

Electric Dipole Moments of Hadrons and Light Nuclei in chiral EFT

Figure about “Doesn’t matter” omitted

Some things that will be not addressed

Figure about “Dark matter and assignment” omitted

Some questions that might hopefully be answered:

- 1 Why is ~~CP~~ beyond the Standard Model expected?
- 2 How can a point-particle (e.g. an electron) support an EDM?
- 3 Why don't the EDMs of certain molecules predict a strong ~~CP~~?
- 4 What is the natural scale of a neutron EDM?
- 5 How large is the EDM window for *New Physics* searches?
- 6 How can the EDM-producing sources be discriminated?
- 7 Why is low-energy Effective Field Theory needed here?
- 8 Why deuteron and helion EDM measurements essential?

HISTORY OF THE UNIVERSE

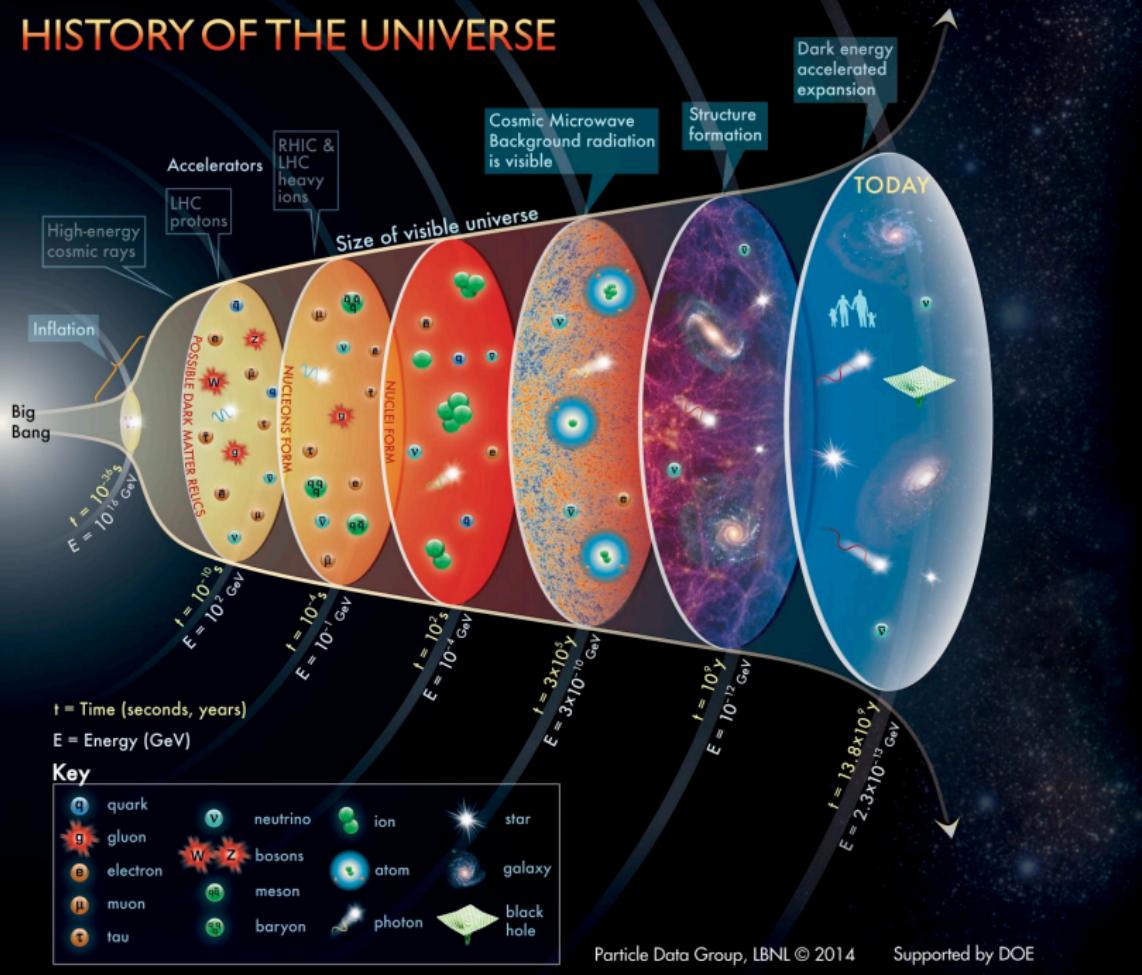


Fig. courtesy of PDG, LBNL © 2014

Matter Excess in the Universe

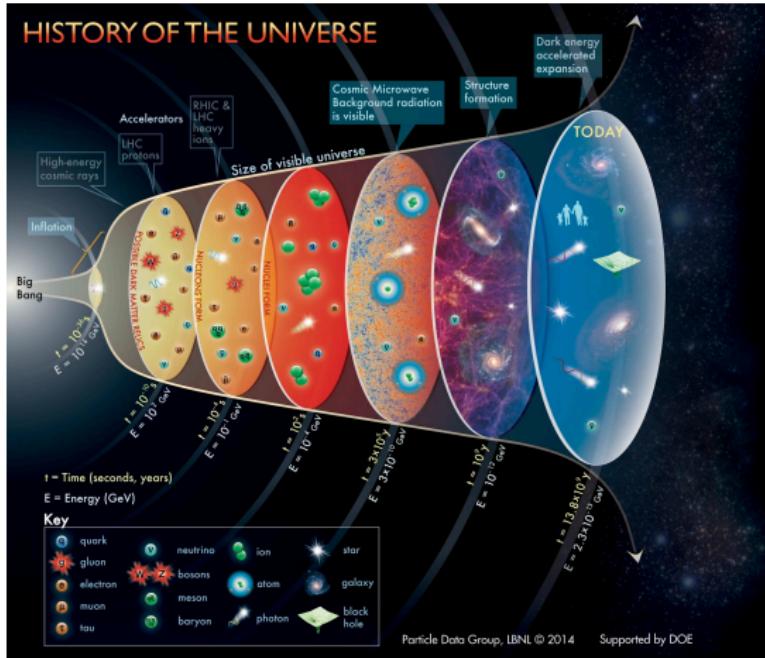


Fig. courtesy of PDG, LBNL © 2014

$$(*) 2J_{\text{Jarlskog}}^{\text{CKM}} (m_t^2 - m_u^2)(m_t^2 - m_c^2)(m_c^2 - m_u^2)(m_b^2 - m_d^2)(m_b^2 - m_s^2)(m_s^2 - m_d^2) \sim 10^{-18} M_{\text{EW}}^{12}$$

► in the SM?

Andreas Wirzba

- 1 End of inflation: $n_B = n_{\bar{B}}$
- 2 Cosmic Microwave Bkgr.
 - SM(s) prediction: $(n_B - n_{\bar{B}})/n_\gamma|_{\text{CMB}} \sim 10^{-18}$
 - WMAP+PLANCK ('13): $n_B/n_\gamma|_{\text{CMB}} = (6.05 \pm 0.07) 10^{-10}$

Sakharov conditions ('67)
for dyn. generation of net B :

- 1 B violation to depart from initial $B=0$
- 2 C & CP violation
to distinguish B from \bar{B} production rates
- 3 Either CPT violation or
out of thermal equilibrium
to dist. B production from back reaction
and to escape $\langle B \rangle = 0$ if CPT holds

CP violation and the Electric Dipole Moment (EDM)

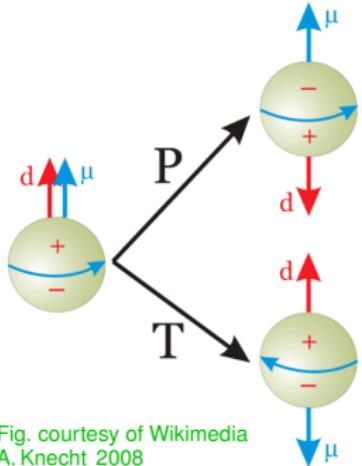


Fig. courtesy of Wikimedia
A. Knecht 2008

$$\text{EDM: } \vec{d} = \sum_i \vec{r}_i e_i \xrightarrow[\text{(polar)}]{\substack{\text{subatomic} \\ \text{particles}}} d \cdot \vec{S}/|\vec{S}| \xrightarrow[\text{(axial)}]{} d \cdot \vec{S}/|\vec{S}|$$

$$\mathcal{H} = -\mu \frac{\vec{S}}{S} \cdot \vec{B} - d \frac{\vec{S}}{S} \cdot \vec{E}$$

$$P: \quad \mathcal{H} = -\mu \frac{\vec{S}}{S} \cdot \vec{B} + d \frac{\vec{S}}{S} \cdot \vec{E}$$

$$T: \quad \mathcal{H} = -\mu \frac{\vec{S}}{S} \cdot \vec{B} + d \frac{\vec{S}}{S} \cdot \vec{E}$$

Any *non-vanishing EDM* of a **non-degenerate**
(e.g. subatomic) particle violates **P & T**

- Assuming **CPT** to hold, **CP** is violated as well (flavor-diagonally)
→ subatomic EDMs: “rear window” to CP violation in early universe
- Strongly suppressed in SM (CKM-matrix): $|d_n| \sim 10^{-31} \text{ ecm}$, $|d_e| \sim 10^{-38} \text{ ecm}$
- Current bounds: $|d_n| < 3^\diamond / 1.6^* \cdot 10^{-26} \text{ ecm}$, $|d_p| < 2 \cdot 10^{-25} \text{ ecm}$, $|d_e| < 1 \cdot 10^{-28} \text{ ecm}$
n: Baker et al.(2006)[◊], *p* prediction: Dimitriev & Sen'kov (2003)*, *e*: Baron et al.(2013)[†]

* from $|d_{^{199}\text{Hg}}| < 7.4 \cdot 10^{-30} \text{ ecm}$ bound of Graner et al. (2016)

† from polar ThO: $|d_{\text{ThO}}| \lesssim 10^{-21} \text{ ecm}$

Theorem: Permanent EDMs of *non-selfconjugate** particles with spin $j \neq 0$

Let $\langle j^P | \vec{d} | j^P \rangle = \textcolor{red}{d} \langle j^P | \vec{J} | j^P \rangle$ with $\vec{d} \equiv \int \vec{r} \rho(\vec{r}) d^3 r$ be an EDM operator in a stationary state $|j^P\rangle$ of specified parity P and nonzero spin j and

$\vec{d} \rightarrow \mp \vec{d}$ & $\vec{J} \rightarrow \pm \vec{J}$ under $\begin{cases} \text{space reflection,} \\ \text{time reversal.} \end{cases}$

If $d \neq 0$ and $|j^P\rangle$ has no degeneracy (besides rotational), then \cancel{P} & $\cancel{\chi}$.

* *non-selfconjugate particle* is not its own antiparticle \Rightarrow at least one "charge" non-zero

Werner Bernreuther (2012)

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It can be interpreted as a special case of the theorem:

Any *finite* quantum system without *explicit* symmetry breaking cannot have a spontaneously broken groundstate.

Keywords: *symmetric double-well* potential and *quantum tunneling* (instantons)

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How can such a particle be polarized and support an EDM?’

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There are always vacuum polarizations with rich short-distance structure
($g-2$ of the electron and muon aren't exactly zero either)

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The ground states of these molecules at non-zero temperatures or strong E -fields are mixtures of at least 2 opposite parity states:

The theorem doesn't apply for *degenerate states*: neither $\not T$ nor $\not P$!

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'But what about the induced EDM (polarization)?'

The induced EDM is *quadratic* in the electric field and *neither P nor T*

induced EDM	\longleftrightarrow	quadratic Stark effect ($\propto E^2$)
permanent EDM	\longleftrightarrow	linear Stark effect ($\propto E$)

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If the interactions are described by an action which is

local, Lorentz-invariant, and hermitian

then CPT invariance holds: thus $\not T \iff \not CP$

A *naive* estimate of the scale of the nucleon EDM

Khriplovich & Lamoreaux (1997); Kolya Nikolaev (2012)

- CP & P conserving magnetic moment \sim nuclear magneton μ_N

$$\mu_N = \frac{e}{2m_p} \sim 10^{-14} \text{ ecm}.$$

- A nonzero EDM requires

parity P violation: the price to pay is $\sim 10^{-7}$

$$(G_F \cdot F_\pi^2 \sim 10^{-7} \text{ with } G_F \approx 1.166 \cdot 10^{-5} \text{ GeV}^{-2}),$$

and additionally **CP violation:** the price to pay is $\sim 10^{-3}$

$$(|\eta_{+-}| \equiv |\mathcal{A}(K_L^0 \rightarrow \pi^+ \pi^-)| / |\mathcal{A}(K_S^0 \rightarrow \pi^+ \pi^-)| = (2.232 \pm 0.011) \cdot 10^{-3}).$$

- In summary: $|d_N| \sim 10^{-7} \times 10^{-3} \times \mu_N \sim 10^{-24} \text{ ecm}$
- In SM (without θ term): extra $G_F F_\pi^2$ factor to *undo* flavor change

$$\hookrightarrow |d_N^{\text{SM}}| \sim 10^{-7} \times 10^{-24} \text{ ecm} \sim 10^{-31} \text{ ecm}$$

\hookrightarrow *The empirical window* for search of physics BSM($\theta=0$) is

$$10^{-24} \text{ ecm} > |d_N| > 10^{-30} \text{ ecm}.$$

Chronology of upper bounds on the neutron EDM

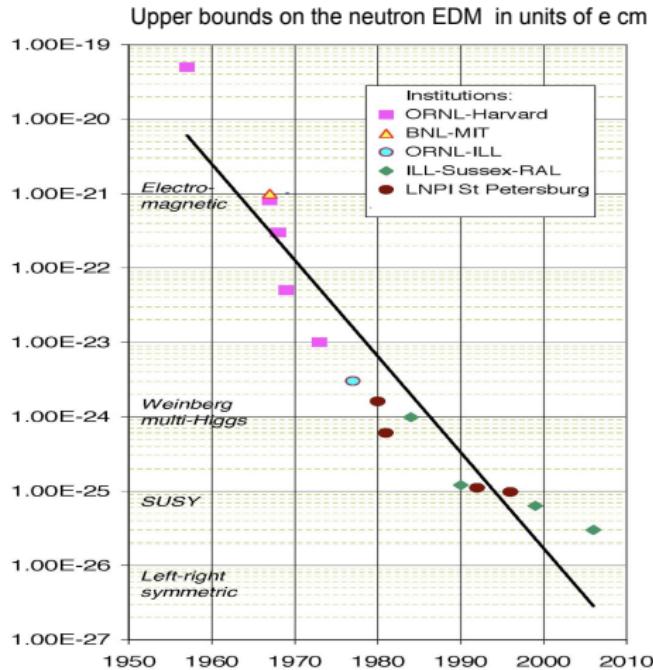


Fig. courtesy of N.N. Nikolaev

Smith, Purcell, Ramsey (1957) Baker et al. (2006)

→ 5 to 6 orders above SM predictions which are out of reach !

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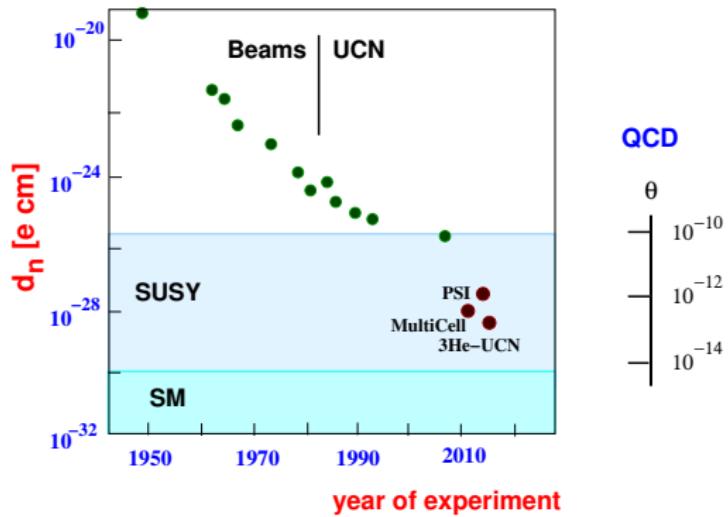


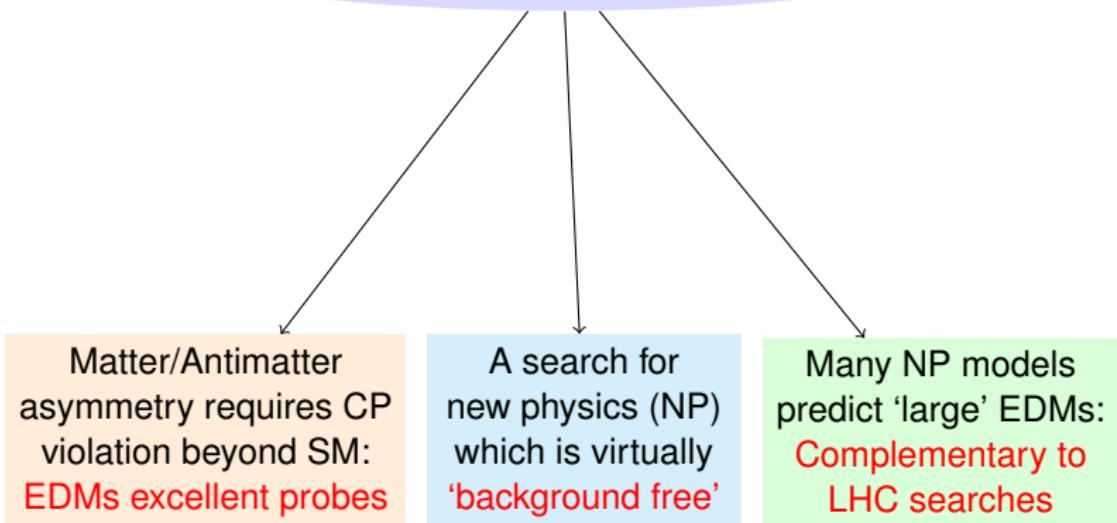
Fig. courtesy of U.-G. Meißner

Smith, Purcell, Ramsey (1957) Baker et al. (2006)

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Three motivations for EDM searches

Why are EDMs interesting?



EDM bounds from neutral particles

- Modern neutron EDM experiments at ILL, SNS, PSI, TRIUMF

current $d_n = (-0.21 \pm 1.82) \cdot 10^{-26} \text{ ecm}$

Baker et al. *PRL* '06 (ILL); Pendlebury et al. *PRD*'15

proposed $\sim 10^{-28} \text{ ecm}$

- Proton (and neutron) EDM inferred from diamagnetic atoms

current $|d(^{199}\text{Hg})| < 7.4 \cdot 10^{-30} \text{ ecm}$ (95% C.L.)

Graner et al. *PRL* '16(UW)

→ $|d_p| < 2 \cdot 10^{-25} \text{ ecm}$ & $|d_n| < 1.6 \cdot 10^{-26} \text{ ecm}$

Theory input from: Dimitriev & Sen'kov *PRL* '03

ongoing experiments on Xe, Ra, (Rn) ...

- Electron EDM inferred from paramagnetic atoms or non-generate molecules:

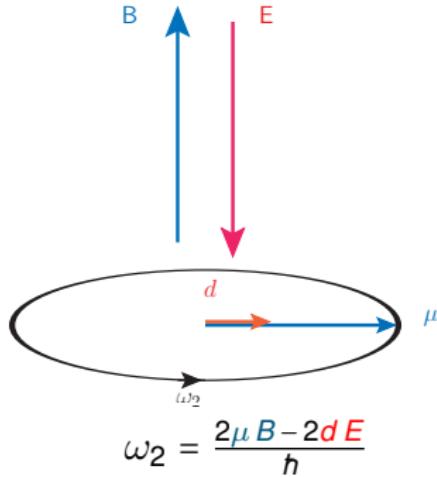
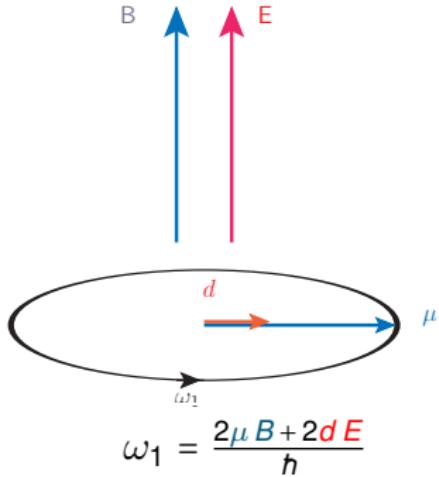
current $|d_e| < 8.7 \cdot 10^{-29} \text{ ecm}$ (90% C.L.)

from polar ThO

Baron et al. *Science* '14 (ACME)

EDM measurement of neutral particles in a nutshell

ground state (here with $s = 1/2$):

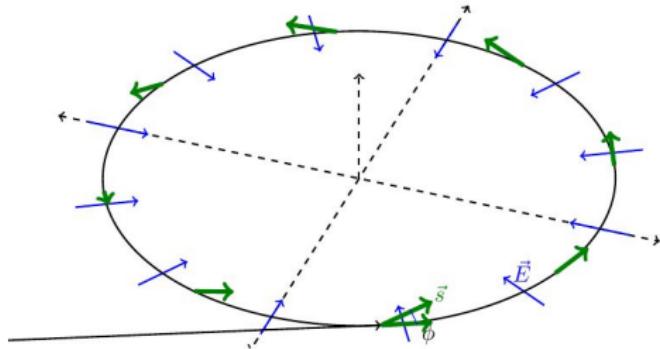


$$d = \frac{\hbar(\omega_1 - \omega_2)}{4E}$$

Direct EDM searches with charged particles in storage rings

General idea:

Farley et al. *PRL* '04



Initially **longitudinally** polarized particles interact with **radial \vec{E}** field
 ↳ **build-up of vertical polarization** (measured with a polarimeter)

Limit on muon EDM: $d_\mu < 1.8 \cdot 10^{-19} \text{ e cm}$ (95 % C. L.)

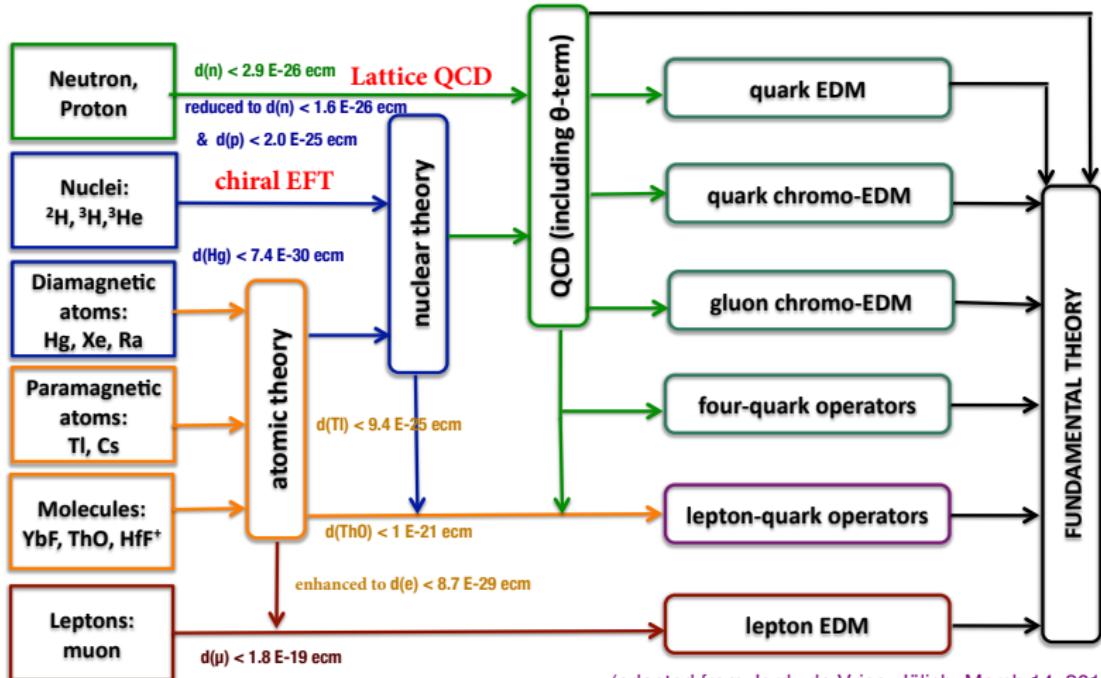
Bennett et al. (BNL *g-2*) *PRL* '09:

- Sensitivity of storage ring experiments $\sim 10^{-29} \text{ e cm}$
- But **systematical** errors $\sim ?$
- Precursor experiment $\gtrsim 10^{-(20\cdots 24)} \text{ e cm}$ for p or D planned at COSY@Jülich

Road map from EDM Measurements to EDM Sources

Experimentalist's point of view →

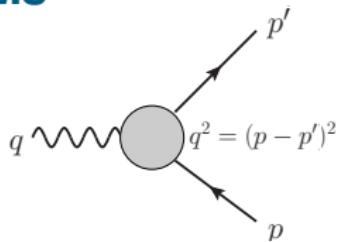
← Theorist's point of view



(adapted from Jordy de Vries, Jülich, March 14, 2013)

Calculation: from form factors to EDMs

$$\langle f(p') | J_{\text{em}}^\mu | f(p) \rangle = \bar{u}_f(p') \Gamma^\mu(q^2) u_f(p)$$



$$\Gamma^\mu(q^2) = \gamma^\mu F_1(q^2) - i\sigma^{\mu\nu} q_\nu \frac{F_2(q^2)}{2m_f} + \sigma^{\mu\nu} q_\nu \gamma_5 \frac{F_3(q^2)}{2m_f} + (\not{q} q^\mu - q^2 \gamma^\mu) \gamma_5 \frac{F_a(q^2)}{m_f^2}$$

Dirac FF

Pauli FF

electric dipole FF (\mathcal{OP})

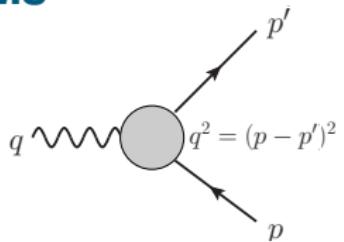
anapole FF (\mathcal{P}')

$$\hookrightarrow \quad d_f := \lim_{q^2 \rightarrow 0} \frac{F_3(q^2)}{2m_f} \quad \text{for } s = 1/2 \text{ fermion}$$



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Dirac FF

Pauli FF

electric dipole FF ($\mathcal{Q}\mathcal{P}$)

anapole FF (\mathcal{P})

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Nucleus A

$$\langle \uparrow | J_{PT}^0(q) | \uparrow \rangle \text{ in Breit frame}$$

$$\begin{aligned}
 & \langle \uparrow | J_{PT}^0(q) | \uparrow \rangle = \langle \uparrow | J_{PT}^{total} | \uparrow \rangle = \langle \uparrow | J_{PT} | \uparrow \rangle + \langle \uparrow | V_{PT} | \uparrow \rangle = -iq^3 \frac{F_3^A(\vec{q}^2)}{2m_A} \hookrightarrow d_A
 \end{aligned}$$

CP violation in the Standard Model

The conventional source: Kobayashi-Maskawa mechanism

Empirical facts: 3 generations of u/d quarks (& e/ν leptons)

- quarks & leptons in **mass basis** \neq quarks & leptons in **weak-int. basis**
- $\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{gauge-fermion}} + \mathcal{L}_{\text{gauge-Higgs}} + \mathcal{L}_{\text{Higgs-fermion}}$ is CP inv.,
 - with the exception of the θ term of QCD (see later)

and the **charged-weak-current interaction** ($\subset \mathcal{L}_{\text{gauge-fermion}}$)

$$\mathcal{L}_{\text{c-w-c}} = -\frac{g_w}{\sqrt{2}} \sum_{ij=1}^3 \bar{d}_{Li} \gamma^\mu \mathbf{V}_{ij} u_{Lj} W_\mu^- - \frac{g_w}{\sqrt{2}} \sum_{ij=1}^3 \bar{\ell}_{Li} \gamma^\mu \mathbf{U}_{ij} \nu_{Lj} W_\mu^- + \text{h.c.}$$

- \mathbf{V} : 3×3 unitary quark-mixing matrix

► (Cabibbo-Kobayashi-Maskawa matrix)

- \mathbf{U} : 3×3 unitary lepton-mixing matrix

(Pontecorvo-Maki-Nakagawa-Sakata m.)

3 angles + 1 **CP** phase δ_{KM}

3 angles + 1(3) **CP** phase(s) for Dirac (**Majorana**) ν_i 's

\mathcal{CP} and EDMs and in the SM with $J_{\text{KM}} = \text{Im}(V_{tb} V_{td}^* V_{cd} V_{cb}^*) \simeq 3 \cdot 10^{-5}$

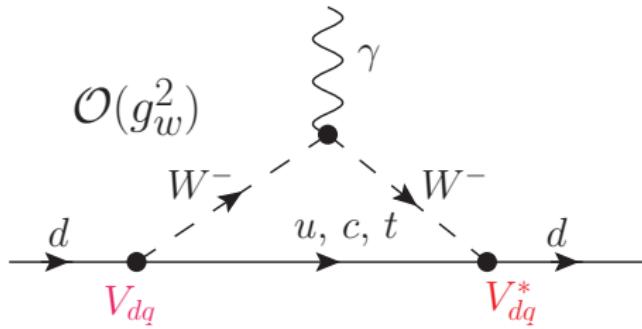
$$\propto \left(\frac{m_t^2 - m_c^2}{M_{EW}^2} \right) \left(\frac{m_c^2 - m_u^2}{M_{EW}^2} \right) \left(\frac{m_t^2 - m_u^2}{M_{EW}^2} \right) \cdot \left(\frac{m_b^2 - m_s^2}{M_{EW}^2} \right) \left(\frac{m_s^2 - m_d^2}{M_{EW}^2} \right) \left(\frac{m_b^2 - m_d^2}{M_{EW}^2} \right) \cdot J_{\text{KM}} \simeq 10^{-15} J_{\text{KM}},$$

Jarlskog *PRL* '85

↪ $(n_B - n_{\bar{B}})/n_\gamma|_{T \sim 20 \text{ MeV}}^{\text{SM}} \sim 10^{-20}$ and $d_n^{\text{SM}} \sim 10^{-20} \cdot 10^{-14} \text{ e cm} \sim 10^{-34} \text{ e cm}$

EDM flavor-neutral ⇒ KM predictions tiny: $\mathcal{O}(G_F^2) \sim \mathcal{O}(g_W^4)$

1 loop:



↪ \mathcal{CP} phase δ_{KM} cancels → prefactor real ⇒ $d_q^{\text{1-loop}} = 0$

\cancel{CP} and EDMs and in the SM with $J_{KM} = \text{Im}(V_{tb} V_{td}^* V_{cd} V_{cb}^*) \simeq 3 \cdot 10^{-5}$

$$\propto \left(\frac{m_t^2 - m_c^2}{M_{EW}^2} \right) \left(\frac{m_c^2 - m_u^2}{M_{EW}^2} \right) \left(\frac{m_t^2 - m_u^2}{M_{EW}^2} \right) \cdot \left(\frac{m_b^2 - m_s^2}{M_{EW}^2} \right) \left(\frac{m_s^2 - m_d^2}{M_{EW}^2} \right) \left(\frac{m_b^2 - m_d^2}{M_{EW}^2} \right) \cdot J_{KM} \simeq 10^{-15} J_{KM},$$

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2 loops:

$$d_{\text{quark}}^{\text{2-loop}} = d_{\text{chromo q}}^{\text{2-loop}} = 0$$

Shabalin *Sov.J.NP* '78

CP and EDMs and in the SM with $J_{\text{KM}} = \text{Im}(V_{tb} V_{td}^* V_{cd} V_{cb}^*) \simeq 3 \cdot 10^{-5}$

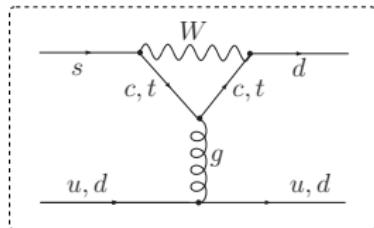
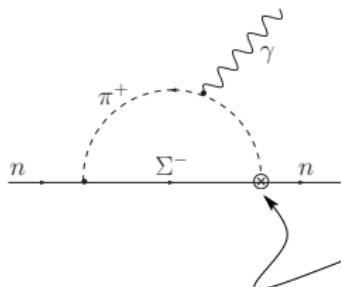
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however:



$$\mathcal{O}(g_W^4 g_s^2)$$

$d_n^{\text{KM}} \simeq 10^{-32} \text{ e cm}$ because of long-range pion & 'strong penguin'

Gavela; Khriplovich & Zhitnitsky ('82)

CP and EDMs and in the SM with $J_{\text{KM}} = \text{Im}(V_{tb} V_{td}^* V_{cd} V_{cb}^*) \simeq 3 \cdot 10^{-5}$

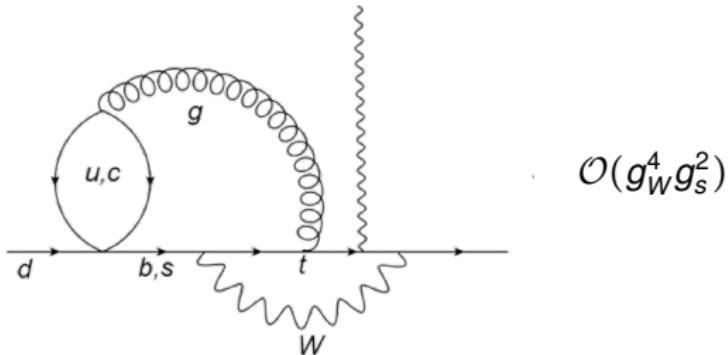
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at ≥ 3 loops:

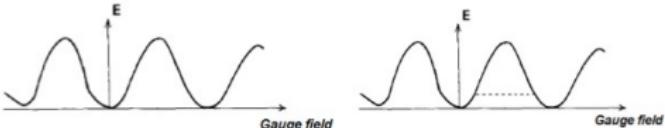


$$d_n^{\text{KM}} \simeq 10^{-34} \dots 10^{-31} \text{ e cm} \quad (d_e^{\text{KM}} \sim 10^{-38} \dots 10^{-40} \text{ e cm} \text{ since 4 loops & } \mathcal{O}(g_W^6 g_s^2))$$

Khriplovich (1986); Czarnecki & Krause ('97) (Khriplovich & Pospelov (1991))

EDM sources: QCD θ -term of the SM

The topologically non-trivial vacuum structure of QCD



- induces a direct $P \& T \sim CP$ interaction with a new parameter θ :

$$\mathcal{L}_{QCD} = \mathcal{L}_{QCD}^{CP} - \theta \frac{g_s^2}{64\pi^2} \epsilon^{\mu\nu\rho\sigma} G_{\mu\nu}^a G_{\rho\sigma}^a \quad (\text{note: } \epsilon^{0123} = -\epsilon_{0123} \text{ & dim = 4})$$

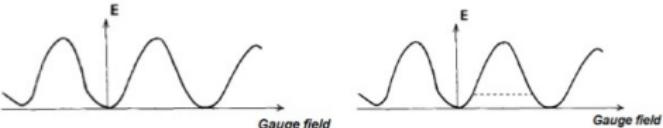
- Anomalous $U_A(1)$ quark-rotations induce mixing with ‘mass’ term

$$-\theta \frac{g_s^2}{64\pi^2} \epsilon^{\mu\nu\rho\sigma} G_{\mu\nu}^a G_{\rho\sigma}^a \xrightarrow{U_A(1)} \bar{\theta} m_q^* \sum_f \bar{q}_f i \gamma_5 q_f \quad (m_q^* = \frac{m_u m_d}{m_u + m_d} \text{ reduced mass})$$

→ additional coupling constant is actually $\bar{\theta} = \theta + \arg \det \mathcal{M}_{\text{quark}}$

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- Naive Dimensional Analysis (NDA) estimate of $\bar{\theta}$ -induced n EDM:

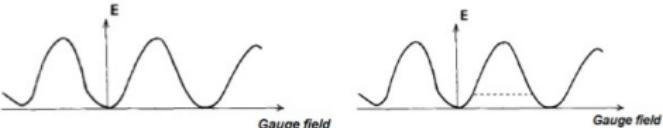
$$|d_n^{\bar{\theta}}| \sim \bar{\theta} \cdot \frac{m_q^*}{m_s} \cdot \frac{e}{2m_n} \sim \bar{\theta} \cdot 10^{-2} \cdot 10^{-14} \text{ ecm} \sim \bar{\theta} \cdot 10^{-16} \text{ ecm} \quad \text{with } \bar{\theta} \sim \mathcal{O}(1).$$

$$|d_n^{\text{emp}}| < 2.9 \cdot 10^{-26} \text{ ecm} \sim |\bar{\theta}| < 10^{-10}$$

► strong CP problem

EDM sources: QCD θ -term of the SM

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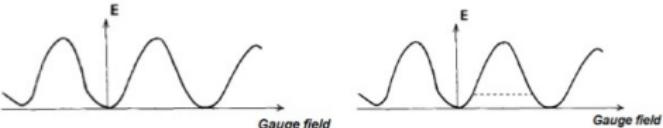
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$10^{-10} > |\bar{\theta}| > 10^{-14}$ eventually measurable via nonzero EDM, but because of $\Lambda_{\chi SB} \ll \Lambda_{EWSB}$ it doesn’t explain the cosmic matter surplus.

EDM sources: QCD θ -term of the SM

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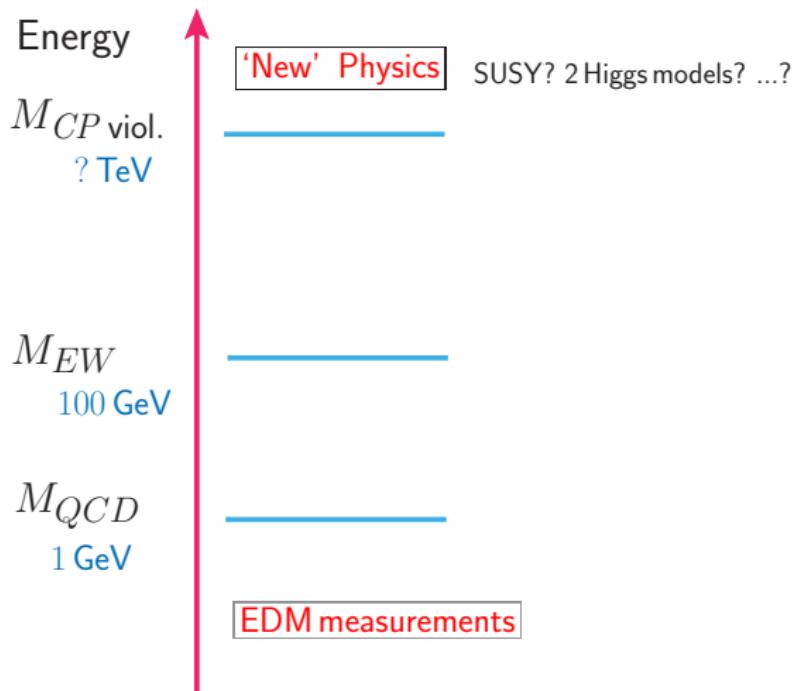
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Thus CP by new physics (NP) (i.e. dimension ≥ 6 sources beyond SM) needed to explain the cosmic matter-antimatter asymmetry.

How to handle CP-violating sources beyond the SM?

Running through the scales

W. Dekens & J. de Vries, *JHEP* '13
recall talk by Vincenzo Cirigliano on 09/13/16

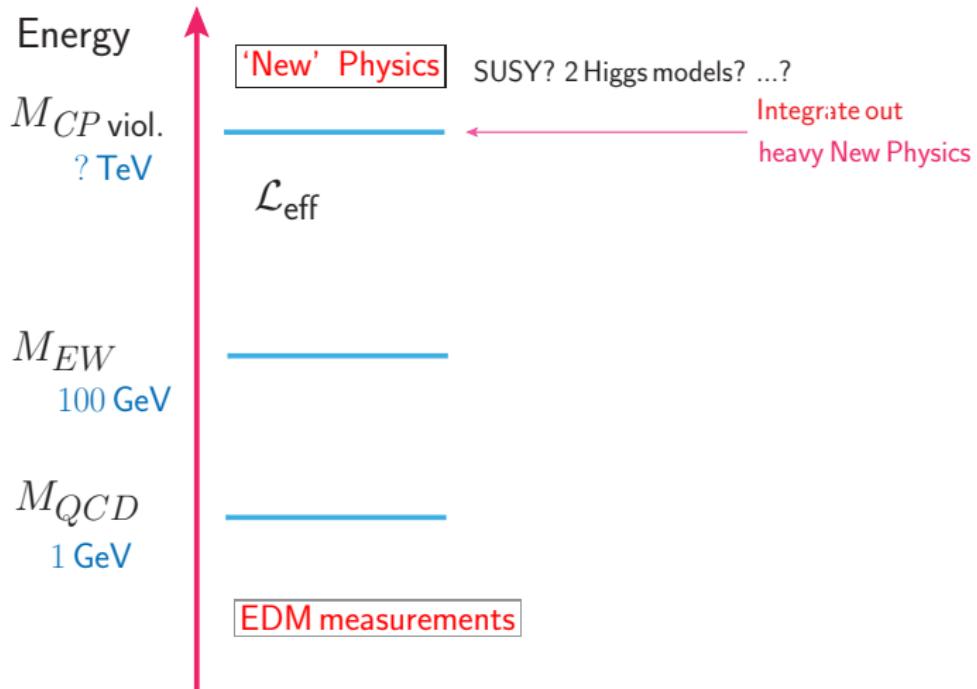


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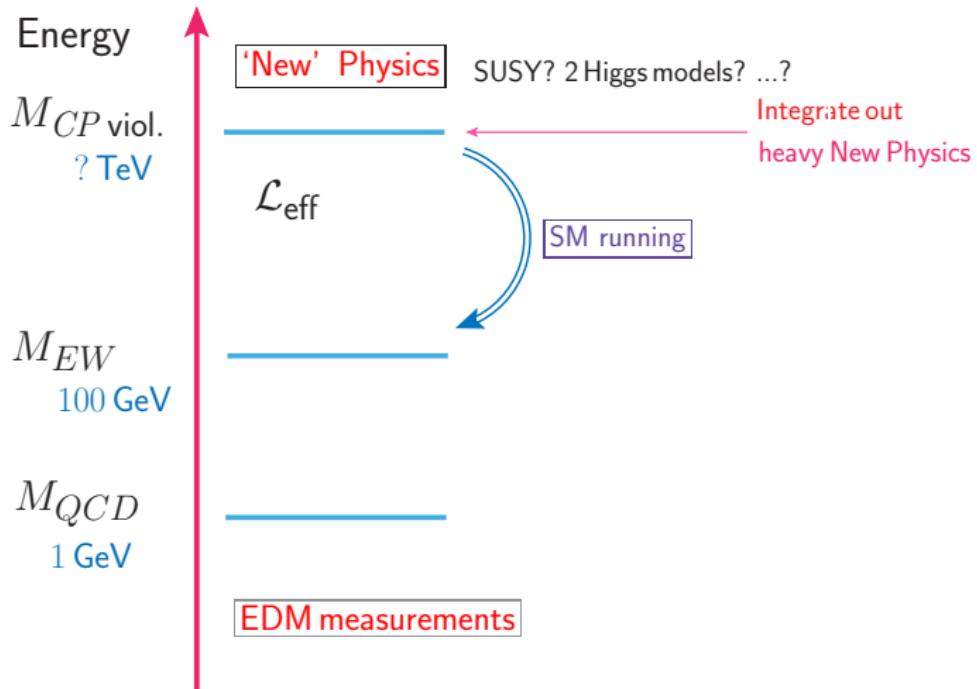


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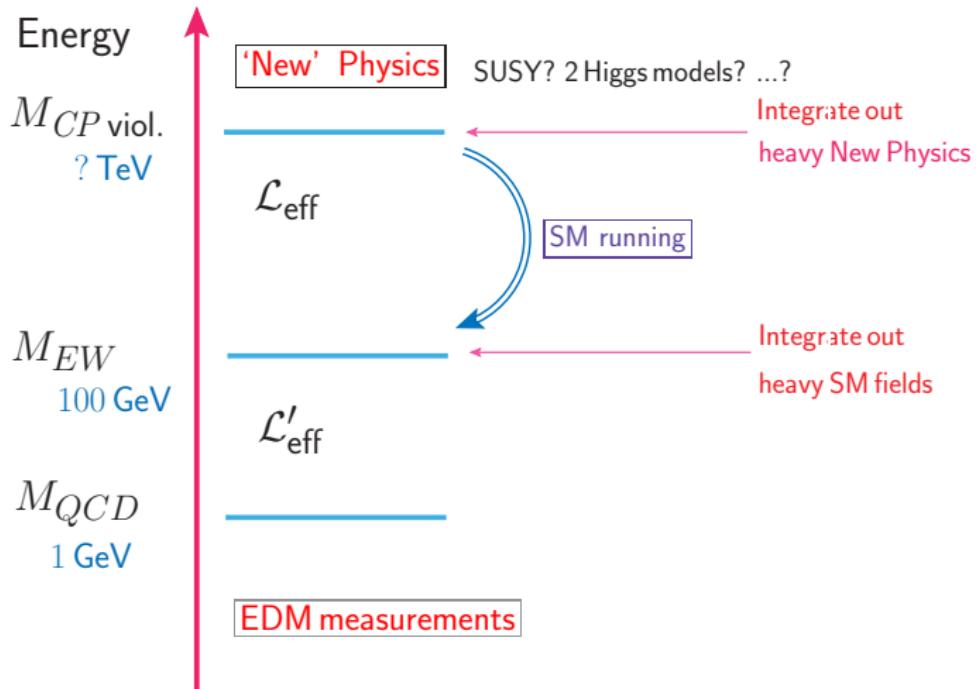


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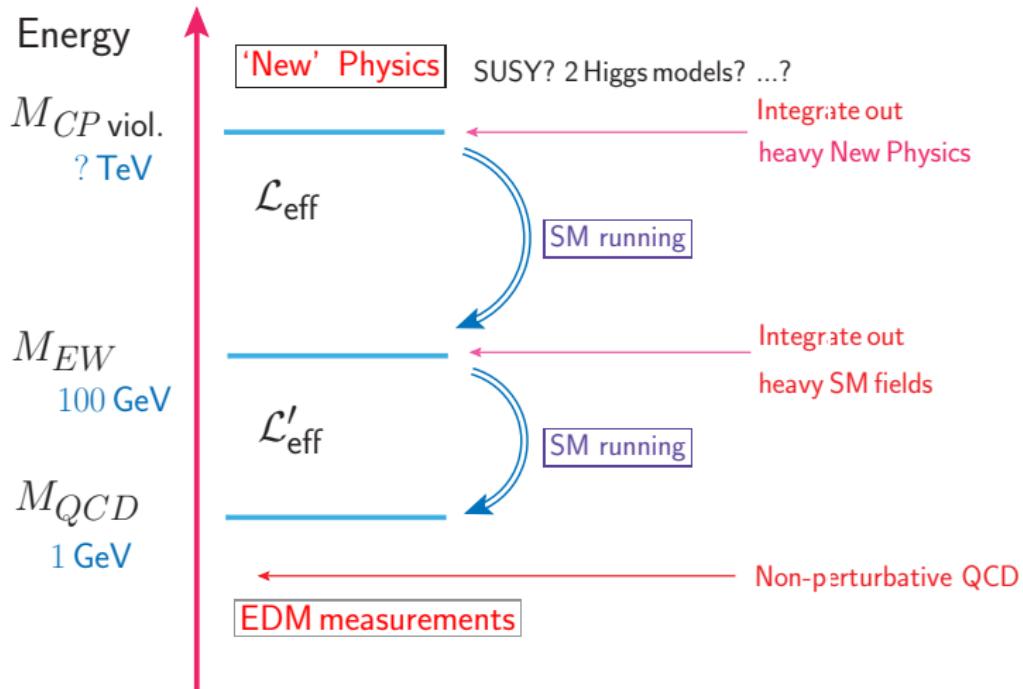


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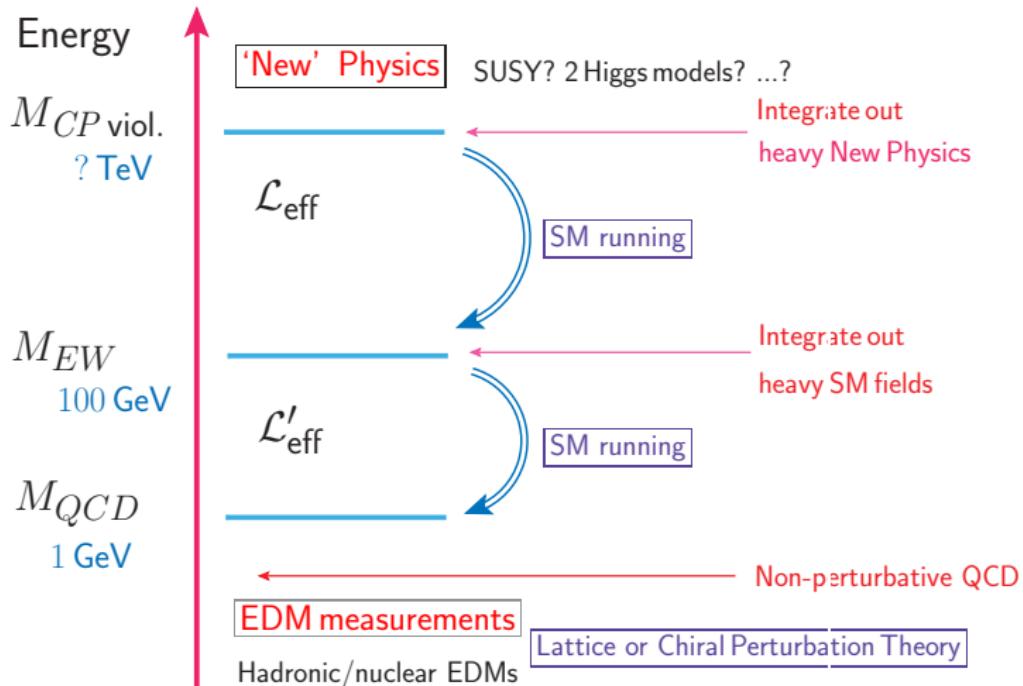


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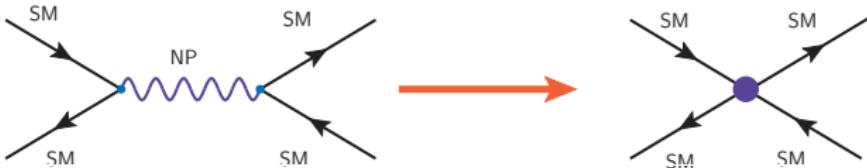
recall talk by Vincenzo Cirigliano on 09/13/16



How to handle CP-violating sources beyond the SM?

New interactions as higher dimensional operators

- Add to the SM **all possible** effective interactions



- The new interactions appear as higher dimensional operators

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_i \frac{c_5^{(i)}}{M_{\gamma}} \mathcal{O}_5^{(i)} + \sum_i \frac{c_6^{(i)}}{M_{\gamma}^2} \mathcal{O}_6^{(i)} + \dots$$

where M_{γ} is the scale of the *New Physics* particles

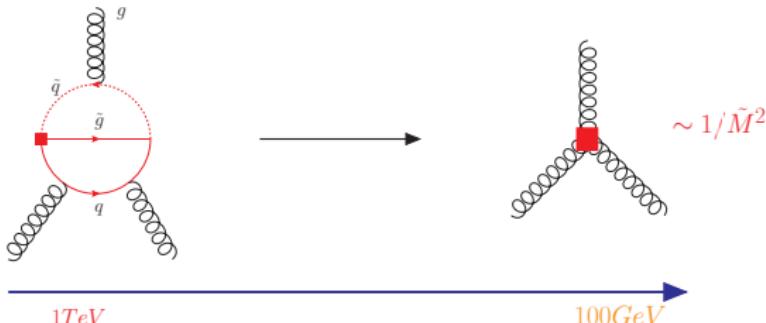
- Only the lowest dimensional operators should be important
- Hadronic EDMs: non-leptonic CP-violating operators of dim. 6
Not of dim. 5 because of Higgs insertion (chiral symm.) at **high** (low) scales

How to handle CP-violating sources beyond the SM?

Evaluation in Effective Field Theory (EFT) approach

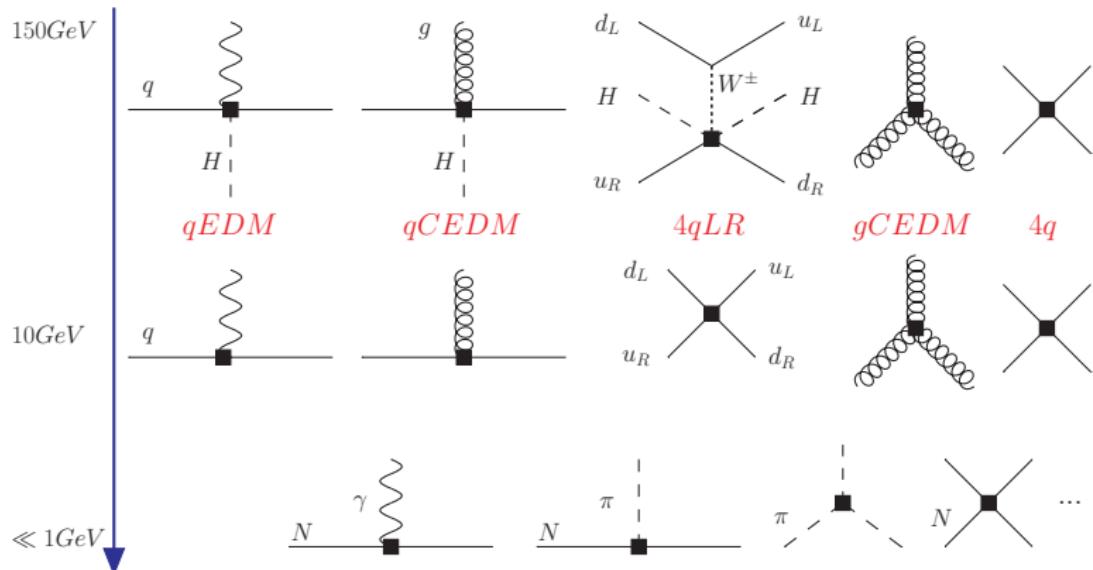
▶ EFT

- All degrees of freedom *beyond NP (EW) scale* are integrated out:
→ Only SM degrees of freedom remain: $q, g, (H, Z, W^\pm, \dots)$
- Write down *all* interactions for these *active degrees of freedom* that *respect the* SM+ Lorentz *symmetries*: here dim. 6 or higher order
- Need a *power-counting scheme* to order these *infinite #* interactions
- Relics of eliminated BSM physics ‘remembered’ by the values of the **low-energy constants (LECs)** of the **CP-violating contact terms**, e.g.



CP-violating BSM sources of dimension 6 from above EW scale to their hadronic equivalents below 1 GeV

W. Dekens & J. de Vries JHEP '13

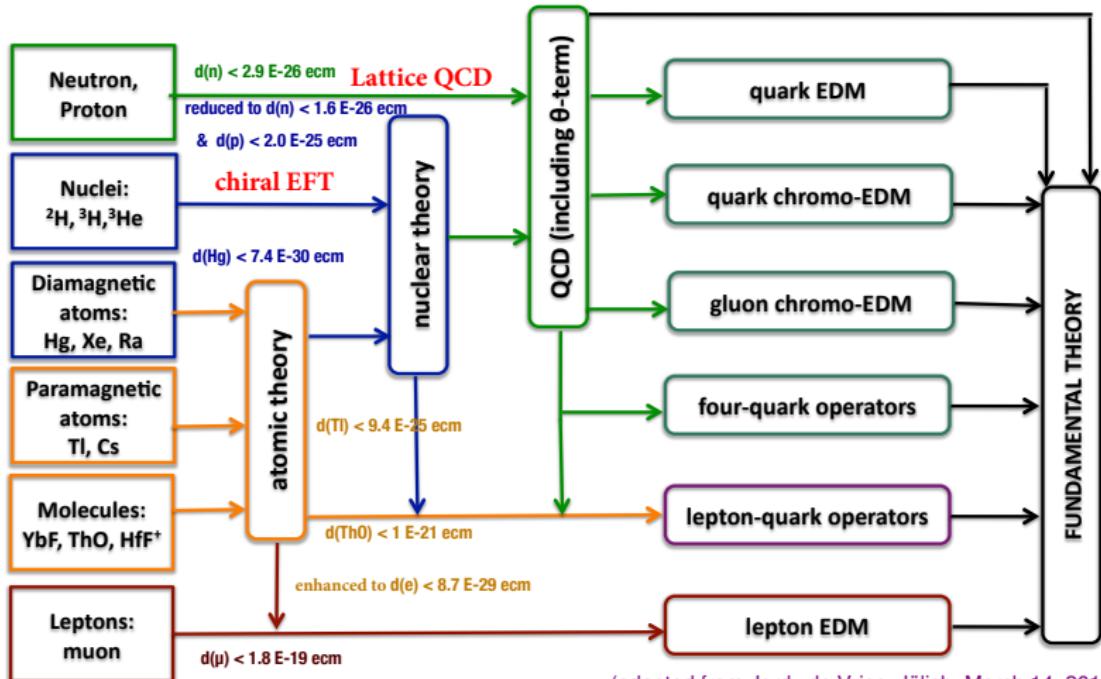


$$\begin{aligned}
 \text{Total #} &= 1(\bar{\theta}) + 2(qEDM) + 2(qCEDM) + 1(4qLR) + 1(gCEDM) + 2(4q) \quad [+3(\text{semi}) + 1(\text{lept})] \\
 &= \underbrace{1(\text{dim-four}) + 8(\text{dim-six})}_{\hookrightarrow 5 \text{ discriminable classes}} \quad [+3+1] \quad [\text{Caveat: } m_s \gg m_u, m_d \text{ (\& } m_\mu \gg m_e \text{ assumed)}]
 \end{aligned}$$

Road map from EDM Measurements to EDM Sources

Experimentalist's point of view →

← Theorist's point of view



(adapted from Jordy de Vries, Jülich, March 14, 2013)

EDM Translator

from ‘quarkish/machine’ to ‘hadronic/human’ language?



3-CPO & R2-D2



Dirk Vorderstraße

EDM Translator

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3-CPO & R2-D2

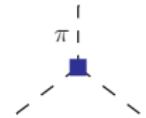
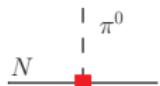
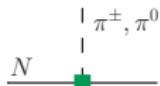


Dirk Vorderstraße

Symmetries (esp. chiral one) plus Goldstone Theorem
→ Low-Energy Effective Field Theory with External Sources
i.e. Chiral Perturbation Theory (suitably extended)

Summary of scalings of \mathcal{CP} hadronic vertices

from θ to BSM sources

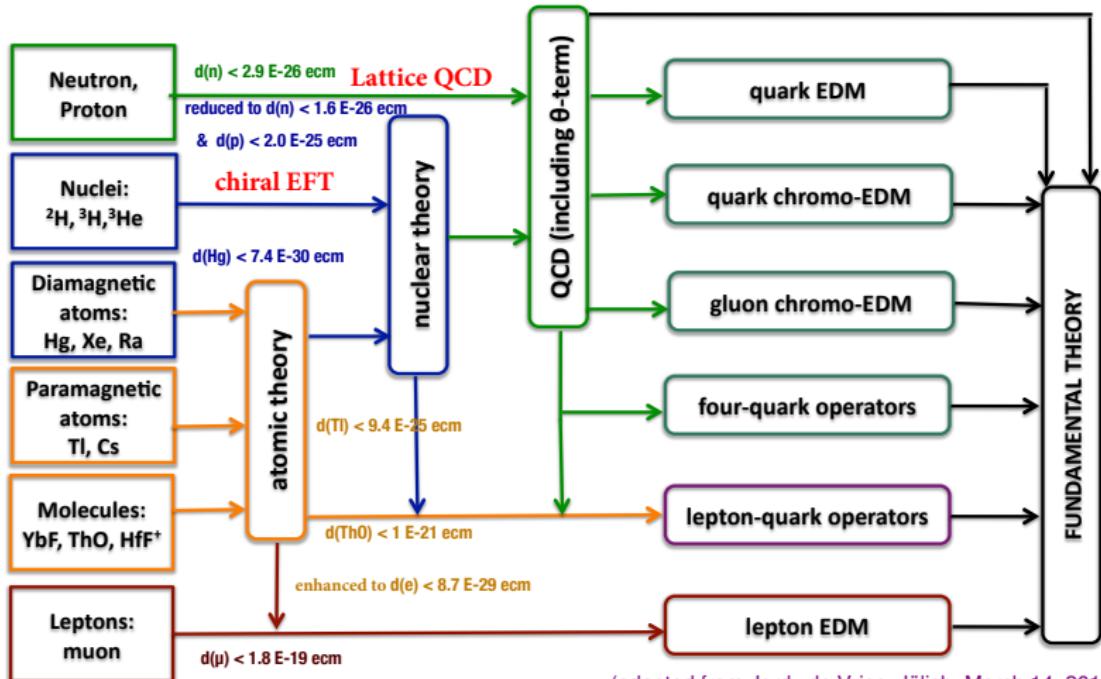
 $g_0: \mathcal{CP}, I$
 $g_1: \mathcal{CP}, I$
 $d_0, d_1: \mathcal{CP}, I + I'$
 $(m_N \Delta): \mathcal{CP}, I$
 $\mathcal{L}_{\text{EFT}}^{\mathcal{CP}}$

 $\theta\text{-term:}$
 $\mathcal{O}(1)$
 $\mathcal{O}(M_\pi/m_N)$
 $\mathcal{O}(M_\pi^2/m_N^2)$
 $\mathcal{O}(M_\pi^2/m_N^2)$
 qEDM:
 $\mathcal{O}(\alpha_{EM}/(4\pi))$
 $\mathcal{O}(\alpha_{EM}/(4\pi))$
 $\mathcal{O}(1)$
 $\mathcal{O}(\alpha_{EM}/(4\pi))$
 qCEDM:
 $\mathcal{O}(1)$
 $\mathcal{O}(1)$
 $\mathcal{O}(M_\pi^2/m_N^2)$
 $\mathcal{O}(M_\pi^2/m_N^2)$
 4qLR:
 $\mathcal{O}(M_\pi^2/m_n^2)$
 $\mathcal{O}(1)$
 $\mathcal{O}(M_\pi^2/m_N^2)$
 $\mathcal{O}(M_\pi/m_n)$
 gCEDM:
 $\mathcal{O}(M_\pi^2/m_N^2)^*$
 $\mathcal{O}(M_\pi^2/m_N^2)^*$
 $\mathcal{O}(1)$
 $\mathcal{O}(M_\pi^2/m_N^2)$
 4q:
 $\mathcal{O}(M_\pi^2/m_N^2)^*$
 $\mathcal{O}(M_\pi^2/m_N^2)^*$
 $\mathcal{O}(1)$
 $\mathcal{O}(M_\pi^2/m_N^2)$

*: Goldstone theorem \rightarrow relative $\mathcal{O}(M_\pi^2/m_n^2)$ suppression of $N\pi$ interactions

Road map from EDM Measurements to EDM Sources

Experimentalist's point of view →

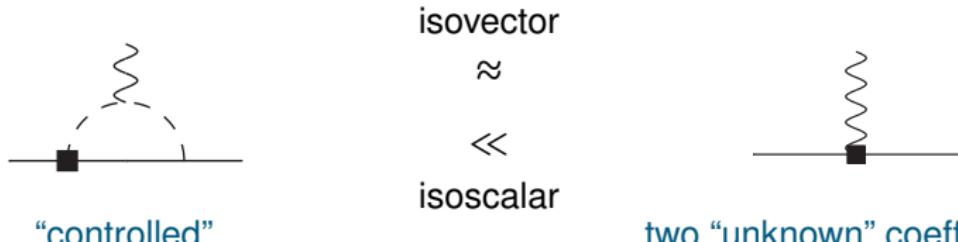
← Theorist's point of view



(adapted from Jordy de Vries, Jülich, March 14, 2013)

θ -Term Induced Nucleon EDM

single nucleon EDM:



Guo & Meißner *JHEP*'12: also in SU(3) case

$$d_N|_{\text{loop}}^{\text{isovector}} = -e \frac{g_{\pi NN} g_0^\theta}{4\pi^2} \frac{\ln(m_N^2/M_\pi^2)}{2m_N} \sim \bar{\theta} M_\pi^2 \ln M_\pi^2 \quad (e > 0)$$

Crewther, di Vecchia, Veneziano & Witten *PLB*'79; Pich & de Rafael *NPB*'91; Ott nad et al. *PLB*'10

$$g_0^\theta = \frac{(m_n - m_p)^{\text{strong}} (1 - \epsilon^2)}{4F_\pi \epsilon} \bar{\theta} \approx (-0.016 \pm 0.002) \bar{\theta} \quad (\text{where } \epsilon \equiv \frac{m_u - m_d}{m_u + m_d})$$

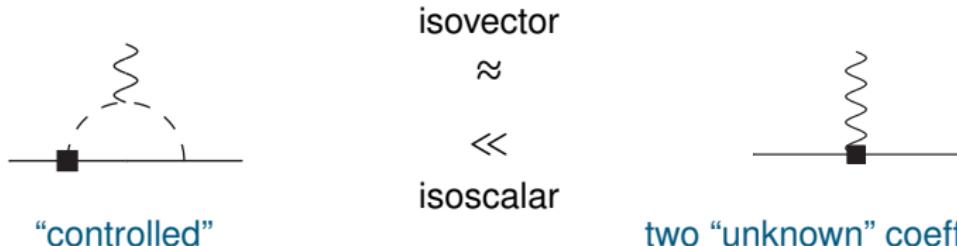
→ $d_N|_{\text{loop}}^{\text{isovector}} \sim (1.8 \pm 0.3) \cdot 10^{-16} \bar{\theta} \text{ ecm}$

Bsaisou et al., *EPJA*'13 & *JHEP*'15



θ -Term Induced Nucleon EDM

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Bsaisou et al., *EPJA*'13 & *JHEP*'15

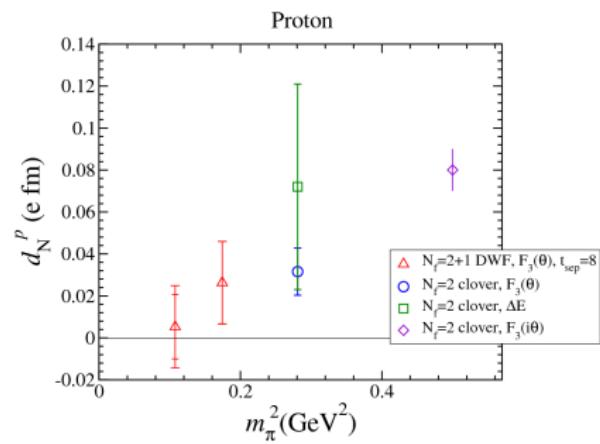
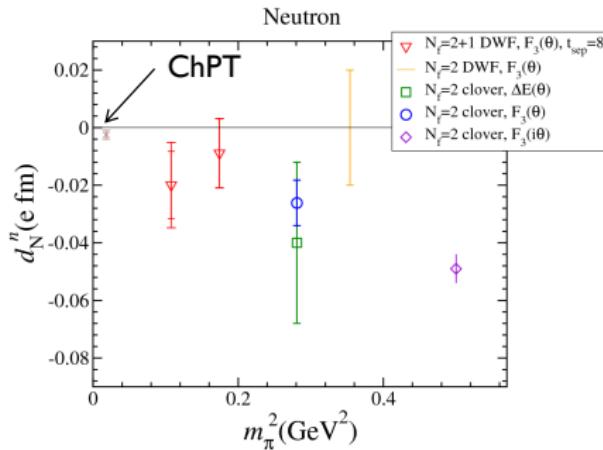
▶ details

But what about the two “unknown” coefficients of the contact terms?

Preliminary Lattice (full QCD) results

neutron EDM and

proton EDM



$$\theta \equiv 1 !$$

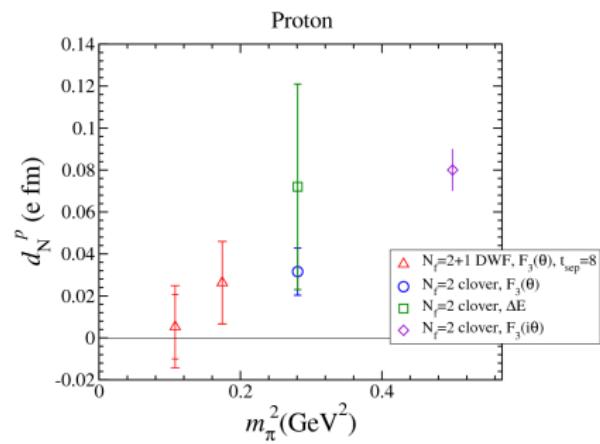
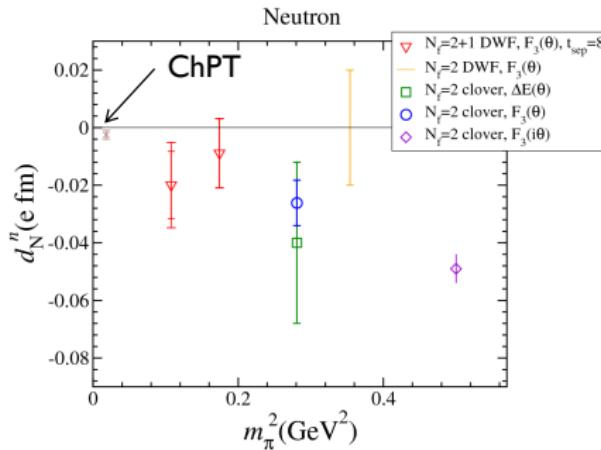
(adapted from Eigo Shintani (Mainz), *Lattice calculation of nucleon EDM*, Hirschegg, Jan. 14, 2014)

no systematical errors!

Preliminary Lattice (full QCD) results

neutron EDM and

proton EDM



$$\theta \equiv 1 !$$

(adapted from Eigo Shintani (Mainz), *Lattice calculation of nucleon EDM*, Hirschegg, Jan. 14, 2014)

no systematical errors!

$$\rightarrow d_n = \bar{\theta} (-2.7 \pm 1.2) \cdot 10^{-3} \cdot \text{efm} \quad \text{and} \quad d_p = \bar{\theta} (2.1 \pm 1.2) \cdot 10^{-3} \cdot \text{efm}$$

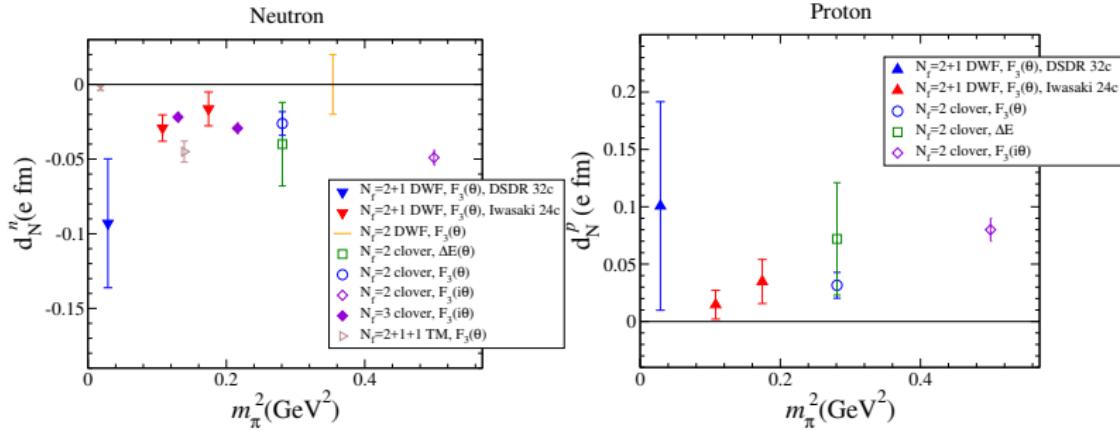
Akan, Guo & Meißner, *PLB 736* (2014); see also $d_n = \bar{\theta} (-3.9 \pm 0.2 \pm 0.9) 10^{-3} \text{efm}$ Guo et al., *PRL 115* (2015)

Preliminary Lattice (full QCD) results

neutron EDM

and

proton EDM



$$\bar{\theta} \equiv 1 !$$

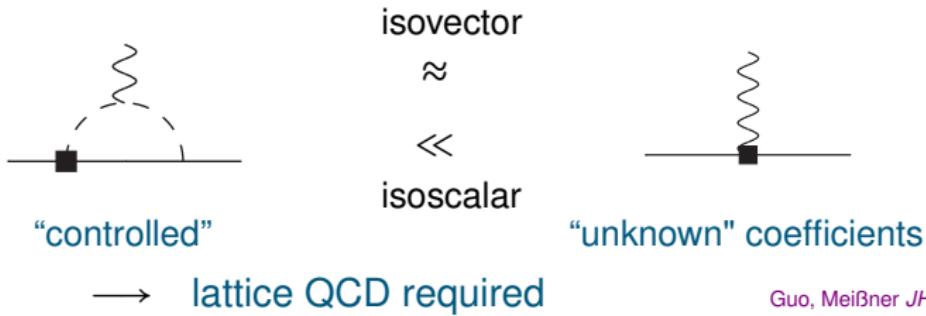
Eigo Shintani et al., *Phys. Rev. D* **93**, 094503 (2016) $M_\pi = 170, 330, 420, 530$ MeV

Don't mention the ... light nuclei

Single Nucleon Versus Nuclear EDM

Crewther, di Vecchia, Veneziano, Witten *PLB'79*; Pich, de Rafael *NPB'91*; Ott nad et al. *PLB'10*

single nucleon EDM:



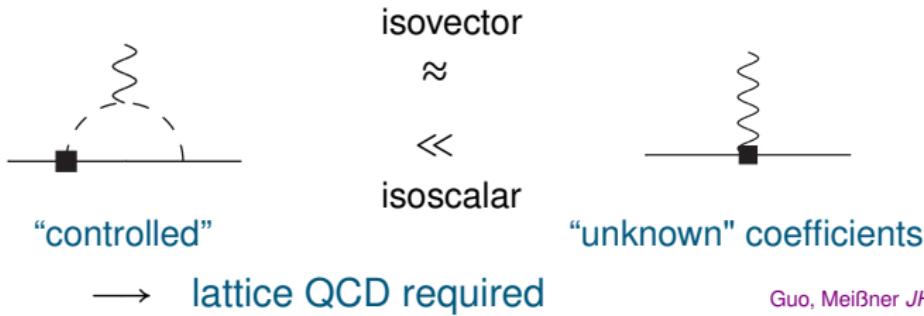
two nucleon EDM:



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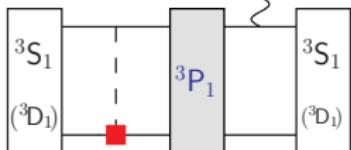


two nucleon EDM:



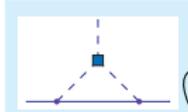
EDM of the Deuteron at LO: CP-violating π exchange

$$\begin{aligned} \mathcal{L}_{CP}^{\pi N} = & -d_n N^\dagger (1 - \tau^3) S^\mu v^\nu N F_{\mu\nu} - d_p N^\dagger (1 + \tau_3) S^\mu v^\nu N F_{\mu\nu} \\ & + (m_N \Delta) \pi^2 \pi_3 + \cancel{g_0 N^\dagger \vec{\pi} \cdot \vec{\tau} N} + \cancel{g_1 N^\dagger \pi_3 N} \\ & + \cancel{C_1 N^\dagger N D_\mu (N^\dagger S^\mu N)} + \cancel{C_2 N^\dagger \vec{\tau} N \cdot D_\mu (N^\dagger \vec{\tau} S^\mu N)} + \dots \end{aligned}$$



LO: $\cancel{g_0 N^\dagger \vec{\pi} \cdot \vec{\tau} N}$ (CP, I) $\rightarrow 0$ (Isospin filter!)

NLO: $g_1 N^\dagger \pi_3 N$ (CP, I) \rightarrow "LO" in D case



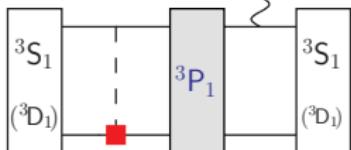
term	$N^2\text{LO ChPT}$	Δv_{18}	CD-Bonn	units
d_n^D	0.939 ± 0.009	0.914	0.927	d_n
d_p^D	0.939 ± 0.009	0.914	0.927	d_p
g_1	0.183 ± 0.017	0.186	0.186	$g_1 \text{ e fm}$
Δf_{g_1}	-0.748 ± 0.138	-0.703	-0.719	$\Delta \text{ e fm}$

Bsaisou, dissertation, Univ. Bonn (2014); Bsaisou et al., JHEP 03 (2015)

BSM CP sources: $g_1 \pi NN$ vertex is of LO in qCEDM and 4qLR case

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Yamanaka & Hiyama, PRC 91 (2015):

$$d_N^D = \left(1 - \frac{3}{2} P_{^3D_1}\right) d_N$$

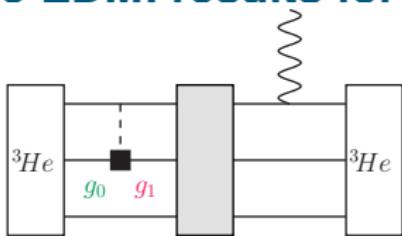
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^3He EDM: results for CP-violating π exchange



$g_0 N^\dagger \vec{\pi} \cdot \vec{\tau} N$ (\cancel{CP}, I)

LO: θ -term, qCEDM

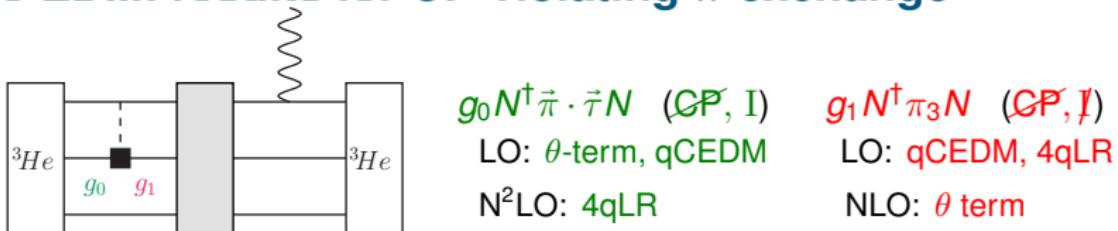
$N^2\text{LO}$: 4qLR

$g_1 N^\dagger \pi_3 N$ (\cancel{CP}, I)

LO: qCEDM, 4qLR

NLO: θ term

^3He EDM: results for CP-violating π exchange



term	A	$N^2\text{LO ChPT}$	$\text{Av}_{18} + \text{UIX}$	CD-Bonn+TM	units
d_n	^3He ^3H	0.904 ± 0.013 -0.030 ± 0.007	0.875 -0.051	0.902 -0.038	d_n
d_p	^3He ^3H	-0.029 ± 0.006 0.918 ± 0.013	-0.050 0.902	-0.037 0.876	d_p
Δ	^3He ^3H	-0.017 ± 0.006 -0.017 ± 0.006	-0.015 -0.015	-0.019 -0.018	$\Delta \text{ e fm}$
g_0	^3He ^3H	0.111 ± 0.013 -0.108 ± 0.013	0.073 -0.073	0.087 -0.085	$g_0 \text{ e fm}$
g_1	^3He ^3H	0.142 ± 0.019 0.139 ± 0.019	0.142 0.142	0.146 0.144	$g_1 \text{ e fm}$
Δf_{g_1}	^3He ^3H	-0.608 ± 0.142 -0.598 ± 0.141	-0.556 -0.564	-0.586 -0.576	$\Delta \text{ e fm}$
C_1	^3He ^3H	-0.042 ± 0.017 0.041 ± 0.016	-0.0014 0.0014	-0.016 0.016	$C_1 \text{ e fm}^{-2}$
C_2	^3He ^3H	0.089 ± 0.022 -0.087 ± 0.022	0.0042 -0.0044	0.033 -0.032	$C_2 \text{ e fm}^{-2}$

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$$\begin{aligned}\mathcal{L}_{\text{CP}}^{\pi N} = & -d_n N^\dagger (1 - \tau^3) S^\mu v^\nu N F_{\mu\nu} - d_p N^\dagger (1 + \tau_3) S^\mu v^\nu N F_{\mu\nu} \\ & + (m_N \Delta) \pi^2 \pi_3 + g_0 N^\dagger \vec{\pi} \cdot \vec{\tau} N + g_1 N^\dagger \pi_3 N \\ & + C_1 N^\dagger N \mathcal{D}_\mu (N^\dagger S^\mu N) + C_2 N^\dagger \vec{\tau} N \cdot \mathcal{D}_\mu (N^\dagger \vec{\tau} S^\mu N) + \dots.\end{aligned}$$

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Discriminating between three scenarios at 1 GeV

Dekens et al. *JHEP* 07 (2014); Bsaisou et al. *JHEP* 03 (2015)

1 The Standard Model + $\bar{\theta}$

$$\mathcal{L}_{\text{SM}}^{\bar{\theta}} = \mathcal{L}_{\text{SM}} + \bar{\theta} m_q^* \bar{q} i \gamma_5 q$$

2 The left-right symmetric model — with two 4-quark operators:

$$\mathcal{L}_{LR} = -i \Xi [1.1 (\bar{u}_R \gamma_\mu u_R) (\bar{d}_L \gamma^\mu d_L) + 1.4 (\bar{u}_R t^a \gamma_\mu u_R) (\bar{d}_L t^a \gamma^\mu d_L)] + \text{h.c.}$$

3 The aligned two-Higgs-doublet model — with the dipole operators:

$$\mathcal{L}_{a2HM} = -e \frac{d_d}{2} \bar{d} i \sigma_{\mu\nu} \gamma_5 d F^{\mu\nu} - \frac{\tilde{d}_d}{4} \bar{d} i \sigma_{\mu\nu} \gamma_5 \lambda^a d G^{a\mu\nu} + \frac{d_W}{3} f_{abc} \tilde{G}^{a\mu\nu} G_{\mu\rho}^b G_{\nu}^{c\rho}$$

— with the hierarchy $\tilde{d}_d \simeq 4 d_d \simeq 20 d_W$

matched on

$$\begin{aligned} \mathcal{L}_{\text{CP EFT}}^{\pi N} &= -d_N N^\dagger (1 - \tau^3) S^\mu v^\nu N F_{\mu\nu} - d_p N^\dagger (1 + \tau_3) S^\mu v^\nu N F_{\mu\nu} \\ &\quad + (m_N \Delta) \pi^2 \pi_3 + g_0 N^\dagger \vec{\pi} \cdot \vec{\tau} N + g_1 N^\dagger \pi_3 N \\ &\quad + C_1 N^\dagger N \mathcal{D}_\mu (N^\dagger S^\mu N) + C_2 N^\dagger \vec{\tau} N \cdot \mathcal{D}_\mu (N^\dagger \vec{\tau} S^\mu N) + \dots . \end{aligned}$$

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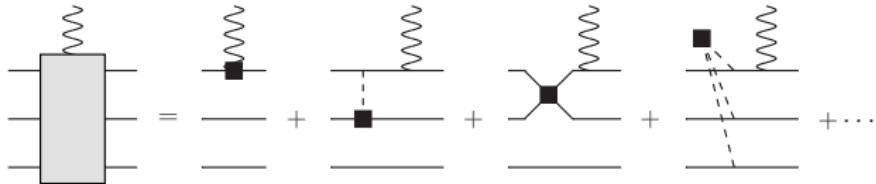
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Testing strategies: SM + $\bar{\theta}$

Dekens et al. *JHEP* **07** (2014); Bsaisou et al. *JHEP* **03** (2015)

Measurement of the helion
and neutron EDMs

Testing strategies: SM + $\bar{\theta}$

Dekens et al. *JHEP* **07** (2014); Bsaisou et al. *JHEP* **03** (2015)

Measurement of the helion
and neutron EDMs

$$d_{^3\text{He}} - 0.9d_n = -\bar{\theta} (1.01 \pm 0.31_{\text{had}} \pm 0.29^*_{\text{nucl}}) \cdot 10^{-16} \text{ ecm}$$

Extraction of $\bar{\theta}$

* includes ± 0.20 uncertainty from 2N contact terms

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Extraction of $\bar{\theta}$

$$d_D - 0.94(d_n + d_p) = \bar{\theta} (0.89 \pm 0.29_{\text{had}} \pm 0.08_{\text{nucl}}) \cdot 10^{-16} \text{ ecm}$$

Prediction for $d_D - 0.94(d_n + d_p)$
(& triton EDM): $d_D^{\text{Nucl}} \approx -d_{^3\text{He}}^{\text{Nucl}} \approx \frac{1}{2} d_{^3\text{H}}^{\text{Nucl}}$

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$$g_1^\theta / g_0^\theta \approx -0.2$$

* includes ± 0.20 uncertainty from 2N contact terms

$$g_0^\theta = \frac{(m_n - m_p)^{\text{strong}} (1 - \epsilon^2)}{4F_\pi \epsilon} \bar{\theta} = (-16 \pm 2) 10^{-3} \bar{\theta}$$

$$\frac{g_1^\theta}{g_0^\theta} \approx \frac{8c_1(M_{\pi^\pm}^2 - M_{\pi^0}^2)^{\text{strong}}}{(m_n - m_p)^{\text{strong}}} , \quad \epsilon \equiv \frac{m_u - m_d}{m_u + m_d}$$

Testing strategies: minimal LR symmetric Model

Dekens et al. *JHEP* 07 (2014); Bsaisou et al. *JHEP* 03 (2015)

Measurement of the deuteron
and nucleon EDMs

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Measurement of the deuteron
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$$d_D - 0.94(d_n + d_p) \simeq d_D = -(2.1 \pm 0.5^*) \Delta^{LR} \text{ e fm}$$

Extraction of Δ^{LR}

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Extraction of Δ^{LR}

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Prediction for the helion EDM
(& triton EDM): $d_D \approx d_{^3\text{He}} \approx d_{^3\text{H}}$

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Prediction for the helion EDM
(& triton EDM): $d_D \approx d_{^3\text{He}} \approx d_{^3\text{H}}$

$$\begin{aligned} g_1^{LR} &= 8c_1 m_N \Delta^{LR} &= (-7.5 \pm 2.3) \Delta^{LR}, \\ g_0^{LR} &= \frac{(m_n - m_p)^{\text{str}} m_N}{M_\pi^2} \Delta^{LR} &= (0.12 \pm 0.02) \Delta^{LR} \end{aligned}$$

$-g_1^{LR}/g_0^{LR} \gg 1$ (!)

* includes ± 0.1 uncertainty from 2N contact terms

Testing strategies: aligned 2-Higgs Doublet Model

Dekens et al. *JHEP* 07 (2014); Bsaisou et al. *JHEP* 03 (2015)

Measurement of the deuteron
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Dekens et al. *JHEP* 07 (2014); Bsaisou et al. *JHEP* 03 (2015)

Measurement of the deuteron
and nucleon EDMs

$$d_D - 0.94(d_n + d_p) = [(0.18 \pm 0.02)g_1 - (0.75 \pm 0.14)\Delta] \text{ e fm}$$

Extraction of g_1^{eff} (including Δ correction)

Testing strategies: aligned 2-Higgs Doublet Model

Dekens et al. *JHEP* 07 (2014); Bsaisou et al. *JHEP* 03 (2015)

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$$\begin{aligned} d_{^3\text{He}} - 0.9d_n \\ = [(0.11 \pm 0.02^*)g_0 + (0.14 \pm 0.02^*)g_1 - (0.61 \pm 0.14)\Delta] \text{efm} \end{aligned}$$

Extraction of g_0

* includes ± 0.01 uncertainty from 2N contact terms

Testing strategies: aligned 2-Higgs Doublet Model

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Extraction of g_0

Prediction of $d_{^3\text{H}}$ (or $d_{^3\text{He}}$)

* includes ± 0.01 uncertainty from 2N contact terms

Summary

- D EDM might **distinguish** between $\bar{\theta}$ and other scenarios and allows **extraction** of the g_1 coupling constant via $d_D - 0.94(d_n + d_p)$. (The prefactor of $(d_n + d_p)$ stands for a 4% probability of the 3D_1 state.)
- 3He (or 3H) EDM necessary for a **proper test** of $\bar{\theta}$ and LR scenarios:
- Deuteron & helion work as complementary **isospin filters** of EDMs
- 2N contact terms **cannot be neglected** for nuclei beyond D
- **a2HDM case:** 3He and 3H EDMs would be needed for a proper test
- **pure qCEDM:** similar to a2HDM scenario
- **pure qEDM:** $d_D = 0.94(d_n + d_p)$ and $d_{{}^3He/{}^3H} = 0.9d_{n/p}$
- **gCEDM, 4quark χ singlet:** controlled calculation difficult (lattice ?)
- Ultimate progress may eventually come from **Lattice QCD**
→ $GP N\pi$ couplings g_0 & g_1 may be accessible even for dim-6 case

Traditional atomic EDMs

- Why can't we get **this info** from EDMs of Hg, Ra, Rn, ... ?

Strong bound on atomic EDM: $|d_{^{199}\text{Hg}}| < 7.4 \cdot 10^{-30} \cdot e \cdot \text{fm}$

Graner et al. *PRL*'16

- The **atomic** part of the calculation is well under control

$$d_{^{199}\text{Hg}} = (2.8 \pm \underbrace{0.6}_{0.3}) S_{\text{Hg}} \cdot 10^{-4} \cdot \text{fm}^{-2}$$

Dzuba et al. *PRA*'02, '09, *IJMPE*'12

S_{Hg} : Nuclear Schiff moment

- But the **nuclear** part isn't ...

$$S_{^{199}\text{Hg}} = [(0.4 \pm 0.4)g_0 + (0.4 \pm 0.8)g_1] e \cdot \text{fm}^3$$

Engel et al. *PPNP*'13

- There is **no power counting** for nuclei with so many nucleons
- Short-range 4N contributions not even considered
- The CP-violating 3π contribution Δf_{g_1} to g_1 grows linearly with momentum transfer \sim more important for heavier systems
- Hadronic uncertainties of g_0 and g_1 are underestimated too

Conclusions

- EDMs **probe New CP-odd Physics** (at similar energy scales as LHC)
- The **first** non-vanishing EDM might be detected in a **charge-neutral** case: *neutrons* or *dia-/ paramagnetic atoms* or *molecules* ...
- However, measurements of **light ion EDMs** can play a key role in **disentangling the sources of (flavor-diagonal) CP**
- EDM measurements are characteristically of **low-energy nature**:
 - non-leptonic predictions have to be in the *language of hadrons*
 - only systematical methods: *ChPT/EFT* and *Lattice QCD*
- EDMs of light nuclei provide **independent information** to nucleon ones and may be even larger and, moreover, even **simpler**

At least the EDMs of p , n , D , and 3He would be needed to have a **realistic** chance to disentangle the underlying physics

Many thanks to my colleagues

in Jülich: **Jan Bsaisou**, Christoph Hanhart, Susanna Liebig, Ulf-G. Meißner,
David Minossi, Andreas Nogga, and **Jordy de Vries**

in Bonn: Feng-Kun Guo, Bastian Kubis, Ulf-G. Meißner

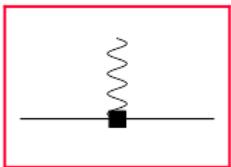
and: Werner Bernreuther, Bira van Kolck, and Kolya Nikolaev

References:

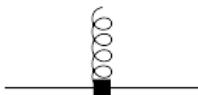
- 1 J. Bsaisou, U.-G. Meißner, A. Nogga and A.W.,
P- and T-Violating Lagrangians in Chiral Effective Field Theory and Nuclear Electric Dipole Moments, Annals of Physics **359**, 317-370 (2015), arXiv:1412.5471 [hep-ph].
- 2 J. Bsaisou, C. Hanhart, S. Liebig, D. Minossi, U.-G. Meißner, A. Nogga and A.W.,
Electric dipole moments of light nuclei, JHEP **03**, 104 (05, 083) (2015), arXiv:1411.5804.
- 3 A.W., *Electric dipole moments of the nucleon and light nuclei*
Nuclear Physics A **928**, 116-127 (2014), arXiv:1404.6131 [hep-ph].
- 4 W. Dekens, J. de Vries, J. Bsaisou, W. Bernreuther, C. Hanhart, U.-G. Meißner,
A. Nogga and A.W.,
Unraveling models of CP violation through electric dipole moments of light nuclei,
JHEP **07**, 069 (2014), arXiv:1404.6082 [hep-ph].
- 5 J. Bsaisou, C. Hanhart, S. Liebig, U.-G. Meißner, A. Nogga and A.W.,
The electric dipole moment of the deuteron from the QCD θ -term,
Eur. Phys. J. A **49**, 31 (2013), arXiv:1209.6306 [hep-ph].

Backup slides

If $\bar{\theta}$ -term tests fail, then effective BSM dim. 6 sources de Vries et al.(2011)



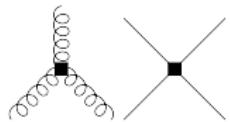
$qEDM$



$qCEDM$



$4qLR$



$gCEDM + 4qEDM$

$$d_D \approx d_p + d_n$$

$$d_{^3He} \approx d_n$$

$$d_D > d_p + d_n$$

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$$d_D > d_p + d_n$$

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$$d_D \sim d_p + d_n$$

$$d_{^3He} \sim d_n$$

→ $g_0, g_1 \propto \alpha/(4\pi)$

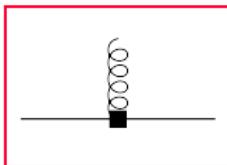
2N contribution suppressed by photon loop!

here: only absolute values considered

If $\bar{\theta}$ -term tests fail, then effective BSM dim. 6 sources de Vries et al.(2011)



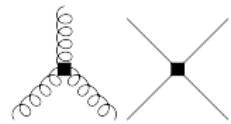
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→ g_0 , g_1 dominant and of the same order

$2N$ contribution enhanced!

here: only absolute values considered

If $\bar{\theta}$ -term tests fail, then effective BSM dim. 6 sources

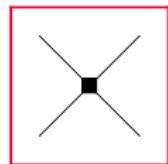
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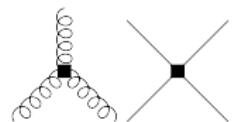
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$$d_D \sim d_p + d_n$$

$$d_{^3He} \sim d_n$$

→ $g_1 \gg g_0$; 3π -coupling (unsuppressed in 3He)

isospin-breaking $2N$ contribution enhanced!

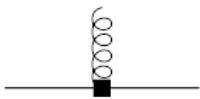
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If $\bar{\theta}$ -term tests fail, then effective BSM dim. 6 sources

de Vries et al.(2011)



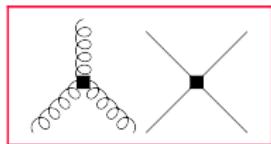
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$$d_{^3He} \sim d_n$$

→ g_1 , g_0 , $4N$ – coupling on the same footing

$2N$ contribution difficult to asses!

here: only absolute values considered

Generic features of a permanent EDM

$$\mathcal{H}_{\text{eff}} = i \frac{d_f}{2} \bar{f} \sigma^{\mu\nu} \gamma_5 f F_{\mu\nu} \xrightarrow{\text{non-rel.}} -d_f \langle \sigma \rangle \cdot \mathbf{E} \longrightarrow \text{linear Stark effect}$$

Generic features of a permanent EDM

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- 1 For any non-zero EDM, \cancel{CP} must be *flavor diagonal*!
- 2 Sum of the mass dimension of the fields: $\frac{3}{2} + \frac{3}{2} + 2 = 5$:
 $\hookrightarrow \dim(d_f) = e \times \text{mass}^{-1} = e \times \text{length}$ (such that $\int d^4x \mathcal{L} \sim \text{mass}^0$)
 \hookrightarrow non-renormalizable *effective* interaction
- 3 fermion EDMs flip chirality: $\frac{1}{2}(\mathbf{1} - \gamma_5)f \equiv f_L \longleftrightarrow f_R \equiv \frac{1}{2}(\mathbf{1} + \gamma_5)f$

$$\mathcal{L}_{\text{EDM}} = -i \frac{d_f}{2} \bar{f} \sigma_{\mu\nu} \gamma_5 f F^{\mu\nu} = -i \frac{d_f}{2} \bar{f}_L \sigma_{\mu\nu} f_R F^{\mu\nu} + i \frac{d_f}{2} \bar{f}_R \sigma_{\mu\nu} f_L F^{\mu\nu}$$

\Rightarrow fermion mass m_f insertion (e.g. via Higgs mechanism) needed:

$$d_f \propto m_f^n, \quad n = 1, 2, 3 \quad (\text{depending on the model of } \cancel{CP})$$

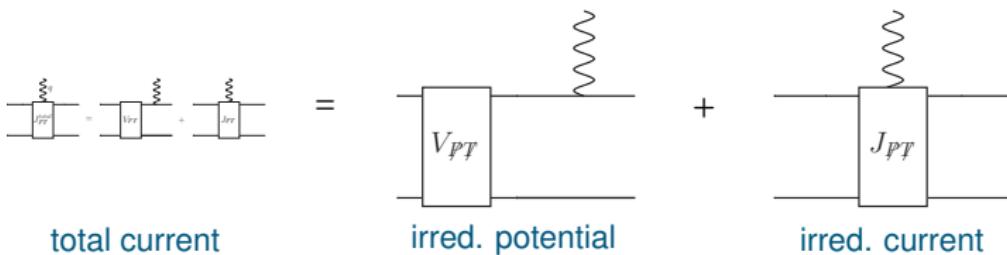
$$\hookrightarrow \cancel{CP} \text{ beyond SM: } \mathcal{L}_{\text{BSM}}^{\cancel{CP}} = \frac{1}{M_{\text{BSM}}} \cancel{\mathcal{L}_{\text{dim 5}}} + \frac{1}{M_{\text{BSM}}^2} \cancel{\mathcal{L}_{\text{dim 6}}} + \dots$$

EDM of the Deuteron:

Deuteron (D) as Isospin Filter

note:  = $\frac{ie}{2}(1 + \tau_3)$

2N-system: $I + S + L = \text{odd}$



$$I = 0$$

$$I = 0$$

$$I = 0 \rightarrow I = 1 \rightarrow I = 0$$

$$I = 0$$

$$I = 0$$

isospin selection rules!

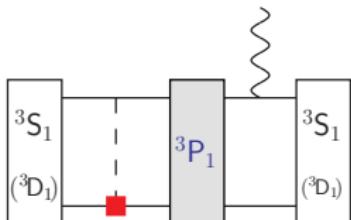


~~$g_0^\theta N^\dagger \vec{\pi} \cdot \vec{\tau} N$~~ at leading order (LO)



subleading (NLO) $g_1^\theta N^\dagger \pi_3 N$ acts as 'new' leading order (LO) for D

EDM of the Deuteron at LO: CP-violating π exchange



LO: ~~$g_0^\theta N^\dagger \vec{\pi} \cdot \vec{\tau} N$~~ (\cancel{CP}, I) $\rightarrow 0$ (Isospin filter!)

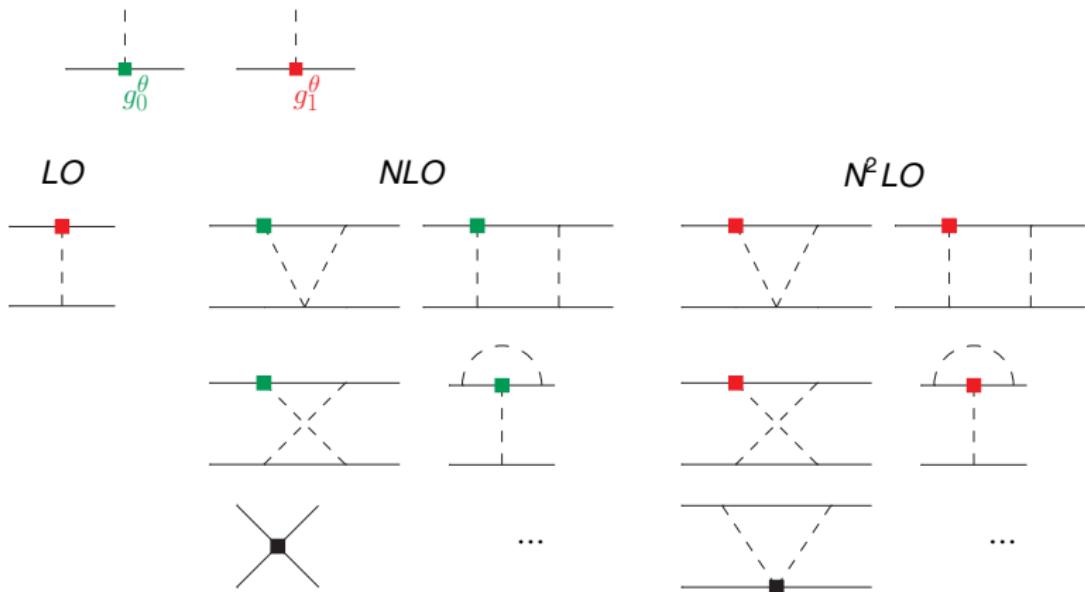
NLO: $g_1^\theta N^\dagger \pi_3 N$ (CP, I) \rightarrow "LO" in D case

Reference	potential	result [$g_1 g_{\pi NN} e \text{fm}$]
Liu & Timmermans (2004)	Λv_{18}	1.43×10^{-2}
Afnan & Gibson (2010)	Reid 93	1.53×10^{-2}
Song et al. (2013)	Λv_{18}	1.45×10^{-2}
JBC (2014)*	Λv_{18}	1.45×10^{-2}
Bsaisou et al. (2013)◊	CD Bonn	1.52×10^{-2}
JBC (2014)*	ChPT (N^2LO)	$(1.43 \pm 0.13) \times 10^{-2}$

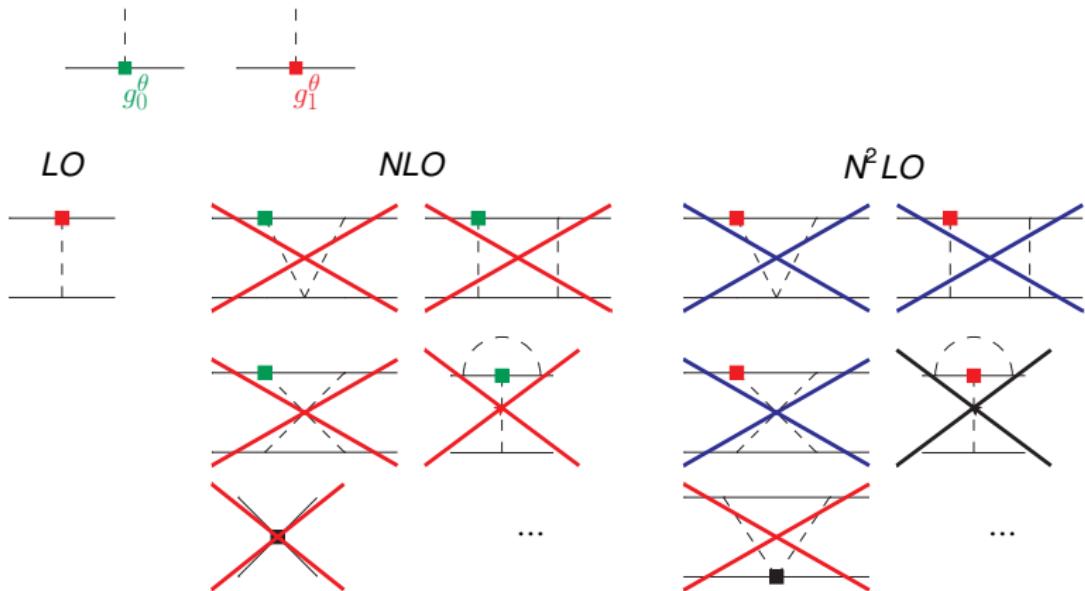
* unpublished, ◊ Eur. Phys. J. A **49** (2013) 31 [arXiv:1209.6306]

BSM \cancel{CP} sources: $g_1 \pi NN$ vertex is of LO in qCEDM and 4qLR case

EDM of the Deuteron: NLO - and N^2LO -Potentials

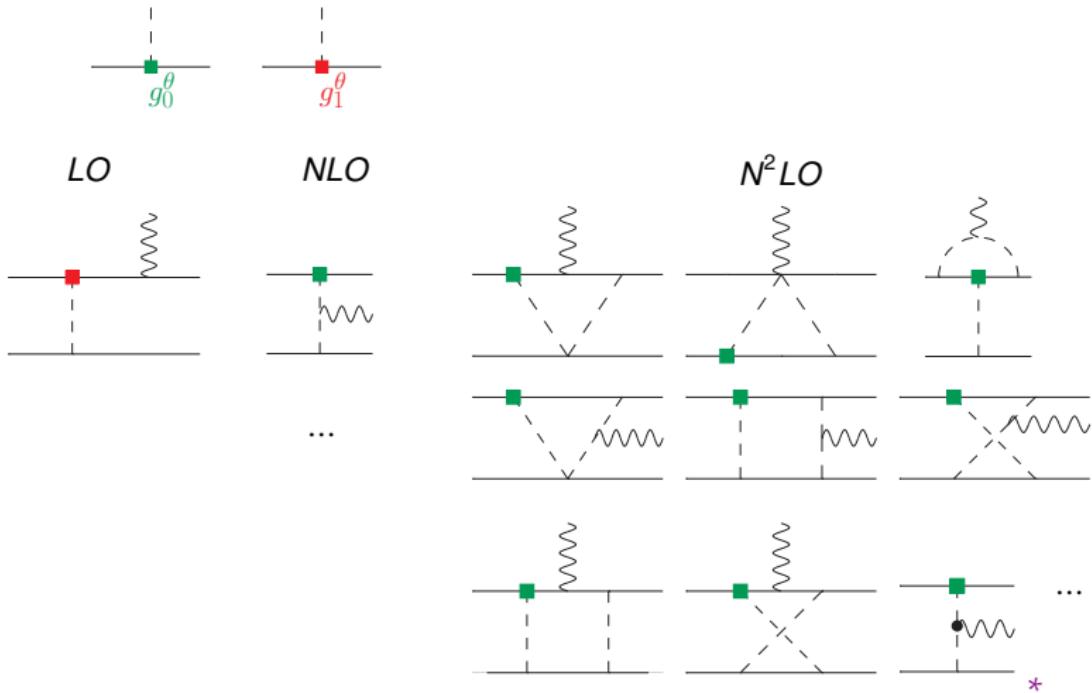


EDM of the Deuteron: NLO - and N^2LO -Potentials



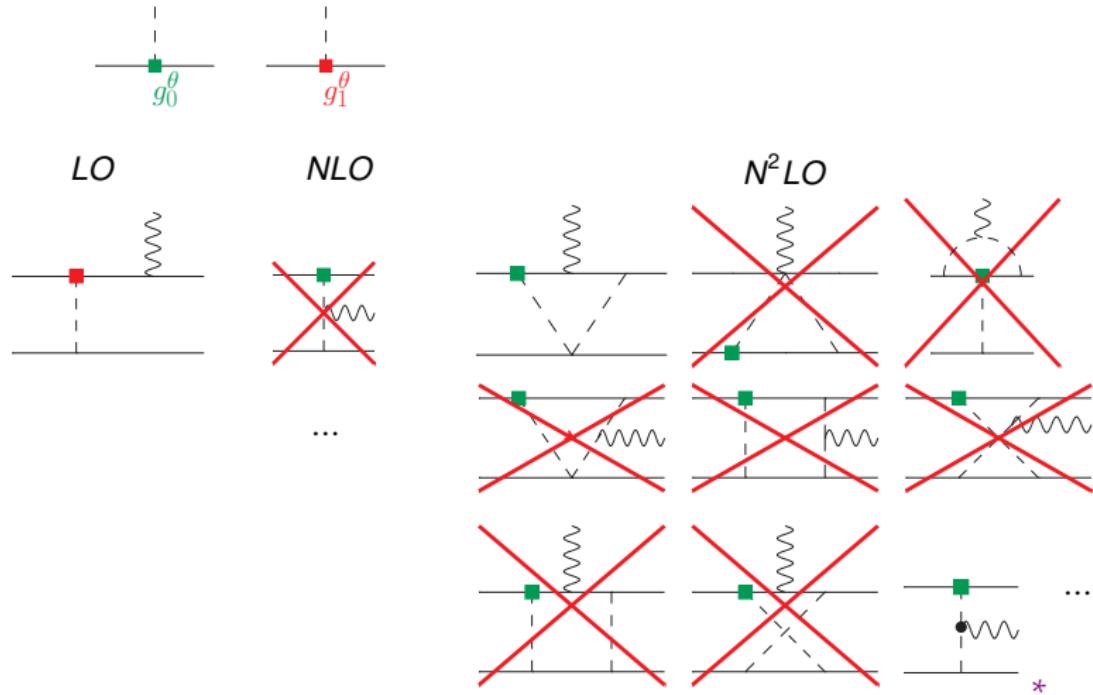
- X : vanishing by selection rules, \times : sum of diagrams vanishes
 \times : vertex correction

EDM of the Deuteron: NLO - and N^2LO -Currents



*: de Vries et al. (2011), Bsaisou et al. (2013)

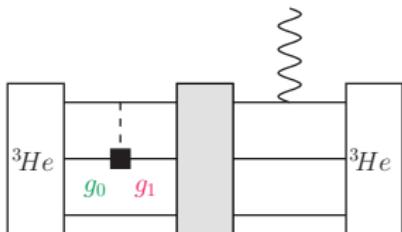
EDM of the Deuteron: NLO - and N^2LO -Currents



*: de Vries et al. (2011), Bsaisou et al. (2013)

- **X**: vanishing by selection rules, **X**: sum of diagrams vanishes

^3He EDM: results for CP-violating π exchange


 $g_0 N^\dagger \vec{\pi} \cdot \vec{\tau} N$ (CP, I)

 LO: θ -term, qCEDM

 $N^2\text{LO}$: 4qLR

 $g_1 N^\dagger \pi_3 N$ (CP, I)

LO: qCEDM, 4qLR

 NLO: θ term

Reference	potential	result [$g_0 g_{\pi NN}$ e fm]	result [$g_1 g_{\pi NN}$ e fm]
Stetcu et al.(2008)	Λv_{18} UIX	$1.20 \times 10^{-2} / 2^\diamond$	$2.20 \times 10^{-2} / 2^\diamond$
Song et al.(2013)	Λv_{18} UIX	0.55×10^{-2}	1.06×10^{-2}
JBC (2014) [◊]	Λv_{18} UIX	0.57×10^{-2}	1.11×10^{-2}
JBC (2014) [◊]	CD BONN TM	0.68×10^{-2}	1.14×10^{-2}
JBC (2014) [◊]	ChPT ($N^2\text{LO}$)	$(0.86 \pm 0.11) \times 10^{-2}$	$(1.11 \pm 0.14) \times 10^{-2}$

[◊] unpublished

 Results for ${}^3H(g_i) \approx (-1)^{1+i} \times {}^3He(g_i)$ ones

Jump slides

EW Baryogenesis: Standard Model

Conservation of the EM current under weak ($L - R$) interactions:

$$\begin{aligned}
 & \partial_\mu B_{EM}^\mu \text{---} q_L \text{---} W^\pm \quad + \quad \partial_\mu L_{EM}^\mu \text{---} \ell_L \text{---} W^\mp \\
 & \qquad\qquad\qquad \propto N_c \cdot (Q_u + Q_d) + (0 - 1) = 1 - 1 = 0 \\
 & \hookrightarrow \Delta(Q_B + Q_L) = 0 \text{ (charge conservation!)}
 \end{aligned}$$

EW Baryogenesis: Standard Model

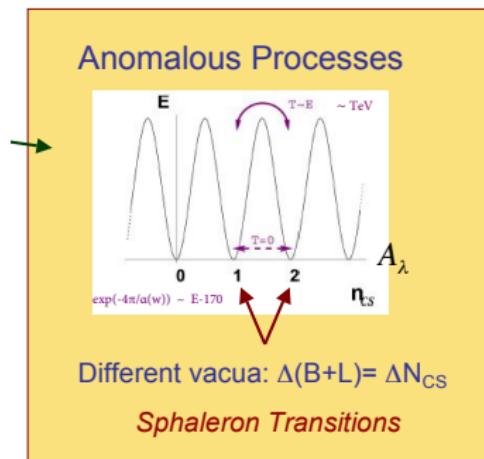
Conservation of the Baryon–Lepton current under ($L - R$) interactions:

$$\partial_\mu B^\mu - \partial_\mu L^\mu = q_L W^+ - \ell_L W^- \propto N_c \cdot 1/3 - 1 = 1 - 1 = 0$$

$\rightarrow \Delta(B - L) = 0 \quad \text{but} \quad \Delta(B + L) \neq 0 !$

Sakharov criteria

- 1 B violation ✓
($\Delta(B+L) \neq 0$ sphaleron transitions)
- 2 C & CP violation ✗
(CKM determinant)
- 3 Nonequilibrium dynamics ✗
(only fast cross over for $\mu_{chem} = 0$)



Naive Quark Model results for a nucleon with N_c quarks

A proton (neutron) with isospin $I = \frac{1}{2}$ and spin $J = \frac{1}{2}$ contains $\frac{N_c+1}{2}$ quarks of u (d) flavor and $\frac{N_c-1}{2}$ quarks of d (u) flavor.

Because of spin-flavor *symmetry* the total spin \vec{J}_u (\vec{J}_d) of all u (d) quarks satisfies $J_u = \frac{N_c+1}{4}$ ($J_d = \frac{N_c-1}{4}$) s.t. $J_z = \pm(J_u - J_d) = \pm\frac{1}{2}$ and

$$\begin{aligned} \langle n | \vec{J}_d | n \rangle &\equiv \langle p | \vec{J}_u | p \rangle \equiv \lambda_u^p \langle p | \vec{J} | p \rangle = \frac{N_c+5}{6} \langle p | \vec{J} | p \rangle \xrightarrow{N_c=3} \frac{4}{3} \langle p | \vec{J} | p \rangle \\ \langle n | \vec{J}_u | n \rangle &\equiv \langle p | \vec{J}_d | p \rangle \equiv \lambda_d^p \langle p | \vec{J} | p \rangle = -\frac{N_c-1}{6} \langle p | \vec{J} | p \rangle \xrightarrow{N_c=3} -\frac{1}{3} \langle p | \vec{J} | p \rangle \end{aligned}$$

$$\rightarrow \frac{\mu_n}{\mu_p} = \frac{\left[\frac{2e}{3} \lambda_u^n - \frac{e}{3} \lambda_d^n \right]}{\left[\frac{2e}{3} \lambda_u^p - \frac{e}{3} \lambda_d^p \right]} = \frac{\left[\frac{2e}{3} \frac{-1}{3} - \frac{e}{3} \frac{4}{3} \right]}{\left[\frac{2e}{3} \frac{4}{3} - \frac{e}{3} \frac{-1}{3} \right]} = -\frac{2}{3} \quad (!) \quad \left(-\frac{(N_c+1)^2-4}{(N_c+1)^2+2} \text{ in general} \right),$$

$$g_A^p = \lambda_u^p - \lambda_d^p = \frac{4}{3} - \frac{-1}{3} = \lambda_u^n - \lambda_d^n = g_A^n = \frac{5}{3} \quad \left(\frac{N_c+2}{3} \text{ in general} \right),$$

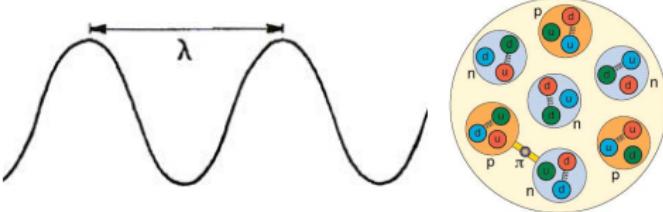
$$d_p = \lambda_u^p d_u + \lambda_d^p d_d = \frac{4}{3} d_u - \frac{1}{3} d_d \quad \text{such that } \boxed{d_p - d_n = \frac{N_c+2}{3} (d_u - d_d)}$$

$$d_n = \lambda_u^n d_u + \lambda_d^n d_d = -\frac{1}{3} d_u + \frac{4}{3} d_d \quad \text{and only } \boxed{d_p + d_n = d_u + d_d} .$$

[◀ back](#)

What are Effective Field Theories (EFT)?

- Different areas in physics describe phenomena at very disparate **scales** (of length, time, energy, mass)
 - Very intuitive idea: scales **much smaller / much bigger** than the ones of interest shouldn't matter much
 - e.g. masses in particle physics: $m_e \approx 0.511\text{MeV} \dots m_t \sim 180\text{GeV}$ range nearly six orders of magnitude (even without neutrinos)
 - still hydrogen atom spectrum can be calculated very precisely without knowing m_t at all
- Separation of scales: $1/k = \lambda \gg R_{\text{substructure}}$



▶ back

Effective Field Theory: Weinberg's conjecture

Quantum Field Theory has no content besides unitarity, analyticity, cluster decomposition and symmetries

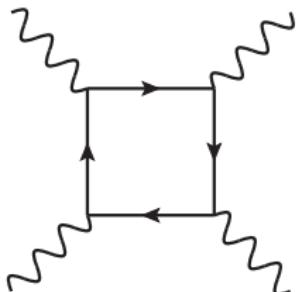
Weinberg 1979

To calculate the S-matrix for any theory below some scale, simply use the most general effective Lagrangian consistent with these principles in terms of the appropriate asymptotic states (i.e. the general S-matrix can be obtained by perturbation theory using some effective lagrangian from the free theory — Witten (2001))

Power-law expand the amplitudes in energy(momentum) / scale.

- Physics at specific energy scale described by active d.o.f.s
- Unresolved substructure incorporated via low-energy const(s)
- Systematic approach \leadsto estimate of uncertainty possible

EFT example: light-by-light scattering



Euler, Heisenberg, Kockel (1935/6)

- only one scale: m_e
- consider energies $\omega \ll m_e$
- $\mathcal{L}_{QED}[\underbrace{\psi, \bar{\psi}}_{\text{matter}}, \underbrace{A_\mu}_{\text{light}}] \rightarrow \mathcal{L}_{\text{eff}}[A_\mu]$
- invariants: $F_{\mu\nu}F^{\mu\nu} \propto \vec{E}^2 - \vec{B}^2$ & $F_{\mu\nu}\tilde{F}^{\mu\nu} \propto \vec{E} \cdot \vec{B}$

$$\mathcal{L}_{\text{eff}} = \frac{1}{2} (\vec{E}^2 - \vec{B}^2) + \frac{e^4}{16\pi^2 m_e^4} \left[a (\vec{E}^2 - \vec{B}^2)^2 + b (\vec{E} \cdot \vec{B})^2 \right] + \dots$$

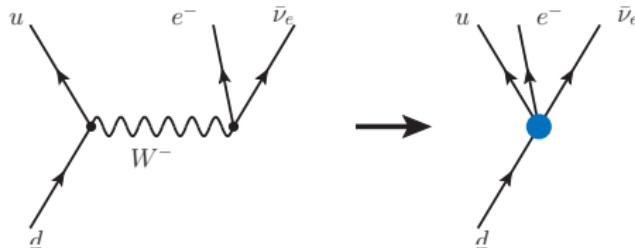
- calculation from the underlying theory, QED, yields $7a = b = 14/45$
- energy power law expansion: $(\omega/m_e)^{2n}$
- \mathcal{L}_{eff} more efficient than full *QED* for calculating cross sections etc.

back

EFT example: weak interactions for $E \ll M_W$

Weak decays:

- mediated by the W^\pm boson, $M_W \approx 80 \text{ GeV}$
- energy release in neutron decay: $\approx 1 \text{ MeV}$
- energy release in kaon decays: $\approx \text{few } 100 \text{ MeV}$



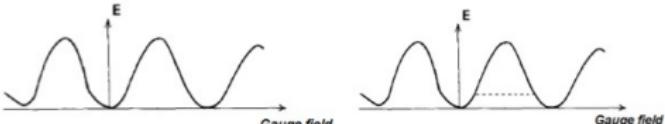
$$\frac{e^2}{8 \sin \theta_W} \times \frac{1}{M_W^2 - q^2} \xrightarrow{q^2 \ll M_W^2} \underbrace{\frac{e^2}{8 M_W^2 \sin \theta_W} \left(1 + \frac{q^2}{M_W^2} + \dots\right)}_{G_F/\sqrt{2}} + \mathcal{O}(q^2/M_W^2)$$

↪ Fermi's current-current interaction

◀ back

θ vacua in strong interaction physics

The topologically non-trivial vacuum structure of QCD



induces **winding number n** and **strong gauge transformation (instanton)**

$$\Omega_1 : |n\rangle \rightarrow |n+1\rangle$$

Naive vacuum therefore *unstable* (and violates *cluster decomposition*).

Thus true vacuum must be a superposition of the various $|n\rangle$ vacua

~ **Theta vacuum:**

$$|vac\rangle_\theta = \sum_{n=-\infty}^{+\infty} e^{in\theta} |n\rangle \quad \text{with} \quad \Omega_1 : |vac\rangle_\theta \rightarrow e^{-i\theta} |vac\rangle_\theta \quad (\text{only phase shift !})$$

Note

$${}_{\theta'} \langle vac | e^{-iHt} | vac \rangle_\theta = \delta_{\theta-\theta'} \times {}_\theta \langle vac | e^{-iHt} | vac \rangle_\theta$$

such that **θ unique parameter of strong interaction physics** which can be incorporated into the effective Lagrangian

$$\mathcal{L}_{\text{eff}} = \mathcal{L} - \frac{\theta}{16\pi^2 g^2} \text{Tr} (G_{\mu\nu} \tilde{G}^{\mu\nu})$$

θ term

$$\mathcal{L}_{QCD} = \mathcal{L}_{QCD}^{\text{CP}} - \theta \frac{g_s^2}{32\pi^2} \tilde{G}_{\mu\nu}^a G^{a,\mu\nu} = \mathcal{L}_{QCD}^{\text{CP}} - \theta \frac{g_s^2}{32\pi^2} \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} G_{\mu\nu}^a G_{\alpha\beta}^a$$

- Under $U_A(1)$ rotation of the quark fields $q_f \rightarrow e^{-i\alpha\gamma_5/2} q_f \approx (1 - i\frac{1}{2}\alpha\gamma_5)q_f$:

$$\mathcal{L}_{QCD} \rightarrow \mathcal{L}_{QCD}^{\text{CP}} + \alpha \sum_f m_f \bar{q}_f i\gamma_5 q_f - (\theta - N_f \alpha) \frac{g_s^2}{32\pi^2} \tilde{G}_{\mu\nu}^a G^{a,\mu\nu}$$

$$\hookrightarrow \mathcal{L}_{SM}^{\text{strCP}} = \mathcal{L}_{SM}^{\text{CP}} + \bar{\theta} m^* \sum_f \bar{q}_f i\gamma_5 q_f \quad \text{with } \bar{\theta} = \theta + \arg \det \mathcal{M} \text{ and } m^* = \frac{m_u m_d}{m_u + m_d}$$

◀ back

Strong CP problem: Peccei-Quinn symmetry and axions

R. Peccei & H. Quinn (1977)

Consider adding a new field a (the axion field) to the QCD action

$$\mathcal{L}_{\text{axion}} = \bar{\psi} (\cancel{M} e^{-ia/f_a}) \psi + \frac{1}{2} \partial_\mu a \partial^\mu a$$

- The axion arises as Goldstone boson of the new broken U(1) symmetry of the quark sector and the Higgs sector.
- Perform further axial U(1) transformation on quark fields to eliminate the $G\tilde{G}$ term entirely (or to make mass term real again)
 - new phase of quark mass term: $e^{i(\theta + \arg \det M - a/f_a)}$
 - or instead $G\tilde{G}$ term becomes: $(-\theta - \arg \det M + a/f_a) \frac{g_s^2}{16\pi^2} \text{tr } G\tilde{G}$
- Make the trivial U(1) shift $a \rightarrow a + (\theta + \arg \det M) \times f_a$.
The kinetic term is invariant under this shift (axion massless to LO)
- At higher order, the axion acquires its mass as
 $m_a \approx 0.5 m_\pi f_\pi / f_a$ with $f_a \gg \langle H \rangle = (\sqrt{2} G_F)^{-1/2} = 247 \text{ GeV}$
- New Problem: find a (light) axion !

◀ back

Axions and EDMs: generic effective Lagrangian of the axion

Kiwoon Choi (Daejeon, Korea), Bethe-Lectures, Bonn, March 2015

$$\mathcal{L}_{\text{eff}}(a) = \underbrace{\mathcal{L}_0}_{\text{indep. of } a} + \underbrace{\frac{1}{2}(\partial_\mu a)^2 + \frac{\partial_\mu a}{f_a} \tilde{J}^\mu(\bar{\psi} \dots \psi, \phi)}_{\text{PQ-invariant}} + \underbrace{\frac{a}{f_a} \frac{N}{32\pi^2} G\tilde{G}}_{\text{expl. PQ-breaking by QCD anomaly}} + \underbrace{\Delta\mathcal{L}_{UV} \left(= -\epsilon m_{UV}^4 \cos(a/f_a + \delta_{UV}) \right)}_{\text{a coupling from expl. PQ breaking at UV scale}}$$

$\bar{\theta} = \langle a \rangle / f_a$ is calculable in terms of the \mathcal{CP} angles (in presence of a !):

δ_{KM} = Kobayashi-Maskawa phase in the PQ-invariant SM

δ_{BSM} = \mathcal{CP} phase in PQ-invariant BSM at the scale m_{BSM}

δ_{UV} = \mathcal{CP} phase in explicit PQ-breaking sector at $m_{UV} \sim M_{\text{Planck}}$

$$V_{QCD} \sim f_\pi^2 m_\pi^2 \cos(a/f_a) \quad (\text{expl. PQ-breaking by low-energy QCD})$$

$$10^{-14} \quad \text{Jarlskog inv.}$$

$$V_{KM} \sim f_\pi^2 m_\pi^2 \times \overbrace{G_F^2 f_\pi^4}^{10^{-14}} \times \overbrace{10^{-5} \sin \delta_{KM}}^{\text{Jarlskog inv.}} \times \sin(a/f_a)$$

$$V_{BSM} \sim f_\pi^2 m_\pi^2 \times \underbrace{(10^{-2} - 10^{-3})}_{\text{loop suppression}} \times \frac{f_\pi^2}{m_{BSM}^2} \sin \delta_{BSM} \times \sin(a/f_a)$$

$$V_{UV} \sim \epsilon m_{UV}^4 \sin \delta_{UV} \sin(a/f_a)$$

$\bar{\theta} = \langle a \rangle / f_a$ and contributions to the nucleon EDM

$$\begin{aligned}\bar{\theta} &\sim 10^{-19} \sin \delta_{\text{KM}} + \overbrace{(10^{-10} - 10^{-11})}^{(10^{-2} - 10^{-3}) \times f_\pi^2 / \text{TeV}^2} \times \left(\frac{\text{TeV}}{m_{\text{BSM}}} \right)^2 \sin \delta_{\text{BSM}} \\ &+ \epsilon \frac{m_{\text{UV}}^4}{f_\pi^2 m_\pi^2} \sin \delta_{\text{UV}} \quad (\text{with } \epsilon < 10^{-10} f_\pi^2 m_\pi^2 / m_{\text{UV}}^4 \sim 10^{-88} \text{ for } m_{\text{UV}} \sim M_{\text{Pl}})\end{aligned}$$

→ Regardless of the existence of BSM physics near the TeV scale, $\bar{\theta} = \langle a \rangle / f_a$ can have *any value* below the present bound $\sim 10^{-10}$.

$$\begin{aligned}d_N &\sim \frac{e}{m_N} \left[\frac{m^*}{m_N} \bar{\theta} + G_F^2 f_\pi^4 \times 10^{-5} \sin \delta_{\text{KM}} + (10^{-2} - 10^{-3}) \times \frac{f_\pi^2}{m_{\text{BSM}}^2} \sin \delta_{\text{BSM}} \right. \\ &\quad \left. + (10^{-2} - 10^{-3}) \times \frac{f_\pi^2}{m_{\text{UV}}^2} \sin \delta_{\text{UV}} \right] \\ &\sim \underbrace{\frac{e}{m_N} \left[\frac{m^*}{m_N} \times \underbrace{\bar{\theta}_{\text{UV}}}_{\sim 10^{-2}} + (10^{-2} - 10^{-3}) \times \frac{f_\pi^2}{m_{\text{BSM}}^2} \sin \delta_{\text{BSM}} \right]}_{\frac{\epsilon m_{\text{UV}}^4 \sin \delta_{\text{UV}}}{f_\pi^2 m_\pi^2}}\end{aligned}$$

likely dominated by $\bar{\theta}_{\text{UV}}$ induced by CP in the PQ sector @ $m_{\text{UV}} (\sim M_{\text{Pl}})$, and/or by the BSM contribution near the TeV scale.

kHz to MHz Dark Matter Axions or Axion-Like Particles

P.W. Graham & S. Rajendran, PRD 84 (2011) & 88 (2013)

Apply

$$\mathcal{L}_{\text{axion}} = \frac{a}{f_a} \frac{g_s^2}{16\pi^2} \text{tr } G\tilde{G} + \frac{1}{2}\partial_\mu a \partial^\mu a - \frac{1}{2}m_a^2 a^2 \quad \text{with} \quad m_a \approx 0.5 m_\pi f_\pi / f_a$$

and let the axion decay constant f_a be in the window

$$10^{16} \text{ GeV} \sim M_{\text{GUT}} \lesssim f_a \lesssim M_{\text{Planck}} \sim 10^{19} \text{ GeV}.$$

- ~ Axions in our galaxy *spatially* constant over a scale of $\lesssim 500 \text{ km} \times (f_a/M_{\text{GUT}})$:
 - ↪ Ansatz: $a(t, \vec{x}) \approx a(t) = a_0 \cos(m_a t)$ in the lab.
- Equating $\frac{1}{2}m_a^2 a_0^2 \sim \rho_{\text{local DM}} \approx (0.3 \pm 0.1) \text{ GeV/cm}^3$ gives as **axion amplitude**

$$\theta_a \equiv \frac{a_0}{f_a} \sim \frac{\sqrt{\rho_{\text{local DM}}}}{0.5 m_\pi f_\pi} \sim 3 \times 10^{-19} \xrightarrow[\text{independently of } f_a]{d_n \approx 10^{-16} \theta_a \text{ e cm}} d_n \approx 4 \times 10^{-35} \cos(m_a t) \text{ e cm}$$

with $m_a \approx 1 \text{ kHz} [M_{\text{Planck}}/f_a]$ to $1 \text{ MHz} [M_{\text{GUT}}/f_a]$ oscillations.

- ~ Bounds on oscillating Axions or ALPs from storage ring EDM searches ?

Non-relativistic reduction of

$$\mathcal{H}_{\text{eff}} = -\frac{a_f}{2} \bar{f} \sigma^{\mu\nu} f F_{\mu\nu}, \quad a_f = \frac{F_2(0)}{2m_f}$$

$$-\int d^3x \frac{a_f}{2} \bar{\psi}_f \sigma^{ij} \psi_f F_{ij} + \dots$$

$$\rightarrow -\frac{a_f}{2} \int d^3x \bar{\psi}_f \epsilon^{ijk} \begin{pmatrix} \sigma^k & 0 \\ 0 & \sigma^k \end{pmatrix} \psi_f F_{ij}$$

$$= -\frac{a_f}{2} \int d^3x \bar{\psi}_f \begin{pmatrix} \sigma^k & 0 \\ 0 & \sigma^k \end{pmatrix} \psi_f \underbrace{\epsilon^{ijk} F_{ij}}_{-2B^k}$$

$$\rightarrow a_f \int d^3x \bar{\psi}_f \vec{\sigma} \psi_f \cdot \vec{B}$$

$$= a_f \langle \vec{\sigma} \rangle \cdot \vec{B}$$

$$= a_f g \langle \vec{S} \rangle \cdot \vec{B}, \quad g = 2$$

$$\mathcal{H}_{\text{eff}} = i \frac{d_f}{2} \bar{f} \sigma^{\mu\nu} \gamma_5 f F_{\mu\nu}, \quad d_f \equiv \frac{F_3(0)}{2m_f}$$

$$i \int d^3x \frac{d_f}{2} \bar{\psi}_f \sigma^{0i} \gamma_5 \psi_f F_{0i} \times 2 + \dots$$

$$\rightarrow i d_f \int d^3x \bar{\psi}_f i \begin{pmatrix} 0 & \sigma_i \\ \sigma^i & 0 \end{pmatrix} \gamma_5 \psi_f F^{i0}$$

$$= i d_f \int d^3x \bar{\psi}_f i \begin{pmatrix} \sigma^i & 0 \\ 0 & \sigma^i \end{pmatrix} \psi_f \underbrace{F^{i0}}_{E^i}$$

$$\rightarrow i^2 d_f \int d^3x \bar{\psi}_f \vec{\sigma} \psi_f \cdot \vec{E}$$

$$= -d_f \langle \vec{\sigma} \rangle \cdot \vec{E}$$

$$= -d_f \langle \vec{S}/S \rangle \cdot \vec{E} \quad (\text{linear Stark term})$$

$$\mathcal{H}_{\text{eff}} = i \frac{d_f}{2} \bar{f} \sigma^{\mu\nu} \gamma_5 f F_{\mu\nu} = i \frac{d_f}{2} \bar{f}_L \sigma^{\mu\nu} f_R F_{\mu\nu} - i \frac{d_f}{2} \bar{f}_R \sigma^{\mu\nu} f_L F_{\mu\nu} \sim \text{fermion mass insertion}$$

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Transformation Properties of the Form Factor Γ^μ

$A_\mu \langle f(p') | J_{\text{em}}^\mu | f(p) \rangle = A_\mu \bar{u}_f(p') \Gamma^\mu(q^2) u_f(p)$ with

$$\begin{aligned} \Gamma^\mu(q^2) &= \gamma^\mu F_1(q^2) + i\sigma^{\mu\nu} q_\nu \frac{F_2(q^2)}{2m_f} + \sigma^{\mu\nu} q_\nu \gamma_5 \frac{F_3(q^2)}{2m_f} \\ &\quad + (\not{q} q^\mu - q^2 \gamma^\mu) \gamma_5 F_a(q^2) / m_f^2 \end{aligned}$$

\bar{q} Op. q	P	C	CP	T	CPT
A_μ	A^μ	$-A_\mu$	$-A^\mu$	A^μ	$-A_\mu$
γ^μ	γ_μ	$-\gamma^\mu$	$-\gamma_\mu$	γ_μ	$-\gamma^\mu$
$\gamma^\mu \gamma_5$	$-\gamma_\mu \gamma_5$	$\gamma^\mu \gamma_5$	$-\gamma_\mu \gamma_5$	$\gamma_\mu \gamma_5$	$-\gamma^\mu \gamma_5$
$\sigma^{\mu\nu}$	$\sigma_{\mu\nu}$	$-\sigma^{\mu\nu}$	$-\sigma_{\mu\nu}$	$-\sigma_{\mu\nu}$	$\sigma^{\mu\nu}$
$\sigma^{\mu\nu} q_\nu$	$\sigma_{\mu\nu} q^\nu$	$-\sigma^{\mu\nu} q_\nu$	$-\sigma_{\mu\nu} q^\nu$	$-\sigma_{\mu\nu} q^\nu$	$\sigma^{\mu\nu} q_\nu$
$i\sigma^{\mu\nu} q_\nu$	$i\sigma_{\mu\nu} q^\nu$	$-i\sigma^{\mu\nu} q_\nu$	$-i\sigma_{\mu\nu} q^\nu$	$i\sigma_{\mu\nu} q^\nu$	$-i\sigma^{\mu\nu} q_\nu$
$\sigma^{\mu\nu} q_\nu \gamma_5$	$-\sigma_{\mu\nu} q^\nu \gamma_5$	$-\sigma^{\mu\nu} q_\nu \gamma_5$	$\sigma_{\mu\nu} q^\nu \gamma_5$	$-\sigma_{\mu\nu} q^\nu \gamma_5$	$-\sigma^{\mu\nu} q_\nu \gamma_5$

For EDMs of charged particles both $F_1(q^2)$ and $F_3(q^2)$ are present at the same time \leadsto mixing

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Construction of the CKM matrix

Since weak interactions do not respect the global flavor symmetry, there is mixing within the groups of quarks with the same charge:

$$U \equiv \begin{pmatrix} u \\ c \\ t \end{pmatrix} \rightarrow \tilde{U} = M_U U, \quad D \equiv \begin{pmatrix} d \\ s \\ b \end{pmatrix} \rightarrow \tilde{D} = M_D D,$$

where M_U & M_D are 3×3 unitary matrices

$$\hookrightarrow \text{charged weak current: } J_\mu = \tilde{U}^\mu \gamma_\mu (1 - \gamma_5) \tilde{D}^\mu = \tilde{U} \gamma_\mu (1 - \gamma_5) \underbrace{M_U^\dagger M_D}_\text{CKM matrix } M D.$$

- M unitary $n_G \times n_G$ matrix for n_G quark generations $\sim n_G^2$ real parameters
- $2n_G - 1$ of these can be absorbed by the relative phases of the quark wave functions $\sim (n_G - 1)^2$ remaining parameters:
 - $n_G = 2$: one remaining real parameter: Cabibbo angle
 - $n_G = 3$: $O(3)$ matrix with $\frac{1}{2}3 \cdot (3 - 1) = 3$ angles plus 1 CP phase
- Lepton case: neutrinos may be Majoranas: ~ 3 angles plus 3 CP phases
- If phase(s) present, M complex matrix, whereas CP invariance $\sim M^* = M$!

Hierarchy among the sources at the hadronic EFT level

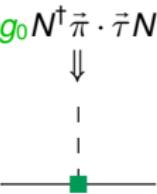
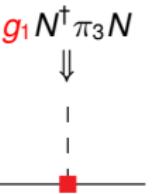
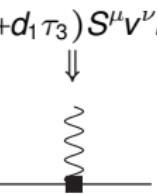
Each source transforms differently under chiral and isospin symmetry

\mathcal{CP}, I	\mathcal{CP}, I	$\mathcal{CP}, I + I'$
$\mathcal{L}_{\text{EFT}}^{\mathcal{CP}} = g_0 N^\dagger \vec{\pi} \cdot \vec{\tau} N$ \downarrow  dominant for $\bar{\theta}$ term	$g_1 N^\dagger \pi_3 N$ \downarrow  suppressed for $\bar{\theta}$ term	$N^\dagger (d_0 + d_1 \tau_3) S^{\mu\nu} F_{\mu\nu} N$ \downarrow  suppressed by $\mathcal{O}(M_\pi^2)$
...

- $\mathcal{L}_{QCD}^\theta = \bar{\theta} m_q^* \sum_f \bar{q}_f i \gamma_5 q_f$: \mathcal{CP}, I $m_q^* = \frac{m_u m_d}{m_u + m_d}$
 - ↪ $\bar{\theta}$ source **breaks chiral symmetry** ($\propto m_q^*$) but conserves the isospin one:
 - ↪ $|g_0^\theta| \gg |g_1^\theta|$: NDA estimate: $g_1^\theta / g_0^\theta \sim \mathcal{O}(M_\pi^2 / m_n^2)$ de Vries et al. *PRC* '11
 - ChPT LECs predict: $g_1^\theta / g_0^\theta \sim \mathcal{O}(M_\pi / m_n)$! Bsaisou et al. *EPJA* '13

Hierarchy among the sources at the hadronic EFT level

Each source transforms differently under chiral and isospin symmetry

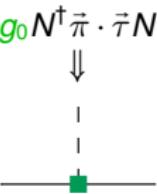
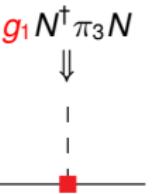
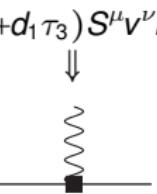
\mathcal{CP}, I	\mathcal{CP}, I	$\mathcal{CP}, I + I$
$\mathcal{L}_{\text{EFT}}^{\mathcal{CP}} = g_0 N^\dagger \vec{\pi} \cdot \vec{\tau} N$ 	$+ g_1 N^\dagger \pi_3 N$ 	$+ N^\dagger (d_0 + d_1 \tau_3) S^{\mu\nu} F_{\mu\nu} N$ 
dominant for chromo qEDM source	dominant for chromo qEDM source	$\mathcal{O}(m_\pi^2)$ suppressed for chromo qEDM source

- **chromo quark EDM:** chiral symmetries are (& isospin ones may be) broken because of quark masses \sim Goldstone theorem respected
- **4quark Left-Right EDM:** **explicit** breaking of **chiral & isospin** symmetries because of underlying W boson exchange \sim Goldstone th. doesn't apply

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Hierarchy among the sources at the hadronic EFT level

Each source transforms differently under chiral and isospin symmetry

\cancel{CP}, I	\cancel{CP}, I	$\cancel{CP}, I + I'$
$\mathcal{L}_{\text{EFT}}^{\cancel{CP}} = g_0 N^\dagger \vec{\pi} \cdot \vec{\tau} N$ 	$+ g_1 N^\dagger \pi_3 N$ 	$+ N^\dagger (d_0 + d_1 \tau_3) S^{\mu\nu} F_{\mu\nu} N$ 
<small>↓</small> <small>↓</small> <small>↓</small> <small>↓</small>	<small>↓</small> <small>↓</small> <small>↓</small> <small>↓</small>	<small>↓</small> <small>↓</small> <small>↓</small> <small>↓</small>
<small>⋮</small> <small>⋮</small> <small>⋮</small> <small>⋮</small>	<small>⋮</small> <small>⋮</small> <small>⋮</small> <small>⋮</small>	<small>⋮</small> <small>⋮</small> <small>⋮</small> <small>⋮</small>
<small>suppressed for quark EDM source</small>	<small>suppressed for quark EDM source</small>	<small>dominating for quark EDM source</small>

- **quark EDM:** $N\pi$ (and NN) interactions are **suppressed** by $\alpha_{\text{em}}/(4\pi)$
- **gluon color EDM (and chiral-4quark EDM):** **relative $\mathcal{O}(M_\pi^2)$ suppression** of $N\pi$ interactions because of Goldstone theorem

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θ -term: ~~CP~~ πNN vertices determined from LECs

Leading g_0^θ coupling (from c_5)

Crewther et al. (1979);
 Ottnad et al. (2010); Mereghetti et al. (2011);
 de Vries et al. (2011); Bsaisou et al. (2013)

g_0^θ : $N^\dagger \vec{\pi} \cdot \vec{\tau} N$ -vertex

$$\mathcal{L}_{\pi N} = \dots + c_5 2B N^\dagger \left((m_u - m_d) \tau_3 + \frac{2m^* \bar{\theta}}{F_\pi} \vec{\pi} \cdot \vec{\tau} \right) N + \dots$$

$$\delta M_{np}^{str} = 4B(m_u - m_d)c_5 \rightarrow g_0^\theta = \bar{\theta} \delta M_{np}^{str} (1 - \epsilon^2) \frac{1}{4F_\pi \epsilon}$$

$$\delta M_{np}^{em} \rightarrow \delta M_{np}^{str} = (2.44 \pm 0.18) \text{ MeV} \quad \text{Walker-Loud ('13); Borsányi et al. ('14)}$$

$$\& \quad m_u/m_d = 0.46 \pm 0.03 \quad \text{Flag Working Group ('14)}$$

$$\rightarrow g_0^\theta = (15.5 \pm 1.9) \cdot 10^{-3} \cdot \bar{\theta} \quad \text{Bsaisou et al. ('15)}$$

$$\epsilon = (m_u - m_d)/(m_u + m_d), \quad 4Bm^* = M_\pi^2(1 - \epsilon^2), \quad m^* = \frac{m_u m_d}{m_u + m_d}$$

θ -term: subleading g_1^θ coupling (from c_1 LEC)

g_1^θ : $\pi_3 NN$ -vertex

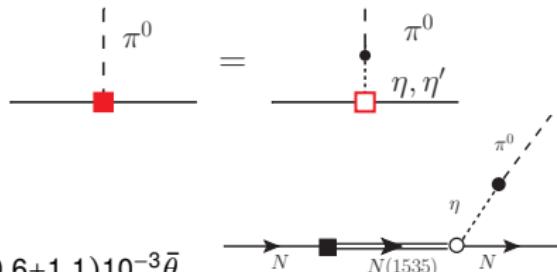
$$\epsilon := (m_u - m_d) / (m_u + m_d)$$

$$\mathcal{L}_{\pi N} = \dots + c_1 4B N^\dagger \left((m_u + m_d) + \frac{(\delta M_\pi^2)_{QCD} (1 - \epsilon^2) \bar{\theta}}{2BF_{\pi\epsilon}} \pi_3 \right) N + \dots$$

1 $c_1 \longleftrightarrow \sigma_{\pi N}$: $c_1 = (-1.0 \pm 0.3) \text{ GeV}^{-1}$

Compilation: Baru et al. (2011)

2 $(\delta M_\pi^2)_{QCD} \approx \frac{\epsilon^2}{4} \frac{M_\pi^4}{M_K^2 - M_\pi^2}$



$$g_1^\theta(c_1) = (2.8 \pm 1.1) 10^{-3} \bar{\theta} \quad \& \quad \bar{g}_1^\theta = (0.6 \pm 1.1) 10^{-3} \bar{\theta}$$

$\sim g_1^\theta = (3.5 \pm 1.5) \cdot 10^{-3} \bar{\theta}$

Bsaisou et al. '13, '15

$$\frac{g_1^\theta}{g_0^\theta} = -0.22 \pm 0.10 \sim \frac{M_\pi}{m_N}$$

Bsaisou et al. '13, '15

$$\gg \epsilon \frac{M_\pi^2}{m_N^2} \sim -0.01 \quad (\text{NDA})$$

de Vries et al. (2011)

$g_0^\theta(\delta M_{np}^{str})$ is unnaturally small!

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Deuteron Quantities in ChPT from NLO to N4LO

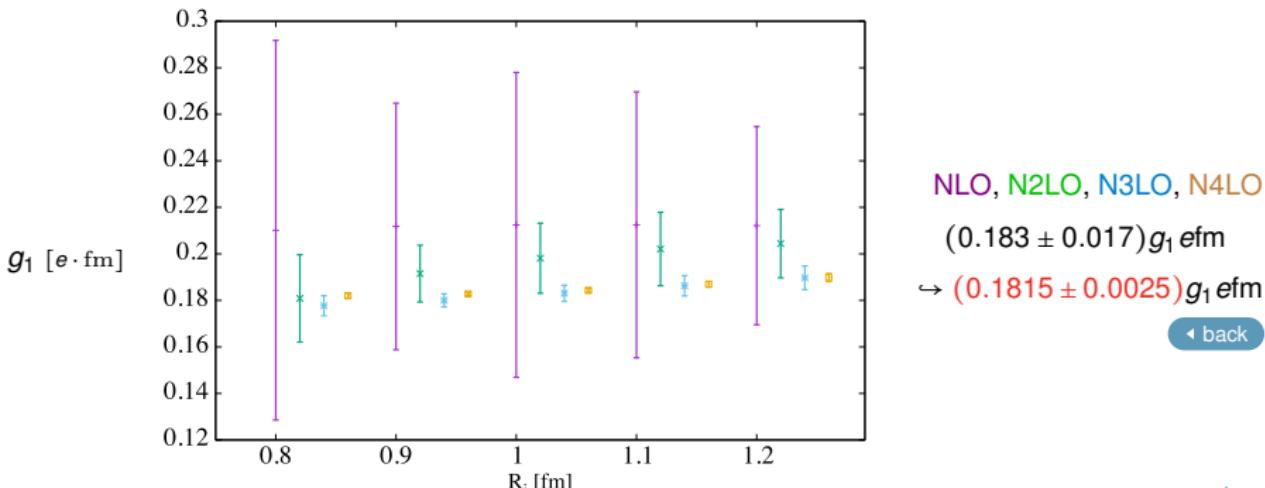
Epelbaum, Krebs, Mei  ner, *EPJA* 51 & *PRL* 115 (2015); Binder et al., *PRC* 93 (2016); and A. Nogga, *priv. comm.*

$$\Delta X^N n\text{LO}(p) = Q^{n+2} \cdot \max \left[\left| X^{\text{LO}}(p) \right|, \frac{|X^{\text{NLO}}(p) - X^{\text{LO}}(p)|}{Q^2}, \frac{|X^{\text{N2LO}}(p) - X^{\text{NLO}}(p)|}{Q^3}, \right. \right.$$

$$\left. \left. \frac{|X^{\text{N3LO}}(p) - X^{\text{N2LO}}(p)|}{Q^4}, \frac{|X^{\text{N4LO}}(p) - X^{\text{N3LO}}(p)|}{Q^5} \right] \quad \text{with} \quad Q = \max \left(\frac{|p|}{\Lambda_b^i}, \frac{M_\pi}{\Lambda_b^i} \right)$$

and $f\left(\frac{r}{R_i}\right) = \left[1 - \exp\left(-\frac{r^2}{R_i^2}\right)\right]^6$ with $\frac{R_i}{\Lambda_b^i}$

	0.8 fm	0.9 fm	1.0 fm	1.1 fm	1.2 fm
$\frac{R_i}{\Lambda_b^i}$	0.6 GeV	0.6 GeV	0.6 GeV	0.5 GeV	0.4 GeV

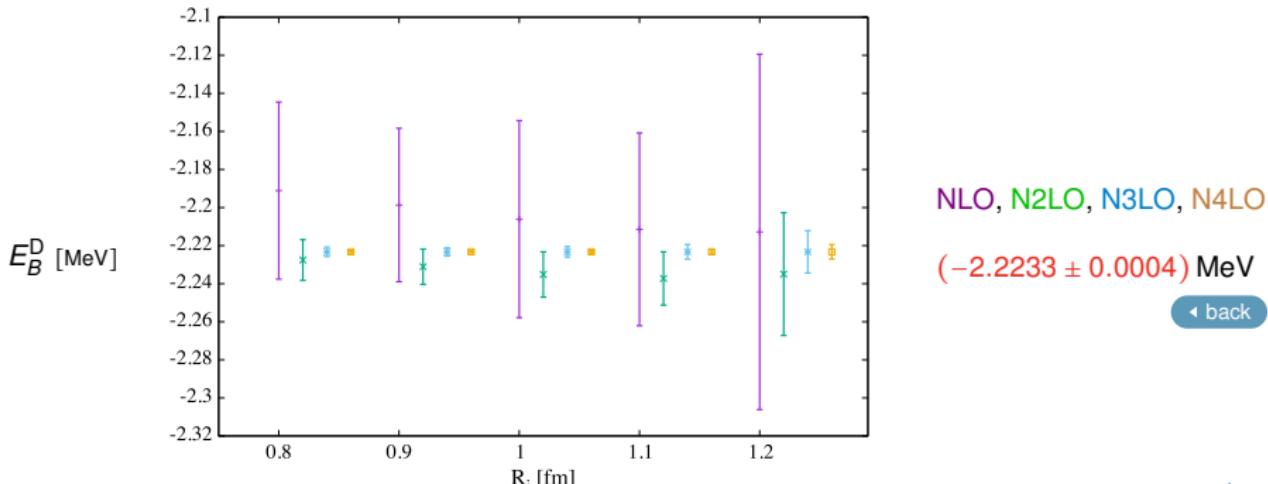


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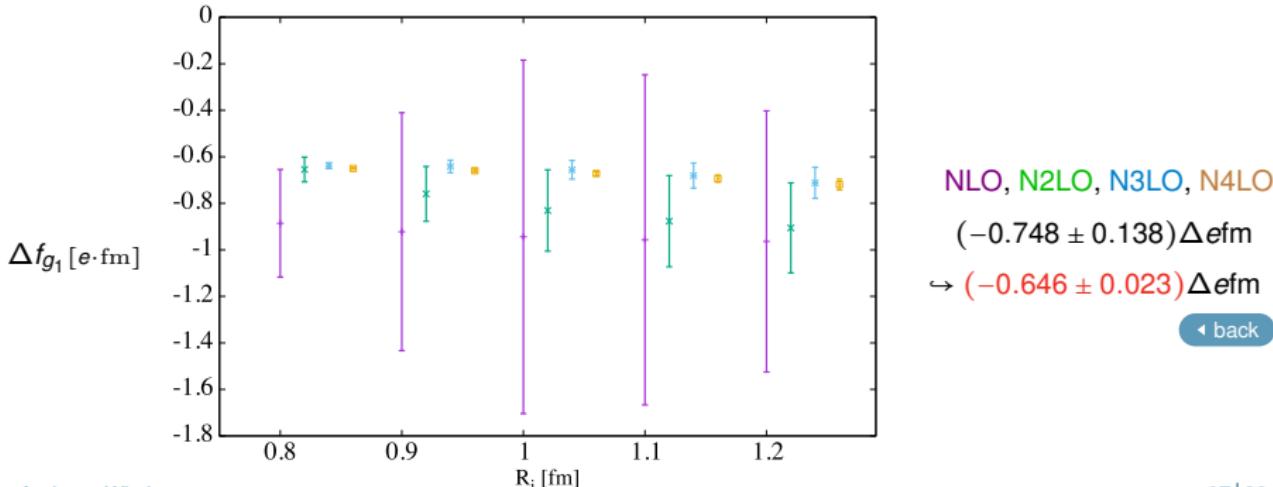
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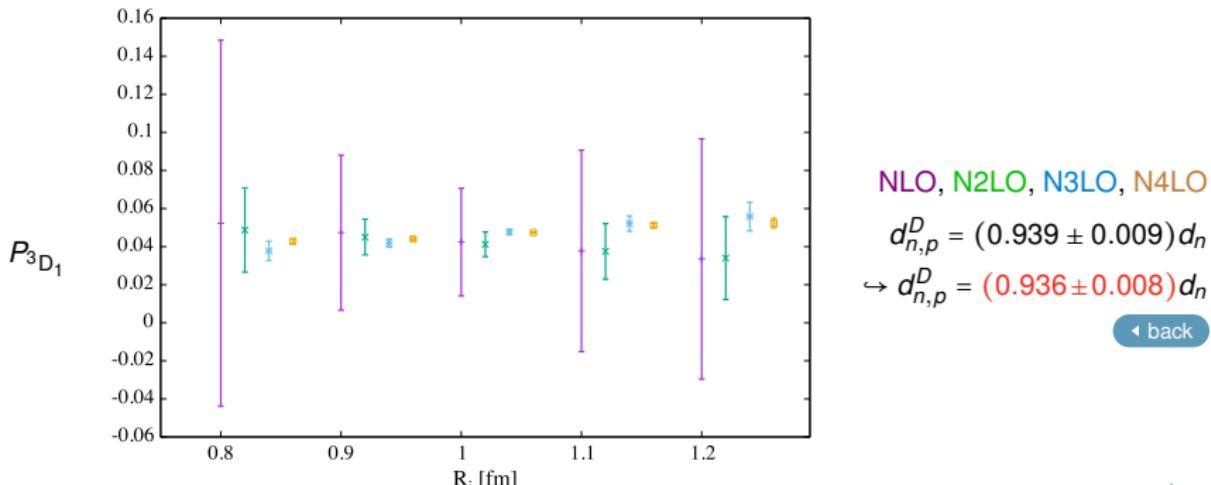
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Deuteron Quantities in ChPT from NLO to N4LO

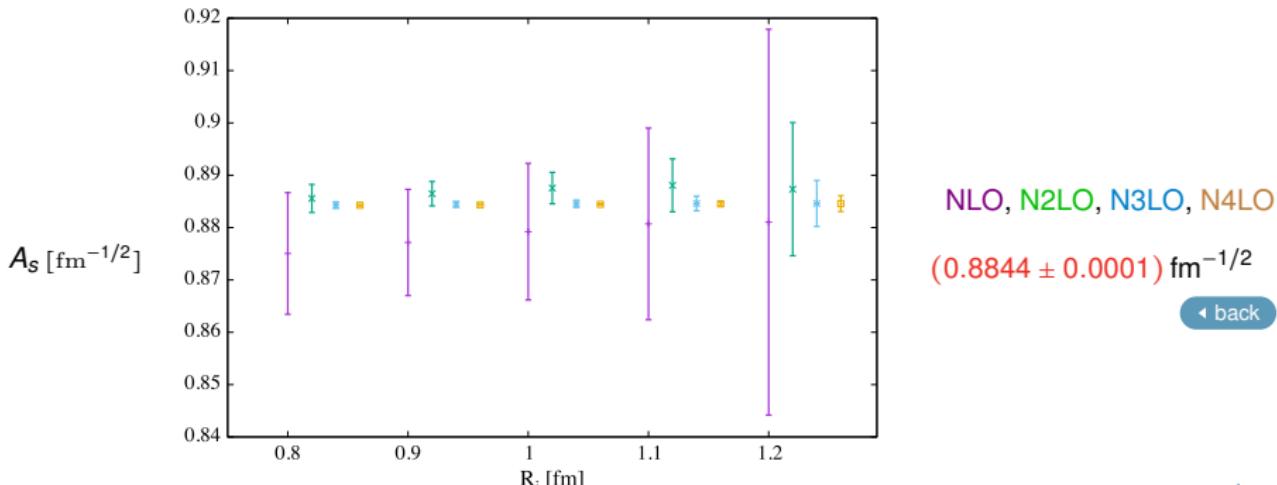
Epelbaum, Krebs, Meißen, *EPJA* 51 & *PRL* 115 (2015); Binder et al., *PRC* 93 (2016); and A. Nogga, *priv. comm.*

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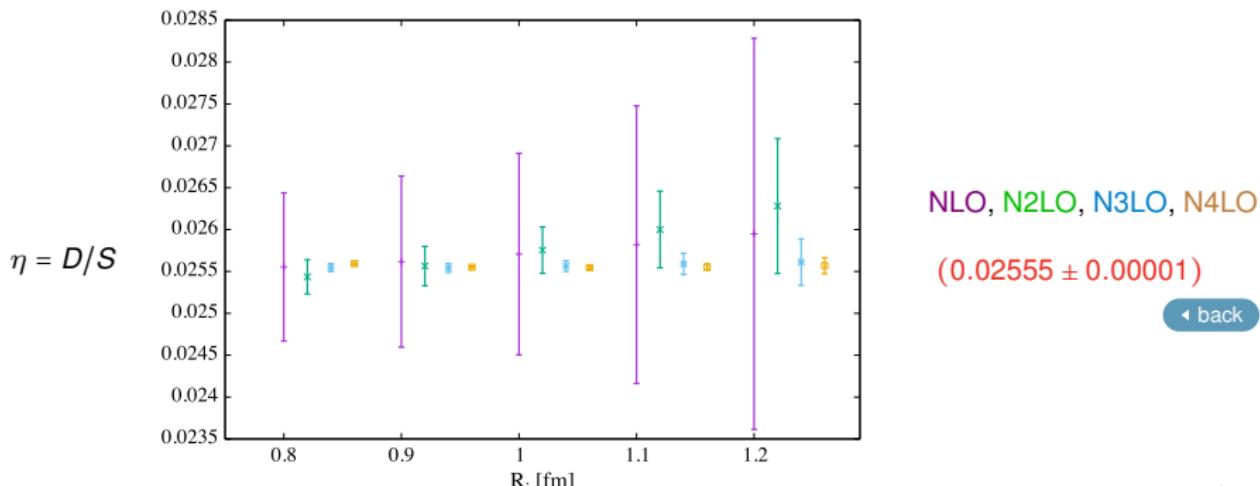
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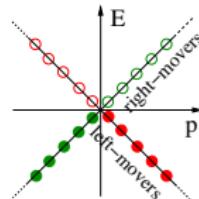
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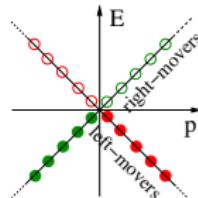
$U_A(1)$ anomaly in 1+1 D

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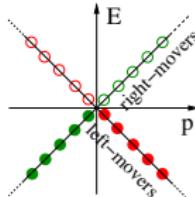
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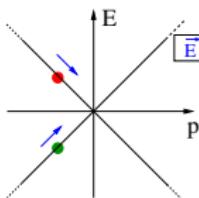
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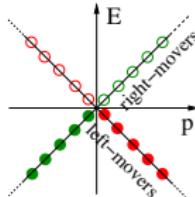
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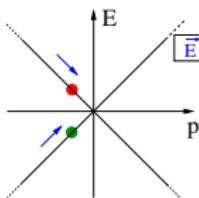
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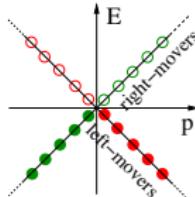
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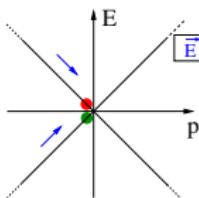
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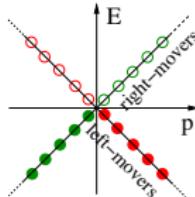
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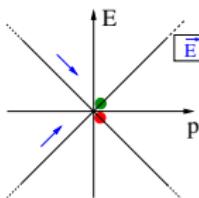
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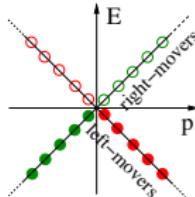
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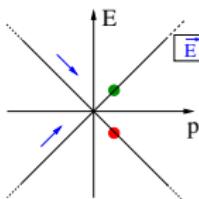
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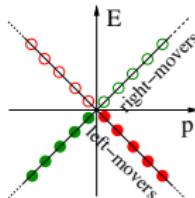
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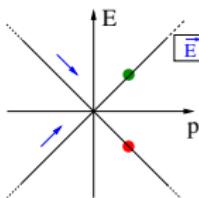
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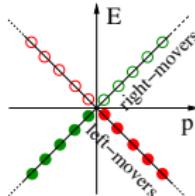
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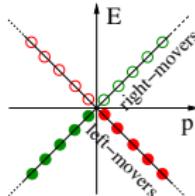


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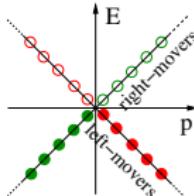
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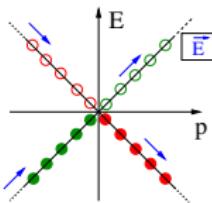
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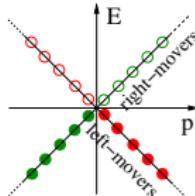
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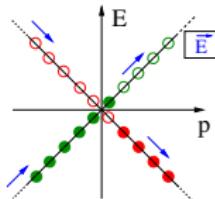
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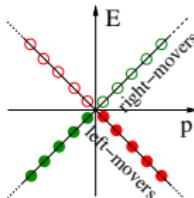
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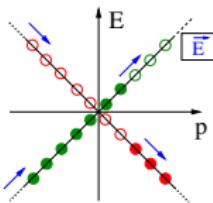
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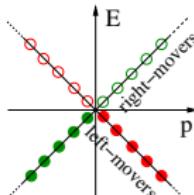
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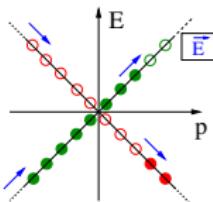
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The symmetries of QCD

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{2} \text{Tr}(G_{\mu\nu} G^{\mu\nu}) + \sum_f \bar{q}_f (\not{D} - m_f) q_f + \dots$$

$$D_\mu = \partial_\mu - ig A_\mu \equiv \partial_\mu - ig A_\mu^a \frac{\lambda^a}{2}, \quad G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]$$

- Lorentz-invariance, P, C, T invariance, $SU(3)_c$ gauge invariance
- The masses of the u , d , s quarks are **small**: $m_{u,d,s} \ll 1 \text{ GeV} \approx \Lambda_{\text{hadron}}$.
- **Chiral** decomposition of quark fields:

$$q = \frac{1}{2}(1 - \gamma_5)q + \frac{1}{2}(1 + \gamma_5)q = q_L + q_R.$$

- For **massless** fermions: left-/right-handed fields do not interact

$$\mathcal{L}[q_L, q_R] = i\bar{q}_L \not{D} q_L + i\bar{q}_R \not{D} q_R - m(\bar{q}_L q_R + \bar{q}_R q_L)$$

and \mathcal{L}_{QCD}^0 invariant under (global) **chiral $U(3)_L \times U(3)_R$** transformations:

↪ rewrite $U(3)_L \times U(3)_R = SU(3)_V \times SU(3)_A \times U(1)_V \times U(1)_A$.

- $SU(3)_V = SU(3)_{R+L}$: still conserved for $m_u = m_d = m_s > 0$
- $U(1)_V = U(1)_{R+L}$: quark or **baryon number** is conserved
- $U(1)_A = U(1)_{R-L}$: broken by quantum effects ($U(1)_A$ anomaly + instantons)

Hidden Symmetry and Goldstone Bosons

$[Q_V^a, H] = 0$, and $e^{-iQ_V^a}|0\rangle = |0\rangle \Leftrightarrow Q_V^a|0\rangle = 0$ (Wigner-Weyl realization)

$[Q_A^a, H] = 0$, but $e^{-iQ_A^a}|0\rangle \neq |0\rangle \Leftrightarrow Q_A^a|0\rangle \neq 0$ (Nambu-Goldstone realiz.)

- Consequence: $e^{-iQ_A^a}|0\rangle \neq |0\rangle$ is not the vacuum, but

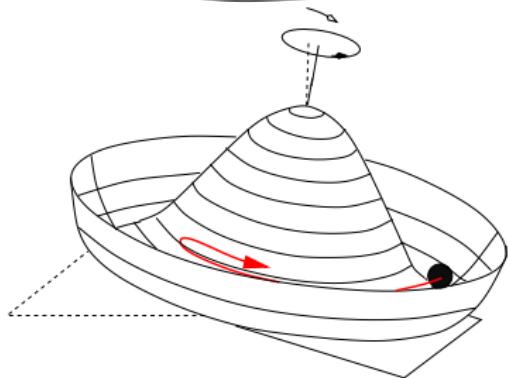
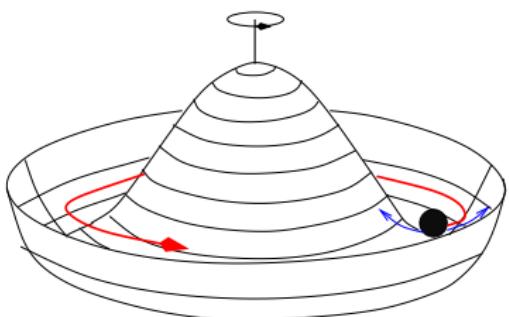
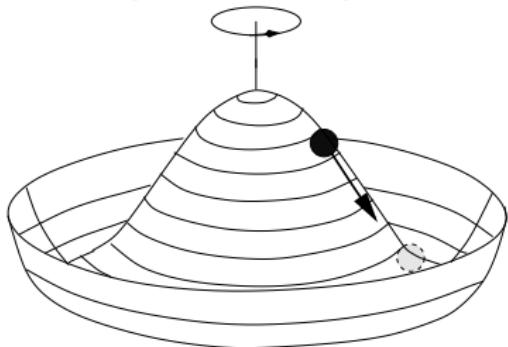
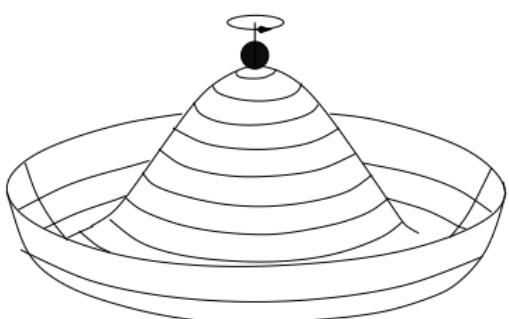
$$H e^{-iQ_A^a}|0\rangle = e^{-iQ_A^a}H|0\rangle = 0 \quad \text{is a massless state!}$$

Goldstone theorem: continuous global symmetry that does *not* leave the ground state invariant ('hidden' or 'spontaneously broken' symm.)

- mass- and spinless particles, "**Goldstone bosons**" (GBs)
- number of GBs = number of broken symmetry generators
- axial** generators broken \Rightarrow GBs should be **pseudoscalars**
- finite masses via (small) quark masses
 \hookrightarrow 8 lightest hadrons: $\pi^\pm, \pi^0, K^\pm, K^0, \bar{K}^0, \eta$ (not η')
- Goldstone bosons **decouple** (non-interacting) at vanishing energy

[◀ back](#)

Illustration: spontaneous symmetry breaking (SSB)



◀ back

Decoupling theorem of Goldstone bosons

Goldstone bosons do not interact at zero energy/momentum

1 $Q_A^a|0\rangle \neq 0 \Rightarrow Q_A^a$ creates GB $\Rightarrow \langle \pi^a | Q_A^a | 0 \rangle \neq 0$.

2 Lorentz invariance $\sim \langle \pi^a(q) | A_b^\mu(x) | 0 \rangle = -if_\pi q^\mu \delta_b^a e^{iq \cdot x} \neq 0$!

A_b^μ axial current

$\rightarrow f_\pi \neq 0$ necessary for SSB (order parameter)

(pion decay constant $f_\pi = 92$ MeV from weak decay $\pi^+ \rightarrow \mu^+ \nu_\mu$)

3 Coupling of axial current A_μ to matter fields (and/or pions)

$$\begin{aligned}
 iA^\mu &= \text{---} \circlearrowleft \quad + \quad \text{---} \circlearrowleft \quad \text{---} \circlearrowright \\
 &= i\mathcal{R}^\mu \text{ (non-sing.)} + -if_\pi q^\mu \frac{i}{q^2 - m_\pi^2 + i\epsilon} V \quad (V: \text{coupling of GB to matter fields})
 \end{aligned}$$

4 Conservation of axial current $\partial_\mu A_b^\mu(x) = 0: \Rightarrow m_\pi^2 = 0$ and $q_\mu A^\mu = 0:$

$$0 = q_\mu \mathcal{R}^\mu - f_\pi \frac{q^2}{q^2} V \xrightarrow{q \rightarrow 0} 0 = -f_\pi \lim_{q \rightarrow 0} V \xrightarrow{f_\pi \neq 0} \lim_{q \rightarrow 0} V = 0 \Rightarrow \text{decoupling!}$$