Electric Dipole Moment Measurements at Storage Rings – Polarisation Measurements

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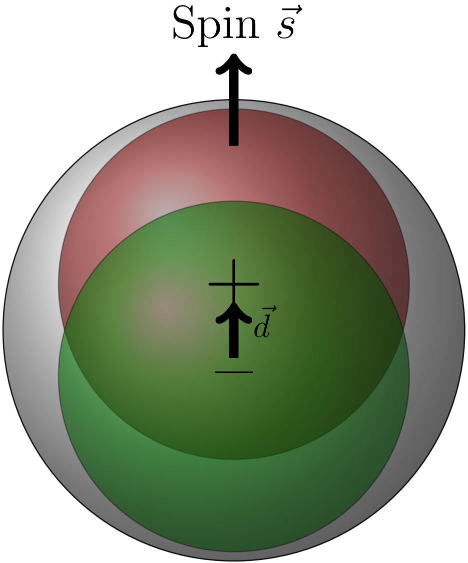
Outline

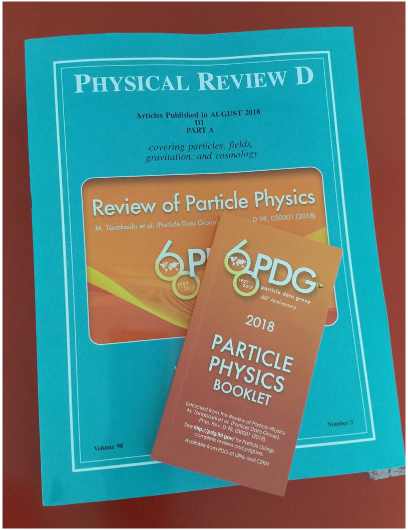
- **Electric Dipole Moments** (EDM)
- $\Rightarrow$ Observable: **Polarisation**
  Optimal Observables, Event Weighting, Maximum Likelihood Method

Electric Dipole Moments (EDM)



- permanent separation of positive and negative charge
- fundamental property of particles (like magnetic moment, mass, charge)
- existence of EDM only possible via violation of time reversal $\mathcal{T} \overset{\mathcal{CPT}}{=} \mathcal{CP}$ and parity $\mathcal{P}$ symmetry
- close connection to “matter-antimatter” asymmetry
- axion field leads to oscillating EDM

Proton EDM



Citation: M. Tanabashi *et al.* (Particle Data Group), Phys. Rev. D **98**, 030001 (2018) and 2019 update

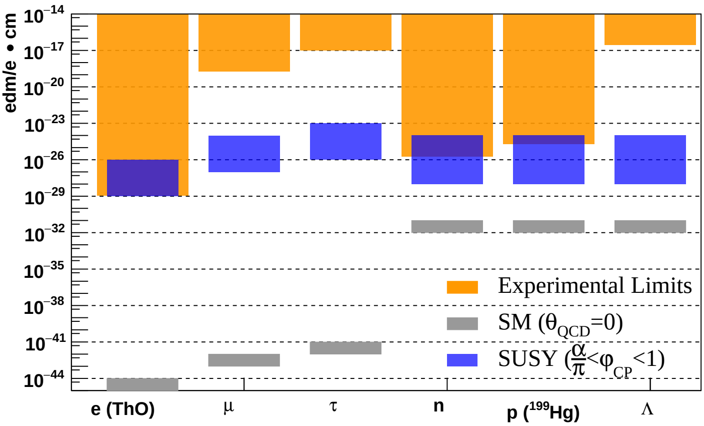
N BARYONS
$(S = 0,\ I = 1/2)$

$p,\ N^+ = uud; \quad n,\ N^0 = udd$

p $\qquad I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$

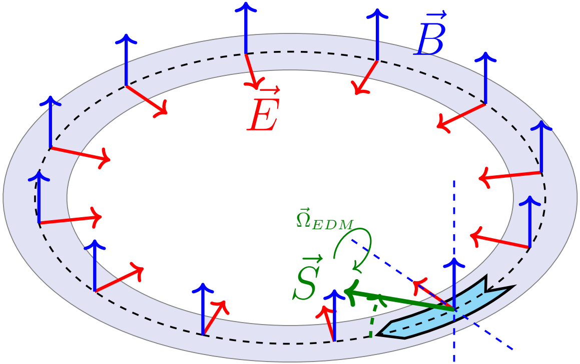
Mass $m = 1.00727646688 \pm 0.00000000009$ u
Mass $m = 938.272081 \pm 0.000006$ MeV [a]
$|m_p - m_{\overline{p}}|/m_p < 7 \times 10^{-10}$, CL = 90% [b]
$|\frac{q_{\overline{p}}}{m_{\overline{p}}}|/(\frac{q_p}{m_p}) = 1.00000000000 \pm 0.00000000007$
$|q_p + q_{\overline{p}}|/e < 7 \times 10^{-10}$, CL = 90% [b]
$|q_p + q_e|/e < 1 \times 10^{-21}$ [c]
Magnetic moment $\mu = 2.7928473446 \pm 0.0000000008\ \mu_N$
$(\mu_p + \mu_{\overline{p}})\ /\ \mu_p = (0.3 \pm 0.8) \times 10^{-6}$
Electric dipole moment $d < 0.021 \times 10^{-23}\ e$ cm
Electric polarizability $\alpha = (11.2 \pm 0.4) \times 10^{-4}$ fm^3
Magnetic polarizability $\beta = (2.5 \pm 0.4) \times 10^{-4}$ fm^3 (S = 1.2)
Charge radius, μp Lamb shift $= 0.84087 \pm 0.00039$ fm [d]
Charge radius, ep CODATA value $= 0.8751 \pm 0.0061$ fm [d]
Magnetic radius $= 0.851 \pm 0.026$ fm [e]
Mean life $\tau > 2.1 \times 10^{29}$ years, CL = 90% [f] ($p \to$ invisible mode)
Mean life $\tau > 10^{31}$ to 10^{33} years [f] (mode dependent)

EDM: Current Upper Limits



storage rings: EDMs of **charged** hadrons: p, d, ^{3}He

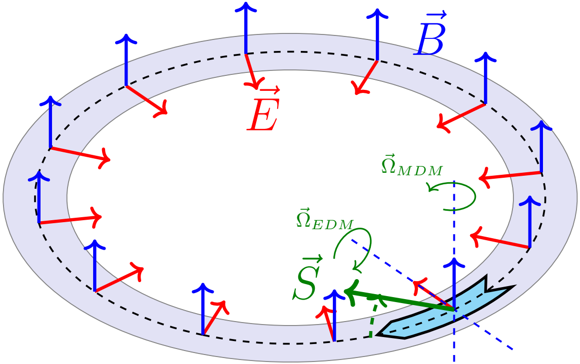
Experimental Method: Generic Idea



$$\frac{\mathrm{d}\vec{s}}{\mathrm{d}t} \propto \underbrace{d(\vec{E} + \vec{v} \times \vec{B})}_{= \vec{\Omega}_{\mathrm{EDM}}} \times \vec{s}$$

build-up of vertical polarization $s_\perp \propto d$, if $\vec{s}_{\mathrm{horz}} || \vec{p}$ (**frozen spin**)

Experimental Method: Generic Idea



$$\frac{\mathrm{d}\vec{s}}{\mathrm{d}t} \propto \underbrace{d(\vec{E} + \vec{v} \times \vec{B})}_{= \vec{\Omega}_{\mathrm{EDM}}} \times \vec{s}$$

In general:

$$\frac{\mathrm{d}\vec{s}}{\mathrm{d}t} = (\vec{\Omega}_{\mathrm{MDM}} + \vec{\Omega}_{\mathrm{EDM}}) \times \vec{s}$$

build-up of vertical polarization $s_\perp \propto d$, if $\vec{s}_{horz} || \vec{p}$ (**frozen spin**)

Spin Precession: Thomas-BMT Equation

$$\frac{\mathrm{d}\vec{s}}{\mathrm{d}t} = \vec{\Omega} \times \vec{s} = \frac{-q}{m}\Bigg[\underbrace{G\vec{B} + \left(G - \frac{1}{\gamma^2 - 1}\right)\vec{v} \times \vec{E}}_{=\vec{\Omega}_{\mathrm{MDM}}} + \underbrace{\frac{\eta}{2}(\vec{E} + \vec{v} \times \vec{B})}_{=\vec{\Omega}_{\mathrm{EDM}}}\Bigg] \times \vec{s}$$

electric dipole moment (EDM): $\vec{d} = \eta \frac{q\hbar}{2mc}\vec{s}$,

magnetic dipole moment (MDM): $\vec{\mu} = 2(G + 1)\frac{q\hbar}{2m}\vec{s}$

Note: $\eta = 2 \cdot 10^{-15}$ for $d = 10^{-29}$ecm, $G \approx 1.79$ for protons

Spin Precession: Thomas-BMT Equation

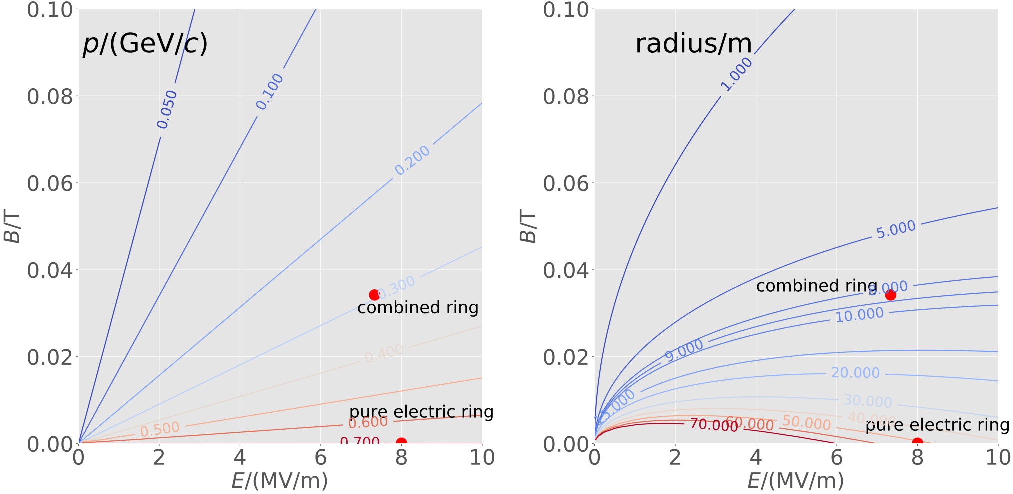
$$\frac{\mathrm{d}\vec{s}}{\mathrm{d}t} = \vec{\Omega} \times \vec{s} = \frac{-q}{m}\left[\underbrace{G\vec{B} + \left(G - \frac{1}{\gamma^2 - 1}\right)\vec{v} \times \vec{E}}_{\vec{\Omega}_{\mathrm{MDM}} = 0\,,\quad \textbf{frozen spin}} + \underbrace{\frac{\eta}{2}(\vec{E} + \vec{v} \times \vec{B})}_{= \vec{\Omega}_{\mathrm{EDM}}}\right] \times \vec{s}$$

achievable with pure electric field if $G = \frac{1}{\gamma^2 - 1}$, works only for $G > 0$, e.g. proton

or with special combination of E, B fields and γ, i.e. momentum

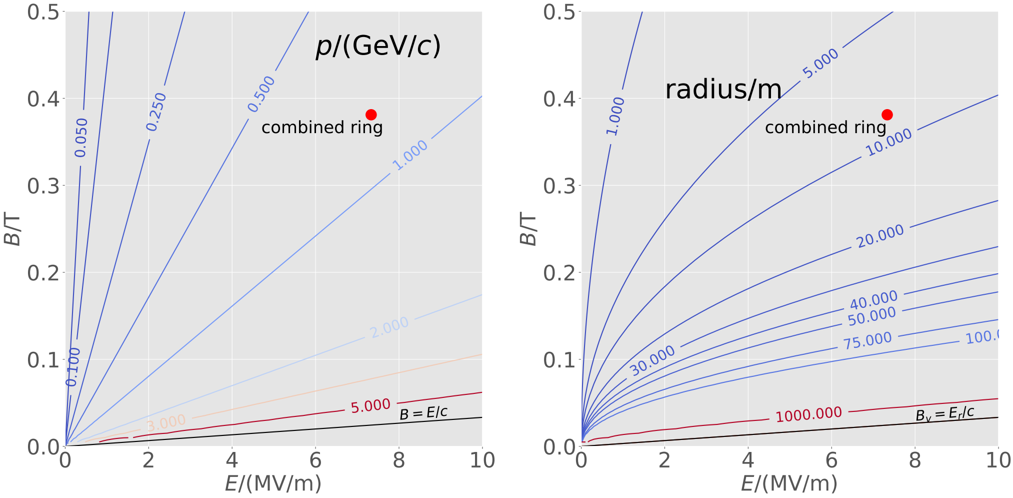
Momentum and ring radius for **proton** in frozen spin condition

$G = 1.7928474$



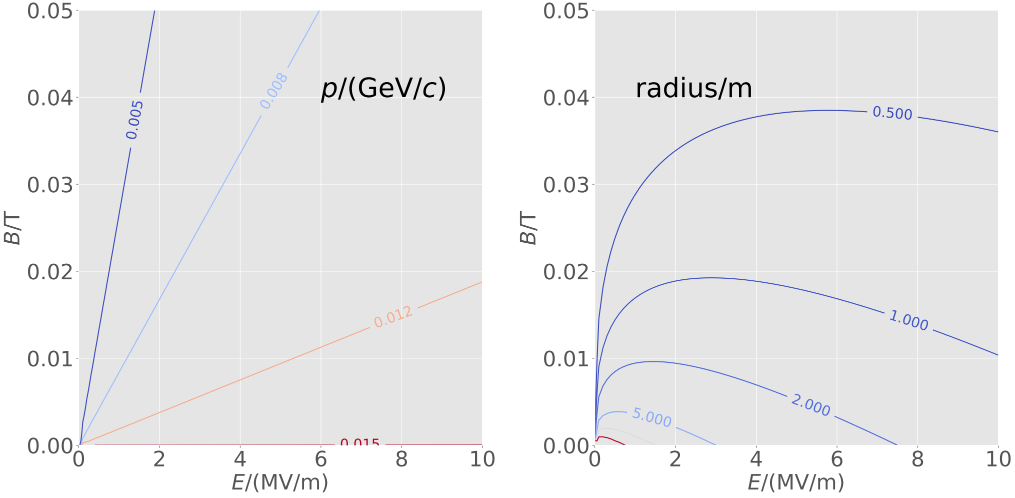
Momentum and ring radius for **deuteron** in frozen spin condition

$G = -0.1425617689$



Momentum and ring radius for **electron** in frozen spin condition

$G = 0.001159652$



Different Options

	☺	☹
3.) pure electric ring	no $\vec{B}$ field needed, ↺, ↻ beams simultaneously	works only for particles with $G > 0$ (e.g. e, p)
2.) combined ring	works for e, p, d, ^{3}He, smaller ring radius	both $\vec{E}$ and $\vec{B}$ B field reversal for ↺, ↻ required
1.) pure magnetic ring	existing (upgraded) COSY ring can be used, shorter time scale	lower sensitivity, precession due to G, i.e. no **frozen spin**

Observable is in all cases a **spin polarization**!

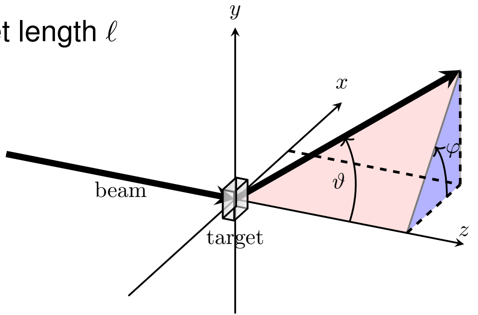
Talk on EDM

Polarization Measurements

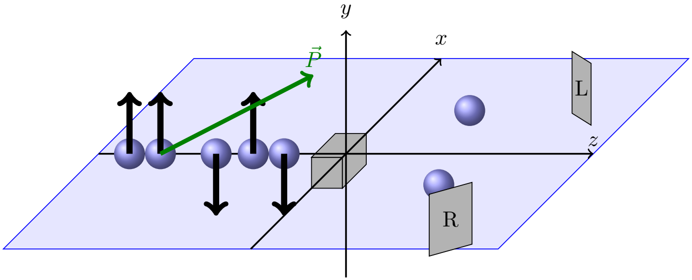
Counting Rates, Cross Section, Polarization

$$N(\vartheta,\varphi) = a(\vartheta,\varphi)\ \mathcal{L}\ \sigma(\vartheta)\ \Big(1 + P\ A(\vartheta)\ \cos(\varphi)\Big)$$

- number of observed events
- acceptance/efficiency
- luminosity
  $\mathcal{L} =$ beam flux $n\times$ target density $\rho\times$ target length ℓ
- unpolarized cross section
- beam polarisation $P = \dfrac{n^{\uparrow} - n^{\downarrow}}{n^{\uparrow} + n^{\downarrow}}$
- analysing power $A = \dfrac{\sigma_L^{\uparrow} - \sigma_R^{\uparrow}}{\sigma_L^{\uparrow} + \sigma_R^{\uparrow}}$



Counting Rates, Cross Section, Polarization



polarisation: $P = \frac{n^{\uparrow} - n^{\downarrow}}{n^{\uparrow} + n^{\downarrow}} = \frac{3-2}{3+2} = 0.2\,,$ analyzing power $A = \frac{\sigma_L^{\uparrow} - \sigma_R^{\uparrow}}{\sigma_L^{\uparrow} + \sigma_R^{\uparrow}}\,.$

$N_L \propto (n^{\uparrow}\sigma_L^{\uparrow} + n^{\downarrow}\sigma_L^{\downarrow})$

Note: $\sigma_L^{\uparrow} \equiv \sigma_R^{\downarrow}$

$$\Rightarrow N(\vartheta,\varphi) = \mathcal{L}a(\vartheta,\varphi)\sigma(\vartheta)\Big(1 + P\,A(\vartheta)\,\cos(\varphi)\Big), \qquad \sigma = \frac{1}{2}(\sigma_L + \sigma_R)$$

Goal

Determine P from counting rate $N(\vartheta, \varphi)$ and analysing power $A(\vartheta)$ with small uncertainty σ_P.

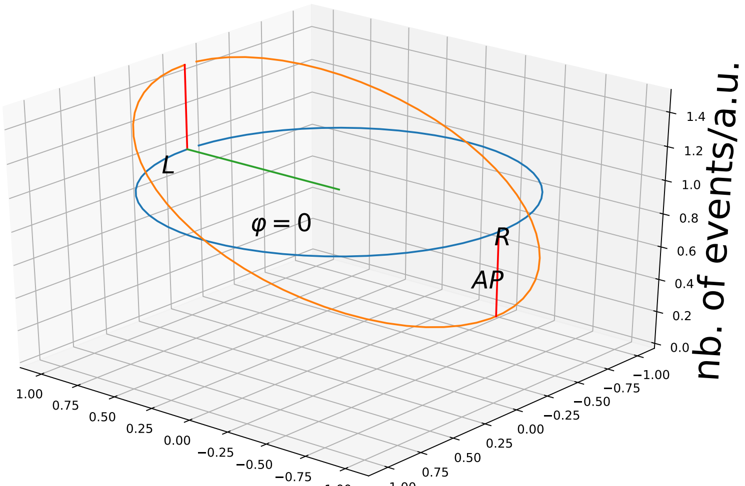
To simplify the discussion

- assume constant acceptance in φ: $\frac{\partial a(\vartheta, \varphi)}{\partial \varphi} = 0$
- detector placed at one polar angle ϑ

We are left with

$$N(\varphi) = \frac{1}{2\pi} N_0 (1 + PA \cos(\varphi)) \quad , N_0 = a\mathcal{L}\sigma$$

Event Distribution



Most easy way to get P

Just consider counts in the left part of the detector $\varphi \approx 0, \cos(\varphi) = 1$ and the right part $\varphi \approx \pi, \cos(\varphi) = -1$.

$$\begin{aligned} \langle N_L \rangle &= N_0 \frac{\Delta\varphi}{2\pi}(1 + AP) \\ \langle N_R \rangle &= N_0 \frac{\Delta\varphi}{2\pi}(1 - AP) \end{aligned}$$

Consider a **counting rate asymmetry**

$$\hat{P} = \frac{1}{A} \frac{N_L - N_R}{N_L + N_R}\,,$$

$\hat{P}$: estimator for P.

If A is known, one can determine P.

Note:

$\langle N_{L,R} \rangle$: expectation value

$N_{L,R}$: actually measured number of events

What about the error?

Error propagation gives: $\sigma_P = \dfrac{1}{A\sqrt{N}}$

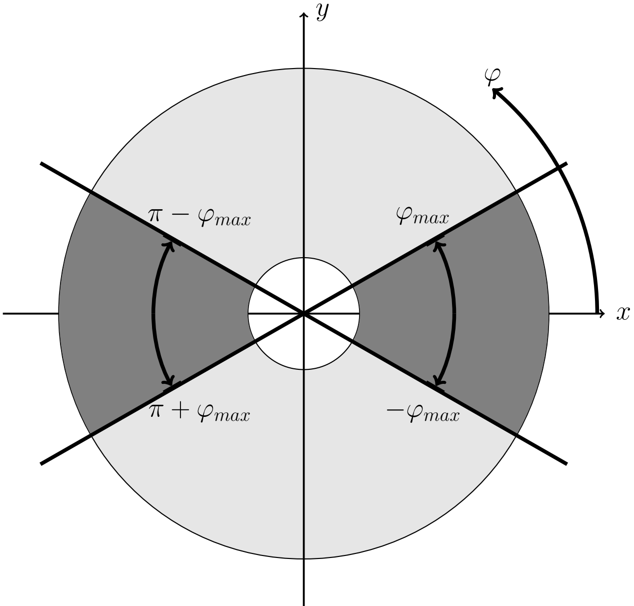
(assuming $PA \ll 1$, i.e. $N_L \approx N_R =: N/2$)
As in any counting experiment the statistical error scales with $1/\sqrt{N}$.
Counting only events in small region $\Delta\varphi$ around $\varphi = 0$ and π results in small $N = N_0 \dfrac{2\Delta\varphi}{2\pi}$ and thus large error.

It's more convenient to work with the Figure of Merit (FOM):

$\text{FOM}_P = \sigma_P^{-2} = N\,A^2$

How does error change if we include more events, i.e. making $\Delta\varphi$ larger?

Enlarge φ range



Enlarge φ range

estimator $$\hat{P} = \frac{1}{A\langle\cos(\varphi)\rangle}\frac{N_L - N_R}{N_L + N_R}$$

$$\sigma_P = \frac{1}{\sqrt{N}}\frac{1}{A\langle\cos(\varphi)\rangle},$$

number of events: $N = \dfrac{4\varphi_{max}}{2\pi}$

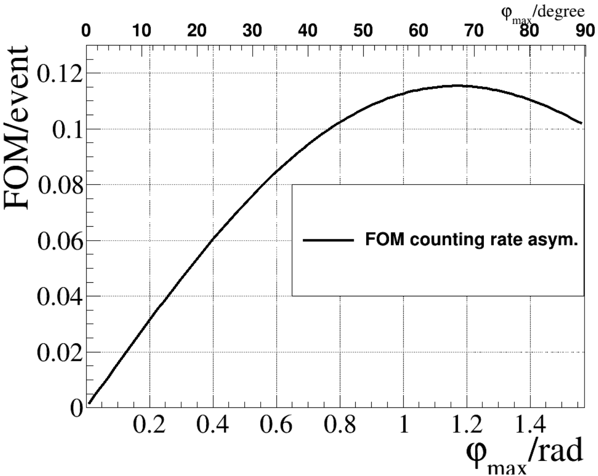
$$\varphi_{max} \nearrow \quad \Rightarrow \quad N \nearrow$$

$$\varphi_{max} \nearrow \quad \Rightarrow \quad \langle\cos(\varphi)\rangle \searrow$$

$$\langle\cos(\varphi)\rangle = \frac{\int_{-\varphi_{max}}^{\varphi_{max}} \cos(\varphi)\mathrm{d}\varphi}{2\varphi_{max}}$$

$$\mathrm{FOM}_P = \sigma_P^{-2} = N\left(A\langle\cos(\varphi)\rangle\right)^2$$

Figure of Merit (FOM)



- strange behavior: Adding data beyond $\varphi_{max} > 67^\circ$ the FOM decreases
- Reason: adding data at larger φ “dilutes” the sample

Can one do better? Yes! **Event Weighting**

Instead of just counting events, weight every event with a weight function $w(\varphi)$.

Estimator for P

$$\hat{P} = \frac{1}{A} \frac{\sum_{L,R} w_i}{\sum_{L,R} w_i \cos(\varphi_i)}$$

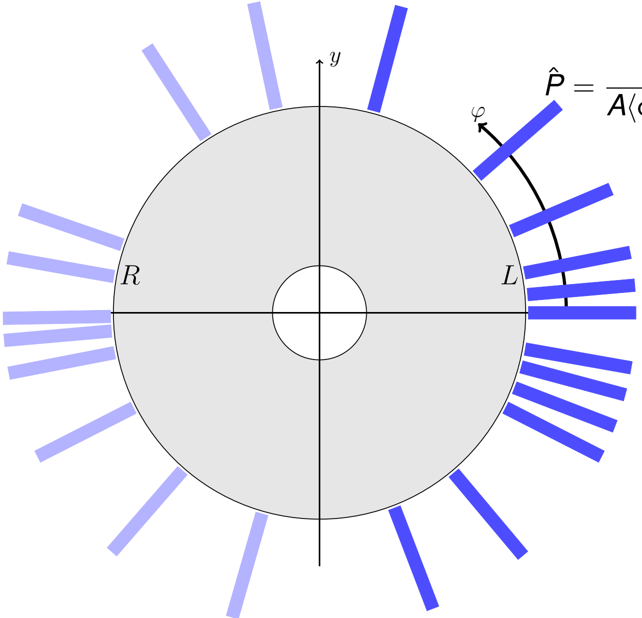
In principle weight w arbitrary, two cases are of interest

- $w = 1$ (left), $w = -1$ (right): $\hat{P} = \frac{1}{A\langle\cos(\varphi)\rangle} \frac{N_L - N_R}{N_L + N_R}$ (counting rate asymmetry)
- $w = A\cos(\varphi)$: $\hat{P} = \frac{1}{A} \frac{\sum_{L,R} \cos(\varphi_i)}{\sum_{L,R} \cos^2(\varphi_i)}$ (optimal weight)[1]

choice $w(\varphi) \equiv A\cos(\varphi)$ leads to smallest statistical error.

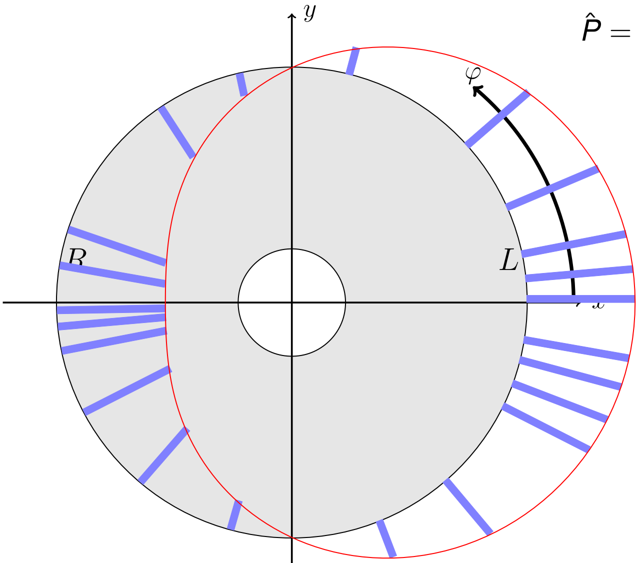
[1] In terms of highest FOM.

Every event weighted with $w = 1$



$$\hat{P} = \frac{1}{A\langle\cos(\varphi)\rangle} \frac{N_L - N_R}{N_L + N_R}$$

Every event weighted with $w = A\cos(\varphi)$



$$\hat{P} = \frac{1}{A}\,\frac{\sum \cos(\varphi_i)}{\sum \cos^2(\varphi_i)}$$

What about the error?

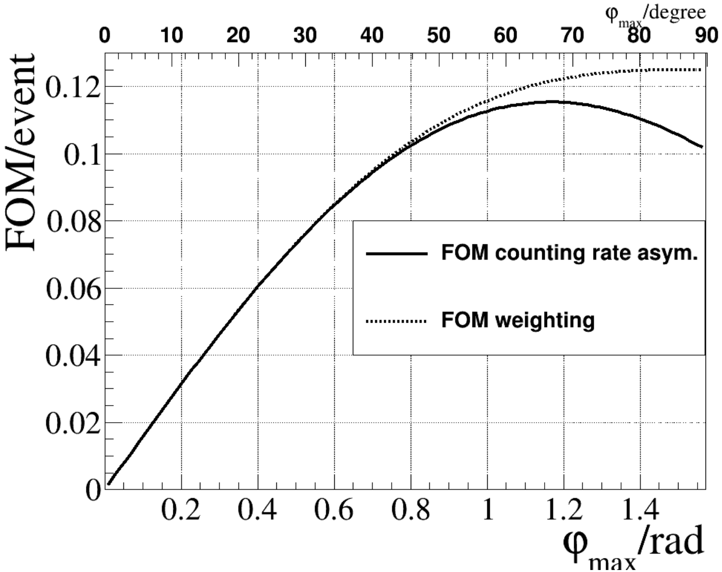
Error Propagation: $\text{FOM}_P = NA^2 \frac{\langle w\cos(\varphi)\rangle^2}{\langle w^2\rangle}$

	counting, $w = 1$	$w = A\cos(\varphi)$, MLH, binning
FOM_P	$NA^2\langle\cos(\varphi)\rangle^2$	$NA^2\langle\cos(\varphi)^2\rangle$

Gain in FOM: $\frac{\langle\cos(\varphi)^2\rangle}{\langle\cos(\varphi)\rangle^2} \geq 1$

An event with a large $\cos(\varphi)$ tells you more about P than an event with lower $\cos(\varphi)$. It should thus enter the analysis with more weight.

FOM



φmax/degree
0
10
20
30
40
50
60
70
80
90
FOM/event
0.12
0.1
0.08
0.06
0.04
0.02
0
0.2
0.4
0.6
0.8
1
1.2
1.4
φmax/rad
FOM counting rate asym.
FOM weighting

Connection to Maximum Likelihood Method

$N(\varphi) \propto (1 + A\cos(\varphi)P) = (1 + \beta(\varphi)P)\,,$
Here: $\beta(\varphi) = A\cos(\varphi)$

Log-likelihood function

$$\ell \;=\; \sum_{i=1}^{N} \ln\left(1 + \beta(\varphi_i)P\right)$$

Connection to Maximum Likelihood Method

MLH estimator for P: Maximize $\ell \Rightarrow \frac{\partial \ell}{\partial P} \stackrel{!}{=} 0$

$$\Rightarrow \frac{\partial \ell}{\partial P} = \sum_i \frac{\beta(\varphi_i)}{1 + \beta(\varphi_i) P} = 0$$

for $\beta(\varphi_i) P \ll 1$:

$$\Rightarrow \sum_i \beta(\varphi_i)(1 - \beta(\varphi_i) P) = 0$$

$$\Rightarrow \hat{P} = \frac{\sum_i \beta(\varphi_i)}{\sum_i \beta^2(\varphi_i)} = \frac{1}{A} \frac{\sum_i \cos(\varphi_i)}{\sum_i \cos^2(\varphi_i)}$$

Estimator of maximum likelihood function coincides with estimator for optimal weight!

It is well known that MLH estimator reach largest FOM (Cramer-Rao-bound).

More general case

events follow distribution $N(\vec{x}) \propto (1 + \beta(\vec{x})P)$

For optimal event weight/MLH FOM is given by

$$\mathrm{FOM}_P = N\langle\beta(\vec{x})^2\rangle$$

Counting rates reach only

$$\mathrm{FOM}_P = N\langle\beta(\vec{x})\rangle^2$$

$$\langle\beta(\vec{x})\rangle = \frac{\int_X \beta(\vec{x})\mathrm{dx^n}}{\int_X \mathrm{dx^n}}\,, \quad X = \text{acc. events}$$

for example $\beta(\vec{x}) = \beta(\vartheta, \varphi) = A(\vartheta)\cos(\varphi)$

Summary

- Polarizations can be extracted from azimuthal dependent event rates, knowing the analyzing power A
- weighting the events with $\cos(\varphi)$ give the largest FOM
- Gain with respect to just counting events is $\frac{\mathrm{FOM}_{w=A\cos(\varphi)}}{\mathrm{FOM}_{cnt}} = \frac{\langle\cos(\varphi)^2\rangle}{\langle\cos(\varphi)\rangle^2}$
- Assumption made on acceptance, $PA \ll 1$, fixed ϑ, ... were only made to simplify dicussions

Literature I

- F. Müller et al., "Measurement of deuteron carbon vector analyzing powers in the kinetic energy range 170-380 MeV," 2020.
- C. Adolph et al., "Longitudinal double spin asymmetries in single hadron quasi-real photoproduction at high p_T," Phys. Lett., vol. B753, pp. 573–579, 2016.
- M. Alekseev et al., "Gluon polarisation in the nucleon and longitudinal double spin asymmetries from open charm muoproduction," Phys. Lett., vol. B676, pp. 31–38, 2009. [Online]. Available: https://doi.org/10.1016/j.physletb.2009.04.059
- G. W. Bennett et al., "Measurement of the negative muon anomalous magnetic moment to 0.7 ppm," Phys. Rev. Lett., vol. 92, p. 161802, 2004.

Literature II

- Pretz, J. and Müller, F., "Extraction of Azimuthal Asymmetries using Optimal Observables," Eur. Phys. J., vol. C79, no. 1, p. 47, 2019. [Online]. Available: https://doi.org/10.1140/epjc/s10052-019-6580-3
- J. Pretz, "Comparison of methods to extract an asymmetry parameter from data," Nucl. Instrum. Meth., vol. A659, pp. 456–461, 2011. [Online]. Available: https://doi.org/10.1016/j.nima.2011.08.036
- J. Pretz and J.-M. Le Goff, "Simultaneous Determination of Signal and Background Asymmetries," Nucl. Instrum. Meth., vol. A602, pp. 594–596, 2009.

Spare

Polarization P, Analyzing Power A

$$
\begin{aligned}
N_L &= a_L\,\rho\ell(n^\uparrow\sigma_{\uparrow,L} + n^\downarrow\sigma_{\downarrow,L}) \\
&\overset{\varphi-\mathrm{sym}}{=} a_L\,\rho\ell(n^\uparrow\sigma_{\uparrow,L} + n^\downarrow\sigma_{\uparrow,R}) \\
N_R &= a_R\,\rho\ell(n^\uparrow\sigma_{\uparrow,R} + n^\downarrow\sigma_{\downarrow,R}) \\
&\overset{\varphi-\mathrm{sym}}{=} a_R\,\rho\ell(n^\uparrow\sigma_{\uparrow,R} + n^\downarrow\sigma_{\uparrow,L})
\end{aligned}
$$

$n^\uparrow(n^\downarrow)$: nb. of beam particles with spin up (down)

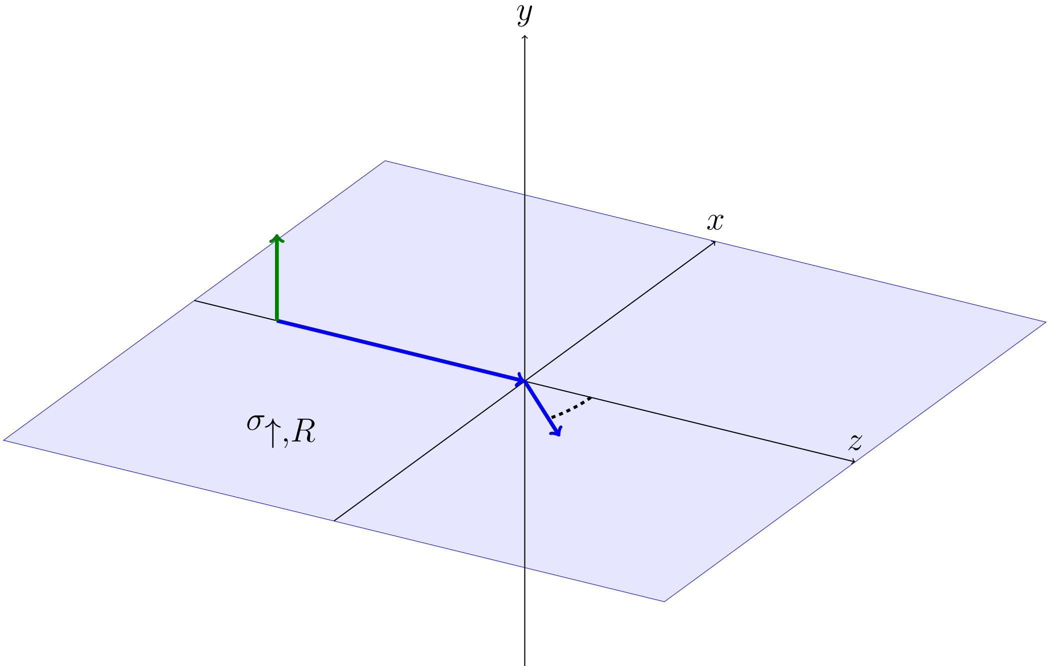
$P = \dfrac{n^\uparrow - n^\downarrow}{n^\uparrow + n^\downarrow}$: Polarization

$\sigma_{\uparrow,R} \equiv \sigma_{\downarrow,L} =: \sigma_R$: cross section for scattering process to

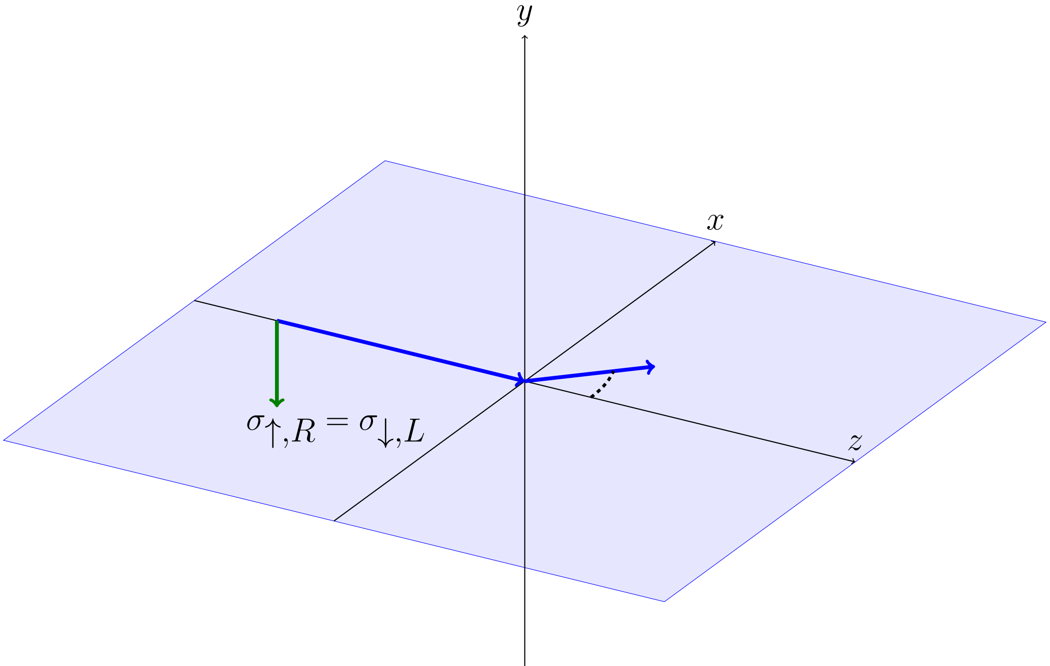
the right (left) if spin is up (down)

$\sigma_{\downarrow,R} \equiv \sigma_{\uparrow,L} =: \sigma_L$:

$A = \dfrac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}$: analyzing power



y
x
z
$\sigma_{\uparrow,R}$


y
x
z
$\sigma_{\uparrow,R} = \sigma_{\downarrow,L}$

Polarization P, Analyzing Power A

With the definitions

$$A = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} \quad \text{and} \quad P = \frac{n^{\uparrow} - n^{\downarrow}}{n^{\uparrow} + n^{\downarrow}}$$

one can write

$$\begin{aligned} N_R &= a_R\, \rho \ell \sigma (1 + AP) \\ N_L &= a_L\, \rho \ell \sigma (1 - AP) \\ & \text{with } \sigma = \frac{1}{2}(\sigma_R + \sigma_L) \end{aligned}$$