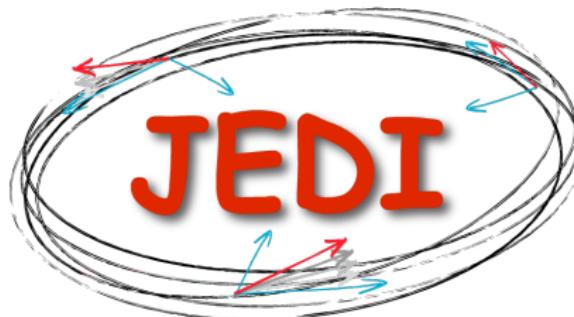
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JEDI Collaboration

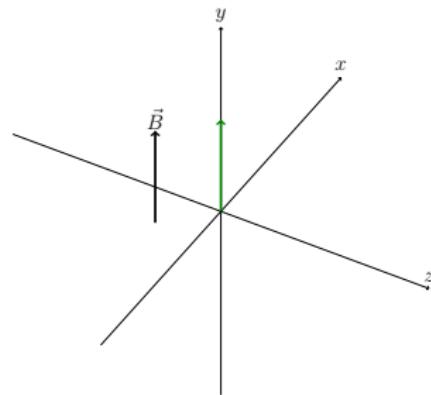
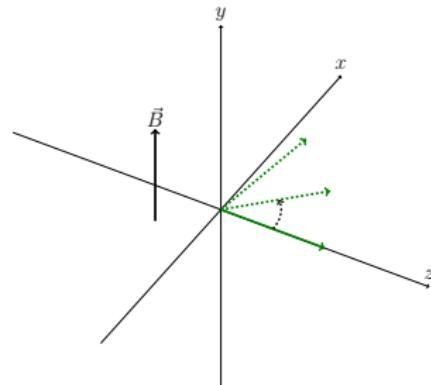
- **JEDI** = Jülich Electric Dipole Moment Investigations
- ≈ 100 members
(Aachen, Dubna, Ferrara, Indiana, Ithaca, Jülich, Krakow, Michigan, Minsk, Novosibirsk, St. Petersburg, Stockholm, Tbilisi, ...)
- ≈ 10 PhD students



Polarization Measurements

Polarization Measurements

- Up to know we looked at the horizontal polarization
- Difficult to measure because spin precesses in horizontal plane
- Let's look at vertical polarization



Vertical polarization P

Measure counting rates in left and right detector:

$$N_R \propto n^\uparrow \sigma_{\uparrow, R} + n^\downarrow \sigma_{\downarrow, R}$$
$$\stackrel{\Phi-\text{sym}}{=} n^\uparrow \sigma_{\uparrow, R} + n^\downarrow \sigma_{\uparrow, L}$$

$$N_L \propto n^\uparrow \sigma_{\uparrow, L} + n^\downarrow \sigma_{\downarrow, L}$$
$$\stackrel{\Phi-\text{sym}}{=} n^\uparrow \sigma_{\uparrow, L} + n^\downarrow \sigma_{\uparrow, R}$$

$n^\uparrow (n^\downarrow)$: nb. of beam particles with spin up (down)

$P = \frac{n^\uparrow - n^\downarrow}{n^\uparrow + n^\downarrow}$: Polarization

$\sigma_{\uparrow, R} \equiv \sigma_R$: cross section for scattering process to the right (R)
if spin is up (\uparrow)

$A = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}$: analyzing power

Vertical polarization

Calculate asymmetry:

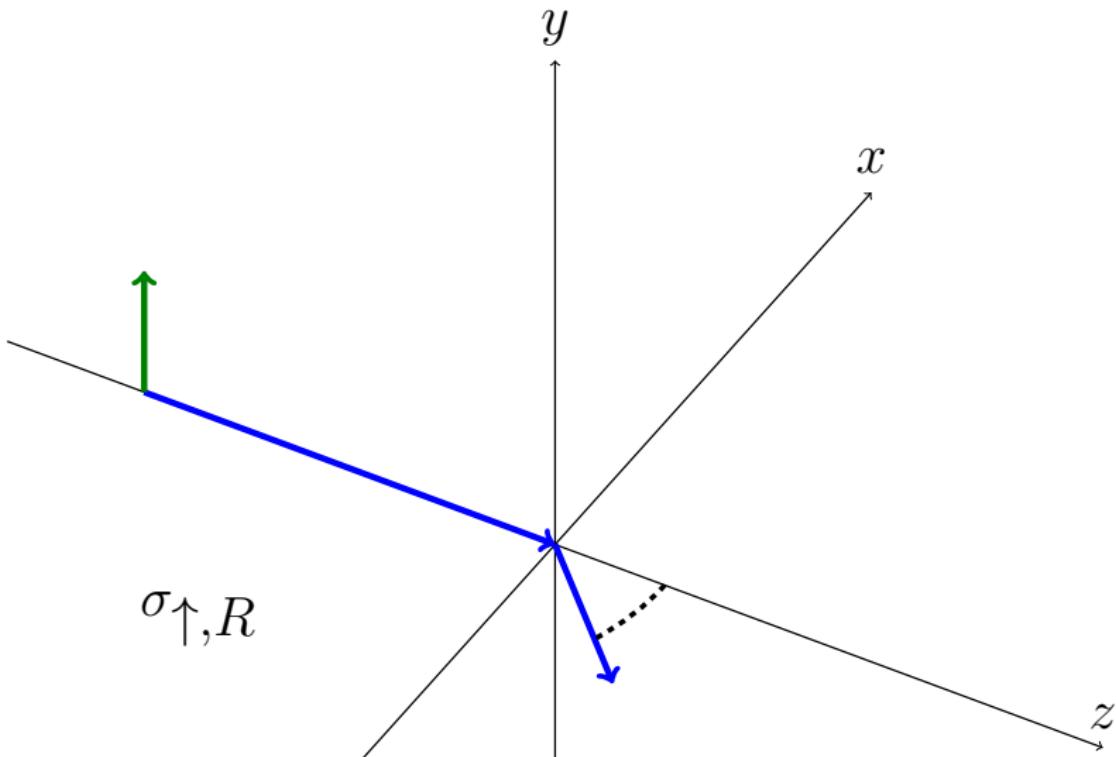
$$\epsilon = \frac{N_R - N_L}{N_R + N_L} = \underbrace{\frac{n^\uparrow - n^\downarrow}{n^\uparrow + n^\downarrow}}_P \underbrace{\frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}}_A = PA$$

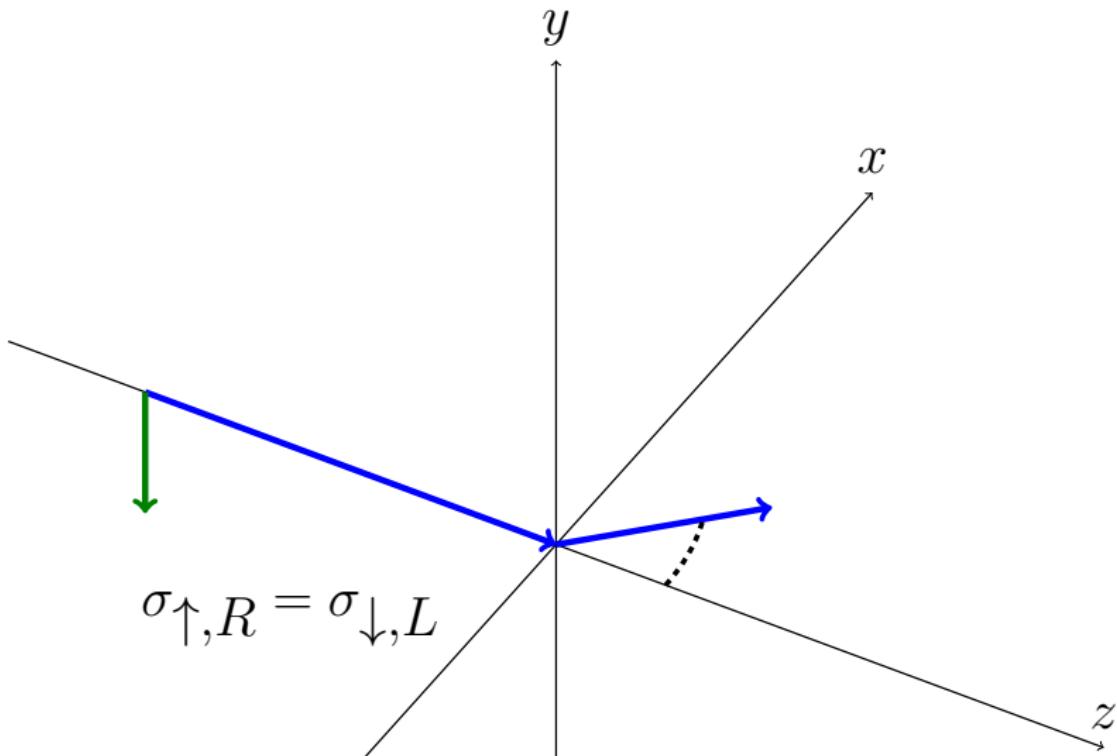
P : Polarization(to be determined), A : analyzing power(known)

$$P = \frac{\epsilon}{A}$$

Statistical error $\sigma_\epsilon = 1/\sqrt{N}$, $N = N_R + N_L$

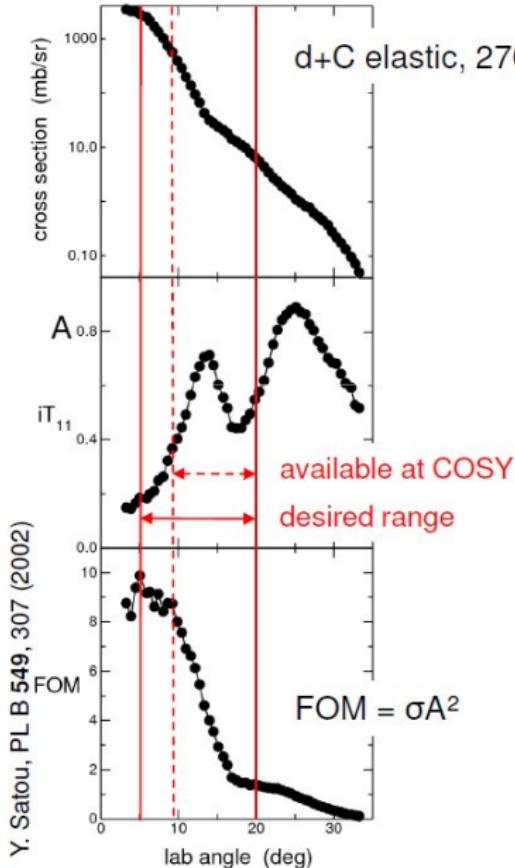
$$\Rightarrow \sigma_P = \frac{1}{A\sqrt{N}}, \quad \text{Figure of merit (FOM)} = \frac{1}{\sigma_P^2} = A^2 N$$





$$\sigma_{\uparrow,R} = \sigma_{\downarrow,L}$$

Polarimetry



d+C elastic, 270 MeV

Cross Section &
Analyzing Power
for deuterons

$$N_{R,L} \propto (1 \pm P A)$$

$$\epsilon = \frac{N_R - N_L}{N_R + N_L} = PA$$

A : analyzing power
 P : beam polarization

Topics for working group

- Derive relation between event rates and cross section
- Estimate event rates in experiment
- Determine polarization from asymmetry
 - Which angular range to choose?
 - Evaluate statistical error?
 - Which systematic errors occur?
- ...

Summary & Outlook

- EDMs of elementary particles are of high interest to disentangle various sources of \mathcal{CP} violation searched for to explain matter - antimatter asymmetry in the Universe
- EDM of charged particles can be measured in storage rings

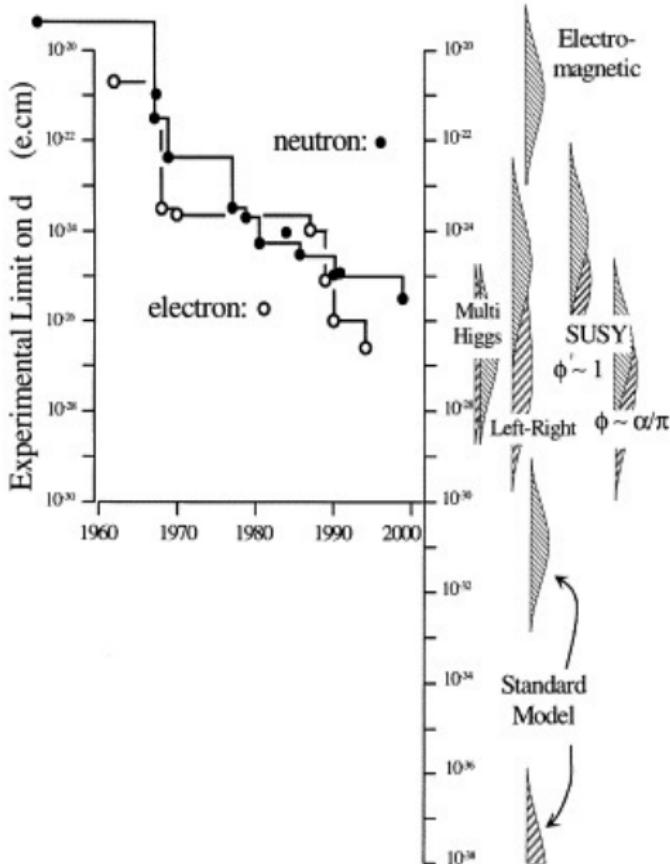
Plans in Jülich: 10^{-24} ecm at COSY

10^{-29} ecm with dedicated ring

- Experimentally very challenging because effect is tiny
- First promising results from test measurements at COSY

Spare

Electron and Neutron EDM



J. M. Pendlebury &
E.A. Hinds,
NIMA 440(2000) 471

EDM: SUSY Limits

electron:

$$\text{MSSM: } \varphi \approx 1 \Rightarrow d = 10^{-24} - 10^{-27} \text{ e}\cdot\text{cm}$$

$$\varphi \approx \alpha/\pi \Rightarrow d = 10^{-26} - 10^{-30} \text{ e}\cdot\text{cm}$$

neutron:

$$\text{MSSM: } d = 10^{-24} \text{ e}\cdot\text{cm} \cdot \sin \phi_{CP} \frac{200 \text{ GeV}}{M_{SUSY}}$$

Electrostatic Deflectors

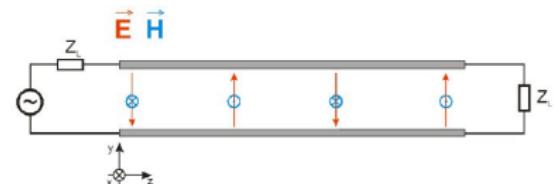


- Electrostatic deflectors from Fermilab ($\pm 125\text{kV}$ at 5 cm
 $\hat{=} 5\text{MV/m}$)
- large-grain Nb at plate separation of a few cm yields $\approx 20\text{MV/m}$

Wien Filter



Conventional design
R. Gebel, S. Mey (FZ Jülich)



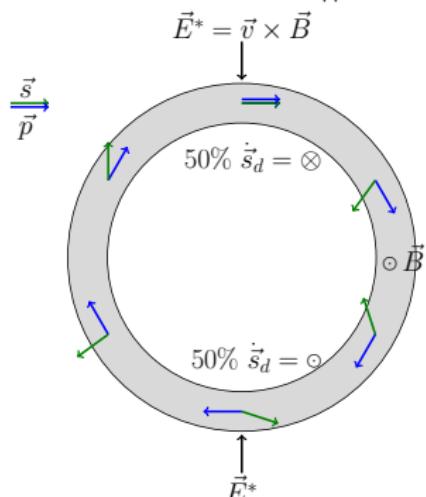
stripline design
D. Hölscher, J. Slim
(IHF RWTH Aachen)

1. Pure Magnetic Ring

$$\vec{\Omega} = \frac{e\hbar}{mc} \left(\textcolor{green}{G}\vec{B} + \frac{1}{2} \textcolor{red}{\eta} \vec{v} \times \vec{B} \right)$$

Problem:

Due to precession caused by magnetic moment, 50% of time longitudinal polarization component is || to momentum, 50% of the time it is anti-||.



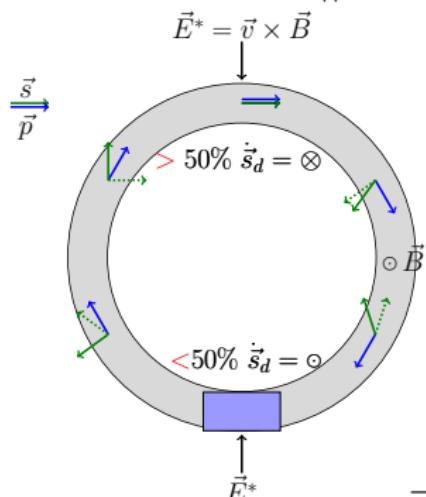
E^* field in the particle rest frame tilts spin due to EDM up and down
⇒ **no net EDM effect**

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Use resonant “magic Wien-Filter” in ring ($\vec{E}_W + \vec{v} \times \vec{B}_W = 0$):

$E_W^* = 0 \rightarrow$ part. trajectory is not affected but

$B_W^* \neq 0 \rightarrow$ mag. mom. is influenced

⇒ **net EDM effect can be observed!**

2. Pure Electric Ring

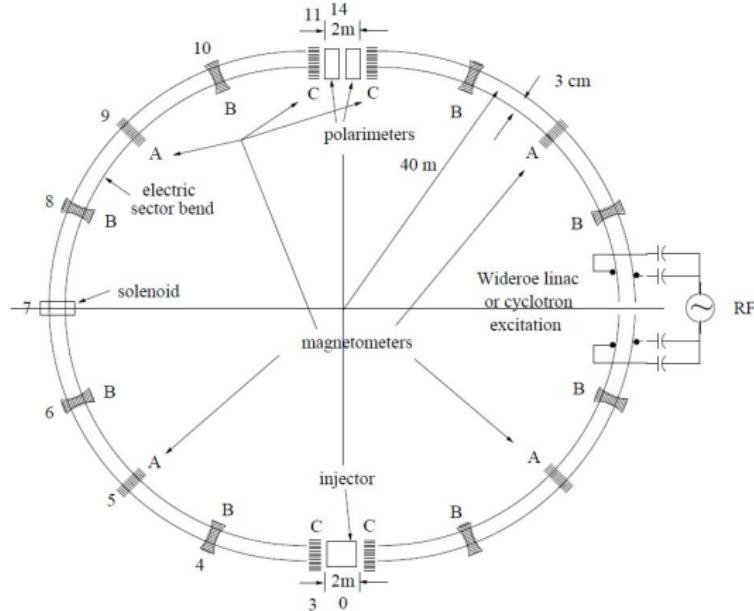


Figure 3: An all-electric storage ring lattice for measuring the electric dipole moment of the proton. Except for having longer straight sections and separated beam channels, the all-in-one lattice of Fig. 1 is patterned after this lattice. Quadrupole and sextupole families, and tunes and lattice functions of the all-in-one lattice of Fig. 1 will be quite close to those given for this lattice in reference[3]. The match will be even closer with magnetic field set to zero for proton operation.

Brookhaven National Laboratory (BNL) Proposal

3. Combined \vec{E}/\vec{B} ring

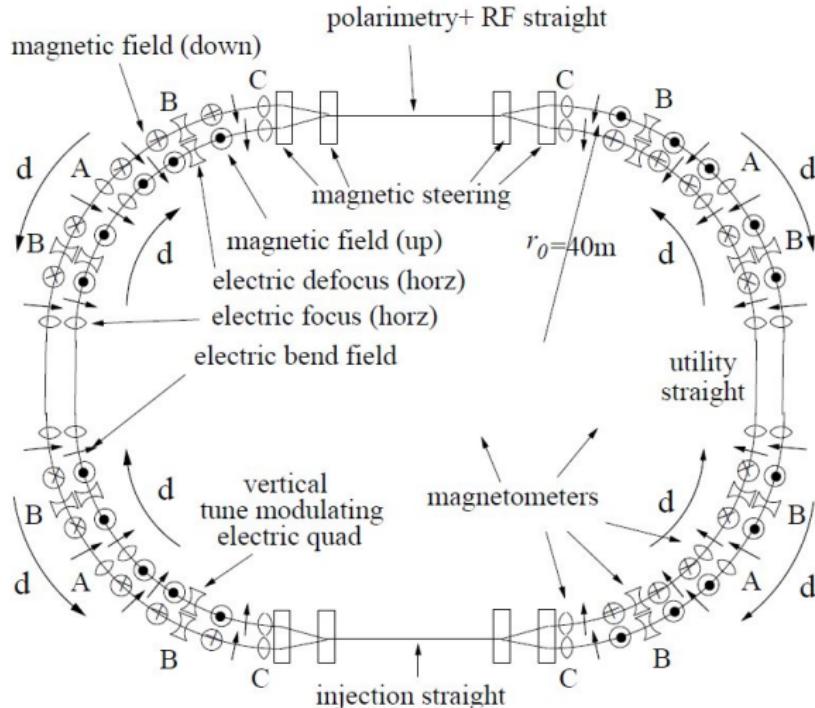


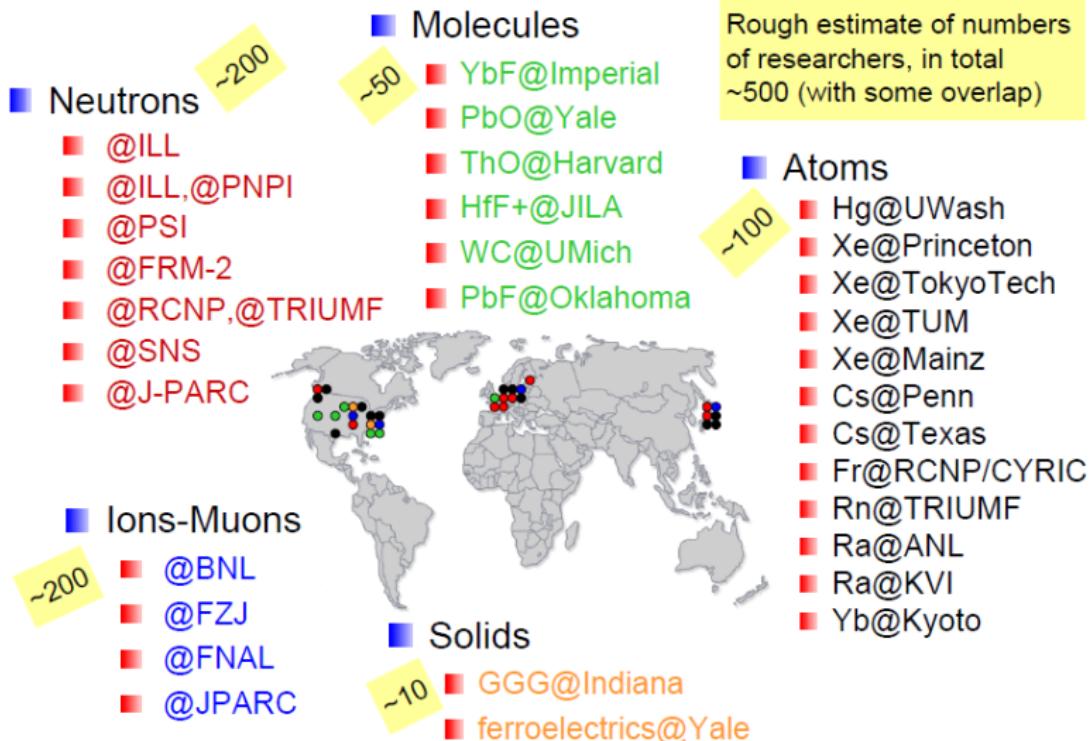
Figure 1: "All-In-One" lattice for measuring EDM's of protons, deuterons, and helions.

Under discussion at Forschungszentrum Jülich (design: R. Talman)

Summary of different options

		
1.) pure magnetic ring (Jülich)	existing (upgraded) COSY ring can be used , shorter time scale	lower sensitivity
2.) pure electric ring (BNL)	no \vec{B} field needed	works only for p
3.) combined ring (Jülich)	works for $p, d, {}^3\text{He}, \dots$	both \vec{E} and \vec{B} required

EDM Activities Around the World



Systematics

- Splitting of beams: $\delta y = \pm \frac{\beta c R_0 B_r}{E_r Q_y^2} = \pm 1 \cdot 10^{-12} \text{ m}$
- $Q_y \approx 0.1$: vertical tune
- Modulate $Q_y = Q_y^0 (1 - m \cos(\omega_m t))$, $m \approx 0.1$
- Splitting causes B field of $\approx 0.4 \cdot 10^{-3} \text{ fT}$
- in one year: 10^4 fills of $1000 \text{ s} \Rightarrow \sigma_B = 0.4 \cdot 10^{-1} \text{ fT}$ per fill needed
- Need sensitivity $1.25 \text{ fT}/\sqrt{\text{Hz}}$

Systematics

