Electric Dipole Moment Measurements at Storage Rings

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Outline

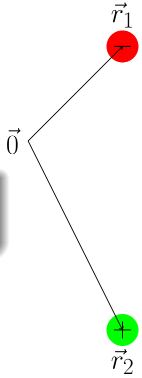
- **Seminar**
  - **Introduction: Electric Dipole Moments (EDMs):**
    What is it?
    Why is it interesting?
    What do we know about EDMs?
  - **Experimental Method:**
    How to measure charged particle EDMs?
  - **Results of first test measurements:**
    Spin Coherence time and Spin tune
- **Lecture**
  - **Polarization Measurement**

What is it?

Electric Dipoles

Classical definition:

$$\vec{d} = \sum_i q_i \vec{r}_i$$



Order of magnitude

	atomic physics	**hadron physics**
charges	e	
$\lvert\vec{r}_1 - \vec{r}_2\rvert$	1 Å= 10^{-8}cm	
EDM		
naive expectation	$10^{-8} e \cdot \mathrm{cm}$	
observed	water molecule	
	$2 \cdot 10^{-8} e\cdot$ cm	

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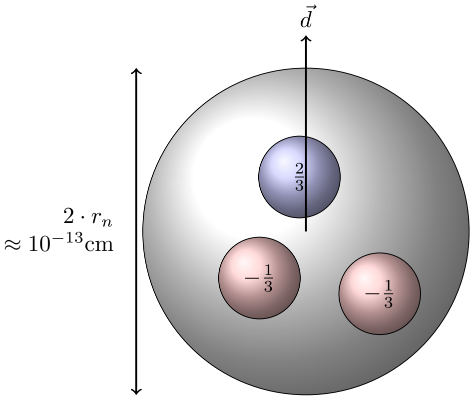
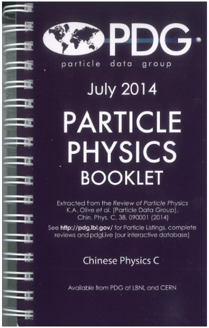
$\|\vec{r}_1 - \vec{r}_2\|$	1 Å= 10^{-8}cm	$1\mathrm{fm} = 10^{-13}$cm
EDM		
naive expectation	$10^{-8} e \cdot \mathrm{cm}$	$10^{-13} e \cdot \mathrm{cm}$
observed	water molecule	neutron
	$2 \cdot 10^{-8} e \cdot$ cm	$< 3 \cdot 10^{-26} e \cdot$ cm

Order of magnitude



neutron EDM of $d_n = 3 \cdot 10^{-26} e{\cdot}\text{cm}$ corresponds to separation of $u-$ from $d-$quarks of $\approx 5 \cdot 10^{-26}\text{cm}$



n

$I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$

Mass $m = 1.0086649160 \pm 0.0000000004$ u
Mass $m = 939.565379 \pm 0.000021$ MeV [a]
$(m_n - m_{\overline{n}})/\ m_n = (9 \pm 6) \times 10^{-5}$
$m_n - m_p = 1.2933322 \pm 0.0000004$ MeV
$= 0.00138844919(45)$ u
Mean life $\tau = 880.3 \pm 1.1$ s (S = 1.9)
$c\tau = 2.6391 \times 10^8$ km
Magnetic moment $\mu = -1.9130427 \pm 0.0000005\ \mu_N$
Electric dipole moment $d < 0.29 \times 10^{-25}$ $e$cm, CL = 90%
Mean-square charge radius $\langle r_n^2 \rangle = -0.1161 \pm 0.0022$ fm^2 (S = 1.3)
Magnetic radius $\sqrt{\langle r_M^2 \rangle} = 0.862^{+0.009}_{-0.008}$ fm
Electric polarizability $\alpha = (11.6 \pm 1.5) \times 10^{-4}$ fm^3
Magnetic polarizability $\beta = (3.7 \pm 2.0) \times 10^{-4}$ fm^3
Charge $q = (-0.2 \pm 0.8) \times 10^{-21}$ e
Mean $n\overline{n}$-oscillation time $> 8.6 \times 10^7$ s, CL = 90% (free n)
Mean $n\overline{n}$-oscillation time $> 1.3 \times 10^8$ s, CL = 90% [f] (bound n)
Mean nn'-oscillation time > 414 s, CL = 90% [g]

Operator $\vec{d} = q\vec{r}$

is odd under parity transformation ($\vec{r} \to -\vec{r}$):

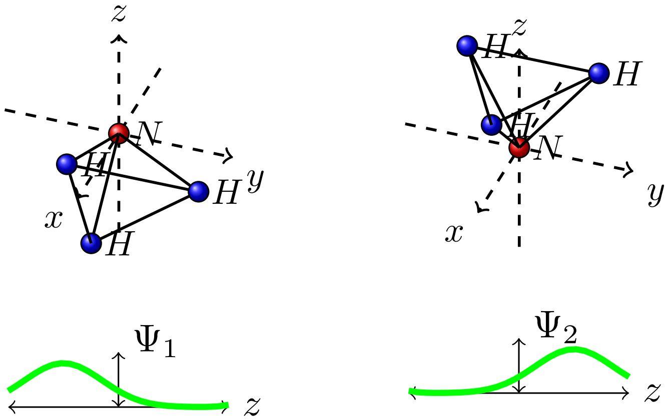
$$\mathcal{P}^{-1}\vec{d}\mathcal{P} = -\vec{d}$$

Consequences:
In a state $|a\rangle$ of given parity the expectation value is 0:

$$\left\langle a|\vec{d}|a\right\rangle = -\left\langle a|\vec{d}|a\right\rangle$$

but if $|a\rangle = \alpha|P = +\rangle + \beta|P = -\rangle$
in general $\left\langle a|\vec{d}|a\right\rangle \neq 0 \Rightarrow$ i.e. molecules

EDM of molecules



ground state: mixture of

$$\Psi_s = \frac{1}{\sqrt{2}}\left(\Psi_1 + \Psi_2\right), \quad P = +$$

$$\Psi_a = \frac{1}{\sqrt{2}}\left(\Psi_1 - \Psi_2\right), \quad P = -$$

Order of magnitude

Molecules can have large EDM because of degenerated ground states with different parity

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Elementary particles (including hadrons) have a definite parity and cannot posses an EDM
$P|\text{had}>= \pm 1|\text{had}>$

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Elementary particles (including hadrons) have a definite parity and cannot posses an EDM
$P|\text{had}>= \pm 1|\text{had}>$

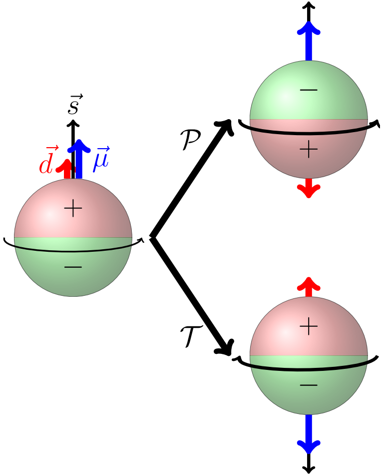
unless

$\mathcal{P}$ and time reversal $\mathcal{T}$ invariance are violated!

$\mathcal{T}$ and $\mathcal{P}$ violation of EDM

$\vec{d}$: EDM
$\vec{\mu}$: magnetic moment
both $||$ to spin

$$
\begin{aligned}
& H = -\mu\vec{\sigma}\cdot\vec{B} - d\vec{\sigma}\cdot\vec{E} \\
\mathcal{T}: \quad & H = -\mu\vec{\sigma}\cdot\vec{B} + d\vec{\sigma}\cdot\vec{E} \\
\mathcal{P}: \quad & H = -\mu\vec{\sigma}\cdot\vec{B} + d\vec{\sigma}\cdot\vec{E}
\end{aligned}
$$



$\Rightarrow$ EDM measurement tests violation of fundamental symmetries $\mathcal{P}$ and $\mathcal{T}$ ($\stackrel{\mathcal{CPT}}{=}$ $\mathcal{CP}$)

Symmetries in Standard Model

	electro-mag.	weak	strong
$\mathcal{C}$	✓	$\lightning$	✓
$\mathcal{P}$	✓	$\lightning$	(✓)
$\mathcal{T} \overset{CPT}{\to} \mathcal{CP}$	✓	($\lightning$)	(✓)

- $\mathcal{C}$ and $\mathcal{P}$ are maximally violated in weak interactions (Lee, Yang, Wu)
- $\mathcal{CP}$ violation discovered in kaon decays (Cronin,Fitch) described by CKM-matrix in Standard Model
- $\mathcal{CP}$ violation allowed in strong interaction but corresponding parameter $\theta_{QCD} \lessapprox 10^{-10}$ (strong $\mathcal{CP}$-problem)

Sources of $\mathcal{CP}-$Violation

Standard Model	
Weak interaction	
CKM matrix	$\rightarrow$ unobservably small EDMs
Strong interaction	
θ_{QCD}	$\rightarrow$ best limit from neutron EDM
beyond Standard Model	
e.g. SUSY	$\rightarrow$ accessible by EDM measurements

Why is it interesting?

Matter-Antimatter Asymmetry

Excess of matter in the universe:

	observed	SM prediction
$\eta = \frac{n_B - n_{\bar{B}}}{n_\gamma}$	$\mathbf{6 \times 10^{-10}}$	$\mathbf{10^{-18}}$

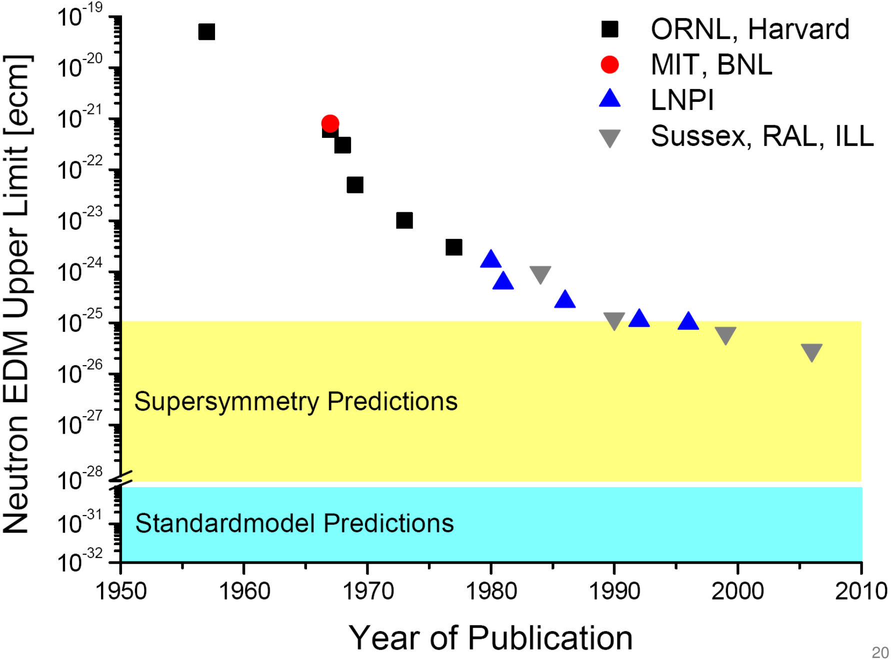
Sakharov (1967): $\mathcal{CP}$ violation needed for baryogenesis

$\Rightarrow$ New $\mathcal{CP}$ violating sources beyond SM needed to explain this discrepancy

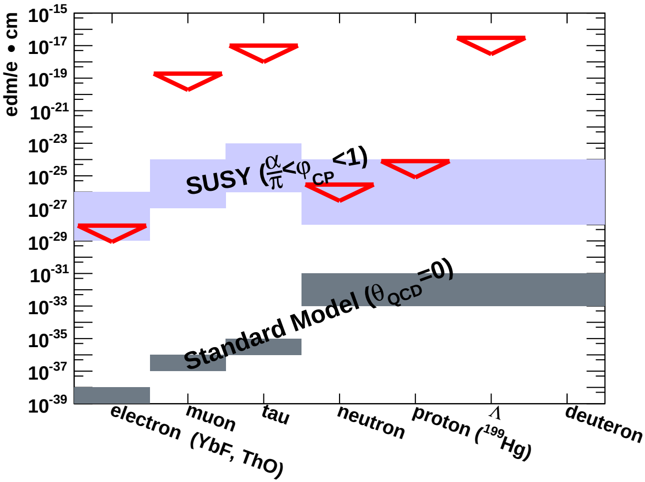
They could manifest in EDMs of elementary particles

What do we know about EDMs?

History of Neutron EDM



EDM: Current Upper Limits



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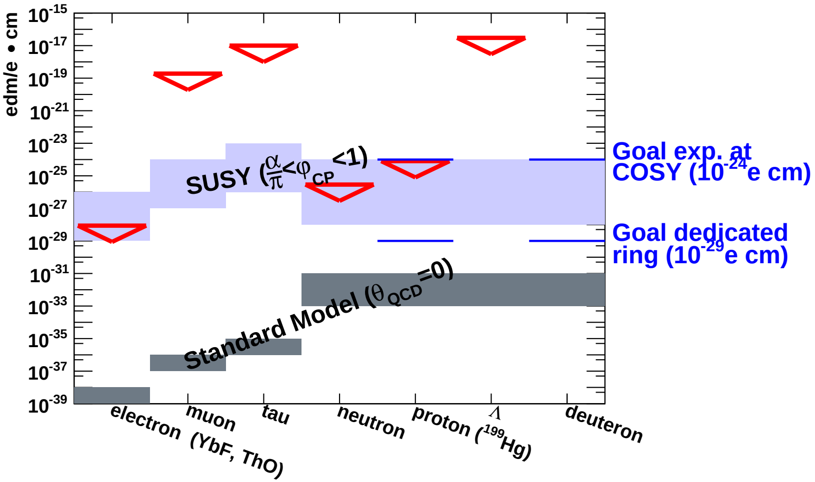
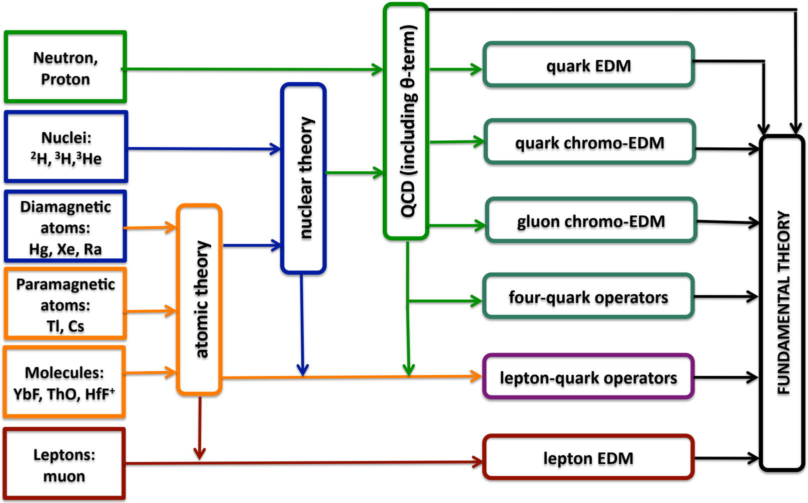


FZ Jülich: EDMs of **charged** hadrons: p, d, ^{3}He

Why Charged Particle EDMs?

- no direct measurements for charged hadrons exist
- potentially higher sensitivity (compared to neutrons):
  - longer life time,
  - more stored protons/deuterons
- complementary to neutron EDM:
  $d_d \stackrel{?}{=} d_p + d_n \Rightarrow$ access to θ_{QCD}
- EDM of one particle alone not sufficient to identify $\mathcal{CP}-$violating source

Sources of $\mathcal{CP}$ Violation

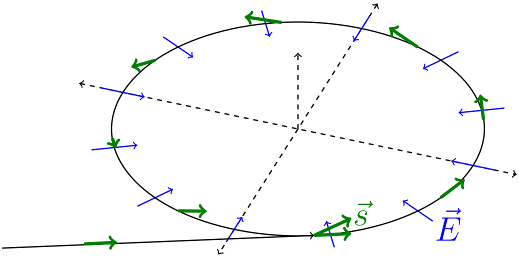


J. de Vries

How to measure charged particle EDMs?

Experimental Method: Generic Idea

For **all** EDM experiments (neutron, proton, atoms, ...):
Interaction of $\vec{d}$ with electric field $\vec{E}$
For charged particles: apply electric field in a storage ring:



$$\frac{\mathrm{d}\vec{s}}{\mathrm{d}t} \propto d\vec{E} \times \vec{s}$$

In general:

$$\frac{\mathrm{d}\vec{s}}{\mathrm{d}t} = \vec{\Omega} \times \vec{s}$$

build-up of vertical polarization $s_{\perp} \propto |d|$

Experimental Requirements

- high precision storage ring
  (alignment, stability, field homogeneity)
- high intensity beams ($N = 4 \cdot 10^{10}$ per fill)
- polarized hadron beams ($P = 0.8$)
- large electric fields ($E = 10$ MV/m)
- long spin coherence time ($\tau = 1000$ s),
- polarimetry (analyzing power $A = 0.6$, acc. $f = 0.005$)

$$\sigma_{\text{stat}} \approx \frac{1}{\sqrt{Nf\tau}PAE} \quad \Rightarrow \sigma_{\text{stat}}(1\text{year}) = 10^{-29}\ e\cdot\text{cm}$$

challenge: get σ_{sys} to the same level

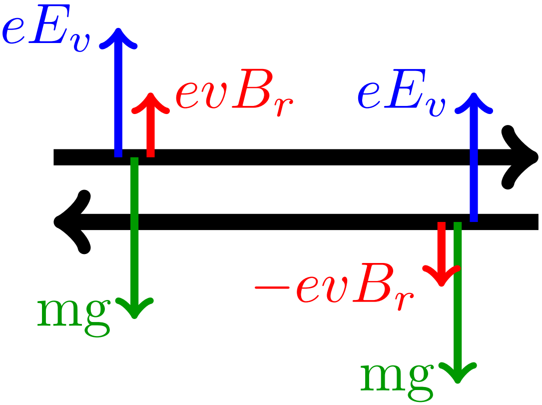
Systematics

Major source:
Radial B field mimics an EDM effect:

- Difficulty: even small radial magnetic field, B_r can mimic EDM effect if :$\mu B_r \approx dE_r$
- Suppose $d = 10^{-29} e{\cdot}\text{cm}$ in a field of $E_r = 10\text{MV/m}$
- This corresponds to a magnetic field:
  $B_r = \frac{dE_r}{\mu_N} = \frac{10^{-22} e\mathrm{V}}{3.1 \cdot 10^{-8} e\mathrm{V}/\mathrm{T}} \approx 3 \cdot 10^{-17}\mathrm{T}$

Solution: Use two beams running clockwise and counter clockwise, separation of the two beams is sensitive to B_r

Systematics



Sensitivity needed: $1.25\,\mathrm{fT}/\sqrt{\mathrm{Hz}}$ for $d = 10^{-29}\,e\,\mathrm{cm}$
(possible with SQUID technology)

Spin Precession: Thomas-BMT Equation

$$\frac{\mathrm{d}\vec{s}}{\mathrm{d}t} = \vec{\Omega} \times \vec{s} = \frac{e}{m}[G\vec{B} + \left(G - \frac{1}{\gamma^2 - 1}\right)\vec{v} \times \vec{E} + \frac{m}{e\,s} d(\vec{E} + \vec{v} \times \vec{B})] \times \vec{s}$$

Ω: angular precession frequency

G: anomalous magnetic moment

d: electric dipole moment

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dedicated ring: pure electric field,

freeze horizontal spin motion $\left(G - \frac{1}{\gamma^2 - 1}\right) = 0$

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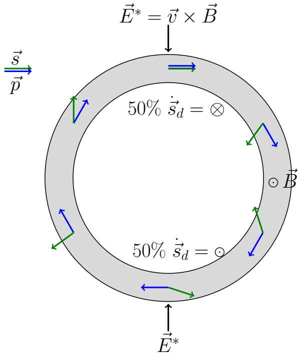
COSY: pure magnetic ring
access to EDM via motional electric field $\vec{v} \times \vec{B}$,
requires additional radio-frequency E and B fields
to suppress $G\vec{B}$ contribution

Pure Magnetic Ring

$$\frac{\mathrm{d}\vec{s}}{\mathrm{d}t} = \vec{\Omega} \times \vec{s} = \frac{e}{m}\left(G\vec{B} + \frac{m}{es} d\vec{v} \times \vec{B}\right) \times \vec{s}$$

Problem:
Due to precession caused by magnetic moment, 50% of time longitudinal polarization component is || to momentum, 50% of the time it is anti-||.



E^* field in the particle rest frame tilts spin due to EDM up and down $\Rightarrow$ **no net EDM effect**

Pure Magnetic Ring

$$\frac{d\vec{s}}{dt} = \vec{\Omega} \times \vec{s} = \frac{e}{m}\left(G\vec{B} + \frac{m}{es} d\vec{v} \times \vec{B}\right) \times \vec{s}$$

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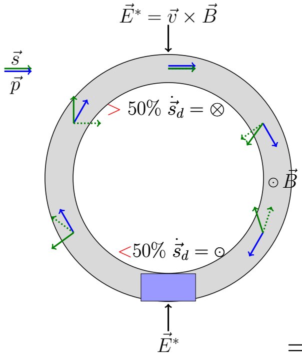

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$\Rightarrow$ **no net EDM effect**

Use resonant "magic Wien-Filter" in ring ($\vec{E}_W + \vec{v} \times \vec{B}_W = 0$):
$E^*_W = 0 \rightarrow$ part. trajectory is not affected but
$B^*_W \neq 0 \rightarrow$ mag. mom. is influenced

$\Rightarrow$ **net EDM effect can be observed!**

Spin Precession: Thomas-BMT Equation

$$\frac{\mathrm{d}\vec{s}}{\mathrm{d}t} = \vec{\Omega} \times \vec{s} = \frac{e}{m}\left[G\vec{B} + \left(G - \frac{1}{\gamma^2 - 1}\right)\vec{v} \times \vec{E} + \frac{m}{e\,s} d(\vec{E} + \vec{v} \times \vec{B})\right] \times \vec{s}$$

Ω: angular precession frequency

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COSY: pure magnetic ring
access to EDM via motional electric field $\vec{v} \times \vec{B}$,
requires additional radio-frequency E and B fields
to suppress $G\vec{B}$ contribution

neglecting EDM term
spin tune: $\nu_s \approx \frac{|\vec{\Omega}|}{|\omega_{\text{cyc}}|} = \gamma G$, $\quad (\vec{\omega}_{cyc} = \frac{e}{\gamma m}\vec{B})$

Results of first test measurements

Cooler Synchrotron COSY

COSY provides (polarized) protons and deuterons with
$p = 0.3 - 3.7\text{GeV}/c$

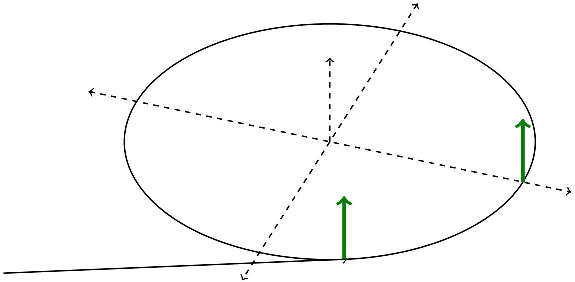
$\Rightarrow$ **Ideal starting point for charged particle EDM searches**

R & D at COSY

- maximize spin coherence time (SCT)
- precise measurement of spin precession (spin tune)
- rf- Wien filter design and construction
- tests of electro static deflectors (goal: field strength > 10 MV/m)
- development of high precision beam position monitors
- polarimeter development
- spin tracking simulation tools

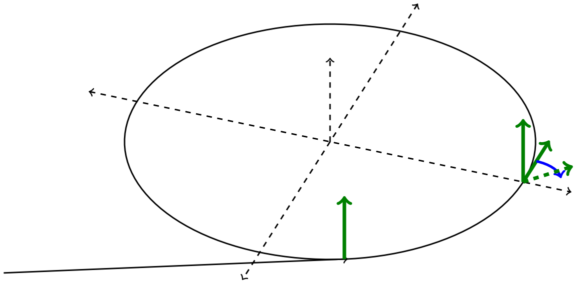
Experimental Setup

- Inject and accelerate vertically polarized deuterons to $p \approx 1$ GeV/c



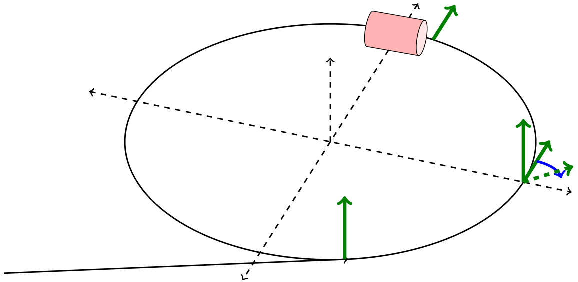
Experimental Setup

- Inject and accelerate vertically polarized deuterons to $p \approx 1$ GeV/c
- flip spin with help of solenoid into horizontal plane



Experimental Setup

- Inject and accelerate vertically polarized deuterons to $p \approx 1$ GeV/c
- flip spin with help of solenoid into horizontal plane
- Extract beam slowly (in 100 s) on target
- Measure asymmetry and determine spin precession



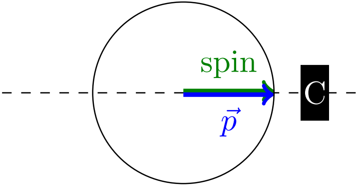
Asymmetry Measurements

- Detector signal $N^{up,dn} \propto (1 \pm P\,A\,\sin(\gamma G f_{rev} t))$

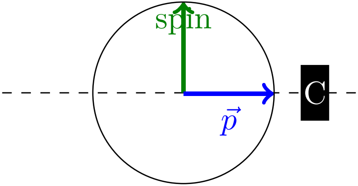
$$A_{up,dn} = \frac{N^{up} - N^{dn}}{N^{up} + N^{dn}} = P\,A\,\sin(\gamma G f_{rev} t) = P\,A\,\sin(\nu_s n_{turn})$$

A: analyzing power, P : polarization

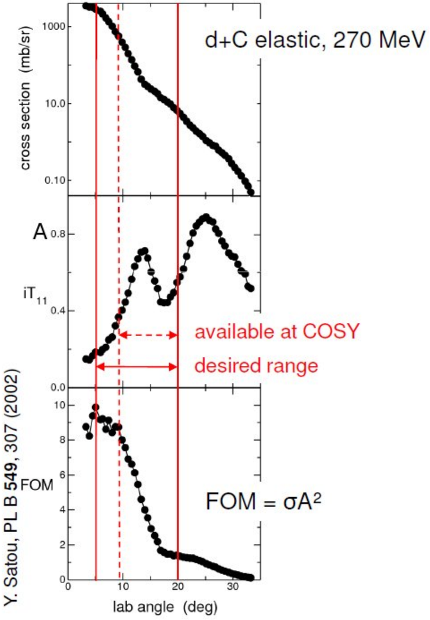
$A_{up,dn} = 0$



$A_{up,dn} = PA$



Polarimetry



Cross Section & Analyzing Power for deuterons

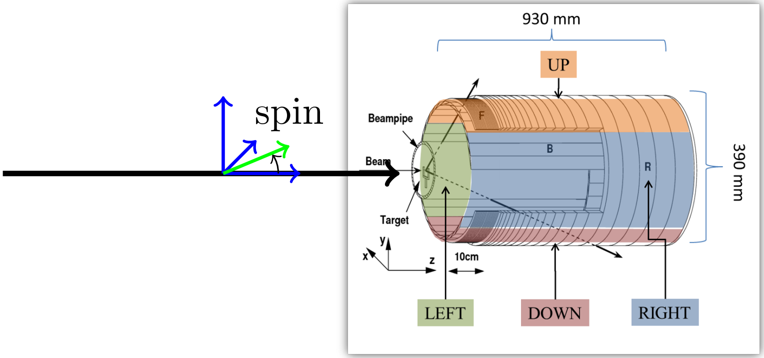
$N_{up,dn} \propto$
$(1 \pm P\,A\,\sin(\nu_s f_{rev} t))$

$$A_{up,dn} = \frac{N^{up} - N^{dn}}{N^{up} + N^{dn}}$$
$$= P\,A\,\sin(\nu_s f_{rev} t)$$
$$= P\,A\,\sin(\nu_s n_{turn})$$

A : analyzing power
P : beam polarization

Polarimeter

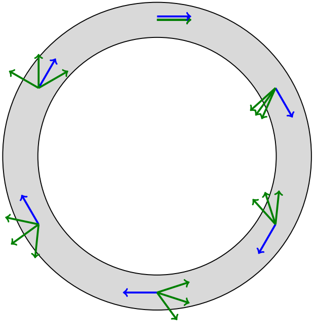
elastic deuteron-carbon scattering
Up/Down asymmetry $\propto$ horizontal polarization $\rightarrow \nu_s = \gamma G$
Left/Right asymmetry $\propto$ vertical polarization $\rightarrow d$

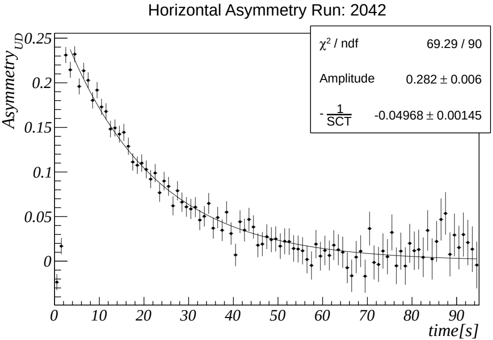


$$N_{up,dn} \propto 1 \pm PA \sin(\nu_s n_{turn}), \quad f_{rev} \approx 750\,\text{kHz}$$

Results: Spin Coherence Time (SCT)

Short Spin Coherence Time





unbunched beam

$\Delta p/p = 10^{-5} \Rightarrow \Delta\gamma/\gamma = 2\cdot 10^{-6}$, $T_{rev} \approx 10^{-6}$ s

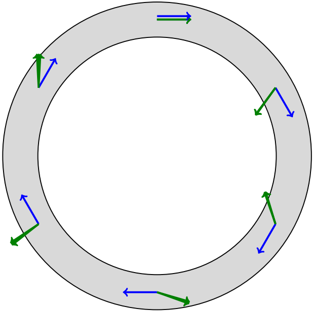
$\Rightarrow$ decoherence after < 1 s

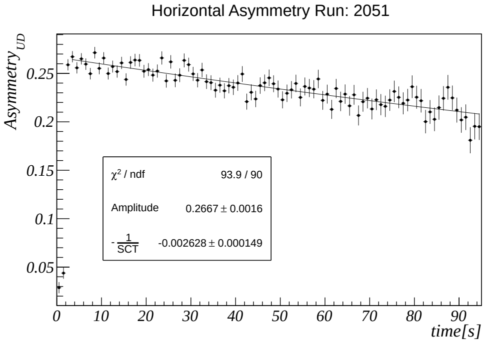
cooled bunched beam eliminates 1st order effects in $\Delta p/p$

$\Rightarrow$ SCT τ = 20 s

Results: Spin Coherence Time (SCT)

Long Spin Coherence Time

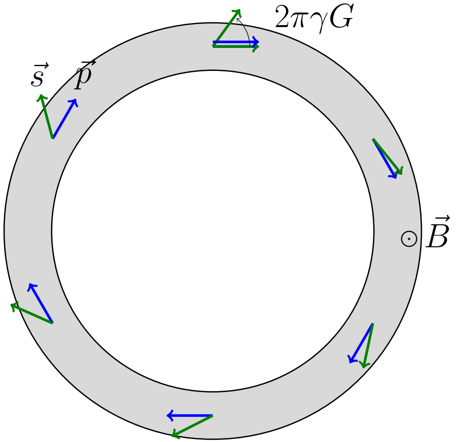




using correction sextupole to correct for higher order effects leads to SCT of $\tau =$400 s

Spin Tune ν_s

Spin tune: $\nu_s = \gamma G = \dfrac{\text{nb. of spin rotations}}{\text{nb. of particle revolutions}}$

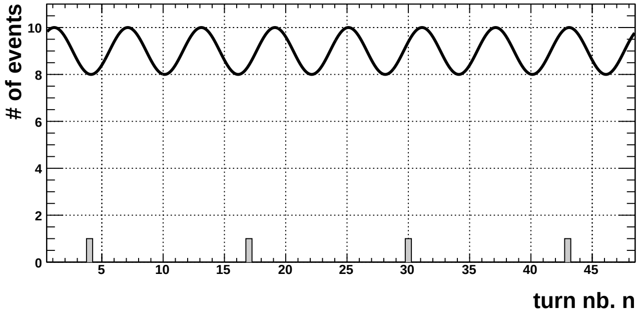


deuterons: $p_d = 1$ GeV/c ($\gamma = 1.13$), $G = -0.14256177(72)$

$\Rightarrow \nu_s = \gamma G \approx -0.161$

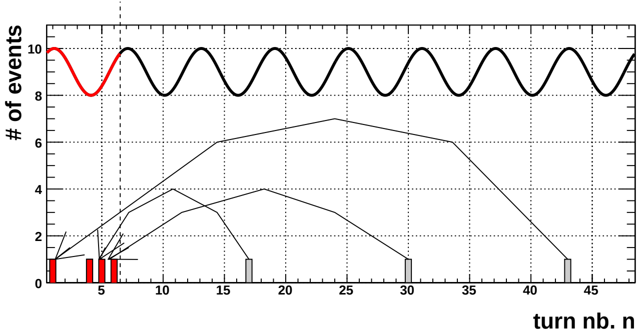
Spin Tune ν_s measurement

- Problem: detector rate $\approx$ 5 kHz, $f_{rev} = 750$kHz
  $\Rightarrow$ only 1 hit every 25th period
- not possible to use usual χ^2-fit
- use unbinned Maximum Likelihood (under investigation)

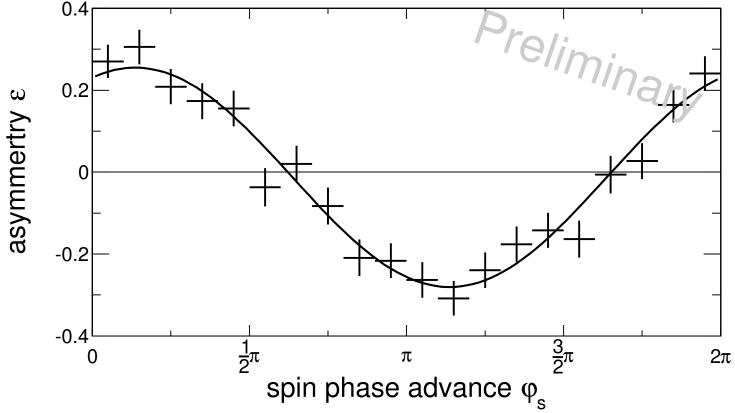


Spin Tune ν_s measurement

- map all events into first period ($T = 1/(\nu_s f_{rev}) \approx 8\mu$s) and perform χ^2-fit
  (requires knowledge of $\nu_s f_{rev}$)
- Analysis is done in macroscopic time bins of 10^6 turns
  (≈ 1.3 s)

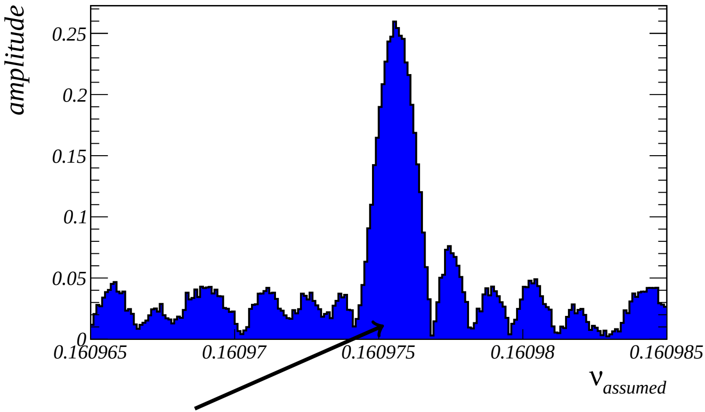


Asymmetry in 1st period



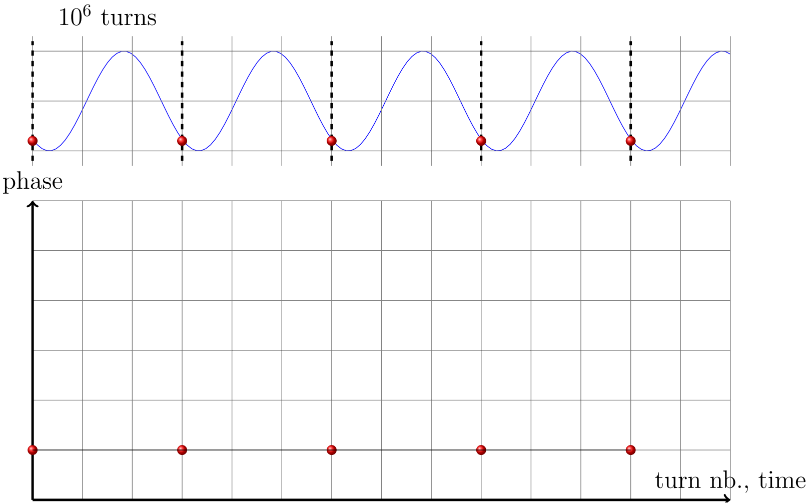
- only works if $T_s = \dfrac{1}{\nu_s f_{rev}}$ is correct.

Scan of ν_s



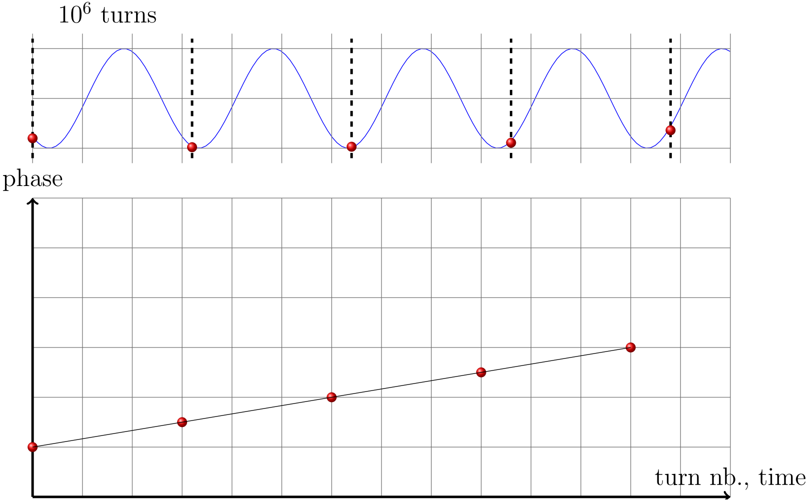
- allows for $\sigma_{\nu_s} \approx 10^{-6}$
- now fix ν_s at maximum and look at phase vs. turn number
  phase is determined for turn intervals of 10^6 turns

Phase Measurements



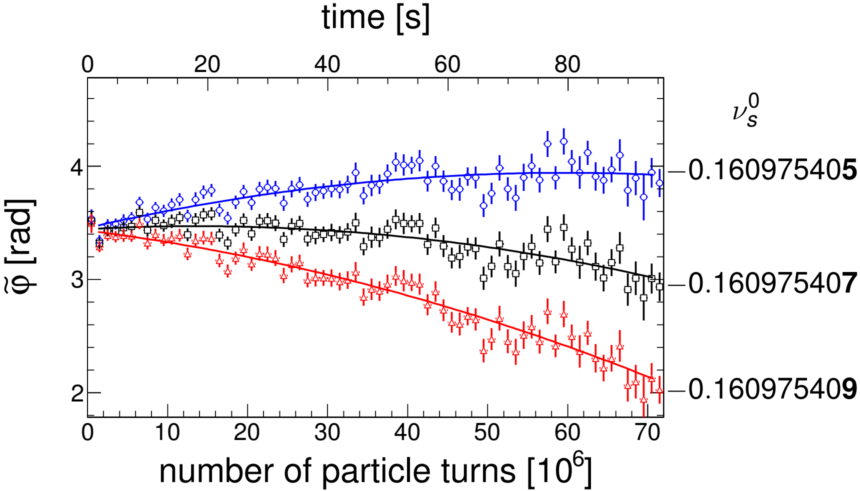
1st derivative gives deviation from assumed spin tune

Phase Measurements



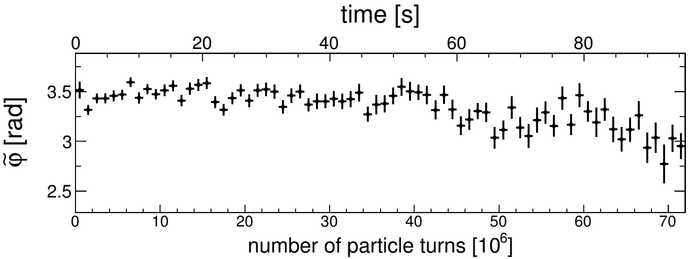
1st derivative gives deviation from assumed spin tune

Phase vs. turn number

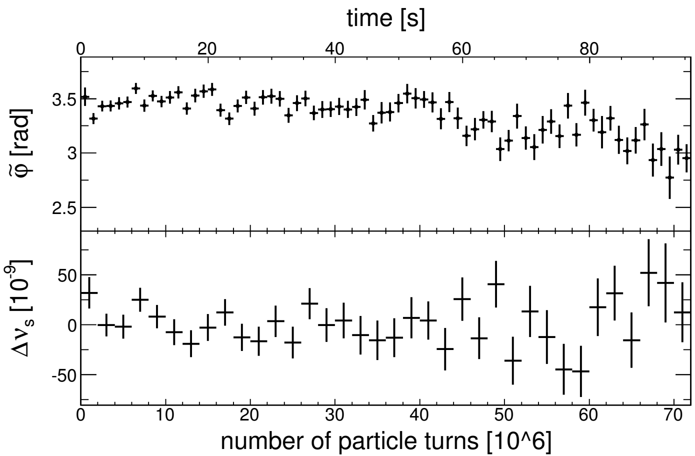


$$\nu_s(n) = \nu_s^0 + \frac{1}{2\pi}\frac{\mathrm{d}\tilde{\varphi}}{\mathrm{d}n}$$

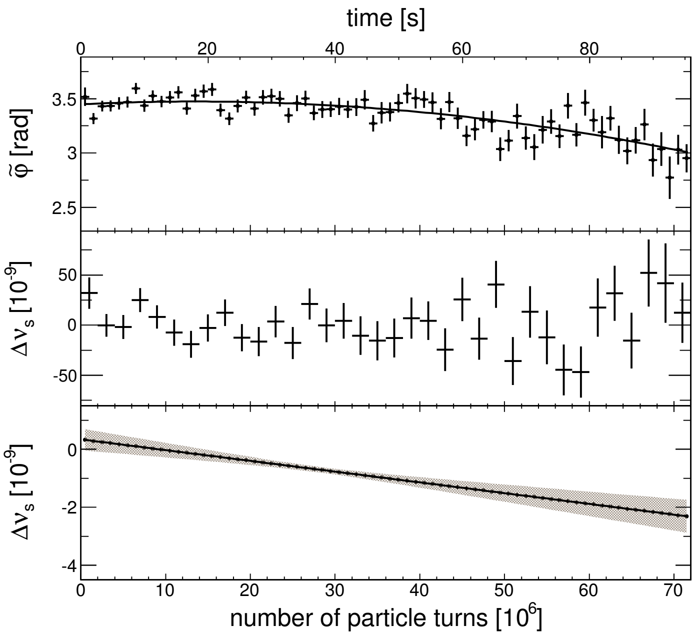
Results: Spin Tune ν_s



Results: Spin Tune ν_s



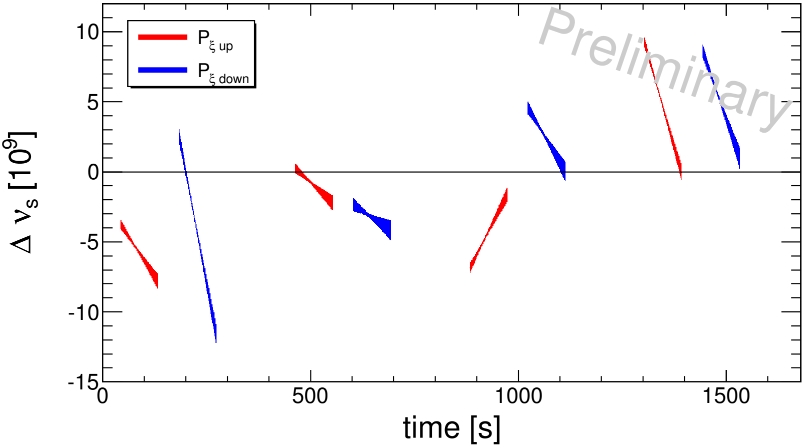
Results: Spin Tune ν_s



Spin Tune Measurement

- precision of spin tune measurement 10^{-10} in one cycle
- spin rotation due to electric dipole moment:
  $\nu_s = \frac{vm\gamma d}{es} = 5 \cdot 10^{-11}$ for $d = 10^{-24} e\,\mathrm{cm}$
  (in addition rotations due to G and imperfections)
- Compare to muon $g-2$: $\sigma_{\nu_s} \approx 3 \cdot 10^{-8}$ per year
  main difference: measurement duration 600μ s compared to 100 s
- spin tune measurement can now be used as tool to investigate systematic errors

Spin Tune as tool to investigate systematics



JEDI Collaboration

- **JEDI** = **J**ülich **E**lectric **D**ipole Moment **I**nvestigations
- $\approx$ 100 members
  (Aachen, Daejeon, Dubna, Ferrara, Grenoble, Indiana, Ithaca, Jülich, Krakow, Michigan, Minsk, Novosibirsk, St. Petersburg, Stockholm, Tbilisi, . . .)
- $\approx$ 10 PhD students

Storage Ring EDM Efforts

Common R&D work

- Spin Coherence Time
- BPMs
- Spin Tracking
- Polarimetry
- electric fields
- ...

BNL

- all electric ring (p)

Jülich

- first direct measurement with upgraded COSY
- dedicated ring (p,d,^{3}He)

Summary & Outlook

- EDMs of elementary particles are of high interest to disentangle various sources of $\mathcal{CP}$ violation searched for to explain matter - antimatter asymmetry in the Universe
- EDM of charged particles can be measured in storage rings
- Experimentally very challenging because effect is tiny
- First promising results from test measurements at COSY:

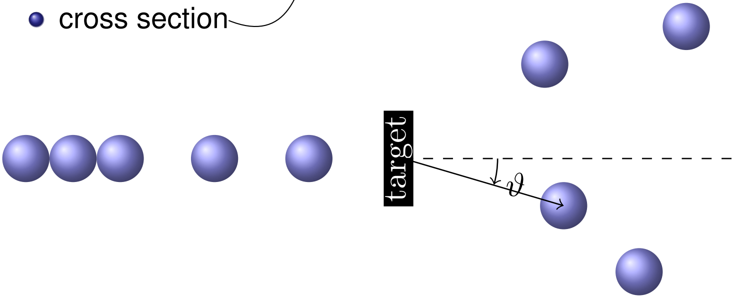
spin coherence time:	few hundred seconds
spin tune precision:	10^{-10} in one cycle

Polarization Measurement

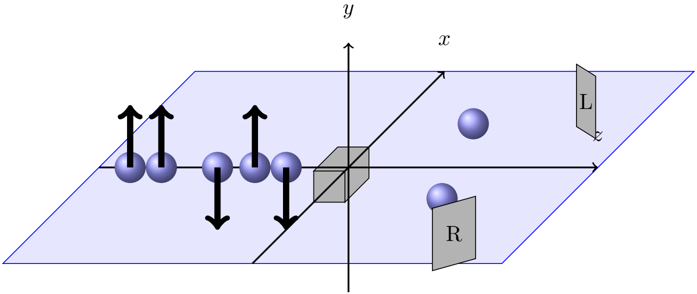
From events to cross section

$$N(\vartheta, \phi) = a(\vartheta, \phi) \quad \mathcal{L} \quad \sigma(\vartheta, \phi)$$

- number of observed events
- acceptance/efficiency
- luminosity
  $\mathcal{L} =$ beam flux $n\times$ target density $\rho\times$ target length ℓ
- cross section



If beam is polarized



$$P = \frac{n^{\uparrow} - n^{\downarrow}}{n^{\uparrow} + n^{\downarrow}} = \frac{3-2}{3+2} = 0.2.$$

Number of particles scattered to the left ($\phi = 0^\circ$):

$$N_L = a_L\,\rho\ell(n^{\uparrow}\sigma_{\uparrow,L} + n^{\downarrow}\sigma_{\downarrow,L})$$

Goal: Determine P, (with small error), knowing $N_L, N_R, \frac{\sigma_{\uparrow,L}}{\sigma_{\uparrow,R}}$

Polarization P, Analyzing Power A

$$
\begin{aligned}
N_L &= a_L\,\rho\ell(n^{\uparrow}\sigma_{\uparrow,L} + n^{\downarrow}\sigma_{\downarrow,L}) \\
&\overset{\Phi-\mathrm{sym}}{=} a_L\,\rho\ell(n^{\uparrow}\sigma_{\uparrow,L} + n^{\downarrow}\sigma_{\uparrow,R}) \\
N_R &= a_R\,\rho\ell(n^{\uparrow}\sigma_{\uparrow,R} + n^{\downarrow}\sigma_{\downarrow,R}) \\
&\overset{\Phi-\mathrm{sym}}{=} a_R\,\rho\ell(n^{\uparrow}\sigma_{\uparrow,R} + n^{\downarrow}\sigma_{\uparrow,L})
\end{aligned}
$$

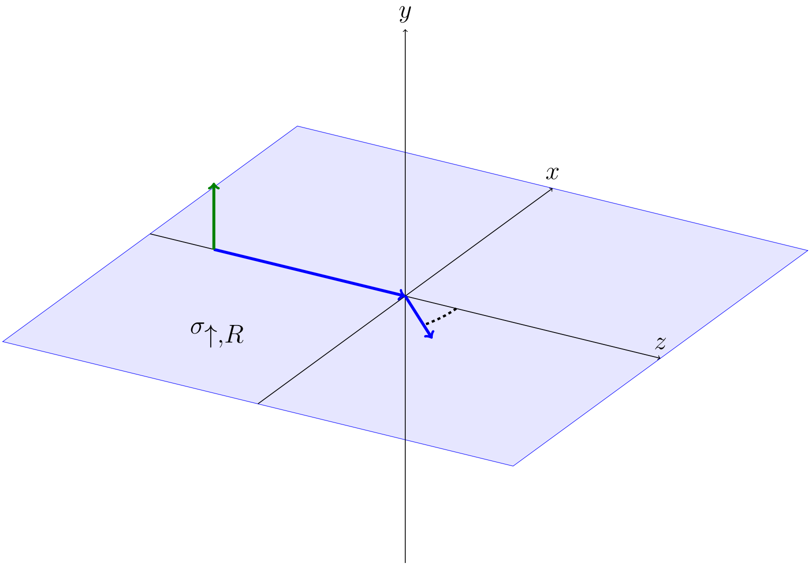
$n^{\uparrow}(n^{\downarrow})$: nb. of beam particles with spin up (down)

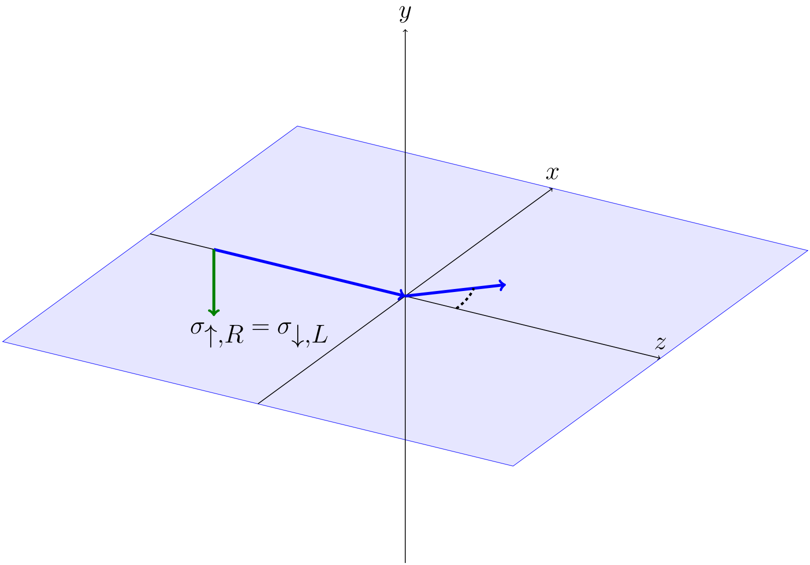
$P = \frac{n^{\uparrow}-n^{\downarrow}}{n^{\uparrow}+n^{\downarrow}}$: Polarization

$\sigma_{\uparrow,R} \equiv \sigma_{\downarrow,L} =: \sigma_R$: cross section for scattering process to the right (left) if spin is up (down)

$\sigma_{\downarrow,R} \equiv \sigma_{\uparrow,L} =: \sigma_L$:

$A = \frac{\sigma_R-\sigma_L}{\sigma_R+\sigma_L}$: analyzing power


y
x
z
σ↑,R



y
x
z
σ↑,R = σ↓,L

Polarization P, Analyzing Power A

With the definitions

$$A = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} \quad \text{and} \quad P = \frac{n^{\uparrow} - n^{\downarrow}}{n^{\uparrow} + n^{\downarrow}}$$

one can write

$$\begin{aligned} N_R &= a_R\, \rho\ell\sigma(1 + AP) \\ N_L &= a_L\, \rho\ell\sigma(1 - AP) \\ & \text{with } \sigma = \frac{1}{2}(\sigma_R + \sigma_L) \end{aligned}$$

To simplify, assume $a_L = a_R$

Now, define asymmetry ϵ:

Asymmetry ϵ

$$\epsilon = \frac{N_R - N_L}{N_R + N_L} = \frac{(1+PA)-(1-PA)}{(1+PA)+(1-PA)} = P\,A \quad \Rightarrow \hat{P} = \frac{\epsilon}{A}$$

$\hat{P}$: estimator for P.
Statistical error $\sigma_\epsilon = 1/\sqrt{N}$, $N = N_R + N_L$
for small asymmetries ($N_L \approx N_R$)

$$\Rightarrow \sigma_P = \frac{1}{A\sqrt{N}}, \quad \text{Figure of merit (FOM)} = \frac{1}{\sigma_P^2} = A^2 N$$

Analysis

Now take into account ϑ dependence

$$\epsilon(\vartheta) = PA(\vartheta)$$

How to extract P, knowing $A(\vartheta)$ and measuring $\epsilon(\vartheta)$?

Straight forward way: Use

$$N = \rho\ell \int_{acc} a(\vartheta, \Phi) n\sigma(\vartheta, \Phi) \mathrm{d}\Omega$$

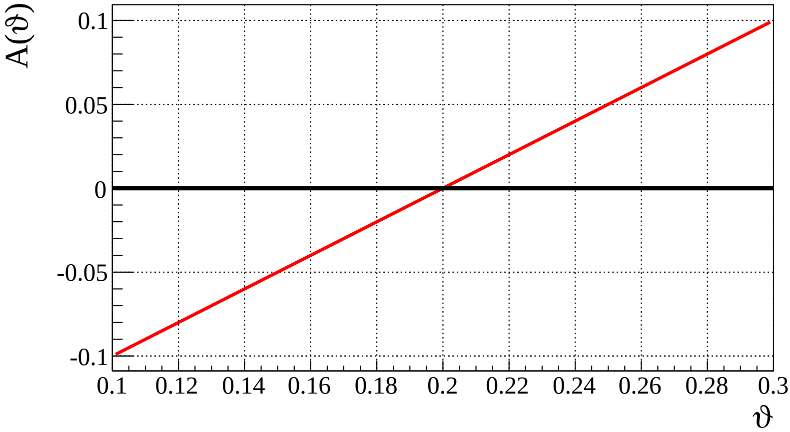
$$\Rightarrow \langle N_{R/L} \rangle \propto (1 \pm \langle A \rangle P), \quad \langle A \rangle = \frac{\int_{acc} a\sigma A \, \mathrm{d}\Omega}{\int_{acc} a\sigma \, \mathrm{d}\Omega} \approx \frac{\sum_i A(\vartheta_i)}{N}$$

and

$$\hat{P} = \frac{1}{\langle A \rangle} \frac{N_R - N_L}{N_R + N_L}, \quad \mathrm{FOM}_{cnt} \overset{\epsilon \lll 1}{=} N \langle A \rangle^2$$

(Academic) Example

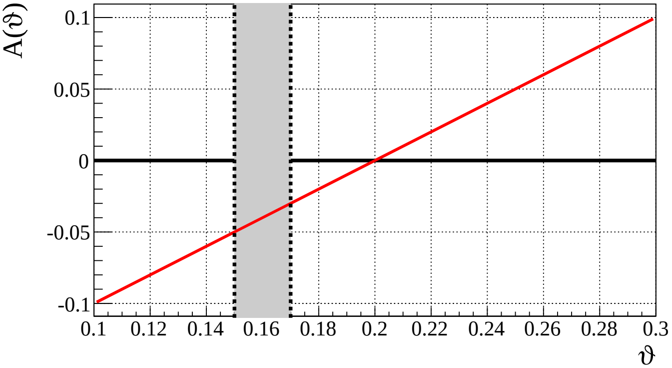
$A(\vartheta) = \vartheta - \overline{\vartheta}$, σ = const. unpolarized cross section and acceptance in region $\vartheta_{min} = 0.1$ to $\vartheta_{max} = 0.3$.
$\overline{\vartheta} = (\vartheta_{max} - \vartheta_{min})/2$



$\langle A \rangle = 0 \Rightarrow \text{FOM} = N\langle A \rangle^2 = 0 \quad \Rightarrow \sigma_P = \infty$

Can one do better?

Analyse in bins of ϑ



In this bin FOM is
$\mathrm{FOM}_i = n_i \langle A_i \rangle^2 \approx n_i A_i^2\,, \quad n_i = N/N_{bin}\,, \quad N_{bin}$:nb. of bins

Note, P does not depend on ϑ.

Analysis in bins

Combine all bins:

$$\hat{P} \;=\; \frac{\sum_i \frac{\hat{P}_i}{\sigma^2_{P_i}}}{\sum_i \frac{1}{\sigma^2_{P_i}}} = \frac{\sum_i \hat{P}_i \,\mathrm{FOM}_i}{\sum_i \mathrm{FOM}_i} = \frac{\sum_i \hat{P}_i \, n_i A_i^2}{\sum_i n_i A_i^2}$$

$$\mathrm{FOM} = \sum_i \mathrm{FOM}_i = \sum_i n_i A_i^2 = N \frac{\sum_i n_i A^2}{\sum_i n_i} \overset{N_{bin}\to\infty}{=} N\langle A^2\rangle$$

many bins		one bin
$N\langle A^2\rangle$	$\geq$	$N\langle A\rangle^2$

Binning

With binning FOM can be improved.
Binning is sometimes inconvenient

- Too few bins $\Rightarrow$ FOM not maximal
- Too many bins $\Rightarrow$ Empty bins

Is there an alternative?

Event weighting

General case:
Consider the following **estimator** for P:

$$\hat{P} = \frac{\sum_R w_i - \sum_L w_i}{\sum_R w_i A_i - \sum_L w_i A_i}$$

where $w_i = w(\vartheta_i)$ as an (arbitrary) weight factor
Easy to show: $\langle \hat{P} \rangle = P$ independet of w
In words: What ever you choose for w, you always get the correct result, **but** with different unceratinties.

$$\mathrm{FOM}_w = N \frac{\langle wA \rangle^2}{\langle w^2 \rangle}$$

Reminder: $\langle wA \rangle = \frac{\sum w_i A_i}{N} = \frac{\int a\sigma wA \mathrm{d}\Omega}{\int a\sigma \mathrm{d}\Omega}$

Examples

Two special cases:

$\underline{w = 1}$

$$\hat{P} = \frac{\sum_R 1 - \sum_L 1}{\sum_R A_i + \sum_L A_i} = \frac{1}{\langle A \rangle} \frac{N_R - N_L}{N_R + N_L}$$

→ like counting rate asymmetry in one bin

$\underline{w = A}$

$$\hat{P} = \frac{\sum_R A_i - \sum_L A_i}{\sum_R A_i^2 + \sum_L A_i^2} = \frac{\sum_j A_j(n_{j,R} - n_{j,L})}{\sum_j A_j^2(n_{j,R} + n_{j,L})}$$

$$= \frac{\sum_j A_J n_j \epsilon_j}{\sum_j A_j^2 n_j} = \frac{\sum_j A_j^2 \, n_j \hat{P}_j}{\sum_j A_j^2 n_j}$$

→ same as infinite number of bins

Best weight

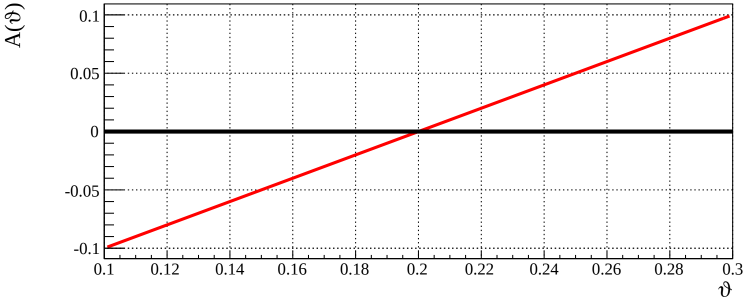
One can show, that among all weight factors, the choice $w = A$ gives the largest FOM.

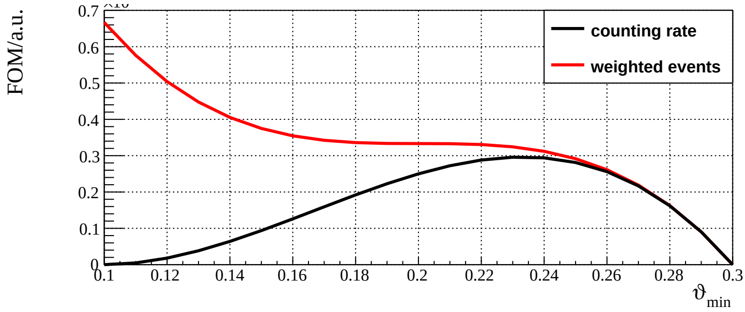
	counting, $w = 1$	Binning, $w = A$, MLH
FOM	$N\langle A\rangle^2$	$N\langle A^2\rangle$

Gain in FOM: $\frac{\langle A^2\rangle}{\langle A\rangle^2}$

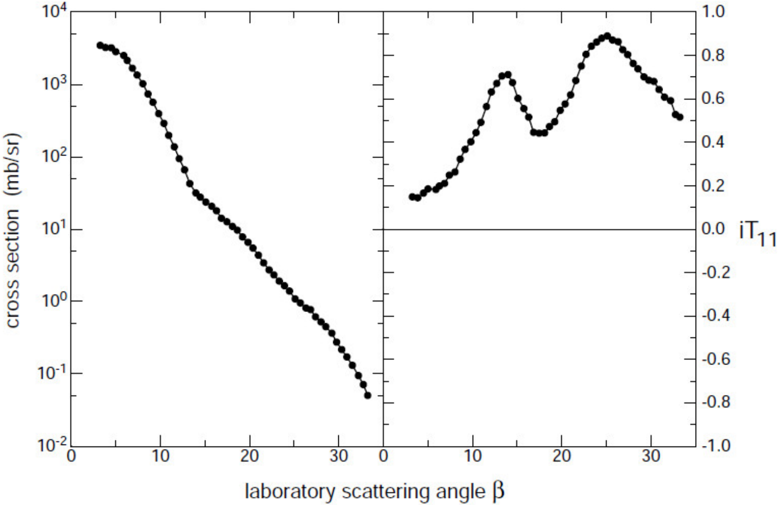
An event with an large analyzing power A tells you more about P than an event with lower A. It should thus enter the analysis with more weight.

(Academic) Example

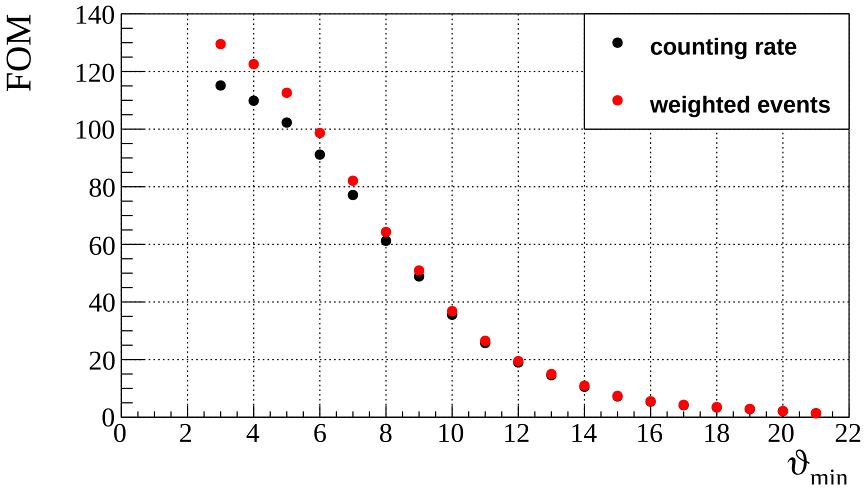




Example: Eleastic deuteron carbon scattering at $T = 270\text{MeV}$



Example



FOM
140
120
100
80
60
40
20
0
0
2
4
6
8
10
12
14
16
18
20
22
ϑ min
counting rate
weighted events

Connection to Maximum Likelihood Method

$N_R \propto a(1 + AP)\,,\ N_L =\propto a(1 - AP)$

Log-likelihood function

$$\begin{aligned}\ell &= \sum_R \ln\left(a_i(1 + A_i P)\right) - \langle N_R\rangle(P) \\ &+ \sum_L \ln\left(a_i(1 - A_i P)\right) - \langle N_L\rangle(P)\,.\end{aligned}$$

Connection to Maximum Likelihood Method

MLH estimator for P: Maximize $\ell \Rightarrow \frac{\partial \ell}{\partial P} \stackrel{!}{=} 0$

$$\Rightarrow \frac{\partial \ell}{\partial P} = \sum_R \frac{A_i}{1 + A_i P} + \sum_L \frac{A_i}{1 - A_i P} = 0$$

for $AP \ll 1$:

$$\Rightarrow \sum_R A_i (1 - A_i P) + \sum_L A_i (1 + A_i P) = 0$$

$$\Rightarrow \hat{P} = \frac{\sum_R A_i - \sum_L A_i}{\sum_R A_i^2 + \sum_L A_i^2}$$

Estimator of maximum likelihood function coincides with estimator for optimal weight!

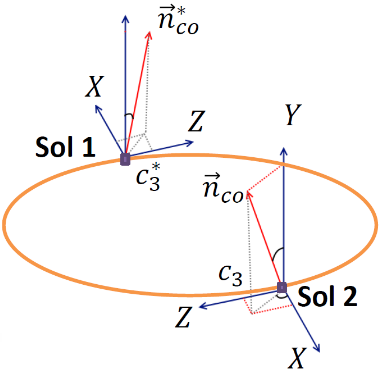
Summary

- Polarizations can be extracted from event rates, knowing the analyzing power A
- weighting the events with their analyzing power A give the largest FOM
- Gain with respect to just counting events is
  $$\frac{\mathrm{FOM}_{w=A}}{\mathrm{FOM}_{cnt}} = \frac{\langle A^2 \rangle}{\langle A \rangle^2}$$
- Details:
  JP, "Comparison of methods to extract an asymmetry parameter from data," Nucl. Instrum. Meth. A **659** (2011) 456 arXiv:1104.1038

Spare

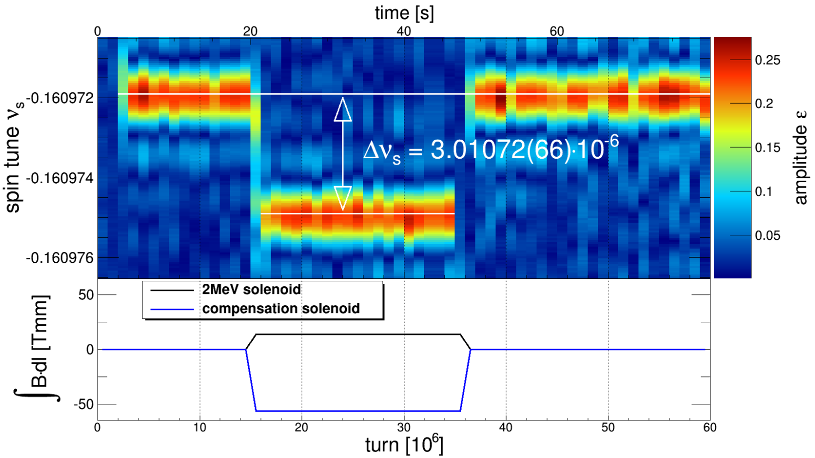
Spin Tune as tool to investigate systematics

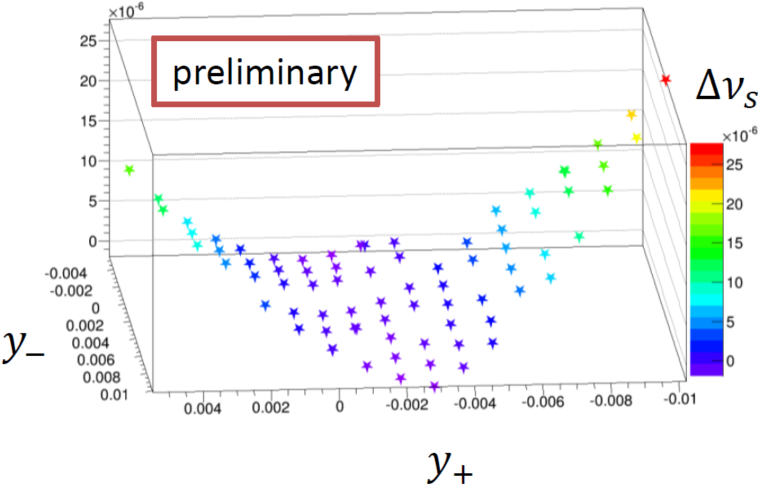
$\nu_s = \gamma G +$ imperfections kicks



- Create artificial imperfections with solenoids/steerers
- measure spin tune change $\Delta\nu_s$
- expectation
  $\Delta\nu_s \propto (y_\pm - a_\pm)^2$
  $a_\pm$: kicks due to imperfections,
  $y_\pm$: kicks due to solenoids

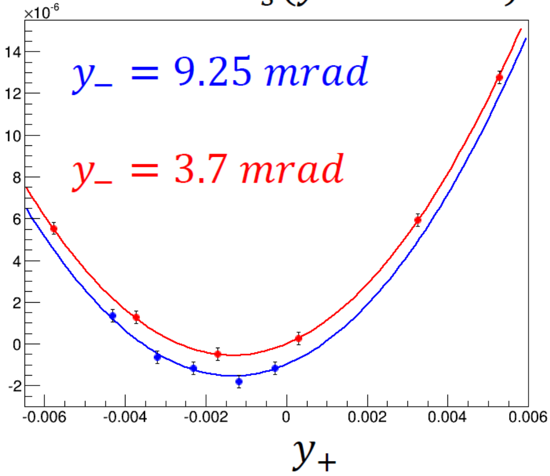
Spin Tune jumps





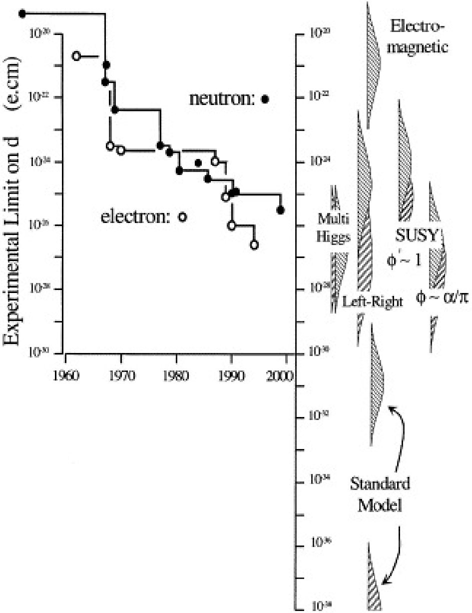
- parabolic behavior expected from simulations
- $y^{\pm} = \dfrac{\chi_1 \pm \chi_2}{2}$, $\chi_{1,2}$: solenoid strength
  for perfect machine, minimum should be at $y^{+} = 0$

$\Delta\nu_s(y_- = const)$



- parabolic behavior expected from simulations
- $y^{\pm} = \dfrac{\chi_1 \pm \chi_2}{2}$, $\chi_{1,2}$: solenoid strength
  for perfect machine, minimum should be at $y^{+} = 0$

Electron and Neutron EDM



J. M. Pendlebury &
E.A. Hinds,
NIMA 440(2000) 471

EDM: SUSY Limits

electron:

MSSM: $\varphi \approx 1 \Rightarrow d = 10^{-24} - 10^{-27} e \cdot \text{cm}$

$\varphi \approx \alpha/\pi \Rightarrow d = 10^{-26} - 10^{-30} e \cdot \text{cm}$

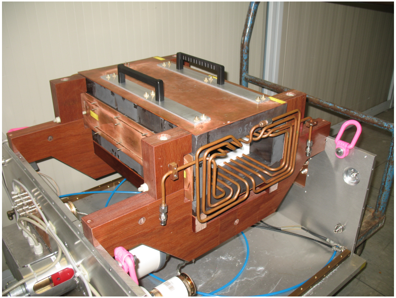
neutron:

MSSM: $d = 10^{-24} e \cdot \text{cm} \cdot \sin \phi_{CP} \frac{200\text{GeV}}{M_{SUSY}}$

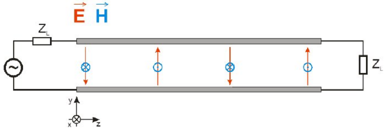
Electrostatic Deflectors

- Electrostatic deflectors from Fermilab ($\pm$125kV at 5 cm $\hat{=}$ 5MV/m)
- large-grain Nb at plate separation of a few cm yields $\approx$ 20MV/m

Wien Filter



Conventional design
R. Gebel, S. Mey (FZ Jülich)



stripline design
D. Hölscher, J. Slim
(IHF RWTH Aachen)

2. Pure Electric Ring

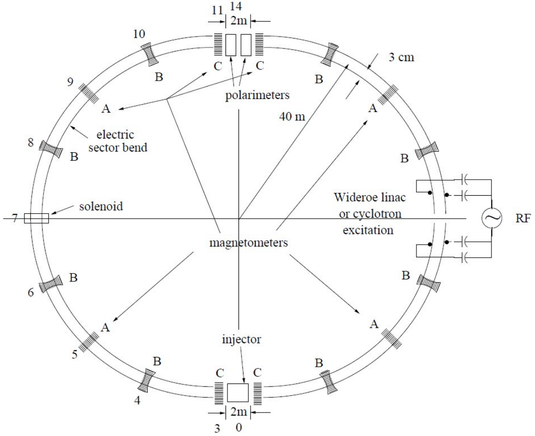


Figure 3: An all-electric storage ring lattice for measuring the electric dipole moment of the proton. Except for having longer straight sections and separated beam channels, the all-in-one lattice of Fig. 1 is patterned after this lattice. Quadrupole and sextupole families, and tunes and lattice functions of the all-in-one lattice of Fig. 1 will be quite close to those given for this lattice in reference[3]. The match will be even closer with magnetic field set to zero for proton operation.

Brookhaven National Laboratory (BNL) Proposal

3. Combined $\vec{E}/\vec{B}$ ring

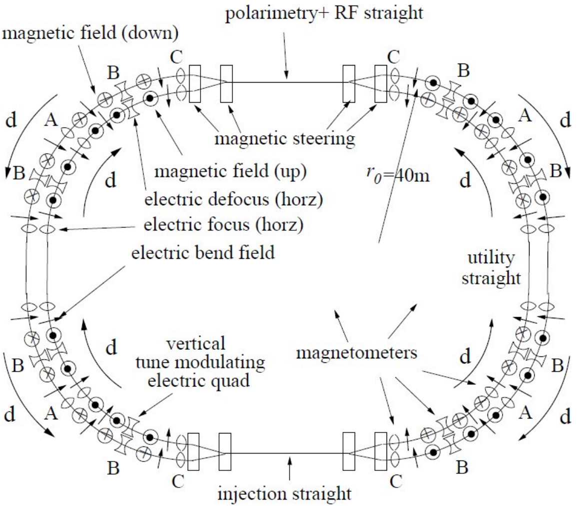


Figure 1: "All-In-One" lattice for measuring EDM's of protons, deuterons, and helions.

Under discussion at Forschungszentrum Jülich (design: R. Talman)

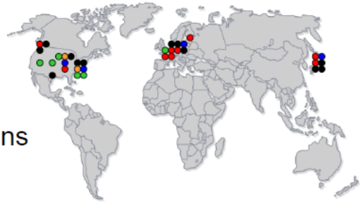
Summary of different options

	☺	☹
1.) pure magnetic ring (Jülich)	existing (upgraded) COSY ring can be used , shorter time scale	lower sensitivity
2.) pure electric ring (BNL)	no $\vec{B}$ field needed	works only for p
3.) combined ring (Jülich)	works for p, d, ^{3}He, ...	both $\vec{E}$ and $\vec{B}$ required

EDM Activities Around the World

Rough estimate of numbers of researchers, in total ~500 (with some overlap)

- **Neutrons** (~200)
  - @ILL
  - @ILL,@PNPI
  - @PSI
  - @FRM-2
  - @RCNP,@TRIUMF
  - @SNS
  - @J-PARC
- **Molecules** (~50)
  - YbF@Imperial
  - PbO@Yale
  - ThO@Harvard
  - HfF+@JILA
  - WC@UMich
  - PbF@Oklahoma
- **Atoms** (~100)
  - Hg@UWash
  - Xe@Princeton
  - Xe@TokyoTech
  - Xe@TUM
  - Xe@Mainz
  - Cs@Penn
  - Cs@Texas
  - Fr@RCNP/CYRIC
  - Rn@TRIUMF
  - Ra@ANL
  - Ra@KVI
  - Yb@Kyoto



- **Ions-Muons** (~200)
  - @BNL
  - @FZJ
  - @FNAL
  - @JPARC
- **Solids** (~10)
  - GGG@Indiana
  - ferroelectrics@Yale

K. Kirch

Systematics

- Splitting of beams: $\delta y = \pm \frac{\beta c R_0 B_r}{E_r Q_y^2} = \pm 1 \cdot 10^{-12}\,\mathrm{m}$
- $Q_y \approx 0.1$: vertical tune
- Modulate $Q_y = Q_y^0\,(1 - m\cos(\omega_m t)),\ m \approx 0.1$
- Splitting causes B field of $\approx 0.4 \cdot 10^{-3}\,\mathrm{fT}$
- in one year: 10^4 fills of $1000\,\mathrm{s} \Rightarrow \sigma_B = 0.4 \cdot 10^{-1}\mathrm{fT}$ per fill needed
- Need sensitivity $1.25\,\mathrm{fT}/\sqrt{\mathrm{Hz}}$

D. Kawall

Systematics