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Comparison of Frequency Domain method with the Koop-Wheel and BNL methods

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Currently, four methods for measuring EDM are considered as working:

-BNL method
-Koop Wheel method
-Frequency Domain method
-Double Magic Ring method
-RF-B (Artem)

The main tasks to be solved by the methods under consideration are:
- what do we measure;
- decoherence suppression mechanism
- implementation of the reverse field at CW and CCW;
- systematic errors.

BNL: magic ring

The main message follows from the T-BMT equations: in order to have a maximum EDM signal, the vector sum of the spin of all particles in a bunch must be non-zero and coincides with the direction of motion (*magic ring*).

However, the magic conditions are feasible only for a synchronous particle due to the difference in the spin frequency of the particles in the bunch.

This effect is called spin-decoherence and its compensation is achieved by introducing sextupoles.

Experiments on the COSY ring (FZJ, IKP) showed that polarization can be preserved for 1000 seconds.

BNL: what does the method measure?

In the frozen spin regime it measures **the vertical projection of the spin**, that is, the amplitude of the changing part of the signal $\tilde{S}_y$ during a long time ~1000 sec, namely

$$\tilde{S}_y = \sqrt{\left(\frac{\Omega_y \Omega_z}{\Omega^2}\right)^2 + \left(\frac{\Omega_x}{\Omega}\right)^2} \sin(\alpha + \varphi)$$

$$\alpha = \Omega \cdot \mathrm{t}$$

the solution at the initial conditions for zero horizontal, vertical Sx,Sy and for longitudinal components Sz=1 in the general form

$$\Omega_x = \Omega_{edm} + \Omega_{B_r} \qquad \Omega = \sqrt{\left(\Omega_{edm} + \Omega_{B_r}\right)^2 + \Omega_y^2 + \Omega_z^2}$$

where $\Omega_x, \Omega_y, \Omega_z$ are frequencies of spin precession in X,Y and Z planes.

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Basic concept of BNL method

Expecting the EDM value at the level of $\tilde{S}_y \approx 10^{-6} rad$ after 1000 sec and assuming that **the systematic errors** arise due to $\Omega_{\mathrm{B_r,E_y}} \neq 0$ plus **the geometric phase** it is necessary to correct all misalignments to such a magnitude

$$\Omega_y, \Omega_z, \Omega_{\mathrm{B_r}} \Rightarrow 0 \text{ up to } \Omega_y, \Omega_z, \Omega_{\mathrm{B_r}} \leq \Omega_{\mathrm{edm}}$$

This mode will be called **the 3-D frozen spin** or zero order spin resonance in 3D space, when the direction of the spin is fixed relative to the pulse in all three space directions.

In that case the contribution will be determined only by the EDM signal, that is the total frequency .

$$\Omega \approx \Omega_{\mathrm{edm}} + o(\Omega_{edm})$$

BNL: requirements to imperfections

first, the accuracy of the installation of magnets should be less than 10^{-9} meters (**super unrealistic !!!);**

second, the amplitude of $\tilde{S}_y$ depends on $\Omega_x, \Omega_y, \Omega_z$,

$$\tilde{S}_y = \sqrt{\left(\frac{\Omega_y \Omega_z}{\Omega}\right)^2 + \left(\frac{\Omega_{B_r} + \Omega_{edm}}{\Omega}\right)^2} \sin(\Omega t + \varphi)$$

$$\alpha = \Omega \cdot \mathrm{t} \quad \text{where } \Omega^2 = (\Omega_{B_r} + \Omega_{edm})^2 + \Omega_y{}^2 + \Omega_z{}^2$$

At $\Omega t \approx 10^{-6}$ $\Rightarrow$ $\tilde{S}_y = \sqrt{(\Omega_y \Omega_z)^2 + (\Omega_{B_r} + \Omega_{edm})^2} \cdot t$

and $\Omega_y, \Omega_z, \Omega_{B_r}$ of the same order with Ω_{edm} we do not know the value of $\tilde{S}_y$.

BNL: the geometric phase

If the frequencies in all three planes are of the same order and close to zero

$$\Omega_y, \Omega_z, \Omega_{B_r} => 0 \text{ up to } \Omega_y, \Omega_z, \Omega_{B_r} \le \Omega_{\mathrm{edm}}$$

then the invariant spin axis is completely undefined, that is, in each element the spin rotates around the arbitrary pronounced axis either X, either Y, or Z with an indefinite amplitude:

$$\sqrt{\left(\frac{\Omega_y \Omega_z}{\Omega}\right)^2 + \left(\frac{\Omega_{B_r} + \Omega_{edm}}{\Omega}\right)^2}$$

BNL: the geometric phase

the GEOMETRIC PHASE occurs when the invariant spin axis changes direction from element to element, and when the total angle of spin rotation in each of the planes

$$\sum_{i}^{2n} \delta_i = 0$$

is zero, nevertheless after **n**-pairs of elements of one turn we have the non-zero MDM deviation:

$$S_{\mathrm{y}}^{\Sigma} = S_y^0 (1 - n\delta^2)$$

This is an real fact!!!

BNL: criterion for minimizing the contribution of the geometric phase

Obviously, this contribution should be less than the EDM angle rotation per turn

$$S_y^{\Sigma} = S_y^0 (1 - n\delta^2)$$

that is

$$n\delta^2 < \alpha_{edm} \approx 10^{-6}/10^{9} = 10^{-15}$$

Thus, at n≈100 elements per one turn we must provide:

$$n\delta^2 < \alpha_{edm} \approx 10^{-6}/10^{9} = 10^{-15} \;\text{ or }\; \delta \approx 10^{-8} \div 10^{-9}$$

This is again **super unreal**!!!

BNL: how to suppress the geometric phase effect

BNL argues that if the beam orbit is stable and the total Lorentz force equals to zero, you can somehow get **δ** closer to this value **δ=>** $10^{-8} \div 10^{-9}$.

But at a total Lorentz force equal to zero, nevertheless the total angle of spin rotation is not equal to zero:

$$F_L = 0 \not\Rightarrow n\delta^2 = 0$$

BNL: how to restore conditions for a polarized beam at passing from CW to CCW

no reasonable ideas

Koop Wheel method: instead CW ⇔ CCW ⟹ Bx ⇔ -Bx

The idea of measuring EDM by introducing a transverse coil and measuring the spin precession in the vertical plane was proposed in the wheel concept by I. Koop [Proceedings of IPAC2013, Shanghai, China TUPWO040].

Ivan Koop proposes to install a transverse solenoid and changing the polarity of the field in the solenoid to use the separation of trajectories to calibrate the magnetic field in solenoid and try to measure the EDM:

$$\Omega_{edm} = \frac{\Omega_x(B_x) + \Omega_{edm} - \Omega_x(-B_x) + \Omega_{edm}}{2}$$

Koop Wheel Method: Systematic errors

First, we have modelled Koop's method in a special structure with a large beta function in the vertical plane of ~ 1000 m, and even in this case the trajectories are separated by the distance of 10^{-9} m.

Second, taking into account the systematic errors in the method, it should look like this:

$$\Omega_x = \frac{\Omega_x(B_x) + \Omega_{edm} + \Omega_x(\text{sys er}) + \Omega_x(-B_x) + \Omega_{edm} + \Omega_x(\text{sys err})}{2} = \Omega_{edm} + \Omega_x(\text{sys err})$$

Thus, the EDM cannot be separated from the systematic errors in the Koop Wheel method, what means that the method does not work.

Koop Wheel Method: spin decoherence

Koop argues in his article that the spin-wheel method allows to reduce the decoherence of the spin in horizontal plane. It is true!

In common case the T-BMT spin equation has a solution:

$$S_x(t) = \frac{\Omega_y \sin(\sqrt{\Omega_x^2 + \Omega_y^2} \cdot t)}{\sqrt{\Omega_x^2 + \Omega_y^2})}; S_y(t) = -\frac{\Omega_x \sin(\sqrt{\Omega_x^2 + \Omega_y^2} \cdot t)}{\sqrt{\Omega_x^2 + \Omega_y^2})}$$

Now we have $\left(\frac{\Omega_y}{\Omega_x}\right) << 10^{-5}$ and $\sqrt{\Omega_x^2 + \Omega_y^2} = \Omega_x \left[1 + 0.5\left(\frac{\Omega_y}{\Omega_x}\right)^2 ....\right]$

The above expression is transferred into:

$$S_x(t) = \frac{\Omega_y}{\Omega_{Bx}} \sin \Omega_{Bx} t; \qquad S_y(t) = -\sin(\Omega_{Bx} + \Omega_{EDM})t$$

Koop Wheel Method: spin decoherence

The spin decoherence in horizontal plane does not grow and oscillates around value of $\dfrac{\Omega_y}{\Omega_{Bx}}$

$$\langle S_x(t)\rangle_{\text{decoh}} \sim \frac{\Omega_y}{\Omega_{Bx}} \sin\langle \Omega_{B_x} \rangle_{decoh} \cdot t$$

In the same time in vertical plane:

$$\langle S_y(t)\rangle_{decoh} = -\sin\langle \Omega_{Bx} \rangle_{decoh} \cdot t$$

The decoherence in the horizontal plane becomes almost zero, but simultaneously the spin decoherence arises in the vertical plane, where we are going to measure EDM, and it plays the major role.

$$\langle \Omega_{Bx} \rangle = \frac{e}{m\gamma}(\langle \gamma \rangle G + 1) B_x$$

Frequency Domain Method: what do we measure?

In Frequency Domain method we measure instead amplitude

$$\tilde{S}_y = \sqrt{\left(\frac{\Omega_y \Omega_z}{\Omega^2}\right)^2 + \left(\frac{\Omega_x}{\Omega}\right)^2}\, \sin[\Omega t + \varphi],$$

the frequency

$$\Omega = \sqrt{\underbrace{\left(\Omega_{edm} + \Omega_{B_r}\right)^2}_{\Omega_x} + \underbrace{\Omega^2_{\mathrm{B_v}, E_r}}_{\Omega_y} + \underbrace{\Omega^2_{\mathrm{B_z}}}_{\Omega_z}},$$

<u>Frequency Domain Method:</u> systematic errors

Let us to consider the case when in

$$\Omega = \sqrt{\left(\Omega_{edm} + \Omega_{B_r}\right)^2 + \Omega^2_{\mathrm{B_v},E_r} + \Omega^2_{\mathrm{B_z}}},$$

We can not realize relation $\Omega^2_{\mathrm{B_v},E_r},\ \Omega^2_{\mathrm{B_z}} << \Omega^2_{edm}$,

but we can $\longrightarrow$ $\Omega^2_{\mathrm{B_v},E_r},\ \Omega^2_{\mathrm{B_z}} << \left(\Omega_{edm} + \Omega_{B_r}\right)^2$

we have so called **2D frozen spin option**:

$$\left(\Omega_{B_r}\right)^2 >> \Omega^2_{\mathrm{B_v},E_r},\ \Omega^2_{\mathrm{B_z}}$$

Frequency Domain Method: basic relationships

$$\Omega = \left(\Omega_{edm} + \Omega_{B_r}\right)\left[1 + \frac{1}{2}\frac{\Omega^2_{B_v,E_r} + \Omega^2_{B_z}}{\left(\Omega_{edm} + \Omega_{B_r}\right)^2}\right] \Rightarrow \Omega = \Omega_{B_r} + \Omega_{edm} + \frac{1}{2}\frac{\Omega^2_{B_v,E_r} + \Omega^2_{B_z}}{\Omega_{edm} + \Omega_{B_r}}$$

$$\Omega_{edm} > \frac{1}{2}\frac{\Omega^2_{B_v,E_r} + \Omega^2_{B_z}}{\Omega_{edm} + \Omega_{B_r}}$$ finally $\Rightarrow$ $$\boxed{\Omega_{B_r} > \frac{1}{2}\frac{\Omega^2_{B_v,E_r} + \Omega^2_{B_z}}{\Omega_{edm}}}$$

Main idea: to make the contribution from EDM frequency into the total frequency Ω bigger than from MDM additions $\Omega^2_{\mathrm{B_v},E_r}, \Omega^2_{\mathrm{B_z}}$

Frequency Domain Method: 2D frozen spin

Thus, we do not require 3D frozen spin when all three frequencies close to zero:

$$\Omega_y, \Omega_z, \Omega_{B_r} \Rightarrow 0 \text{ up to } \Omega_y, \Omega_z, \Omega_{B_r} \leq \Omega_{\mathrm{edm}}$$

Now we realize 2D frozen spin with simple condition:

$$\Omega_{B_r}(\sim 10^2\, rad/\sec) >> \Omega_{edm}(\sim 10^{-9}\, rad/\sec)$$

Having the frequency Ω_{B_r}~50÷100 rad/sec in the vertical plane and making the frequencies Ω_{B_v,E_r} and Ω_{B_z} in other planes much smaller ~10^{-3} rad/sec we realize conditions, when the contribution of other frequencies is less than the contribution of the expected EDM frequency in the vertical frequency.

Frequency Domain Method: geometric phase

Making frequencies Ω_{B_v,E_r} and Ω_{B_z} up to the value of ~10^{-3} rad/sec (**six orders higher of EDM**) it does not mean we need to know them precisely for both CW and CCW.

In Frequency Domain Method there is no problem of the geometric phase, since the spin oscillates around the invariant spin axis under the above condition, and the contribution from frequencies of other planes is less than the EDM frequency.

During a revolution, the invariant axis does not jump, as in the BNL variant, but is oriented all the time along the horizontal direction with a kind of jitter, which on average gives zero contribution.

BNL and Frequency Domain methods

The difference between BNL and Frequency Domain methods is that we do not require the condition of 3D-frozen spin. In one of the plane spin is not frozen and in the other two planes on 6 orders the accuracy requirements are weaker.

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Frequency Domain method: calibration of effective gamma

The second part of FDM strategy is when we calibrate the effective gamma by measuring the spin precession frequency in the horizontal plane, where there is no EDM, and we have the precession frequency of the spin in the horizontal plane much larger than the frequencies in other planes, that is the invariant spin axis coincides with the vertical axis, and we exclude the influence of frequencies in other planes on the calibration in the horizontal plane

Methodology of EDM search

The proposed method of searching for EDM is based on:

- measuring the sum and difference frequency of spin precession in the vertical plane due to EDM and MDM, correspondingly, for the CW and CCW cases with absolute accuracy 10^{-7} rad per second in one fill;

- independence of the absolute error in determining the spin precession frequency from the frequency itself;

- Unchangeable position of the accelerator elements and as a consequence the constant ratio of the leading field By to the component of the field that determine the fake signal Bx;

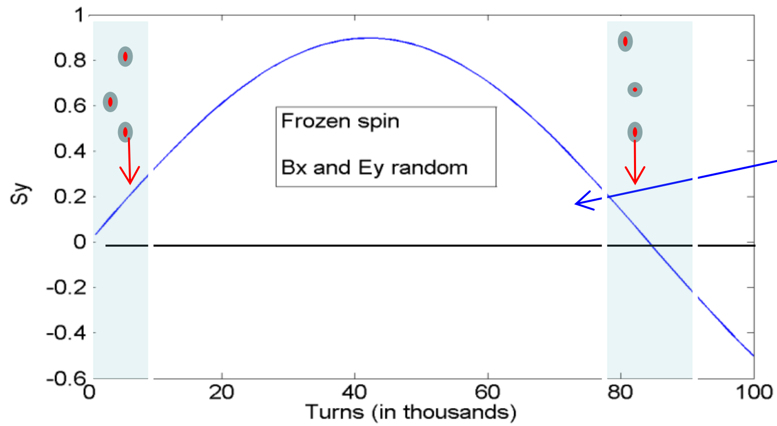
- the non-influence of spin precession frequencies in other planes on the spin precession frequency in the vertical plane. Certain relations are fulfilled between them, having the character of an approximate value.

- For the transition from CW to CCW it is suggested to use the calibration of the equilibrium Lorentz factor in terms of the precession of the spin in the horizontal plane, which is then used in the vertical plane.


Thank you!

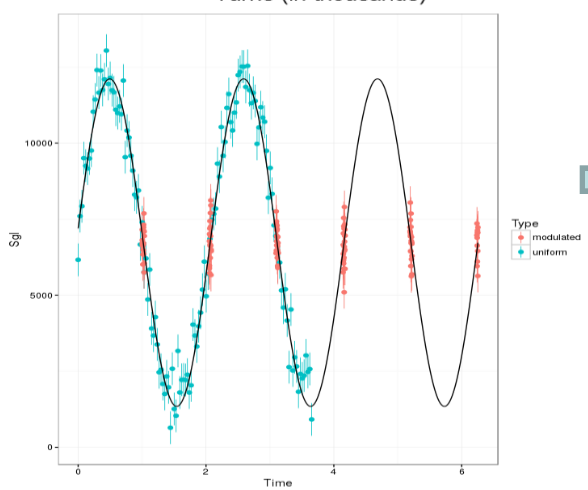
COSY Inf+MODE simulation of systematic errors due to magnet rotation around the longitudinal axis

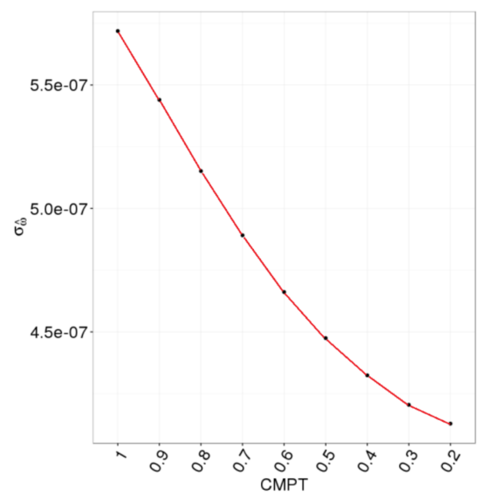
Coherent component

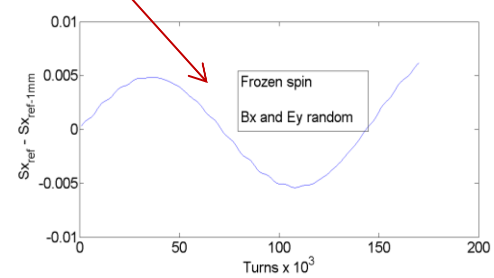


$$S_y(t) \approx -\sin(\Omega_{Bx} + \Omega_{EDM}) \cdot t$$

$$< S_x(t) >= \frac{\langle \delta\Omega_{decoh} \rangle}{\Omega_{Bx}} \cdot \sin\Omega_{Bx} \cdot t$$







Ring with Imperfections

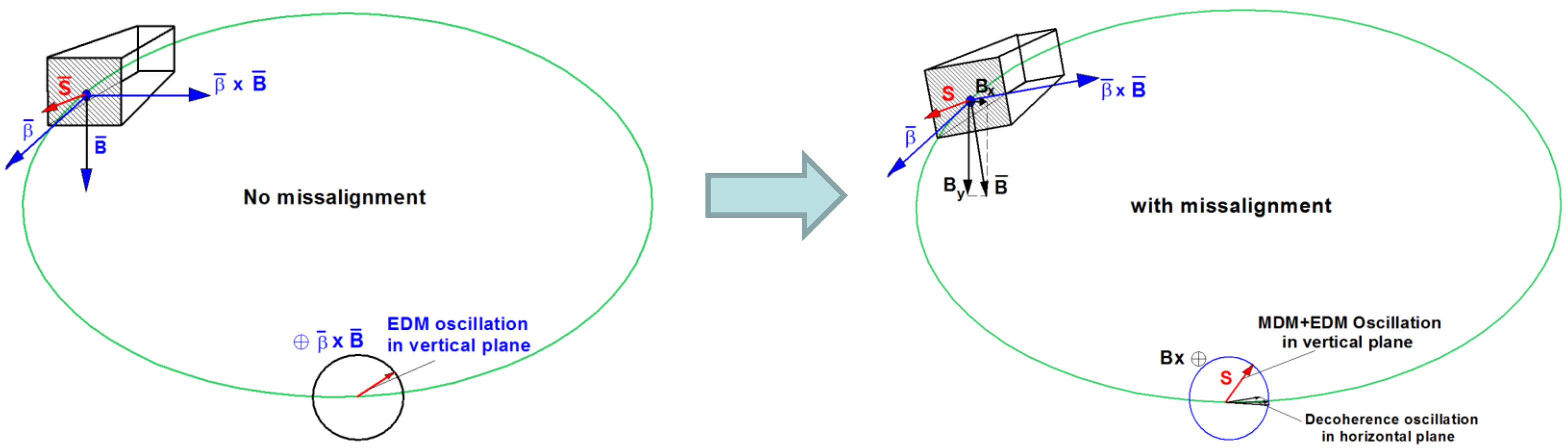
In both proton and deuteron rings it is planned that the concept of frozen spin be realized.

In this sense, there is a general idea of how to construct a ring, but this is realized with the help of different types of deflectors.

These differences do not play an essential role with respect to the spin-orbital motion, so long as the motion of the reference particle in the perfect ring without imperfections is considered.

Radial B_r, vertical E_v fields ⟹ fake EDM signal

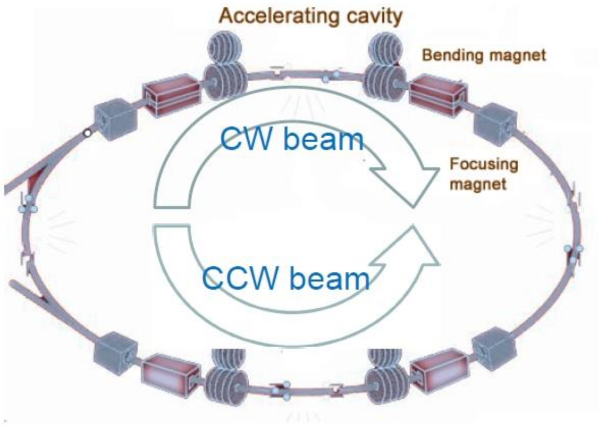
The presence of errors in the installation of the elements (imperfections) of the ring leads to the appearance of vertical and radial components of the electric and magnetic fields, respectively.



They both change the spin components in the vertical plane, in which the EDM signal is expected, and create the systematic errors that initiate <u>the “fake EDM” signal</u>.

CW and CCW procedures

To solve this problem in the case of a proton beam, it was suggested that the procedure of simultaneously injecting two beams in the ring in two opposite directions, clockwise (CW) and counterclockwise (CCW) [BNL], be used.



Adding the CW and CCW results together, the EDM can be separated from a systematic error arising due to MDM.

CW and CCW in pEDM ring

It is assumed that in the electrostatic ring the problem of systematic errors can be solved by simultaneous beam injection in CW and CCW

Although we must note that the simultaneous CW-CCW motion of protons in the pEDM ring does not guarantee an equal contribution of MDM, since they can move along trajectories of different lengths.

MDM spin rotation in pEDM ring

In the Laboratory coordinate system (LCS)for the pure electrostatic ring w/o EDM:

$$\frac{d\vec{S}}{dt} = \vec{S} \times \vec{\Omega}_{mdm};$$
$$\vec{\Omega}_{mdm} = -\frac{e}{m}\left(G + \frac{1}{\gamma+1}\right)\frac{\vec{\beta}\times\vec{E}}{c}$$

In horizontal plane of LCS:

$$\Omega_y = -\frac{e}{m}\left\{\left(G + \frac{1}{\gamma+1}\right)\frac{\beta_z \cdot E_x}{c}\right\}$$

the horizontal frequency of particle momentum precession in the LCS

$$\Omega_p = -\frac{e}{mc}\frac{E \cdot \beta_z}{\gamma\beta^2}$$

the spin tune in the horizontal plane relative to momentum

$$\nu_y = \frac{\Omega_y - \Omega_p}{\Omega_p} = \left(\frac{1}{\gamma^2 - 1} - G\right)$$

in vertical plane of LCS: $\Omega_x = \frac{e}{m}\left(G + \frac{1}{\gamma+1}\right)\frac{\beta_z \cdot E_y}{c}$ or at $\frac{1}{\gamma^2 - 1} = G$ $\Omega_x = G\gamma \cdot \frac{e}{m}\frac{\beta_z \cdot E_y}{c}$

$\Omega_{EDM} \approx 10^{-9} \, rad/\sec$

Spin rotation in vertical plane due to quadrupole misalignment

Parameters of ring:

1. Circumference 399 m
2. Gradient in quadrupole dE/dr=8 kV/cm^2
3. Quadrupole length l_{eff}=0.8 m
4. Quadrupole number N_{quadr}=64
5. Error of installation rms=30 μm

So, now we will substitute these parameters.

Dipole mode value E_y=8*10^7 v/m^2*30·10^{-6} m=2.4·10^3V/m

Taking into account the effective length l_{eff}/L_C=0.8/399

the effective vertical field of quadrupole $E_{y\,eff}$=2.4·0.8/399=4.8 V/m

Thus,

$$\langle \Omega_x \rangle = G\gamma \cdot \frac{e}{m} \frac{\beta_z \cdot \langle E_y \rangle}{c} \cdot \frac{1}{\cdot \sqrt{N_{qudr}}} = 1.4*1.23 \frac{1.6 \cdot 10^{-19} \cdot 0.59 \cdot 4.81}{1.67 \cdot 10^{-27} \cdot 3 \cdot 10^{8} \cdot \sqrt{N_{quadr}}} = 0.2 \, rad/\sec$$

$$\Omega_{EDM} \approx 10^{-9} \, rad/\sec$$

Method of proton Electric Dipole Moment search in storage ring with imperfection

- We are really deprived of ability to measure the accumulated EDM signal by growth of the vertical component of spin suggested in BNL, since the spin rotates in the plane where we expect to see EDM with incomparably higher speed due to MDM.

- In the **Frequency Domain Method (FDM)** we propose to measure not the magnitude of the vertical component of spin, but the total precession frequency of the spin due to the EDM and MDM.

- 1. $\Omega_{CW} = \Omega_{r,mdm}^{CW} + \Omega_{edm}$
- 2. $\Omega_{CCW} = -\Omega_{r,mdm}^{CCW} + \Omega_{edm}$

$$\Longrightarrow \Omega_{edm} = (\Omega_{CW} + \Omega_{CCW})/2 + (\Omega_{r,mdm}^{CCW} - \Omega_{r,mdm}^{CW})/2$$

Four problems

First problem:

the accuracy of the frequency measurement of determines the precision of the EDM measurement.

For an absolute statistical error of measuring a frequency of the spin oscillation, we can use

$$\sigma_\Omega = \delta\varepsilon_A \sqrt{24/N}\,/T$$

N - the total number of recorded events,

$\delta\varepsilon_A \approx 0.03$ - the relative error in measuring the asymmetry

$T \approx 1000$ sec is the measurement duration.

At 10^{11} particles per fill and a polarimeter efficiency of 0.01 an absolute error of frequency measurement is $\sigma_\Omega = 2 \cdot 10^{-7}$.

At an average accelerator beam time of 6,000 hours per year, we can reach $\sigma_\Omega \approx 10^{-9}$ rad/sec using one-year statistics,

that is the accuracy of frequency is satisfactory and sufficient for reaching at a parameter of $d_d \approx 10^{-30} e \cdot cm$

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Second problem is splited into two procedures

From the decoherence investigation:

Two particles which have different orbits and different initial conditions in the transverse planes as well as different initial energy deviations are assumed to be the same – from the point of view of the spin behavior – if they have the same spin tune, and the latter is reached when the length of the particle orbit is equal.

Equation shows this dependence:

$$\Delta\delta_{eq} = \frac{\gamma_s^2}{\gamma_s^2\alpha_0 - 1}\left[\frac{\delta_m^2}{2}\left(\alpha_1 - \frac{\alpha_0}{\gamma_s^2} + \frac{1}{\gamma_s^4}\right) + \left(\frac{\Delta L}{L}\right)_\beta\right]$$

The effective Lorentz factor:

$$\gamma_{eff} = \gamma_s + \beta_s^2\gamma_s \cdot \Delta\delta_{eq}$$

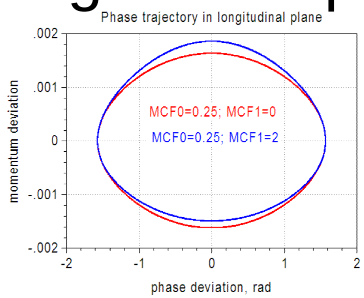
includes three spatial coordinates and completely determines the frequency of spin precession in all three planes.

Effective gamma

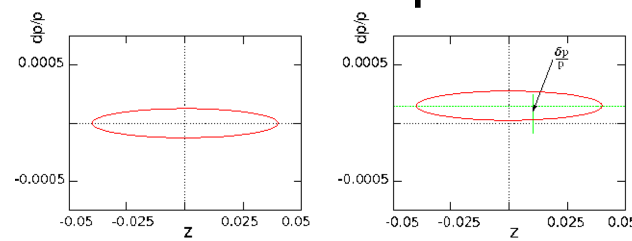
The effective gamma plays an extremely important role. The concept of an effective gamma arises after averaging over synchrotron oscillations.

$$\overline{\frac{\Delta T_{rev}}{T_{rev}}} = \left(\alpha_1 - \frac{\alpha_0}{\gamma^2} + \frac{1}{\gamma^4}\right) \cdot \overline{\delta^2} + \overline{\left(\frac{\Delta L}{L}\right)_\beta}$$

Longitudinal plane



Transverse plane



Synchrotron tune has to be one-two order bigger of the spin tune spread without RF:

$$\nu_{synch} = \frac{1}{\beta}\sqrt{\frac{e\hat{V}h\eta}{2\pi E}} >> \nu_{spin} = \gamma G \cdot \frac{\Delta\gamma}{\gamma}$$

From the decoherence to the beam in CW-CCW

We can now apply this important conclusion to the beams moving in opposite CW and CCW directions.

The beams are identical in terms of spin behaviour if they have the same effective Lorentz factor averaged over all particles in beam.

- this means that the problem of finding the multiparameter dependence of spin precession on fields and 3D trajectories is reduced to the search for a dependence on the effective gamma γ_{eff} ;
- it is no longer necessary to obtain a coincidence of trajectories, but instead only requires the condition of equality for CW and CCW beams;
- this approach saves the whole idea of searching for EDM in a storage ring.

Third problem: rigid structure

Spin tune in vertical plane (around radial axis) is:

$$\Omega^r_{mdm} = -\frac{e}{m\gamma}\left(\gamma G + \frac{\gamma}{\gamma+1}\right)\frac{\beta_z E_v}{c}$$

Due to the frozen spin condition and averaging over whole bunch

$$\nu^r_s = F_E(\gamma_{eff}) \cdot \frac{E_v}{E_r}$$

3D beam dynamic factor

Geometrical factor

If we assume that there are two rings with a direct (CW) and reverse sequence of elements (CCW) with a changed polarity of magnetic field, the similarity of these rings under the beam stability condition is only that the position of all elements on the ring and, consequently, the relation between the values of the vertical and radial components of the field remains unchanged:

$$E_v/\mathrm{E_r} = \mathrm{const}$$

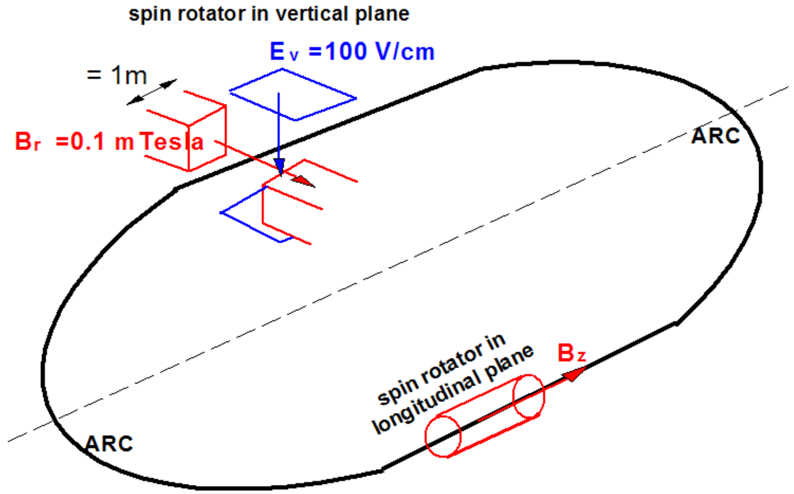
Calibration of γ_{eff} in horizontal plane

Before changing the polarity, we must calibrate the gamma close to the value $\gamma \approx \gamma_s$ using the precession frequency measurements of the spin in the horizontal plane where there is no EDM signal before restoring the same γ_{eff} in the ring with the reverse sequence of elements.

For such a calibration, we need to reduce the spin oscillation in the vertical plane to a low value by introducing a spin rotator 1 m long with ExB transverse magnetic and electric fields in the order of 0.1 mT and 100 V/cm respectively.

Calibration of γ_{eff} in horizontal plane

The transverse spin rotator is switched on only for the time of calibration of the in the CW ring and for the time of its recovery in the CCW ring.



Since we are able to calibrate the frequency, that is γ_{eff} , with the above-mentioned absolute value of errors for one beam fill $\sigma_\Omega \approx 10^{-7}$ rad/sec and $\sigma_\Omega \approx 10^{-9}$rad/sec with one-year statistics, and also taking into account the constant relation between the vertical and the radial components of field, this means that in the case of CCW we have a ring identical to the CW ring in terms of spin behaviour, and we can obtain a zero value of $\Omega^{CCW}_{r,mdm} - \Omega^{CW}_{r,mdm}$ with an accuracy of $\approx 10^{-9}$.

Fourth problem: frequencies mixing

This problem concerns the fact that the spin oscillation in any of the planes includes the uncontrollable spin oscillation in the other two planes.

$\Omega = \sqrt{\Omega_{\mathrm{x}}^2 + \Omega_{\mathrm{y}}^2 + \Omega_{\mathrm{z}}^2}$ is the module of three-dimensional frequency,

where $\Omega_x = \Omega_{E_y} + \Omega_{edm}$, $\Omega_z = \Omega_z$ and $\Omega_y = \Omega_{E_r}$

3D frequency $\Omega = \sqrt{(\Omega_{edm} + \Omega_{E_y})^2 + \Omega_{E_r}^2 + \Omega_z^2}$

Assuming that, in accordance with the frozen spin concept, we maintain the spin along the momentum $\Omega_{E_r} << \Omega_{E_y}$ and $\Omega_z << \Omega_{E_y}$, the latter is realized by installing a longitudinal solenoid one meter long on the straight section with the magnetic field $\sim 10^{-6}$ Tesla, which can be formulated as follows:

$$\Omega = (\Omega_{edm} + \Omega_{E_y}) \left[1 + \frac{1}{2} \frac{\Omega_{E_r}^2 + \Omega_z^2}{(\Omega_{edm} + \Omega_{E_y})^2} \right]$$

Frequencies restriction

The restriction occurs at the values of Ω_{E_r} and Ω_z, which should have less of an effect on the total frequency than the EDM:

$$\frac{\Omega_{E_r}^2 + \Omega_z^2}{2\Omega_{E_y}} < \Omega_{edm}$$

If we evaluate these requirements numerically, we see how difficult it is to implement them technically. For instance, $\Omega_{E_y} \approx$ 1 rad/sec and $\Omega_{edm} \approx 10^{-8}$ rad/sec, then

$$\Omega_{E_r}^2 + \Omega_z^2 < 10^{-8}$$

both must be $\Omega_{B_y,E_r}, \Omega_{B_z} \propto 10^{-4}$.

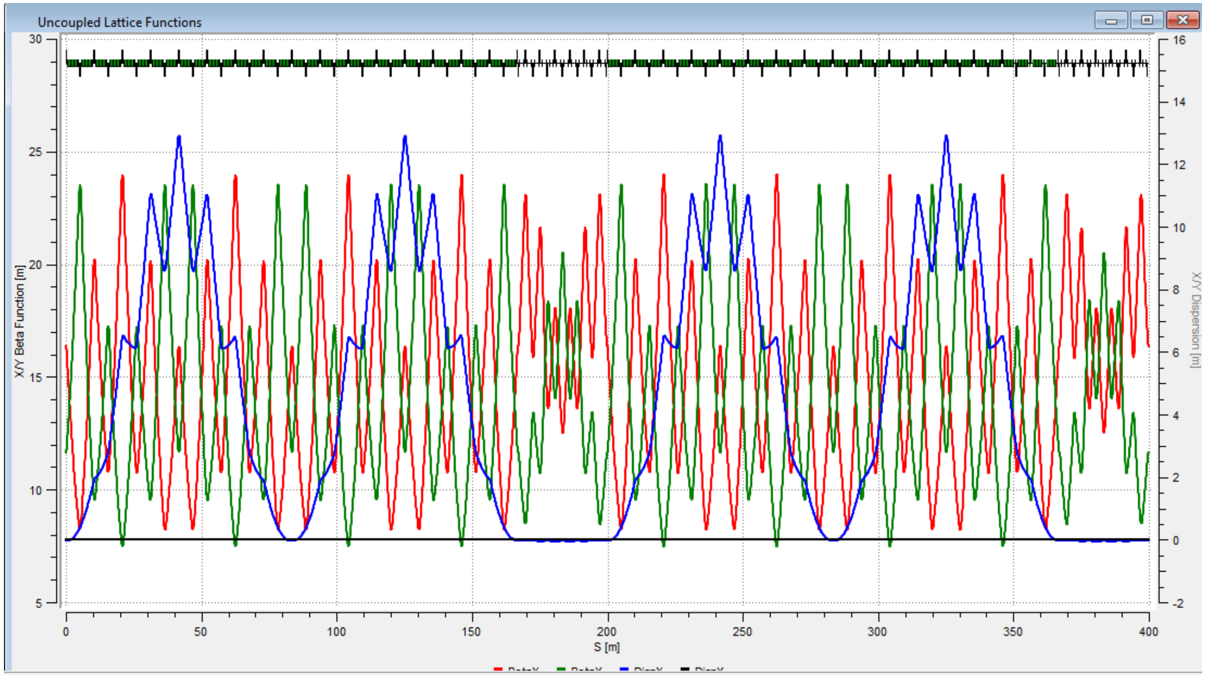
This means that at the spin coherence time ~1000 sec, the spin rotation should not exceed $\Omega_{E_r} \cdot t_{SCT}$ ~ 1rad and $\Omega_z \cdot t_{SCT}$ ~1rad, which is easily achievable both for Ω_{E_r} due to the calibration of energy and for Ω_z due to the introduction of a longitudinal solenoid.

We conclude that the imperfections of ring elements, which previously played a limiting role in the measurement of EDM, now provide a pure precession of the spin in the vertical plane, where we will measure the EDM.

Lattice with one meter straight deflector: total Length=399. m

Tunes: Qx=4.60186 Qy=4.82802

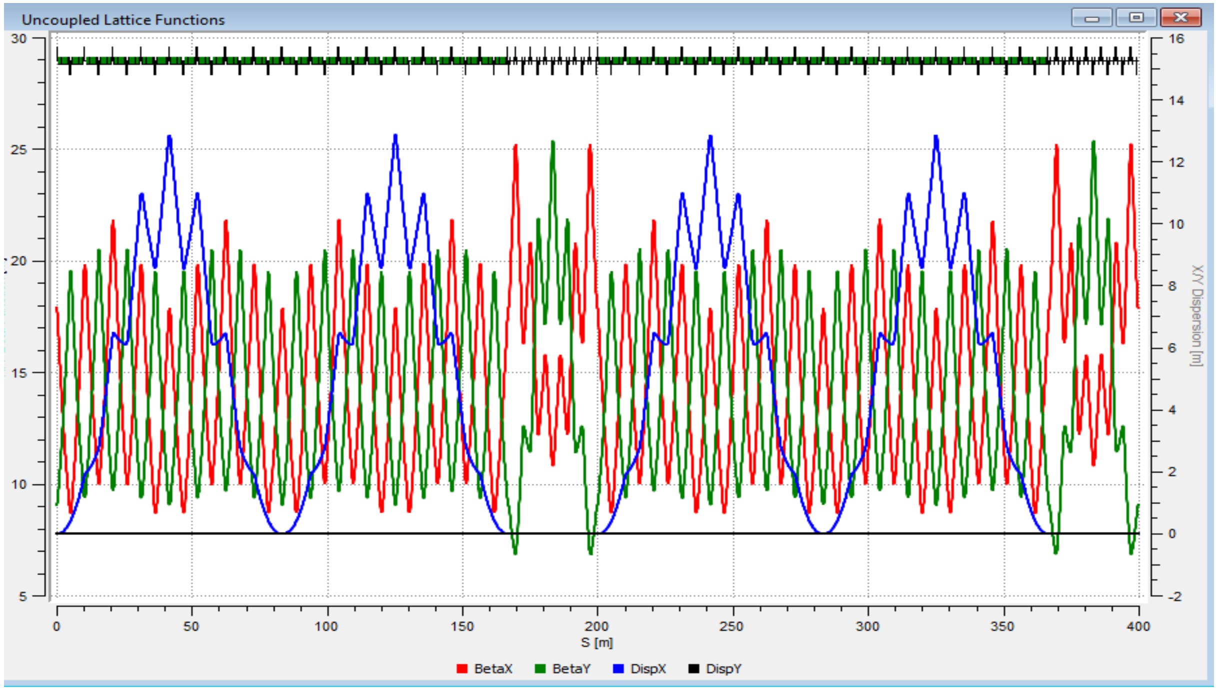
TWISS functions



Lattice with one meter curved deflector: total Length=399. m

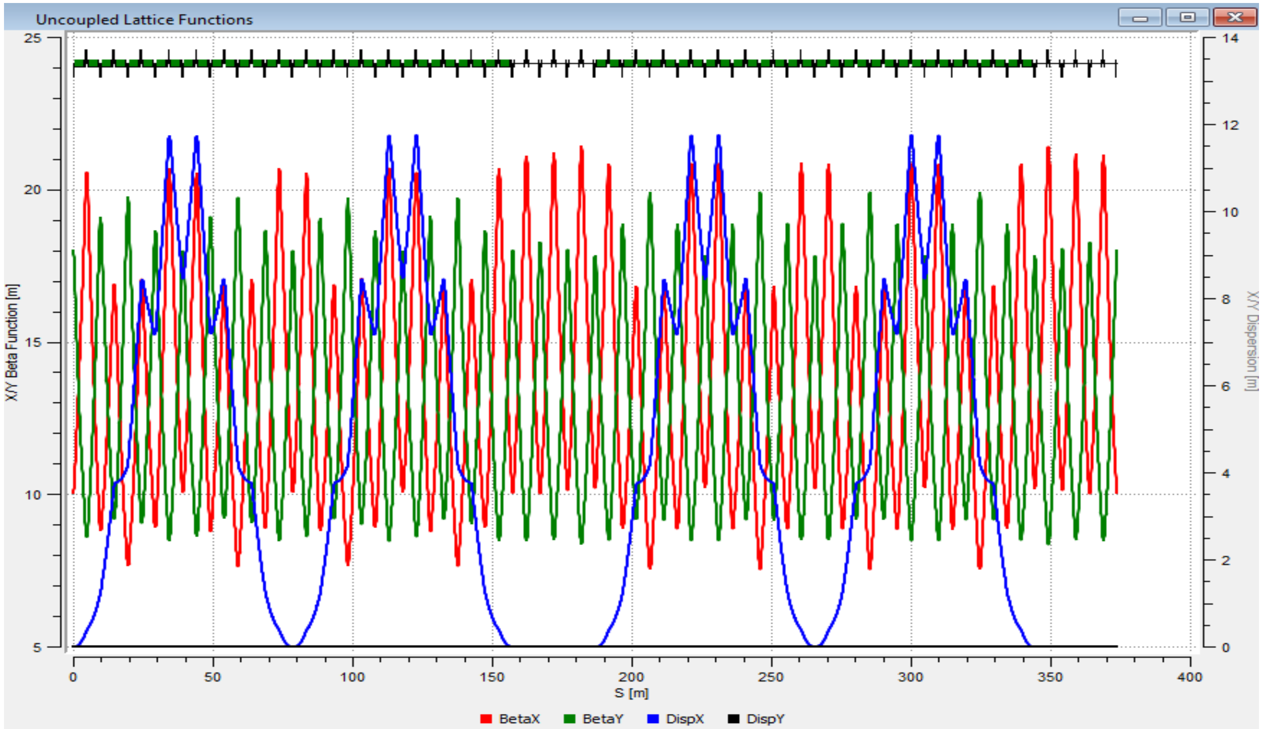
Qx = 4.63434, Qy = 4.84742

TWISS functions



Lattice with 3 meter curved deflector: Total Length : 374 m
Tunes : Qx = 4.65574, Qy = 4.78304

TWISS functions



Conclusion

Three variants have been considered:

- a structure with a cylindrical and direct deflector 1 m long and a cylindrical deflector 3 m long.
- For a case with a deflector length of 1 m, we have an orbit sagitta of the order of 2 mm, which allows us to use the straight deflector, and each deflector rotates about two degrees relative of the previous one.
- For three meters deflector, the sagitta becomes 9 times larger, which is already unacceptable.

Thus, a variant with meter-long deflectors can be attractive in terms of simplified technology and cost.

- The proposed method of searching for EDM is based on measuring the sum and difference frequency of **spin precession in the vertical plane** due to EDM and MDM, respectively, for the CW and CCW cases.

- In order to **exclude the influence of spin precession frequencies** in other planes on the spin precession frequency in the vertical plane, certain relations are fulfilled between them, having the character of an approximate value.

- For the transition from CW to CCW, it is suggested to use **the calibration** of the equilibrium Lorentz factor in terms of the precession of the spin **in the horizontal plane**, which is then used for **EDM search in the vertical plane**.

Methodology of EDM search

The proposed method of searching for EDM is based on:

- measuring the sum and difference frequency of spin precession in the vertical plane due to EDM and MDM, correspondingly, for the CW and CCW cases with absolute accuracy 10^{-7} rad per second in one fill;
- independence of the absolute error in determining the spin precession frequency from the frequency itself;
- Unchangeable position of the accelerator elements and as a consequence the constant ratio of the leading field By to the component of the field that determine the fake signal Bx;
- the non-influence of spin precession frequencies in other planes on the spin precession frequency in the vertical plane. Certain relations are fulfilled between them, having the character of an approximate value.
- For the transition from CW to CCW it is suggested to use the calibration of the equilibrium Lorentz factor in terms of the precession of the spin in the horizontal plane, which is then used in the vertical plane.

Wheel method of Ivan Koop

the idea of measuring EDM by introducing a transverse coil and measuring the spin precession in the vertical plane was proposed in the wheel concept by I. Koop [11].

It differs from the method considered here. The wheel method uses a special horizontal coil, assuming the field is calibrated in the coil, measuring the beam offset versus field values in the coil.

Nevertheless, the problem with systematic errors at the presence of misalignment remained unsolved. In addition, the method requires special optics of the ring with a weak focusing in the vertical plane, which entails certain difficulties in the beam dynamics.

Comparison of the two methods BNL and FZJ

BNL:

- decoherence in horizontal plane;
- super-high requirement to misalignment;
- geometrical phase;
- method is based on measurement of vertical component of spin

FZJ:

- decoherence in vertical plane;
- realistic requirements for misalignment;
- mixing of frequency;
- method is based on measurement of spin frequency in vertical plane;


Thank you!