

# Electric Dipole Moments of Hadrons and Light Nuclei in chiral EFT

# CP violation and the Electric Dipole Moment (EDM)

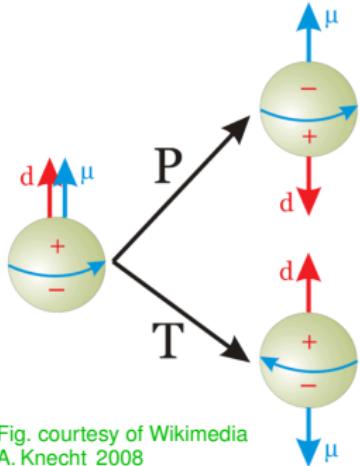


Fig. courtesy of Wikimedia  
A. Knecht 2008

$$\text{EDM: } \vec{d} = \sum_i \vec{r}_i e_i \xrightarrow[\text{(polar)}]{\substack{\text{subatomic} \\ \text{particles}}} d \cdot \vec{S}/|\vec{S}| \xrightarrow[\text{(axial)}]{}$$

$$\mathcal{H} = -\mu \frac{\vec{S}}{S} \cdot \vec{B} - d \frac{\vec{S}}{S} \cdot \vec{E}$$

$$P: \quad \mathcal{H} = -\mu \frac{\vec{S}}{S} \cdot \vec{B} + d \frac{\vec{S}}{S} \cdot \vec{E}$$

$$T: \quad \mathcal{H} = -\mu \frac{\vec{S}}{S} \cdot \vec{B} + d \frac{\vec{S}}{S} \cdot \vec{E}$$

Any *non-vanishing EDM* of a **non-degenerate**  
(e.g. subatomic) particle violates **P & T**

- Assuming **CPT** to hold, **CP** is violated as well (flavor-diagonally)  
→ subatomic EDMs: “rear window” to CP violation in early universe
- Strongly suppressed in SM (CKM-matrix):  $|d_n| \sim 10^{-31} \text{ ecm}$ ,  $|d_e| \sim 10^{-38} \text{ ecm}$
- Current bounds:  $|d_n| < 3^\diamond / 1.6^* \cdot 10^{-26} \text{ ecm}$ ,  $|d_p| < 2 \cdot 10^{-25} \text{ ecm}$ ,  $|d_e| < 1 \cdot 10^{-28} \text{ ecm}$   
*n:* Baker et al.(2006)<sup>◊</sup>, *p* prediction: Dimitriev & Sen'kov (2003)\*, *e:* Baron et al.(2013)<sup>†</sup>

\* from  $|d_{^{199}\text{Hg}}| < 7.4 \cdot 10^{-30} \text{ ecm}$  bound of Graner et al. (2016)

† from polar ThO:  $|d_{\text{ThO}}| \lesssim 10^{-21} \text{ ecm}$

# A *naive* estimate of the scale of the nucleon EDM

Khriplovich & Lamoreaux (1997); Kolya Nikolaev (2012)

- CP & P conserving magnetic moment  $\sim$  nuclear magneton  $\mu_N$

$$\mu_N = \frac{e}{2m_p} \sim 10^{-14} \text{ ecm}.$$

- A nonzero EDM requires

**parity P violation:** the price to pay is  $\sim 10^{-7}$

$$(G_F \cdot F_\pi^2 \sim 10^{-7} \text{ with } G_F \approx 1.166 \cdot 10^{-5} \text{ GeV}^{-2}),$$

and additionally **CP violation:** the price to pay is  $\sim 10^{-3}$

$$(|\eta_{+-}| \equiv |\mathcal{A}(K_L^0 \rightarrow \pi^+ \pi^-)| / |\mathcal{A}(K_S^0 \rightarrow \pi^+ \pi^-)| = (2.232 \pm 0.011) \cdot 10^{-3}).$$

- In summary:  $|d_N| \sim 10^{-7} \times 10^{-3} \times \mu_N \sim 10^{-24} \text{ ecm}$
- In SM (without  $\theta$  term): extra  $G_F F_\pi^2$  factor to *undo* flavor change

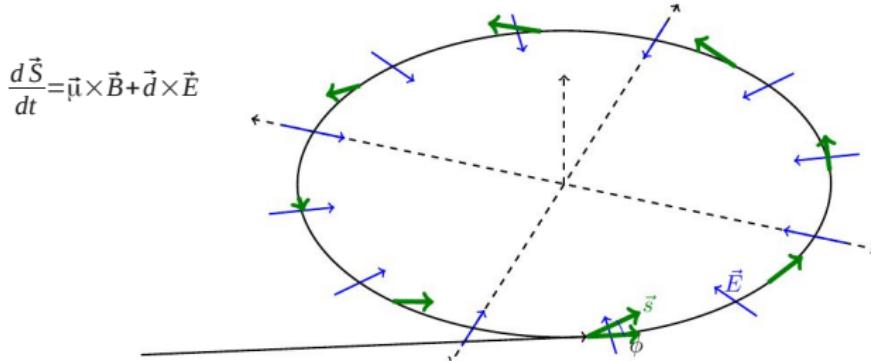
$$\hookrightarrow |d_N^{\text{SM}}| \sim 10^{-7} \times 10^{-24} \text{ ecm} \sim 10^{-31} \text{ ecm}$$

$\hookrightarrow$  *The empirical window* for search of physics BSM( $\theta=0$ ) is

$$10^{-24} \text{ ecm} > |d_N| > 10^{-30} \text{ ecm}.$$

# Search for EDMs of charged particles in storage rings

General idea:



Initially **longitudinally polarized** particles interact with **radial  $\vec{E}$**  field  
 ↳ build-up of vertical polarization (measured with a polarimeter)

The spin precession relative to the momentum direction is given by the **Thomas-BMT equation** (for  $\vec{\beta} \cdot \vec{B} = 0$ ,  $\vec{\beta} \cdot \vec{E} = 0$ ,  $\vec{E} \cdot \vec{B} = 0$ ):

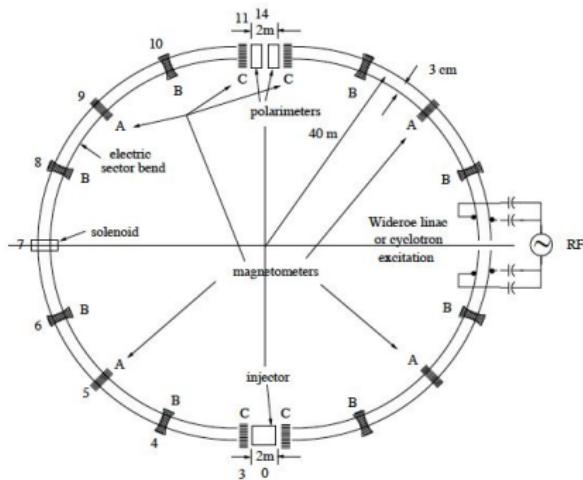
$$\frac{d\vec{S}^*}{dt} = \vec{\Omega} \times \vec{S}^* \quad \text{with} \quad \vec{\Omega} = -\frac{e}{m} (a\vec{B} + (\beta^{-2} - 1 - a)\vec{\beta} \times \vec{E} + \eta(\vec{E} + \vec{\beta} \times \vec{B}))$$

and  $\vec{\mu} = (1 + a) \frac{e}{2m} \vec{S}/S$  and  $\vec{d} = \eta \frac{e}{2m} \vec{S}/S$

## Method 1: pure electrostatic ring

$$\vec{\Omega} = -\frac{e}{m} \left( \underbrace{a\vec{B} + (\beta^{-2} - 1 - a)\vec{\beta} \times \vec{E}}_{:=0, \text{ "Frozen spin method"}}, + \eta(\vec{E} + \vec{\beta} \times \vec{B}) \right) \longrightarrow -\frac{e}{m}\eta\vec{E}$$

only possible for  $a > 0$ , i.e. for  $p$  and  ${}^3\text{H}$  (or  ${}^{19}\text{F}$ ), but not for  $d$  or  ${}^3\text{He}$



### Advantages:

- no magnetic field
- counter rotating beams possible

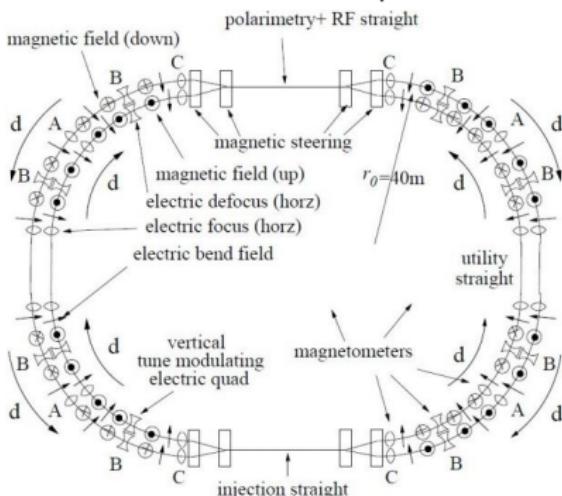
### Disadvantage:

- not possible for deuterons ( $a_D < 0$ )

srEDM BNL / KAIST Korea ( $\gtrsim 2000$ ): design for  $E$ -ring for protons

## Method 2: combined electric & magnetic ring

$$\vec{\Omega} = -\frac{e}{m} \left( \underbrace{a\vec{B} + (\beta^{-2} - 1 - a)\vec{\beta} \times \vec{E}}_{:=0, \text{ "Frozen spin method"}}, + \eta(\vec{E} + \vec{\beta} \times \vec{B}) \right) \rightarrow -\frac{e}{m}\eta(\vec{E} + \vec{\beta} \times \vec{B})$$



### Advantage:

- works for  $p$ , deuterons and  $^3\text{He}$

### Disadvantages:

- requires also magnetic fields
- two beam pipes
- magnetic coils made of copper

JEDI Jülich/Aachen ( $\gtrsim 2011$ ): design for  $E/B$  ring

## Method 3: pure magnetic ring

$$\vec{\Omega} = -\frac{e}{m} \left( \underbrace{a \vec{B}}_{\text{precession in beam plane}} + \left( \frac{1}{\beta^2} - 1 - a \right) \vec{\beta} \times \vec{E} + \underbrace{\eta (\vec{E} + \vec{\beta} \times \vec{B})}_{\substack{\text{+ Wien filter:} \\ \text{accumulation} \\ \text{of vertical spin}}} \right)$$



polarized  $p$  and  $D$  with  $p = (0.3\text{--}3.7)$  GeV/ $c$

### Advantage:

- existing COSY accelerator
- precursor experiment:

First attempt for *direct* measurement  
of an EDM of a charged hadron

### Disadvantage:

- low sensitivity
- $\gtrsim 10^{-21}\text{--}10^{-24}$  e cm JEDI at COSY

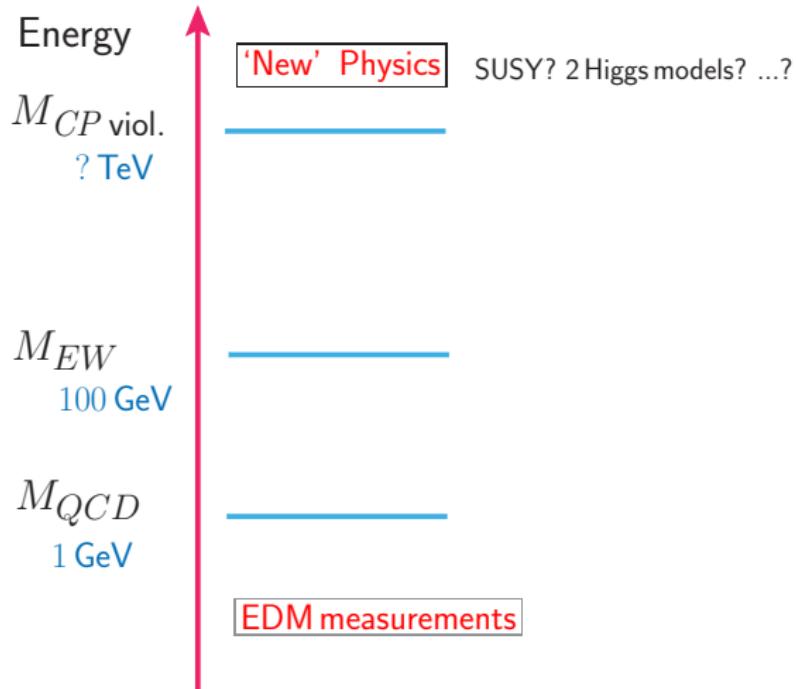
### Recent Achievements PRL 115 ('15), 117 ('16):

- Spin tune  $\bar{\nu}_s = -0.16097\ldots \pm 10^{-10}$  in 100 s
- Spin coherence time  $\tau > 1000$  s
- Spin feedback (pol. vector within 12°)

# How to handle CP-violating sources beyond the SM?

Running through the scales

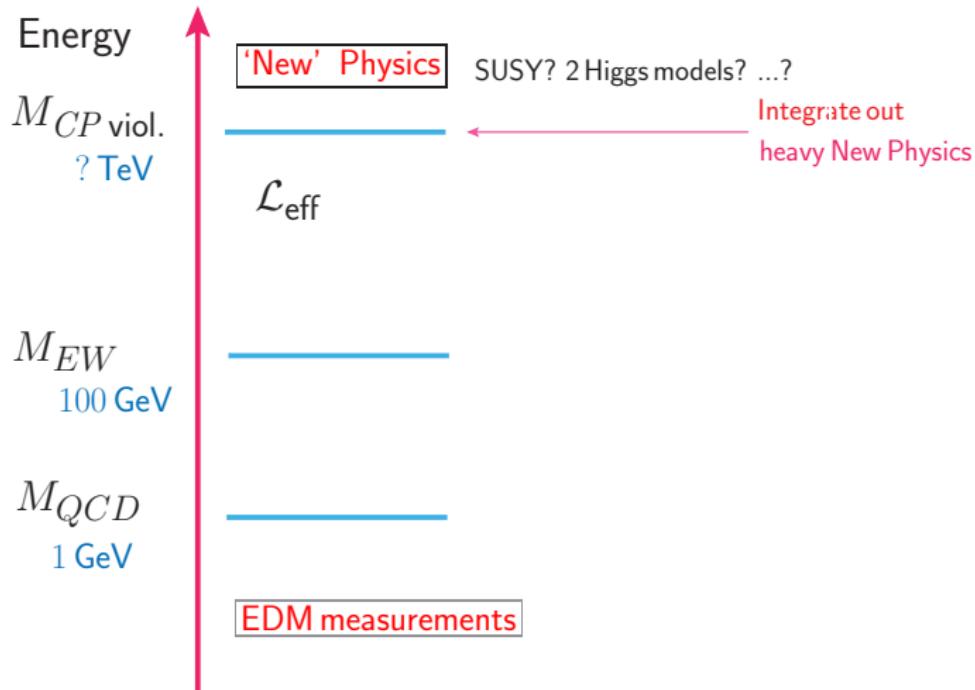
W. Dekens & J. de Vries, *JHEP* '13



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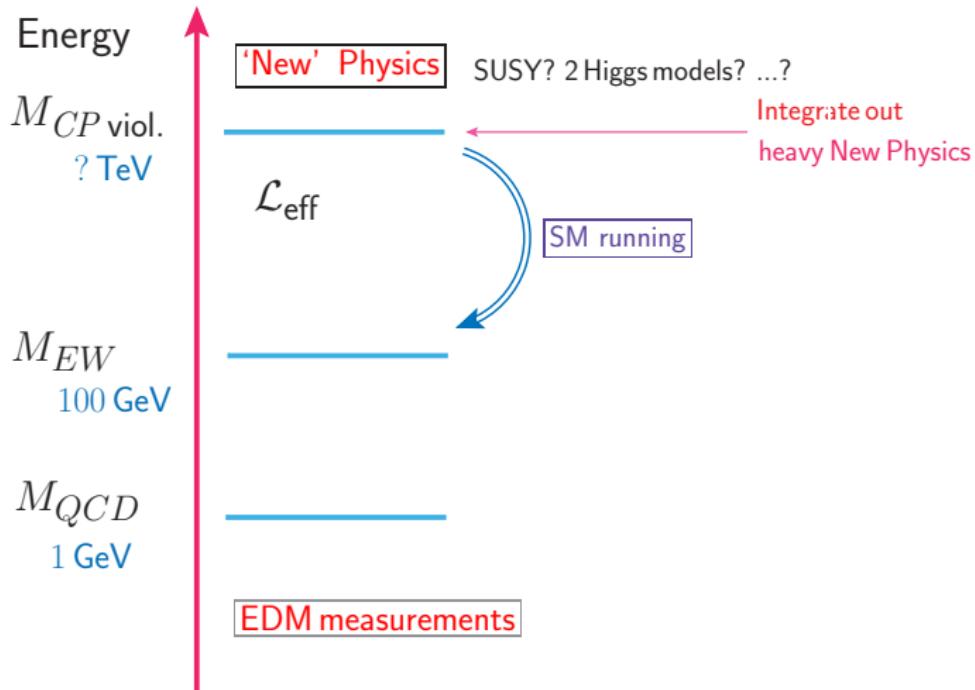
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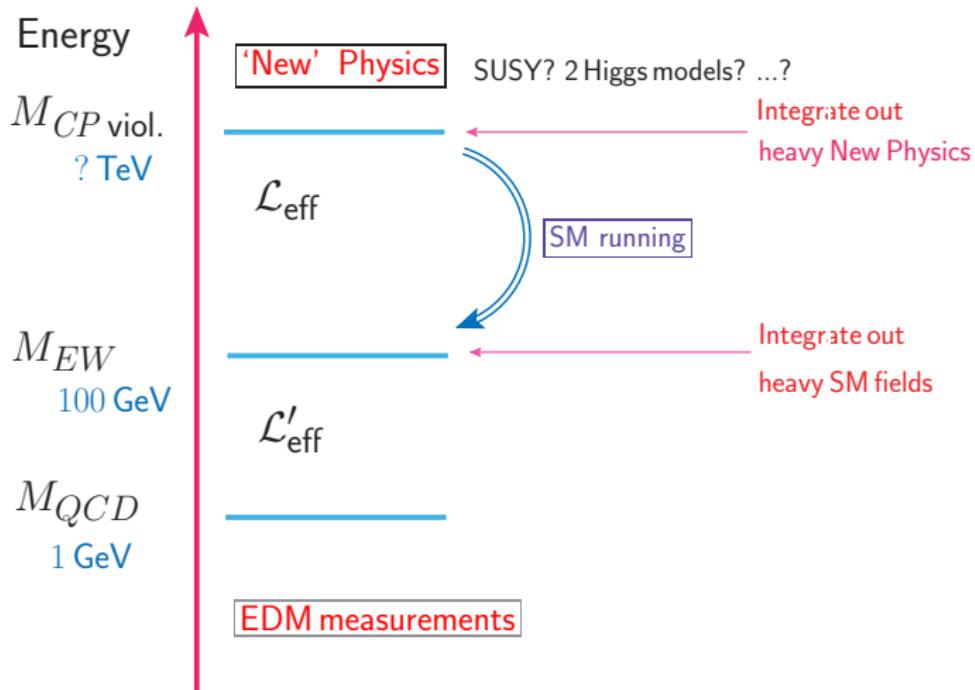
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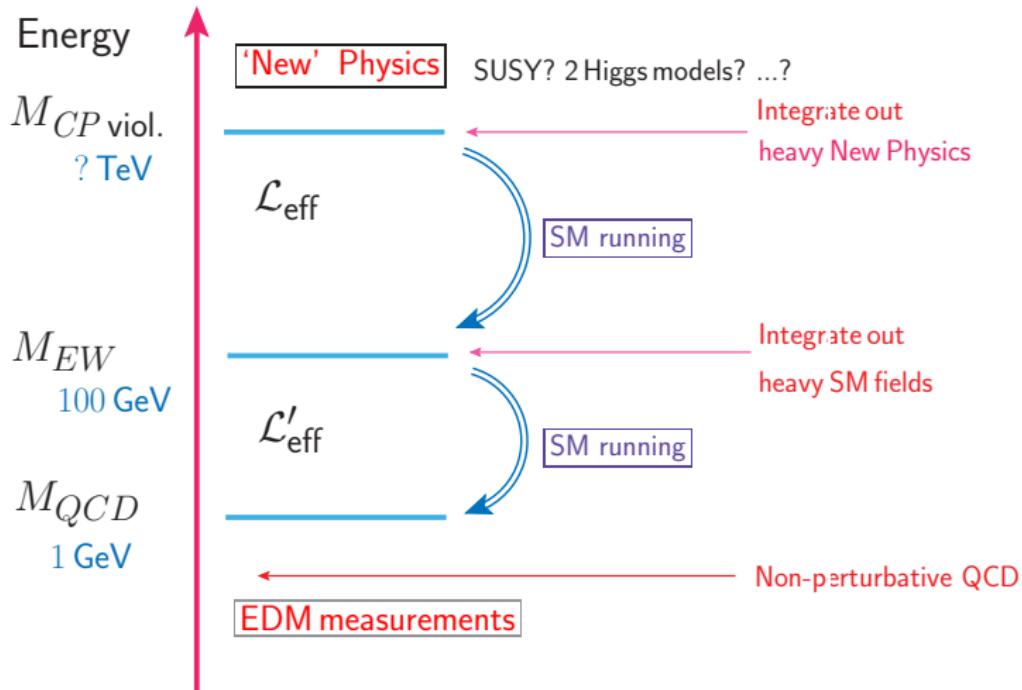
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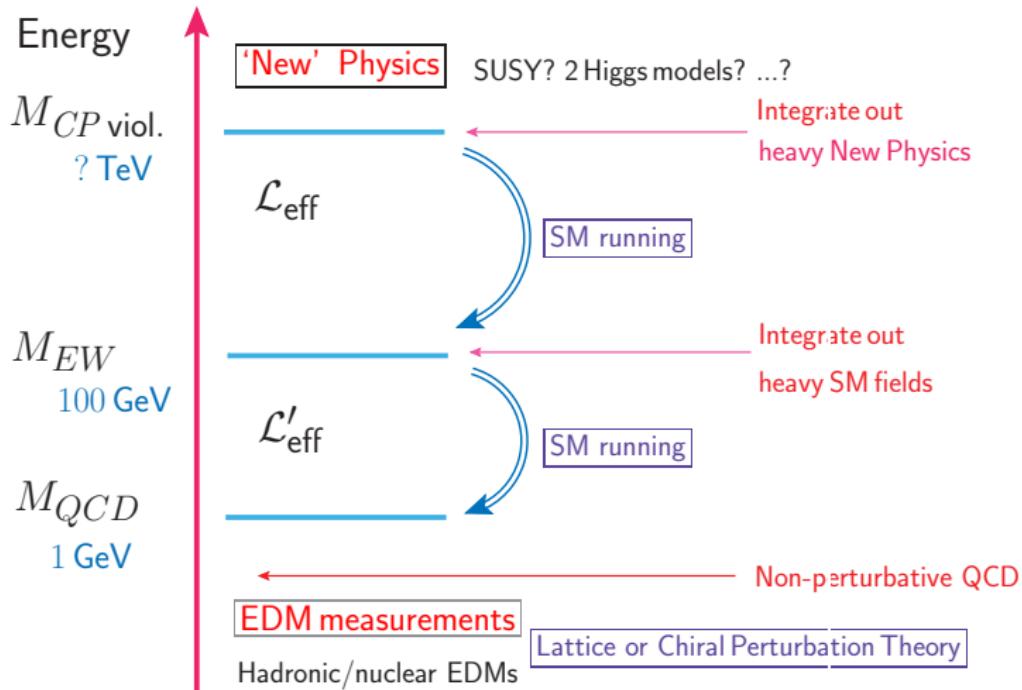
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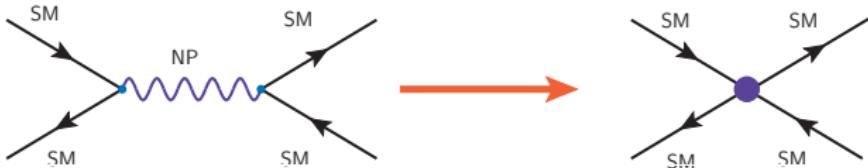
W. Dekens & J. de Vries, *JHEP* '13



# How to handle CP-violating sources beyond the SM?

New interactions as higher dimensional operators

- Add to the SM **all possible** effective interactions



- The new interactions appear as higher dimensional operators

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_i \frac{c_5^{(i)}}{M_{\gamma}} \mathcal{O}_5^{(i)} + \sum_i \frac{c_6^{(i)}}{M_{\gamma}^2} \mathcal{O}_6^{(i)} + \dots$$

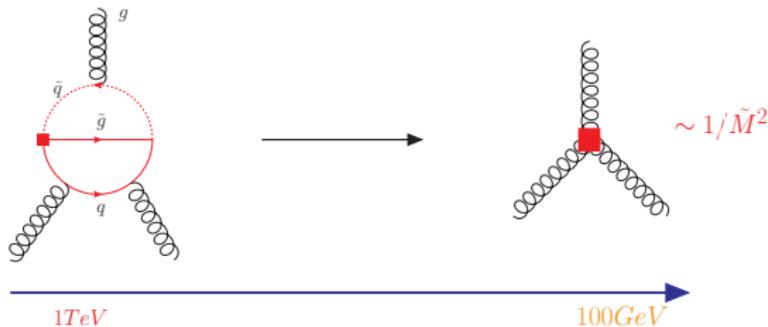
where  $M_{\gamma}$  is the scale of the *New Physics* particles

- Only the lowest dimensional operators should be important
- Hadronic EDMs: non-leptonic CP-violating operators of dim. 6  
Not of dim. 5 because of Higgs insertion / chiral symm. at EW / low scales

## How to handle CP-violating sources beyond the SM?

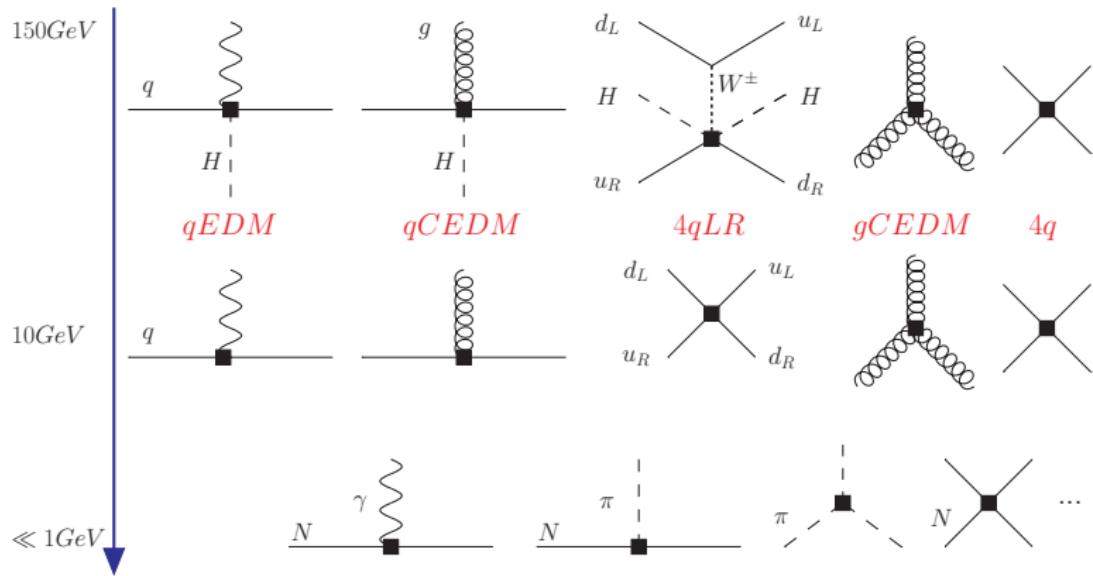
## Evaluation in Effective Field Theory (EFT) approach

- All degrees of freedom *beyond NP (EW) scale* are integrated out:  
→ Only SM degrees of freedom remain:  $q, g, (H, Z, W^\pm, \dots)$
  - Write down *all* interactions for these *active degrees of freedom* that *respect the SM+ Lorentz symmetries*: here dim. 6 or higher order
  - Need a *power-counting scheme* to order these *infinite #* interactions
  - Relics of eliminated BSM physics ‘remembered’ by the values of the *low-energy constants (LECs)* of the **CP-violating contact terms**, e.g.



# CP-violating BSM sources of dimension 6 from above EW scale to their hadronic equivalents below 1 GeV

W. Dekens &amp; J. de Vries JHEP '13

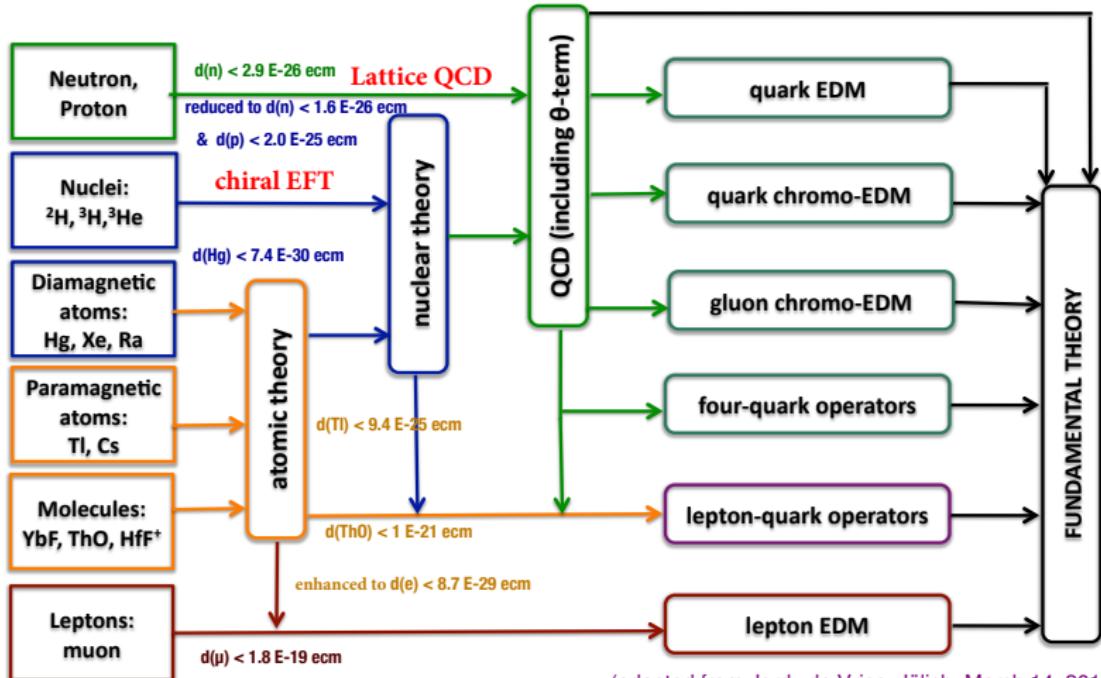


$$\begin{aligned}
 \text{Total #} &= 1(\bar{\theta}) + 2(qEDM) + 2(qCEDM) + 1(4qLR) + 1(gCEDM) + 2(4q) \quad [+3(\text{semi}) + 1(\text{lept})] \\
 &= \underbrace{1(\text{dim-four}) + 8(\text{dim-six})}_{\hookrightarrow 5 \text{ discriminable classes}} \quad [+3+1] \quad [\text{Caveat: } m_s \gg m_u, m_d \text{ (\& } m_\mu \gg m_e \text{ assumed)}]
 \end{aligned}$$

# Road map from EDM Measurements to EDM Sources

Experimentalist's point of view →

← Theorist's point of view



(adapted from Jordy de Vries, Jülich, March 14, 2013)

# EDM Translator

## from ‘quarkish/machine’ to ‘hadronic/human’ language?



3-CPO & R2-D2



Dirk Vorderstraße

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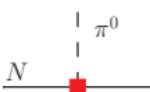
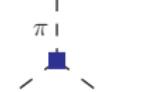


Dirk Vorderstraße

Symmetries (esp. chiral one) plus Goldstone Theorem  
→ Low-Energy Effective Field Theory with External Sources  
i.e. Chiral Perturbation Theory (suitably extended)

# Scalings of $\mathcal{CP}$ hadronic vertices – from $\theta$ to BSM sources

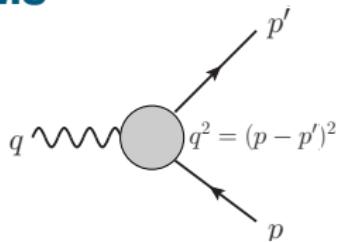
5 discriminable cases: Mereghetti et al., *AP*325 ('10); de Vries et al., *PRC*84('11); Bsaisou et al., *EPJA*49('13)

	$g_0$ $\mathcal{CP}, \text{I}$	$g_1$ $\mathcal{CP}, \text{I}$	$d_0, d_1$ $\mathcal{CP}, \text{I+I'}$	$(m_N \Delta)$ $\mathcal{CP}, \text{I}'$	$C_{1,2} (C_{3,4})$ $\mathcal{CP}, \text{I}(\text{I}')$
$\mathcal{L}_{\text{EFT}}^{\mathcal{CP}}$ :					
$\theta$ -term:	$\mathcal{O}(1)$	$\mathcal{O}(M_\pi/m_N)$	$\mathcal{O}(M_\pi/m_N)$	$\mathcal{O}(M_\pi^2/m_N^2)$	$\mathcal{O}(M_\pi^2/m_N^2)$
qEDM:	$\mathcal{O}(\alpha_{EM}/4\pi)$	$\mathcal{O}(\alpha_{EM}/4\pi)$	$\mathcal{O}(1)$	$\mathcal{O}(\alpha_{EM}/4\pi)$	$\mathcal{O}(\alpha_{EM}/4\pi)$
qCEDM:	$\mathcal{O}(1)$	$\mathcal{O}(1)$	$\mathcal{O}(M_\pi/m_N)$	$\mathcal{O}(M_\pi^2/m_N^2)$	$\mathcal{O}(M_\pi^2/m_N^2)$
4qLR:	$\mathcal{O}(M_\pi^2/m_n^2)$	$\mathcal{O}(1)$	$\mathcal{O}(M_\pi^3/m_N^3)$	$\mathcal{O}(M_\pi/m_n)$	$\mathcal{O}(M_\pi^2/m_N^2)$
gCEDM:	$\mathcal{O}(M_\pi^2/m_N^2)^*$	$\mathcal{O}(M_\pi^2/m_N^2)^*$	$\mathcal{O}(1)$	$\mathcal{O}(M_\pi^2/m_N^2)$	$\mathcal{O}(1)$
4q:	$\mathcal{O}(M_\pi^2/m_N^2)^*$	$\mathcal{O}(M_\pi^2/m_N^2)^*$	$\mathcal{O}(1)$	$\mathcal{O}(M_\pi^2/m_N^2)$	$\mathcal{O}(1)$

\*) Goldstone theorem  $\rightarrow$  relative  $\mathcal{O}(M_\pi^2/m_n^2)$  suppression of  $N\pi$  interactions

## Calculation: from form factors to EDMs

$$\langle f(p') | J_{\text{em}}^\mu | f(p) \rangle = \bar{u}_f(p') \Gamma^\mu(q^2) u_f(p)$$



$$\Gamma^\mu(q^2) = \gamma^\mu F_1(q^2) - i\sigma^{\mu\nu} q_\nu \frac{F_2(q^2)}{2m_f} + \sigma^{\mu\nu} q_\nu \gamma_5 \frac{F_3(q^2)}{2m_f} + (\not{q} q^\mu - q^2 \gamma^\mu) \gamma_5 \frac{F_a(q^2)}{m_f^2}$$

Dirac FF

Pauli FF

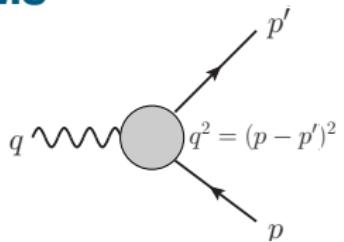
electric dipole FF ( $\mathcal{Q}\mathcal{P}$ )

anapole FF ( $\mathcal{P}'$ )

$$\hookrightarrow \quad d_f := \lim_{q^2 \rightarrow 0} \frac{F_3(q^2)}{2m_f} \quad \text{for } s = 1/2 \text{ fermion}$$

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Dirac FF

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electric dipole FF ( $\mathcal{Q}\mathcal{P}$ )

anapole FF ( $\mathcal{P}$ )

$$\hookrightarrow d_f := \lim_{q^2 \rightarrow 0} \frac{F_3(q^2)}{2m_f} \quad \text{for } s = 1/2 \text{ fermion}$$

**Nucleus A**

$$\langle \uparrow | J_{PT}^0(q) | \uparrow \rangle$$

in Breit frame

$$\begin{aligned}
 & \langle \uparrow | J_{PT}^0(q) | \uparrow \rangle = \langle \uparrow | J_{PT}^{\text{total}} | \uparrow \rangle + \langle \uparrow | V_{PT} | \uparrow \rangle \\
 & = -iq^3 \underbrace{\frac{F_3^A(\vec{q}^2)}{2m_A}}_{\hookrightarrow d_A}
 \end{aligned}$$

# $\theta$ -Term Induced Nucleon EDM

Baluni, *PRD* (1979); Crewther et al., *PLB* (1979); ... Pich & de Rafael, *NPB* (1991); ... Ott nad et al., *PLB* (2010)

Isospin-conserving  $\pi NN$  coupling:

$$g_0^\theta = \frac{(m_n - m_p)^{\text{strong}}(1 - \epsilon^2)}{4F_\pi} \bar{\theta} \approx (-15.5 \pm 1.9) \cdot 10^{-3} \bar{\theta} \quad (\text{where } \epsilon \equiv \frac{m_u - m_d}{m_u + m_d})$$

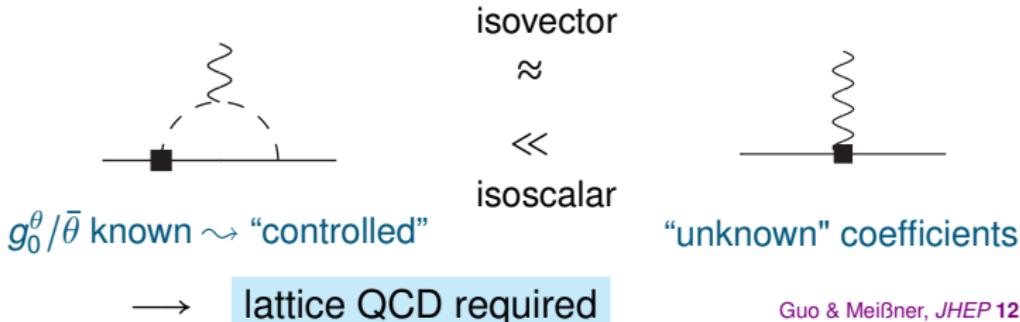
$$\rightarrow dN|_{\text{loop}}^{\text{isovector}} \sim (1.8 \pm 0.3) \cdot 10^{-16} \bar{\theta} \text{ e cm}$$

Bsaisou et al., *EPJA* 49 (2013), *JHEP* 03 (2015)

Note also:  $g_1^\theta = 8c_1 m_N \Delta^\theta + (0.6 \pm 1.1) \cdot 10^{-3} \bar{\theta} = (3.4 \pm 1.5) \cdot 10^{-3} \bar{\theta}$  with the  
 3-pion coupling:  $\Delta^\theta = \frac{\epsilon(1-\epsilon^2)}{16F_\pi m_N} \frac{M_\pi^4}{M_K^2 - M_\pi^2} \bar{\theta} + \dots = (-0.37 \pm 0.09) \cdot 10^{-3} \bar{\theta}$



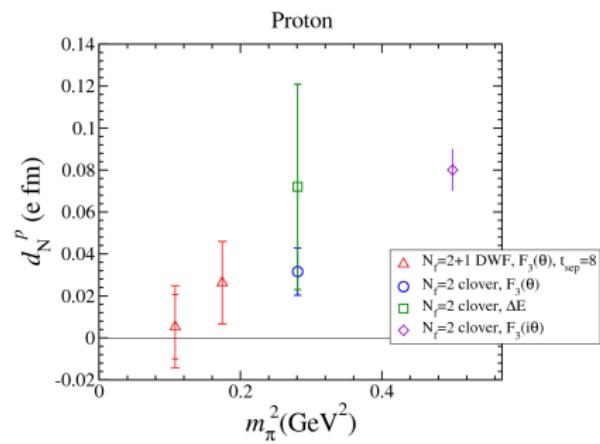
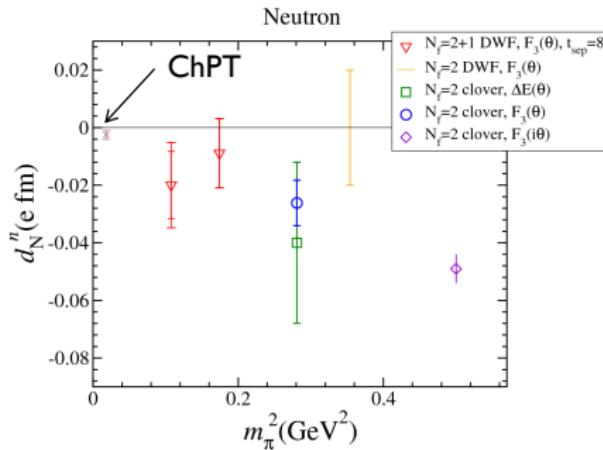
single nucleon EDM:



# Preliminary Lattice (full QCD) results

neutron EDM and

proton EDM



$$\theta \equiv 1 !$$

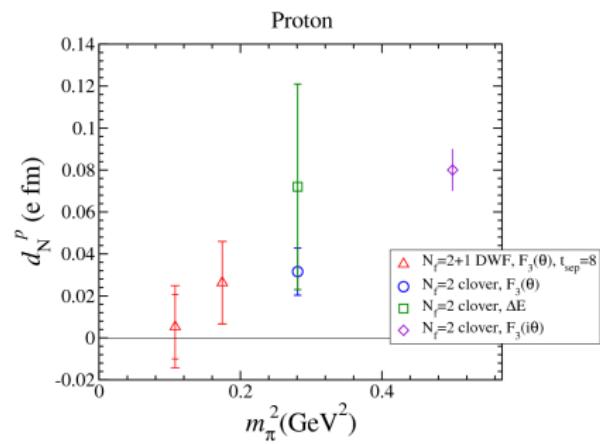
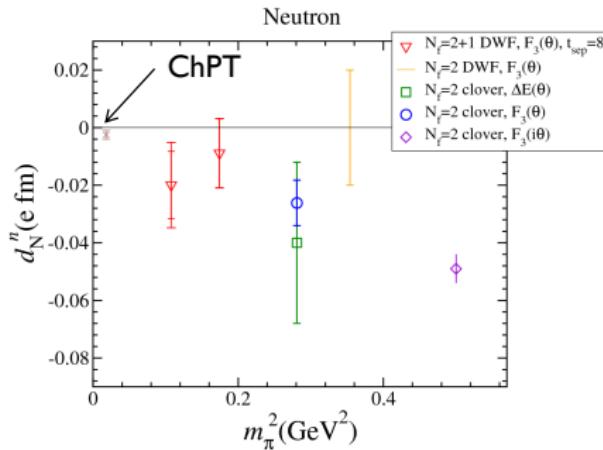
(adapted from Eigo Shintani (Mainz), *Lattice calculation of nucleon EDM*, Hirschegg, Jan. 14, 2014)

no systematical errors!

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no systematical errors!

$$\rightarrow d_n = \bar{\theta} (-2.7 \pm 1.2) \cdot 10^{-3} \cdot \text{efm} \quad \text{and} \quad d_p = \bar{\theta} (2.1 \pm 1.2) \cdot 10^{-3} \cdot \text{efm}$$

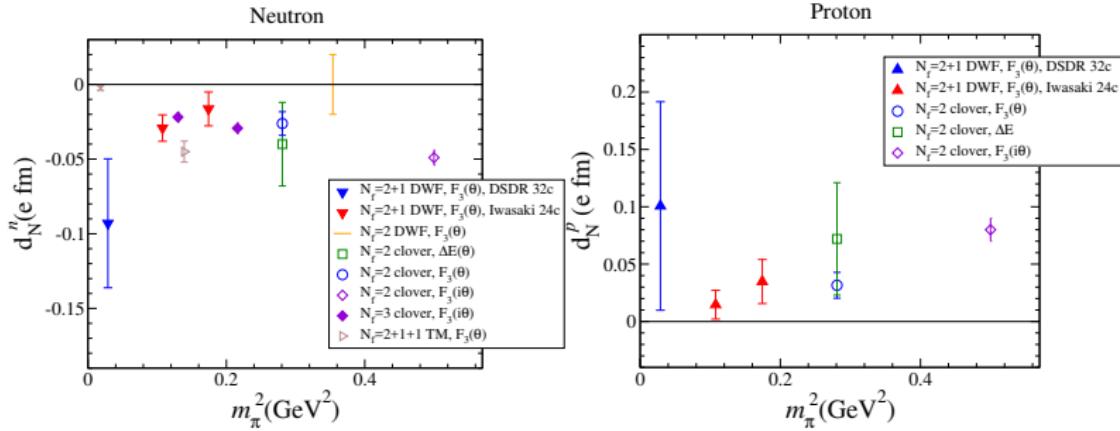
Akan, Guo & Meißner, *PLB 736* (2014); see also  $d_n = \bar{\theta} (-3.9 \pm 0.2 \pm 0.9) 10^{-3} \text{efm}$  Guo et al., *PRL 115* (2015)

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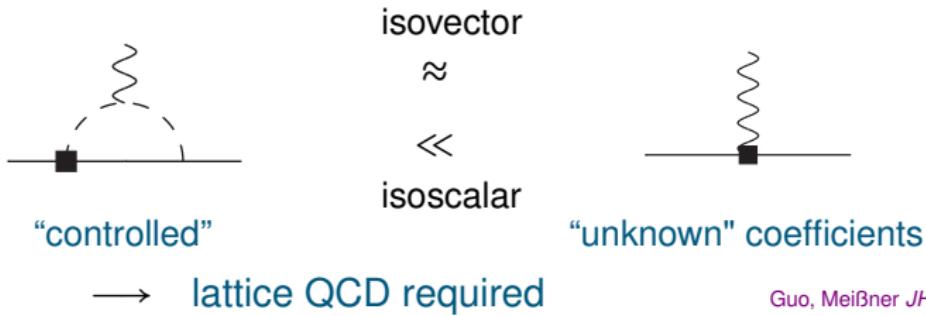
Eigo Shintani et al., *Phys. Rev. D* **93**, 094503 (2016)  $M_\pi = 170, 330, 420, 530 \text{ MeV}$

*Don't mention the ... light nuclei*

# Single Nucleon Versus Nuclear EDM

Baluni, *PRD* (1979); Crewther et al., *PLB* (1979); ... Pich & de Rafael, *NPB* (1991); ... Ott nad et al., *PLB* (2010)

single nucleon EDM:



two nucleon EDM:



Sushkov, Flambaum, Khriplovich *Sov.Phys.JETP*'84

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Baluni, *PRD* (1979); Crewther et al., *PLB*(1979); ... Pich & de Rafael, *NPB*(1991); ... Ottnad et al., *PLB*(2010)

## single nucleon EDM:



“controlled”

→ lattice QCD required

### isovector

22

1

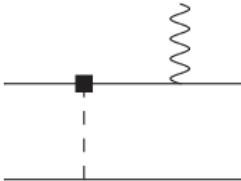
isoscalar

“unknown” coefficients

Guo, Meißner JHEP'12

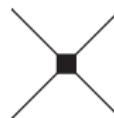
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## controlled

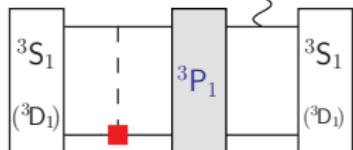
2



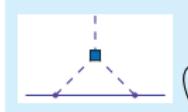
unknown coefficient

# EDM of the Deuteron at LO: CP-violating $\pi$ exchange

$$\begin{aligned} \mathcal{L}_{CP}^{\pi N} = & -d_n N^\dagger (1 - \tau^3) S^\mu v^\nu N F_{\mu\nu} - d_p N^\dagger (1 + \tau_3) S^\mu v^\nu N F_{\mu\nu} \\ & + (m_N \Delta) \pi^2 \pi_3 + \cancel{g_0 N^\dagger \vec{\pi} \cdot \vec{\tau} N} + \cancel{g_1 N^\dagger \pi_3 N} \\ & + \cancel{C_1 N^\dagger N D_\mu (N^\dagger S^\mu N)} + \cancel{C_2 N^\dagger \vec{\tau} N \cdot D_\mu (N^\dagger \vec{\tau} S^\mu N)} + \dots \end{aligned}$$



LO:  $\cancel{g_0 N^\dagger \vec{\pi} \cdot \vec{\tau} N}$  ( $CP, I$ )  $\rightarrow 0$  (Isospin filter!)  
NLO:  $g_1 N^\dagger \pi_3 N$  ( $CP, I$ )  $\rightarrow$  "LO" in D case



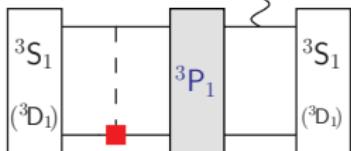
term	$N^2\text{LO } \chi\text{PT}$	$N^4\text{LO } \chi\text{PT}$	$A\gamma_{18}$	CD-Bonn	units
$d_n^D$	0.939(9)	0.936(8)	0.914	0.927	$d_n$
$d_p^D$	0.939(9)	0.936(8)	0.914	0.927	$d_p$
$g_1$	0.183(17)	0.182(2)	0.186	0.186	$g_1 \text{ e fm}$
$\Delta f_{g_1}$	-0.748(138)	-0.646(23)	-0.703	-0.719	$\Delta \text{ e fm}$

Bsaisou et al., JHEP 03 (2015); A.W., Bsaisou, Nogga, IJMP E26 (2017)

BSM  $CP$  sources:  $g_1 \pi NN$  vertex is of LO in qCEDM and 4qLR case

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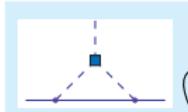


LO:  $\cancel{g_0 N^\dagger \vec{\pi} \cdot \vec{\tau} N}$  ( $CP, I$ )  $\rightarrow 0$  (Isospin filter!)

NLO:  $\cancel{g_1 N^\dagger \pi_3 N}$  ( $CP, I$ )  $\rightarrow$  "LO" in D case

Yamanaka & Hiyama, PRC 91 (2015):

$$d_N^D = \left(1 - \frac{3}{2} P_{^3D_1}\right) d_N$$

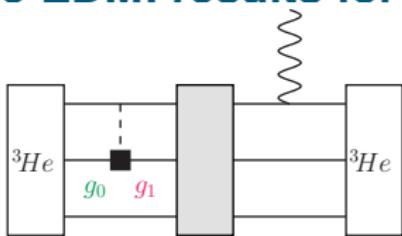


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## $^3\text{He}$ EDM: results for CP-violating $\pi$ exchange



$g_0 N^\dagger \vec{\pi} \cdot \vec{\tau} N$  ( $\cancel{CP}, I$ )

LO:  $\theta$ -term, qCEDM

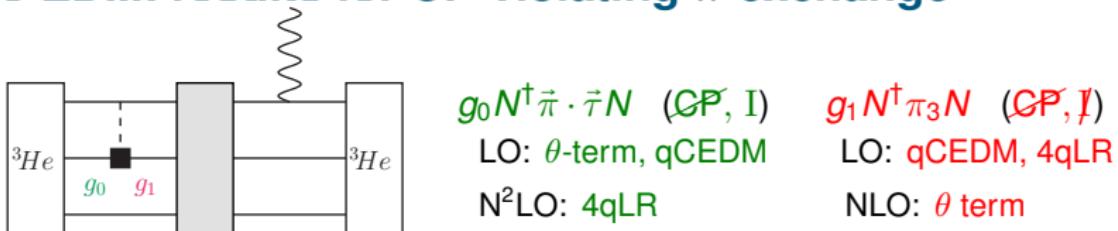
$N^2\text{LO}$ : 4qLR

$g_1 N^\dagger \pi_3 N$  ( $\cancel{CP}, I$ )

LO: qCEDM, 4qLR

NLO:  $\theta$  term

## $^3\text{He}$ EDM: results for CP-violating $\pi$ exchange



term	A	$N^2\text{LO ChPT}$	$\text{Av}_{18} + \text{UIX}$	CD-Bonn+TM	units
$d_n$	$^3\text{He}$ $^3\text{H}$	$0.904 \pm 0.013$ $-0.030 \pm 0.007$	0.875 -0.051	0.902 -0.038	$d_n$
$d_p$	$^3\text{He}$ $^3\text{H}$	$-0.029 \pm 0.006$ $0.918 \pm 0.013$	-0.050 0.902	-0.037 0.876	$d_p$
$\Delta$	$^3\text{He}$ $^3\text{H}$	$-0.017 \pm 0.006$ $-0.017 \pm 0.006$	-0.015 -0.015	-0.019 -0.018	$\Delta \text{ e fm}$
$g_0$	$^3\text{He}$ $^3\text{H}$	$0.111 \pm 0.013$ $-0.108 \pm 0.013$	0.073 -0.073	0.087 -0.085	$g_0 \text{ e fm}$
$g_1$	$^3\text{He}$ $^3\text{H}$	$0.142 \pm 0.019$ $0.139 \pm 0.019$	0.142 0.142	0.146 0.144	$g_1 \text{ e fm}$
$\Delta f_{g_1}$	$^3\text{He}$ $^3\text{H}$	$-0.608 \pm 0.142$ $-0.598 \pm 0.141$	-0.556 -0.564	-0.586 -0.576	$\Delta \text{ e fm}$
$C_1$	$^3\text{He}$ $^3\text{H}$	$-0.042 \pm 0.017$ $0.041 \pm 0.016$	-0.0014 0.0014	-0.016 0.016	$C_1 \text{ e fm}^{-2}$
$C_2$	$^3\text{He}$ $^3\text{H}$	$0.089 \pm 0.022$ $-0.087 \pm 0.022$	0.0042 -0.0044	0.033 -0.032	$C_2 \text{ e fm}^{-2}$

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# Discriminating between three scenarios at 1 GeV

Dekens et al. *JHEP* 07 (2014); Bsaisou et al. *JHEP* 03 (2015)

## 1 The Standard Model + $\bar{\theta}$

$$\mathcal{L}_{\text{SM}}^{\bar{\theta}} = \mathcal{L}_{\text{SM}} + \bar{\theta} m_q^* \bar{q} i \gamma_5 q$$

## 2 The left-right symmetric model — with two 4-quark operators:

$$\mathcal{L}_{LR} = -i \Xi [1.1 (\bar{u}_R \gamma_\mu u_R) (\bar{d}_L \gamma^\mu d_L) + 1.4 (\bar{u}_R t^a \gamma_\mu u_R) (\bar{d}_L t^a \gamma^\mu d_L)] + \text{h.c.}$$

## 3 The aligned two-Higgs-doublet model — with the dipole operators:

$$\mathcal{L}_{a2HM} = -e \frac{d_d}{2} \bar{d} i \sigma_{\mu\nu} \gamma_5 d F^{\mu\nu} - \frac{\tilde{d}_d}{4} \bar{d} i \sigma_{\mu\nu} \gamma_5 \lambda^a d G^{a\mu\nu} + \frac{d_W}{3} f_{abc} \tilde{G}^{a\mu\nu} G_{\mu\rho}^b G_{\nu}^{c\rho}$$

— with the hierarchy  $\tilde{d}_d \simeq 4 d_d \simeq 20 d_W$

matched on

$$\begin{aligned} \mathcal{L}_{\text{CP EFT}}^{\pi N} &= -d_N N^\dagger (1 - \tau^3) S^\mu v^\nu N F_{\mu\nu} - d_p N^\dagger (1 + \tau_3) S^\mu v^\nu N F_{\mu\nu} \\ &\quad + (m_N \Delta) \pi^2 \pi_3 + g_0 N^\dagger \vec{\pi} \cdot \vec{\tau} N + g_1 N^\dagger \pi_3 N \\ &\quad + C_1 N^\dagger N \mathcal{D}_\mu (N^\dagger S^\mu N) + C_2 N^\dagger \vec{\tau} N \cdot \mathcal{D}_\mu (N^\dagger \vec{\tau} S^\mu N) + \dots . \end{aligned}$$

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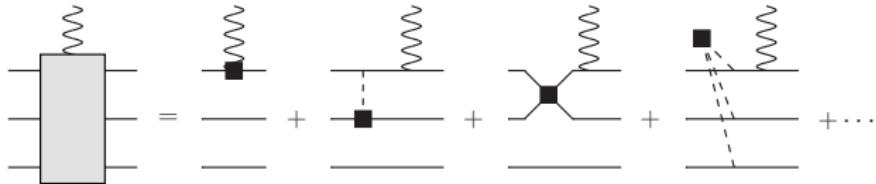
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# Testing strategies: SM + $\bar{\theta}$

Dekens et al. *JHEP* **07** (2014); Bsaisou et al. *JHEP* **03** (2015)

Measurement of the helion  
and neutron EDMs

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$$d_{^3\text{He}} - 0.9d_n = -\bar{\theta} (1.01 \pm 0.31_{\text{had}} \pm 0.29^*_{\text{nucl}}) \cdot 10^{-16} \text{ ecm}$$

Extraction of  $\bar{\theta}$

\* includes  $\pm 0.20$  uncertainty from 2N contact terms

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Extraction of  $\bar{\theta}$

$$d_D - 0.94(d_n + d_p) = \bar{\theta} (0.89 \pm 0.29_{\text{had}} \pm 0.08_{\text{nucl}}) \cdot 10^{-16} \text{ ecm}$$

Prediction for  $d_D - 0.94(d_n + d_p)$   
(& triton EDM):  $d_D^{\text{Nucl}} \approx -d_{^3\text{He}}^{\text{Nucl}} \approx \frac{1}{2} d_{^3\text{H}}^{\text{Nucl}}$

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$$(\& triton EDM): d_D^{\text{Nucl}} \approx -d_{^3\text{He}}^{\text{Nucl}} \approx \frac{1}{2} d_{^3\text{H}}^{\text{Nucl}}$$

$$g_1^\theta / g_0^\theta \approx -0.2$$

$$g_0^\theta = \frac{(m_n - m_p)^{\text{strong}} (1 - \epsilon^2)}{4F_\pi \epsilon} \bar{\theta} = (-16 \pm 2) 10^{-3} \bar{\theta}$$

$$\frac{g_1^\theta}{g_0^\theta} \approx \frac{8c_1(M_{\pi^\pm}^2 - M_{\pi^0}^2)^{\text{strong}}}{(m_n - m_p)^{\text{strong}}} , \quad \epsilon \equiv \frac{m_u - m_d}{m_u + m_d}$$

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# Testing strategies: minimal LR symmetric Model

Dekens et al. *JHEP* 07 (2014); Bsaisou et al. *JHEP* 03 (2015)

Measurement of the deuteron  
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Dekens et al. *JHEP* 07 (2014); Bsaisou et al. *JHEP* 03 (2015)

Measurement of the deuteron  
and nucleon EDMs

$$d_D - 0.94(d_n + d_p) \simeq d_D = -(2.1 \pm 0.5^*) \Delta^{LR} \text{ e fm}$$

Extraction of  $\Delta^{LR}$

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Extraction of  $\Delta^{LR}$

$$d_{^3\text{He}} - 0.9d_n \simeq d_{^3\text{He}} = -(1.7 \pm 0.5^*) \Delta^{LR} \text{efm}$$

Prediction for the helion EDM  
(& triton EDM):  $d_D \approx d_{^3\text{He}} \approx d_{^3\text{H}}$

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$$\begin{aligned} g_1^{LR} &= 8c_1 m_N \Delta^{LR} &= (-7.5 \pm 2.3) \Delta^{LR}, \\ g_0^{LR} &= \frac{(m_n - m_p)^{\text{str}} m_N}{M_\pi^2} \Delta^{LR} &= (0.12 \pm 0.02) \Delta^{LR} \end{aligned}$$

$-g_1^{LR}/g_0^{LR} \gg 1$  (!)

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Dekens et al. *JHEP* 07 (2014); Bsaisou et al. *JHEP* 03 (2015)

Measurement of the deuteron  
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$$d_D - 0.94(d_n + d_p) = [(0.18 \pm 0.02)g_1 - (0.75 \pm 0.14)\Delta] \text{ e fm}$$

Extraction of  $g_1^{\text{eff}}$  (including  $\Delta$  correction)

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Extraction of  $g_1^{\text{eff}}$  (including  $\Delta$  correction)

+ Measurement of  $d_{^3\text{He}}$  (or  $d_{^3\text{H}}$ )

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$$\begin{aligned} d_{^3\text{He}} - 0.9d_n \\ = [(0.11 \pm 0.02^*)g_0 + (0.14 \pm 0.02^*)g_1 - (0.61 \pm 0.14)\Delta] \text{efm} \end{aligned}$$

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Extraction of  $g_0$

Prediction of  $d_{^3\text{H}}$  (or  $d_{^3\text{He}}$ )

\* includes  $\pm 0.01$  uncertainty from 2N contact terms

## Summary

- $D$  EDM might **distinguish** between  $\bar{\theta}$  and other scenarios and allows **extraction** of the  $g_1$  coupling constant via  $d_D - 0.94(d_n + d_p)$ . (The prefactor of  $(d_n + d_p)$  stands for a 4% probability of the  ${}^3D_1$  state.)
- ${}^3He$  (or  ${}^3H$ ) EDM necessary for a **proper test** of  $\bar{\theta}$  and LR scenarios:
- Deuteron & helion work as complementary **isospin filters** of EDMs
- 2N contact terms **cannot be neglected** for nuclei beyond  $D$
- **a2HDM case:**  ${}^3He$  and  ${}^3H$  EDMs would be needed for a proper test
- **pure qCEDM:** similar to a2HDM scenario
- **pure qEDM:**  $d_D = 0.94(d_n + d_p)$  and  $d_{{}^3He/{}^3H} = 0.9d_{n/p}$
- **gCEDM, 4quark  $\chi$  singlet:** controlled calculation difficult (lattice ?)
- Ultimate progress may eventually come from **Lattice QCD**  
→  $GP N\pi$  couplings  $g_0$  &  $g_1$  may be accessible even for dim-6 case

## Many thanks to my colleagues

in Jülich: **Jan Bsaisou**, Christoph Hanhart, Susanna Liebig, Ulf-G. Meißner,  
David Minossi, Andreas Nogga, and **Jordy de Vries**

in Bonn: Feng-Kun Guo, Bastian Kubis, Ulf-G. Meißner

and: Werner Bernreuther, Bira van Kolck, and Kolya Nikolaev

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- 1 A.W., J. Bsaisou, A. Nogga,  
*Permanent Electric Dipole Moments in Single-, Two- and Three-Nucleon Systems*  
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- 2 J. Bsaisou, U.-G. Meißner, A. Nogga and A.W.,  
*P- and T-Violating Lagrangians in Chiral Effective Field Theory and Nuclear Electric Dipole Moments*, Annals of Physics **359**, 317-370 (2015), arXiv:1412.5471 [hep-ph].
- 3 J. Bsaisou, C. Hanhart, S. Liebig, D. Minossi, U.-G. Meißner, A. Nogga and A.W.,  
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- 4 W. Dekens, J. de Vries, J. Bsaisou, W. Bernreuther, C. Hanhart, U.-G. Meißner,  
A. Nogga and A.W., JHEP **07**, 069 (2014), arXiv:1404.6082 [hep-ph].
- 5 J. Bsaisou, C. Hanhart, S. Liebig, U.-G. Meißner, A. Nogga and A.W.,  
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# Jump slides

## $\theta$ -term: CP $\pi NN$ vertices determined from LECs

Leading  $g_0^\theta$  coupling (from  $c_5$ )

Baluni (1979); Crewther et al. (1979);  
 Ott nad et al. (2010); Mereghetti et al. (2011);  
 de Vries et al. (2011); Bsaisou et al. (2013)

$g_0^\theta$ :  $N^\dagger \vec{\pi} \cdot \vec{\tau} N$ -vertex

$$\mathcal{L}_{\pi N} = \dots + c_5 2B N^\dagger \left( (m_u - m_d) \tau_3 + \frac{2m^* \bar{\theta}}{F_\pi} \vec{\pi} \cdot \vec{\tau} \right) N + \dots$$

$$\delta M_{np}^{str} = 4B(m_u - m_d)c_5 \rightarrow g_0^\theta = \bar{\theta} \delta M_{np}^{str} (1 - \epsilon^2) \frac{1}{4F_\pi \epsilon}$$

$$\delta M_{np}^{em} \rightarrow \delta M_{np}^{str} = (2.44 \pm 0.18) \text{ MeV} \quad \text{Walker-Loud ('13); Borsányi et al. ('14)}$$

$$\& \quad m_u/m_d = 0.46 \pm 0.03 \quad \text{Flag Working Group ('14)}$$

$$\rightarrow g_0^\theta = (15.5 \pm 1.9) \cdot 10^{-3} \cdot \bar{\theta} \quad \text{Bsaisou et al. ('15)}$$

$$\epsilon = (m_u - m_d)/(m_u + m_d), \quad 4Bm^* = M_\pi^2(1 - \epsilon^2), \quad m^* = \frac{m_u m_d}{m_u + m_d}$$

## $\theta$ -term: subleading $g_1^\theta$ coupling (from $c_1$ LEC)

$g_1^\theta$ :  $\pi_3 NN$ -vertex

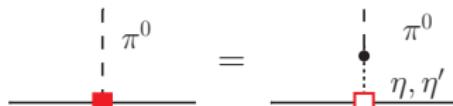
$$\epsilon := (m_u - m_d) / (m_u + m_d)$$

$$\mathcal{L}_{\pi N} = \dots + c_1 4B N^\dagger \left( (m_u + m_d) + \frac{(\delta M_\pi^2)_{QCD} (1 - \epsilon^2) \bar{\theta}}{2BF_\pi \epsilon} \pi_3 \right) N + \dots$$

1  $c_1 \longleftrightarrow \sigma_{\pi N}$ :  $c_1 = (-1.0 \pm 0.3) \text{ GeV}^{-1}$

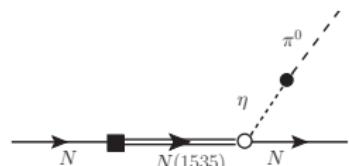
Compilation: Baru et al. (2011)

2  $(\delta M_\pi^2)_{QCD} \approx \frac{\epsilon^2}{4} \frac{M_\pi^4}{M_K^2 - M_\pi^2}$



$$\rightarrow g_1^\theta(c_1) = 8c_1 m_N \Delta^\theta \text{ in terms of } \Delta^\theta = \underbrace{\frac{\epsilon(1-\epsilon^2)}{4F_\pi m_N} \frac{M_\pi^4}{M_K^2 - M_\pi^2} \bar{\theta}}_{CP \text{ 3-pion coupling}}$$

$$g_1^\theta(c_1) = (2.8 \pm 1.1) 10^{-3} \bar{\theta} \quad \& \quad \bar{g}_1^\theta = (0.6 \pm 1.1) 10^{-3} \bar{\theta}$$



Bsaisou et al. '13, '15

$$\frac{g_1^\theta}{g_0^\theta} = -0.22 \pm 0.10 \sim \frac{M_\pi}{m_N}$$

Bsaisou et al. '13, '15

$$\gg \epsilon \frac{M_\pi^2}{m_N^2} \sim -0.01 \quad (\text{NDA})$$

de Vries et al. (2011)

$g_0^\theta(\delta M_{np}^{str})$  is unnaturally small!

◀  $\pi NN$

# Deuteron Quantities in ChPT from NLO to N4LO

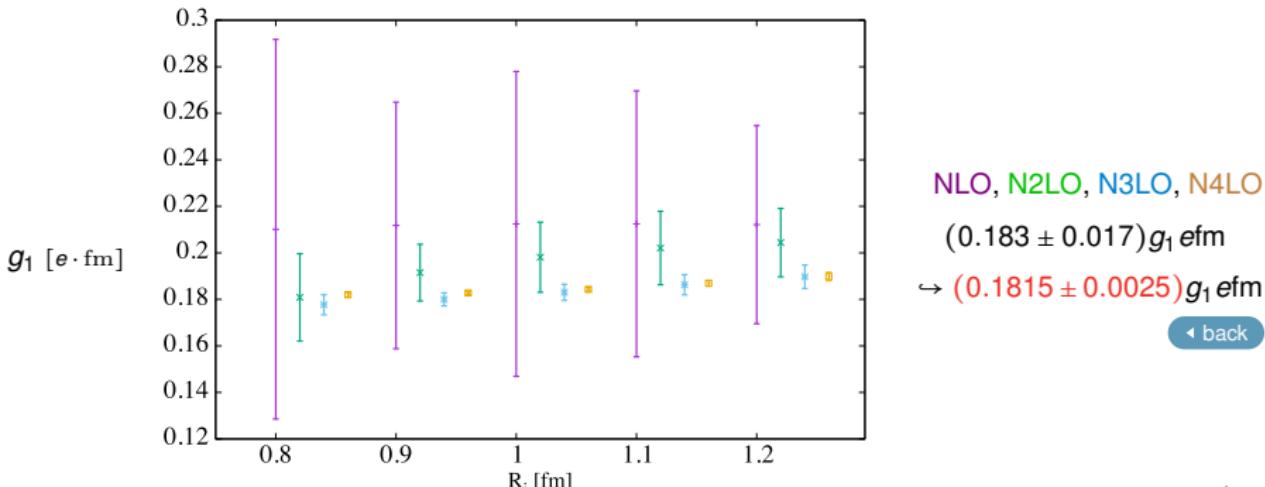
Epelbaum, Krebs, Meißen, *EPJA* 51 & *PRL* 115 (2015); Binder et al., *PRC* 93 (2016); and A. Nogga, *priv. comm.*

$$\Delta X^N n\text{LO}(p) = Q^{n+2} \cdot \max \left[ \left| X^{\text{LO}}(p) \right|, \frac{|X^{\text{NLO}}(p) - X^{\text{LO}}(p)|}{Q^2}, \frac{|X^{\text{N2LO}}(p) - X^{\text{NLO}}(p)|}{Q^3}, \right. \right.$$

$$\left. \left. \frac{|X^{\text{N3LO}}(p) - X^{\text{N2LO}}(p)|}{Q^4}, \frac{|X^{\text{N4LO}}(p) - X^{\text{N3LO}}(p)|}{Q^5} \right] \quad \text{with} \quad Q = \max \left( \frac{|p|}{\Lambda_b^i}, \frac{M_\pi}{\Lambda_b^i} \right)$$

and  $f\left(\frac{r}{R_i}\right) = \left[1 - \exp\left(-\frac{r^2}{R_i^2}\right)\right]^6$  with  $\frac{R_i}{\Lambda_b^i}$

$\frac{R_i}{\Lambda_b^i}$	0.8 fm	0.9 fm	1.0 fm	1.1 fm	1.2 fm
	0.6 GeV	0.6 GeV	0.6 GeV	0.5 GeV	0.4 GeV

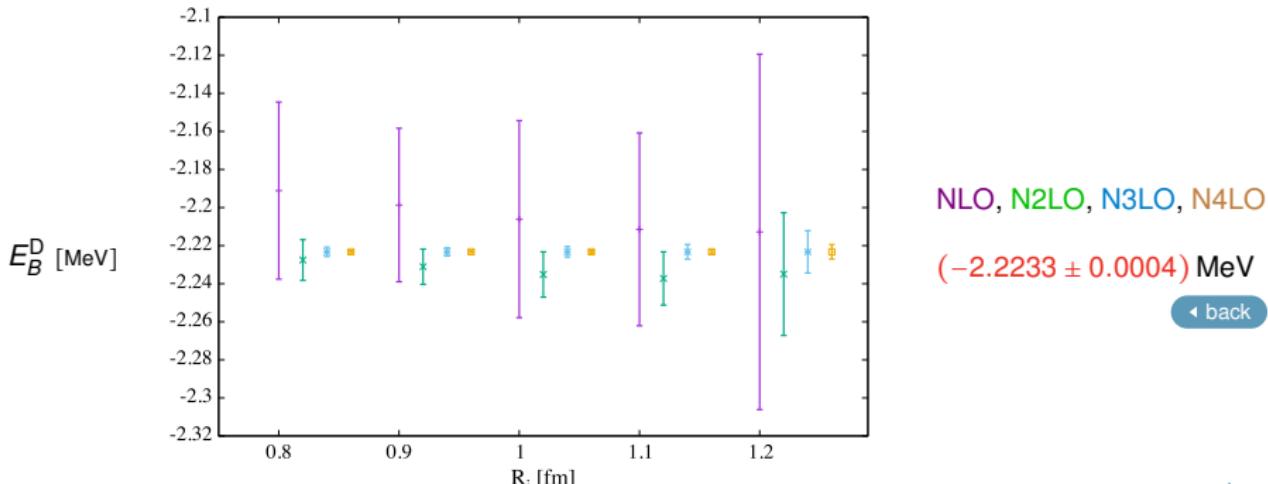


# Deuteron Quantities in ChPT from NLO to N4LO

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and  $f\left(\frac{r}{R_i}\right) = \left[1 - \exp\left(-\frac{r^2}{R_i^2}\right)\right]^6$  with  $\frac{R_i}{\Lambda_b^i}$  | 0.8 fm & 0.9 fm & 1.0 fm & 1.1 fm & 1.2 fm  
 0.6 GeV & 0.6 GeV & 0.6 GeV & 0.5 GeV & 0.4 GeV



# Deuteron Quantities in ChPT from NLO to N4LO

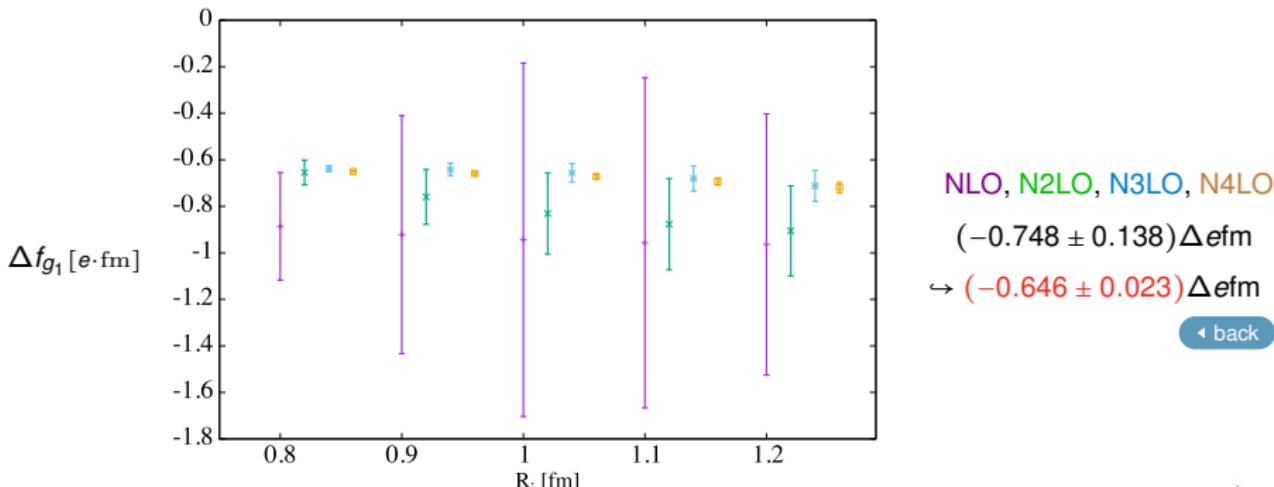
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# Deuteron Quantities in ChPT from NLO to N4LO

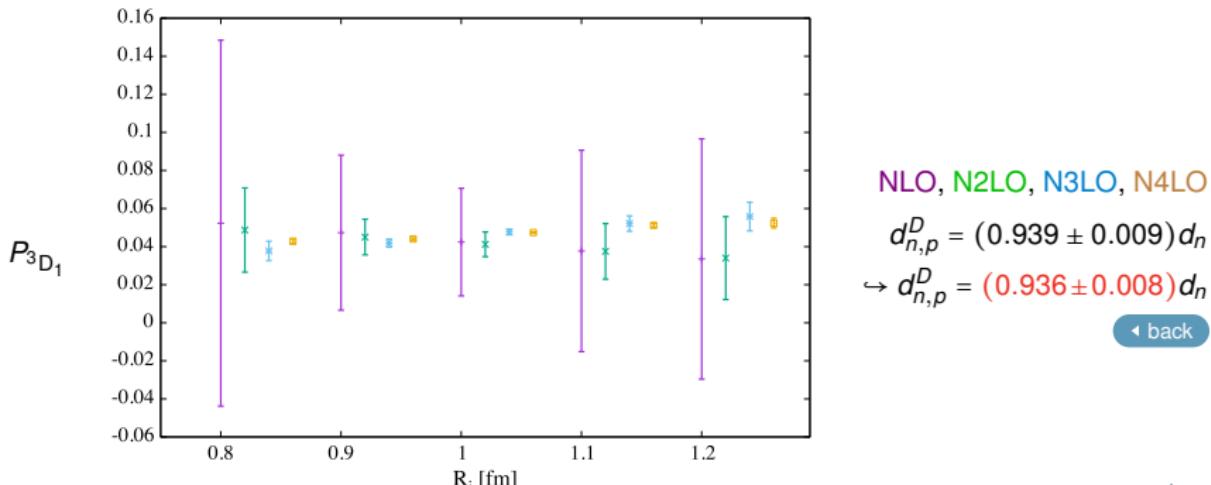
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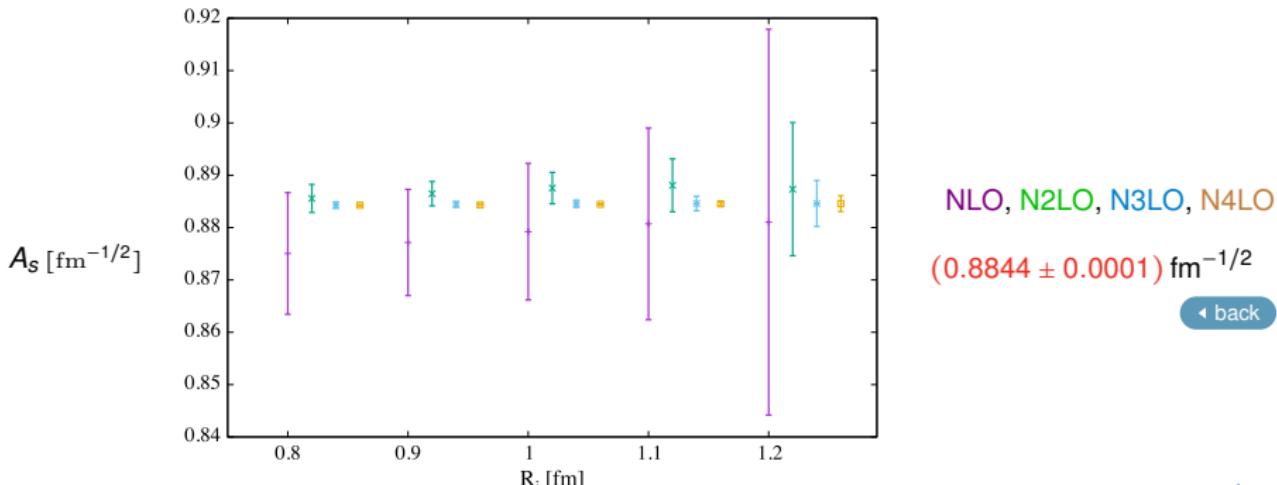
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