Development of a Rogowski coil Beam Position Monitor for Electric Dipole Moment measurements at storage rings

PHD defense talk
Physics Institute III B | Nuclear Physics Institute (IKP-II)

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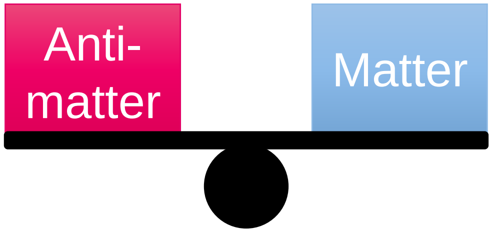
JÜLICH FORSCHUNGSZENTRUM JEDI Physics Institute III B RWTH AACHEN UNIVERSITY

Content

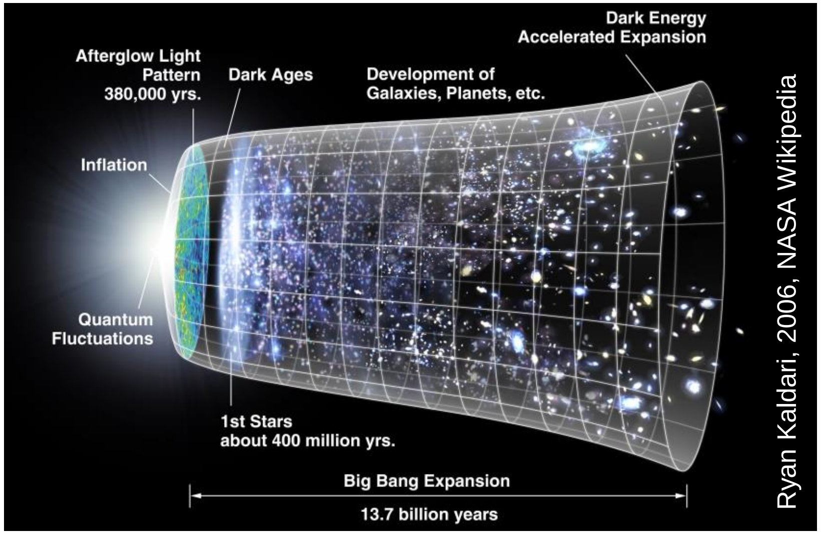
- Motivation for Electric Dipole Moment (EDM) measurements
- Spin dynamics in storage rings
- EDM measurement at the accelerator facility COSY
- Beam position measurements
  1. Common Beam Position Monitors (BPMs) at COSY
  2. Development of a Rogowski coil BPM
  3. Estimation of a deuteron EDM limit for a measurement at COSY
- Summary and future developments

Baryogenesis: *Why does the universe contains more matter than antimatter?*



<u>Big Bang</u>

Symmetry between matter and antimatter



<u>Early Universe</u>

Sakharov* conditions:

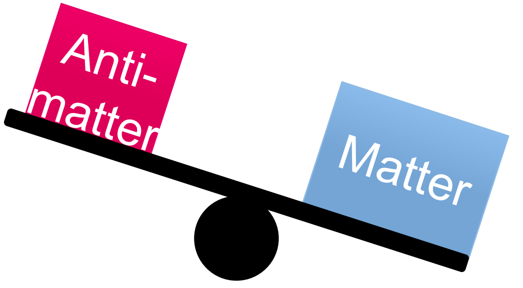
1. Baryon number violating interactions
2. Thermal non-equilibrium
3. **Violation of C, CP symmetry**

*Andrej Sakharov 21 May 1921 – 14 December 1989

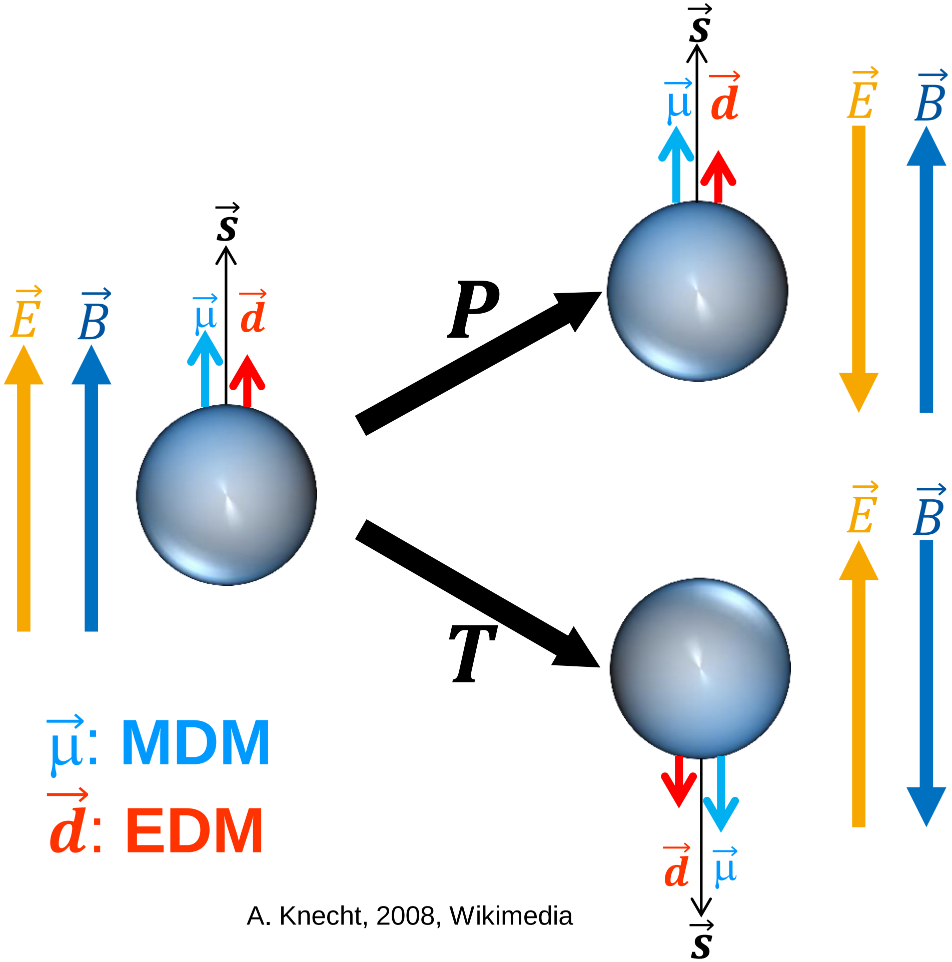
Possible sources:
- Strong CP violation (SM)
- Electroweak CP violation (SM)

<u>Today</u>

	Measurement (WMAP)	**Cosmological SM Expectation**
$(\eta_B - \eta_{\bar{B}})/\eta_\gamma$	$(6.14 \pm 0.25) \cdot 10^{-10}$	10^{-18}



Electric Dipole Moments (EDMs) as a new source of CP violation



A. Knecht, 2008, Wikimedia

$$\vec{\mu} = g \cdot \frac{e}{2m} \vec{s} \qquad \vec{d} = \eta \cdot \frac{e}{2mc} \vec{s}$$

- $$\mathcal{H} = -\vec{\mu} \cdot \vec{B} - \vec{d} \cdot \vec{E}$$
  $$\boldsymbol{P}: \mathcal{H} = -\vec{\mu} \cdot \vec{B} + \vec{d} \cdot \vec{E}$$
  $$\boldsymbol{T}: \mathcal{H} = -\vec{\mu} \cdot \vec{B} + \vec{d} \cdot \vec{E}$$
- Permanent EDMs of light hadrons are T-violating
  - $\boldsymbol{CPT}$ theorem ⇒ $\boldsymbol{CP}$ violation
- Standard Model expectation: $d \approx 10^{-31} \mathrm{e} \cdot \mathrm{cm}$ (Estimated by the neutron EDM limit)

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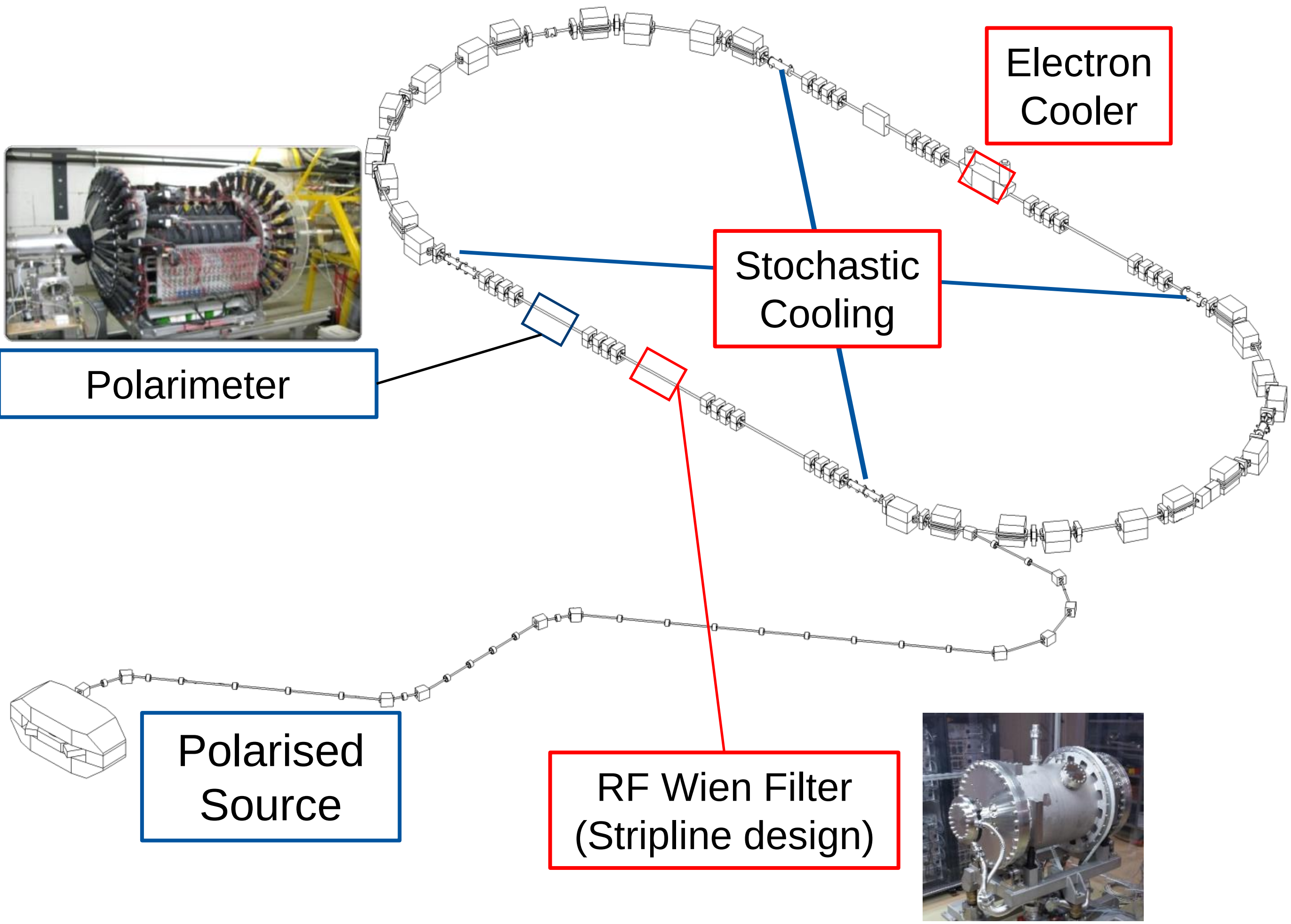
EDM measurement at the accelerator facility COSY (COoler SYnchrotron)



COSY facts	
Provides polarized protons and deuterons	
Circumference	184 m
Momentum	3.7 GeV/c
Intensity	10^9 to 10^{10} particles
Four experimental areas	
Beam diagnostic systems	
Spin manipulators	

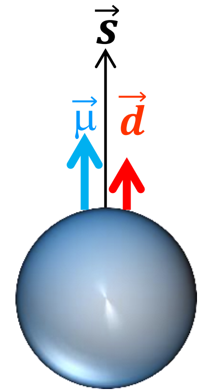
Spin dynamics in storage rings

Thomas-BMT-Equation:

- $\frac{d\vec{S}}{dt} = \vec{S} \times \vec{\Omega}_{\mathrm{MDM}} + \vec{S} \times \vec{\Omega}_{\mathrm{EDM}}$
- $\vec{\Omega}_{\mathrm{MDM}} = \frac{q}{m\gamma}\left(\gamma G \vec{B} + \left(G - \frac{1}{\gamma^2 - 1}\right)\frac{\vec{\beta}\times\vec{E}}{c}\right)$
- $\vec{\Omega}_{\mathrm{EDM}} = \frac{q\eta}{2m}\left(\frac{\vec{E}}{c} + \vec{\beta}\times\vec{B}\right)$

$\vec{\mu} = 2(G+1)\frac{q}{2m}\vec{S}$

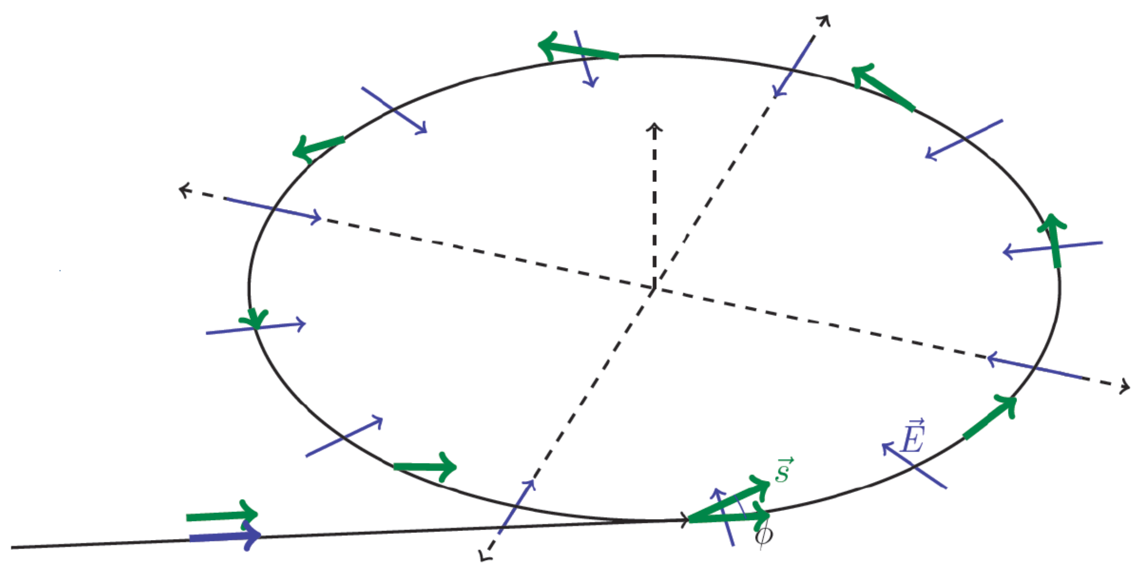
$\vec{d} = \frac{q\eta}{2mc}\vec{S}$



$\vec{\beta}\cdot\vec{E} = 0$

$\vec{\beta}\cdot\vec{B} = 0$

	G
Proton	1.792847357
Deuteron	-0.142561769



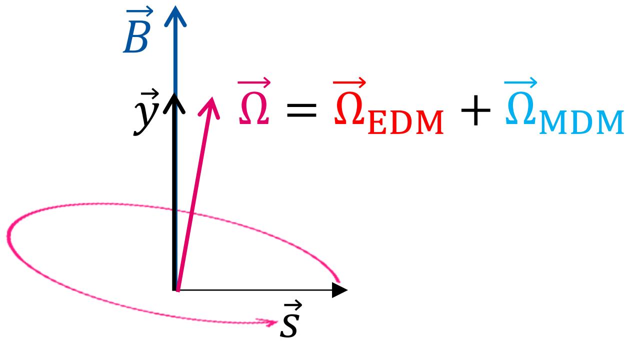
EDM measurement principle in a pure magnetic storage ring (COSY)

- EDMs introduce vertical polarization component of a horizontal polarized beam
- Measure vertical polarization

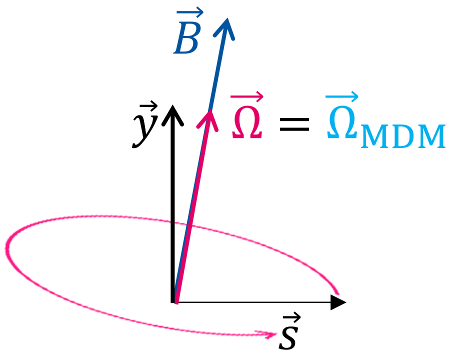
$$\vec{\Omega}_{\mathrm{MDM}} = \frac{q}{m\gamma}\left(\gamma G \vec{B}\right)$$

$$\vec{\Omega}_{\mathrm{EDM}} = \frac{q\eta}{2m}\left(\vec{\beta} \times \vec{\mathrm{B}}\right)$$

Perfect accelerator and an EDM



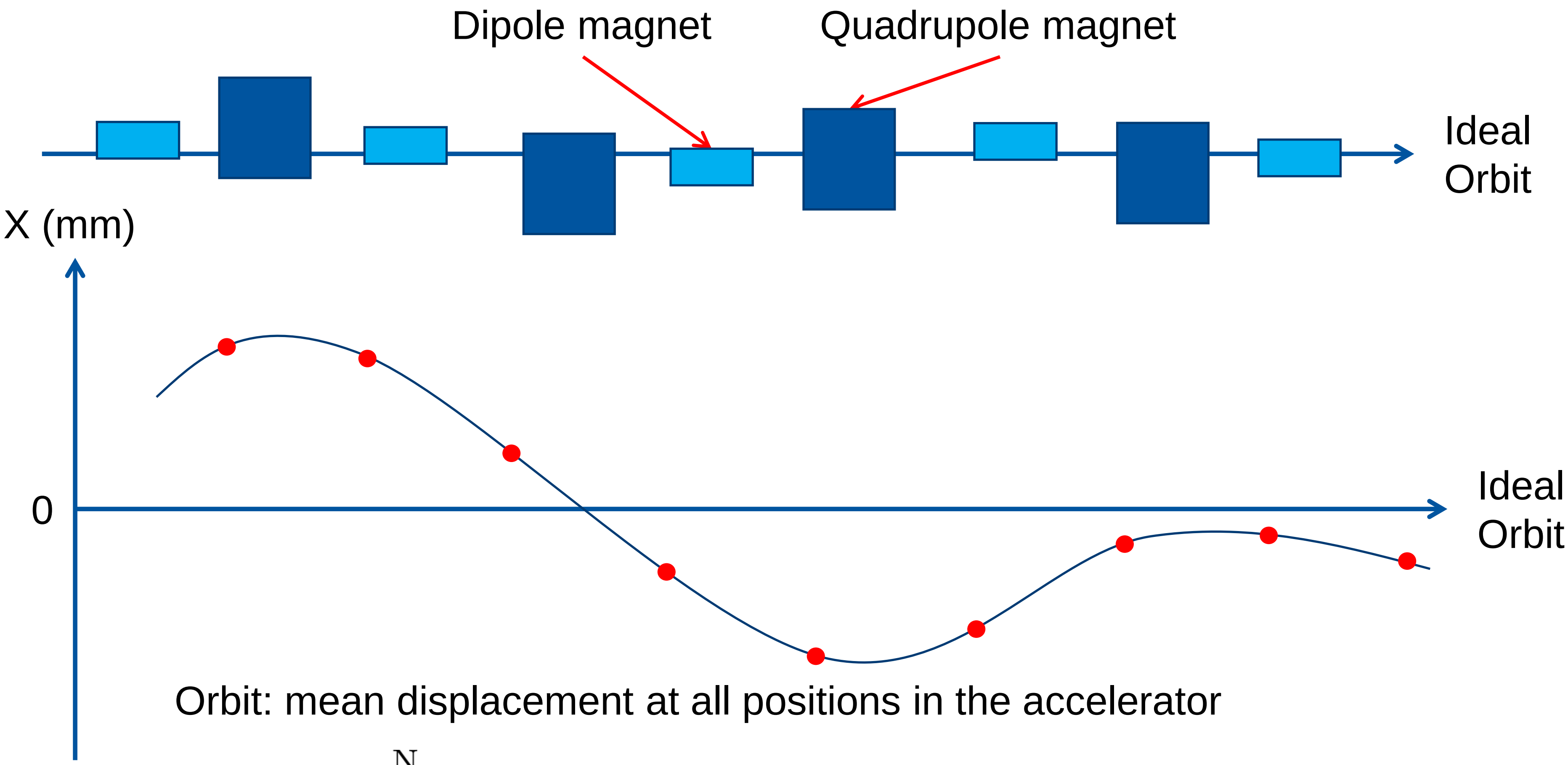
Realistic accelerator without an EDM



Scenario	d (e·cm)	Orbit RMS (mm)	$\Delta S_y/n$
Perfect accelerator with an EDM	$5 \cdot 10^{-19}$	0	$1.7 \cdot 10^{-9}$
Realistic accelerator without an EDM	0	1.3	$1.7 \cdot 10^{-9}$

Source: Simulation M. Rosenthal, 2016, PhD thesis [3], Phys. Rev. ST Accel. Beams 16, 114001 2013 [6]

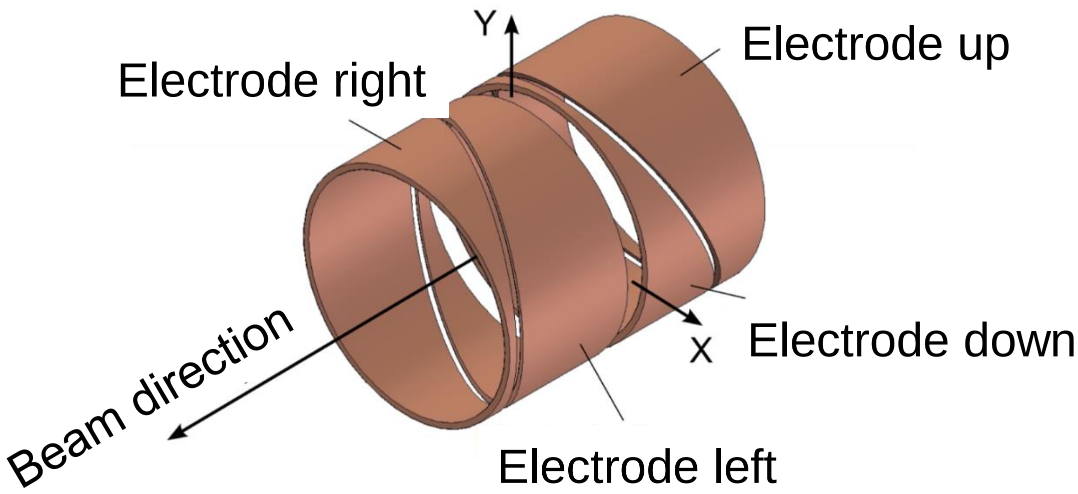
Particle orbit

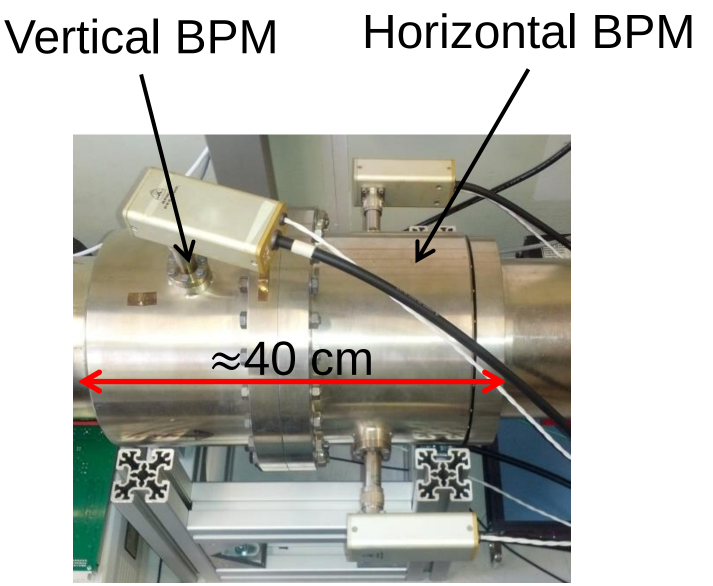


Orbit: mean displacement at all positions in the accelerator

$$\overline{x} = \frac{1}{N} \sum_{\mathrm{i}=1}^{\mathrm{N}} x_{\mathrm{i}}$$

Common Beam Position Monitor (BPM) system at COSY





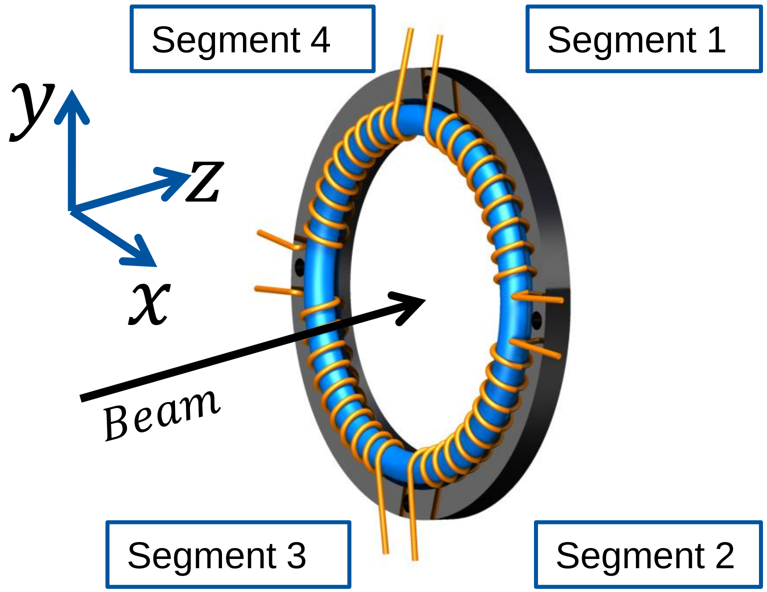
Beam position determination

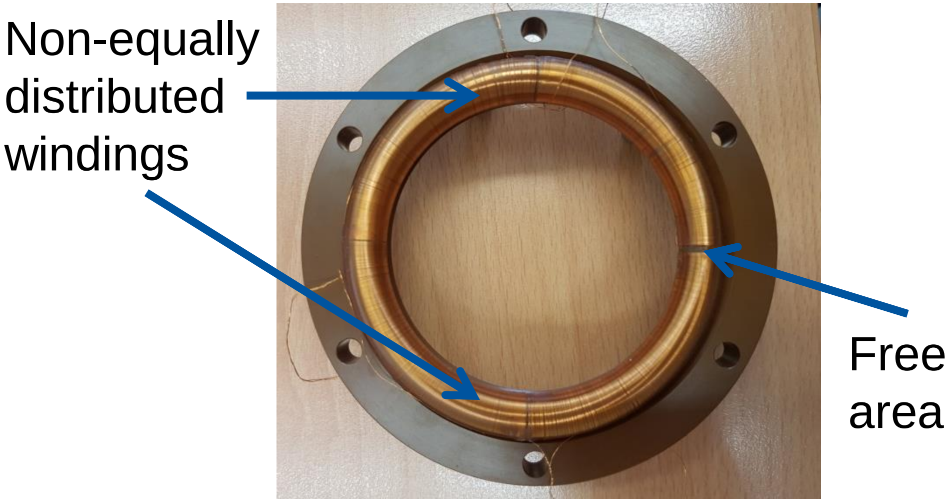
$$x = \frac{d}{2}\frac{U_L - U_R}{U_L + U_R}$$

$$y = \frac{d}{2}\frac{U_U - U_D}{U_U + U_D}$$

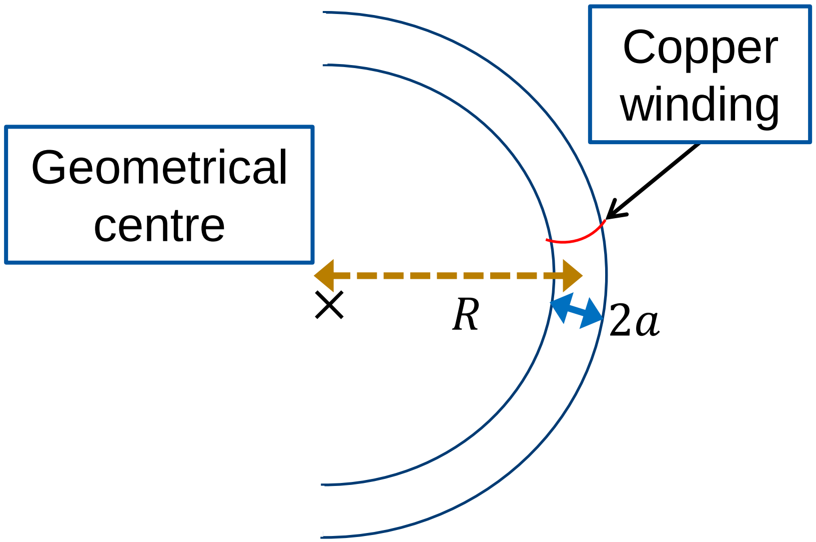
σ_{Pos} (µm)	Source
≈ 0.2	Thermal noise
≈ 10.0	Resolution
≈ 100.0	Accuracy

Development of a Rogowski coil Beam Position Monitor





Parameters	
R (mm)	40.0
a (mm)	5.0
N	366
s (µm)	150



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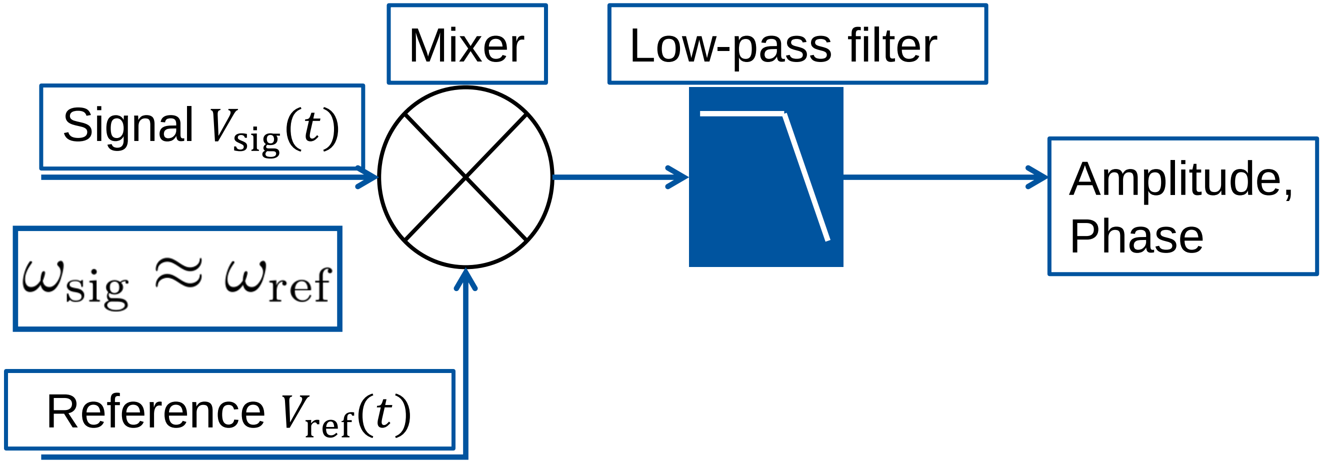
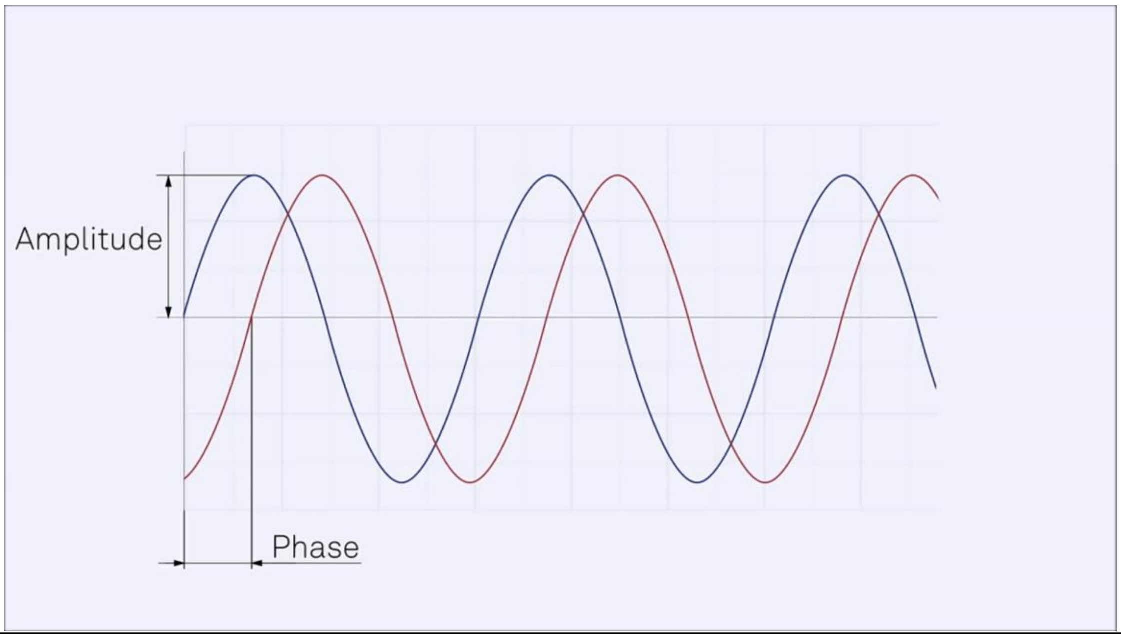
Voltage measurement principle with a lock-in amplifier

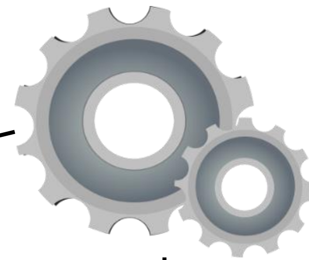


Zurich Instruments HF2LI Lock-in amplifier

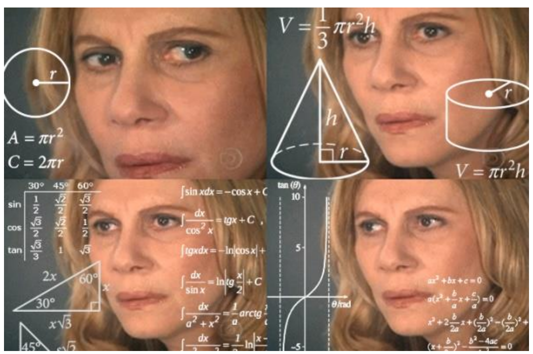


Development of a Rogowski coil BPM

Development process



Theoretical calculations & simulation

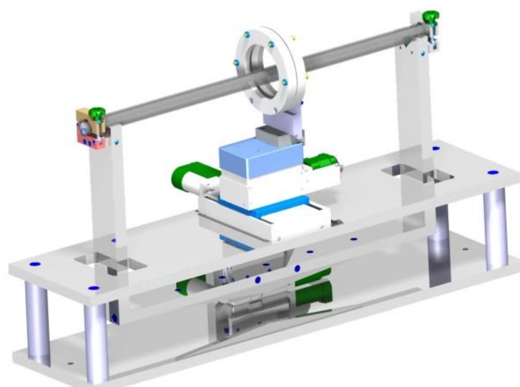


$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0} \qquad \vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{j} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Construction, manufacturing & test measurements with a testbench in the laboratory



Test measurements at the accelerator COSY

https://imgur.com/gallery/glxgY3L

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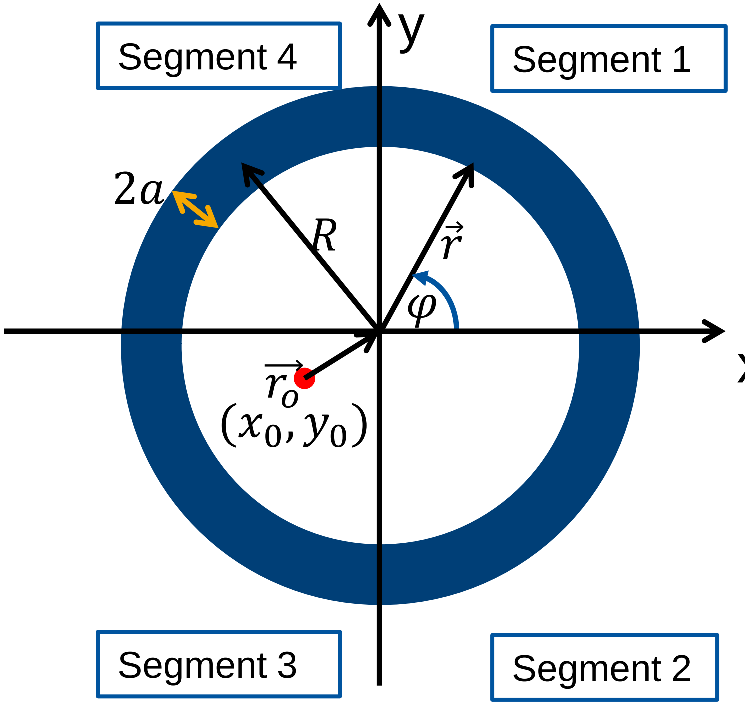
Induced voltage calculation

Model:
Pencil-current with constant velocity at position (x_0, y_0)



Particle Beam

Coil

Current: $\vec{I} = I_0 \cdot \vec{e_z}$

Position: $\vec{r_0} = \begin{pmatrix} x_0 \\ y_0 \\ 0 \end{pmatrix}$ $\vec{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

Magnetic field: $\vec{B} = \frac{\mu_0}{2\pi}\vec{I} \times \frac{\vec{r}-\vec{r_0}}{|\vec{r}-\vec{r_0}|^2}$

Induced voltage for N windings:

$$\mathrm{U_{ind}} = -N\frac{d\Phi}{dt}$$

$$= \frac{-N}{(\varphi_2 - \varphi_1)} \frac{d\mathrm{I}_0}{dt} \int_{\varphi_1}^{\varphi_2} \int_{-\mathrm{a}}^{\mathrm{a}} \int_{R-\sqrt{a^2-z^2}}^{R+\sqrt{a^2-z^2}} \mathrm{B}(r,\varphi)\, dr\, dz\, d\varphi.$$

Voltage ratio calculation

- Definition of the horizontal and vertical voltage ratio:

$$\frac{\Delta \mathrm{U}_{\mathrm{hor}}}{\Sigma \mathrm{U}_i} = \frac{(\mathrm{U}_1 + \mathrm{U}_2) - (\mathrm{U}_3 + \mathrm{U}_4)}{\mathrm{U}_1 + \mathrm{U}_2 + \mathrm{U}_3 + \mathrm{U}_4} \qquad \frac{\Delta \mathrm{U}_{\mathrm{ver}}}{\Sigma \mathrm{U}_i} = \frac{(\mathrm{U}_1 + \mathrm{U}_4) - (\mathrm{U}_2 + \mathrm{U}_3)}{\mathrm{U}_1 + \mathrm{U}_2 + \mathrm{U}_3 + \mathrm{U}_4}$$

- Calculation of the horizontal and vertical voltage ratio:

$$\frac{\Delta \mathrm{U}_{\mathrm{hor}}}{\Sigma \mathrm{U}_i} = c_1 x_0 - c_3 \left(x_0^3 - 3y_0^2 x_0\right) + c_4 \left(x_0^5 - 10 y_0^2 x_0^3 + 5 y_0^4 x_0\right) + \mathcal{O}\left(x_0^n \cdot y_0^m\right)_{n+m \geq 6}$$

$$\frac{\Delta \mathrm{U}_{\mathrm{ver}}}{\Sigma \mathrm{U}_i} = c_1 y_0 - c_3 \left(y_0^3 - 3x_0^2 y_0\right) + c_4 \left(y_0^5 - 10 x_0^2 y_0^3 + 5 x_0^4 y_0\right) + \mathcal{O}\left(x_0^n \cdot y_0^m\right)_{n+m \geq 6}$$

- Calculation of the sensitivities: ($R = 40.000$ mm and $a = 5.075$ mm)

$$c_1 = \frac{2}{\pi\sqrt{R^2 - a^2}} = 16.0 \cdot 10^{-3} \frac{1}{\mathrm{mm}}$$

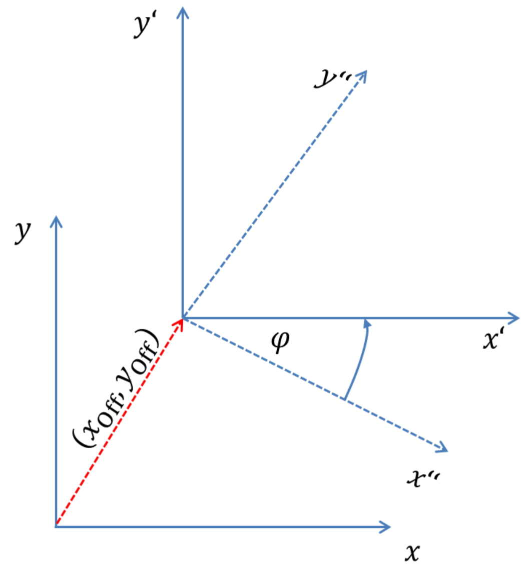
$$c_3 = \frac{a^2 R}{3\pi (R^2 - a^2)^{5/2} (R - \sqrt{R^2 - a^2})} = 3.4353 \cdot 10^{-6} \frac{1}{\mathrm{mm}^3}$$

$$c_4 = \frac{a^2 R (4R^2 + 3a^2)}{20\pi (R^2 - a^2)^{9/2} (R - \sqrt{R^2 - a^2})} = 1.3451 \cdot 10^{-9} \frac{1}{\mathrm{mm}^5}$$

Development of a calibration method

- Offset and rotation of the coil:

$$\begin{pmatrix} x_{\text{rot}} \\ y_{\text{rot}} \end{pmatrix} = \begin{pmatrix} \cos(\varphi)(x_0 - x_{\text{off}}) - \sin(\varphi)(y_0 - y_{\text{off}}) \\ \sin(\varphi)(x_0 - x_{\text{off}}) + \cos(\varphi)(y_0 - y_{\text{off}}) \end{pmatrix}$$



xy-coordinate system: Geometrical centre of the Rogowski coil BPM

x'y'-coordinate system: Offset from the beam to geometrical centre

x''y''-coordinate system: Offset of the beam to geometrical centre and rotation of the coil itself to this centre

- Different signal strength:

$$\frac{N_2}{N_1} = g_2, \quad \frac{N_3}{N_1} = g_3 \quad \text{and} \quad \frac{N_4}{N_1} = g_4$$

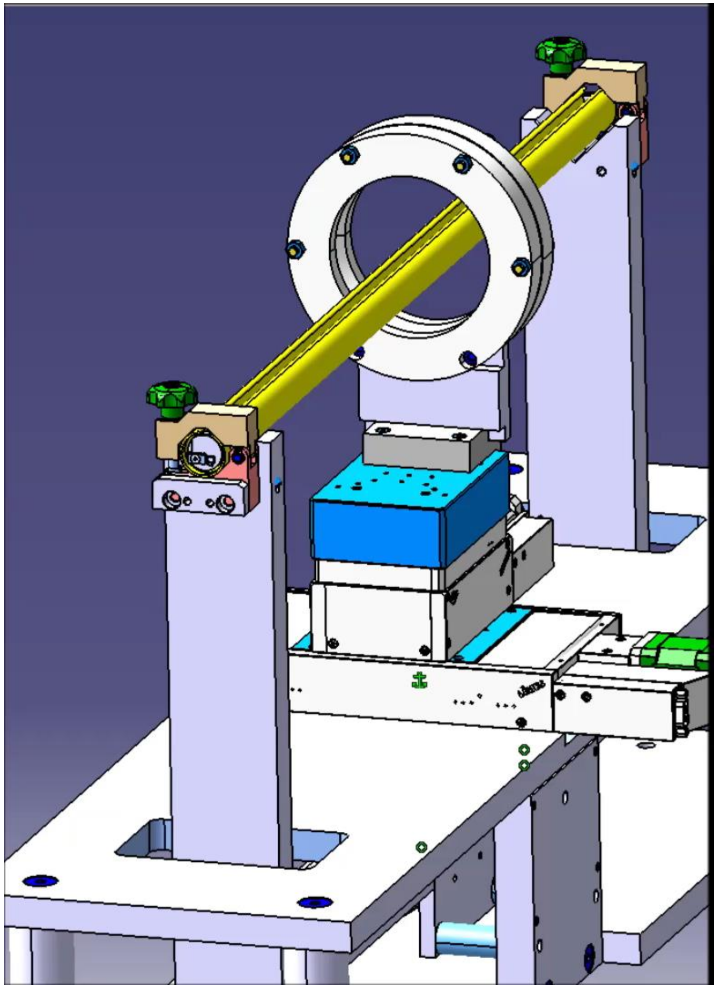
- Calculation of single voltage ratio:

$$\frac{\mathrm{U}_{1,\text{model}}}{\Sigma \mathrm{U}_{i,\text{model}}} = \frac{1}{1 + g_2 + g_3 + g_4} \left[1 + c_1 \left(x_{\text{rot}} + y_{\text{rot}}\right) + 2c_2 x_{\text{rot}} y_{\text{rot}} + c_3 \left(-{x_{\text{rot}}}^3 - {y_{\text{rot}}}^3 + 3{y_{\text{rot}}}^2 x_{\text{rot}} + 3{x_{\text{rot}}}^2 y_{\text{rot}}\right) \right.$$
$$\left. + c_4 \left({x_{\text{rot}}}^5 + {y_{\text{rot}}}^5 - 10{y_{\text{rot}}}^3 {x_{\text{rot}}}^2 - 10{x_{\text{rot}}}^3 {y_{\text{rot}}}^2 + 5{x_{\text{rot}}}^4 y_{\text{rot}} + 5{x_{\text{rot}}}^4 y_{\text{rot}}\right)\right]$$

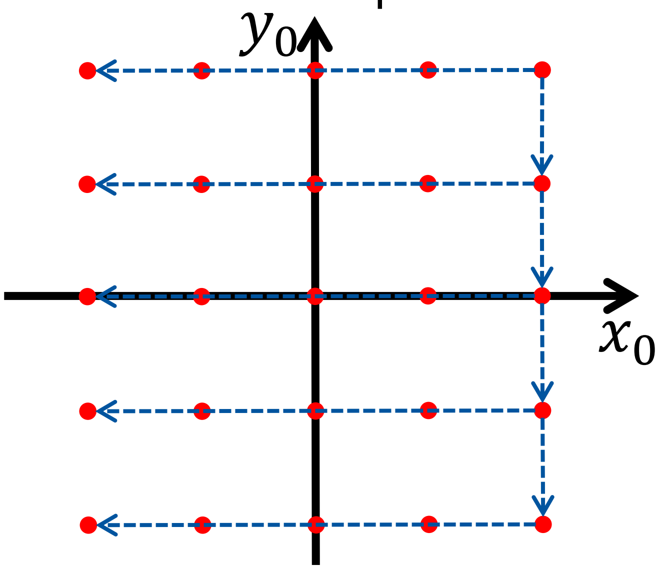
- Definition of a minimization function (6 free parameters):

$$\chi^2 = \frac{\chi^2_{\mathrm{R}_1}}{\sigma^2_{\mathrm{R}_{1,\text{meas}}}} + \frac{\chi^2_{\mathrm{R}_2}}{\sigma^2_{\mathrm{R}_{2,\text{meas}}}} + \frac{\chi^2_{\mathrm{R}_3}}{\sigma^2_{\mathrm{R}_{3,\text{meas}}}} + \frac{\chi^2_{\mathrm{R}_4}}{\sigma^2_{\mathrm{R}_{4,\text{meas}}}} \qquad \chi^2_{\mathrm{R}_i} = \left(\frac{\mathrm{U}_{i,\text{meas}}}{\Sigma \mathrm{U}_{i,\text{meas}}} - \frac{\mathrm{U}_{i,\text{model}}}{\Sigma \mathrm{U}_{i,\text{model}}}\right)^2$$

Laboratory measurements with the Rogowski coil BPM



Measurement procedure

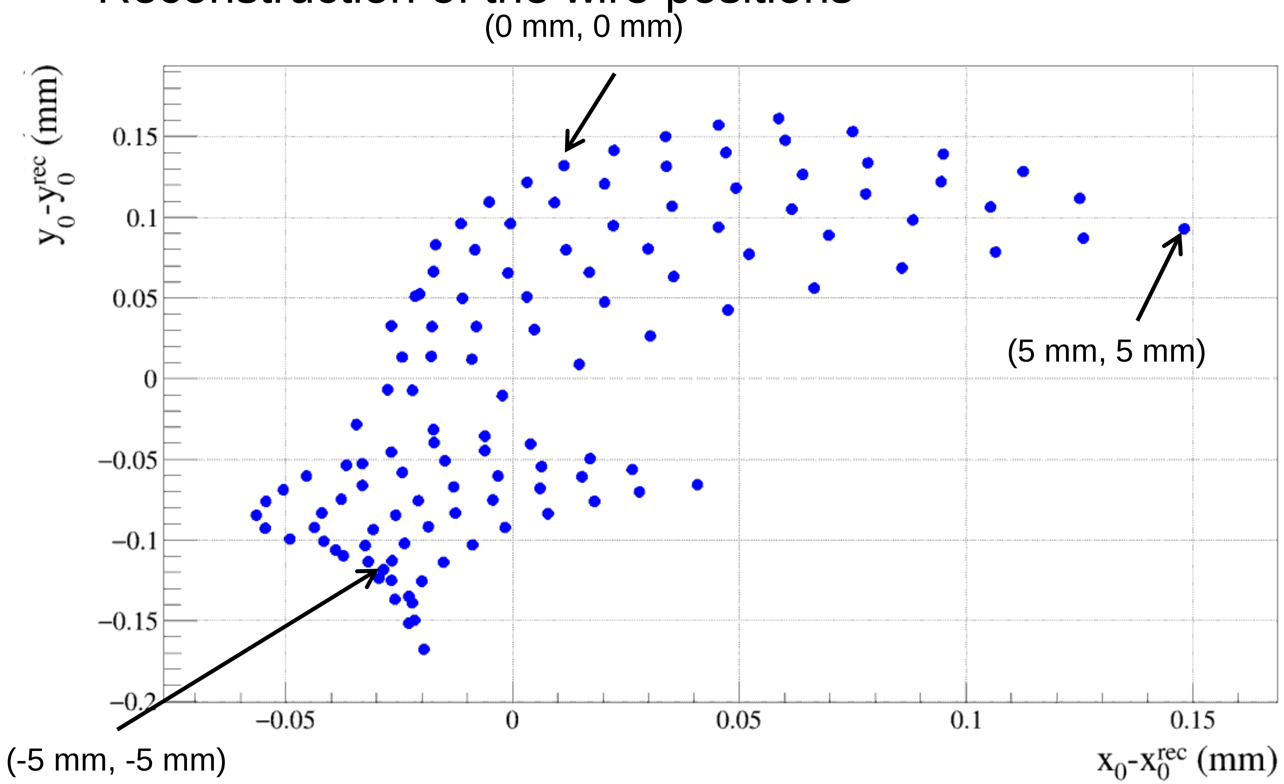


Measurement parameters	
Frequency	750 kHz
Particles	10^{10}
Number of measurements	10^5
Range	-5 mm to 5 mm

Laboratory measurements with the Rogowski coil BPM

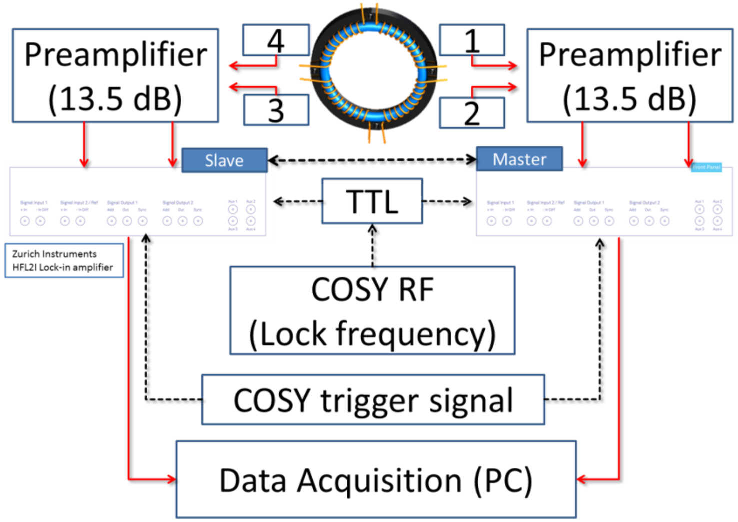
x_{off} (mm)	y_{off} (mm)	φ (°)	$\frac{1}{\Sigma g_i}$ (%)	$\frac{g_2}{\Sigma g_i}$ (%)	$\frac{g_3}{\Sigma g_i}$ (%)	$\frac{g_4}{\Sigma g_i}$ (%)
3.4	3.3	−0.40	20.9	28.3	28.7	22.1

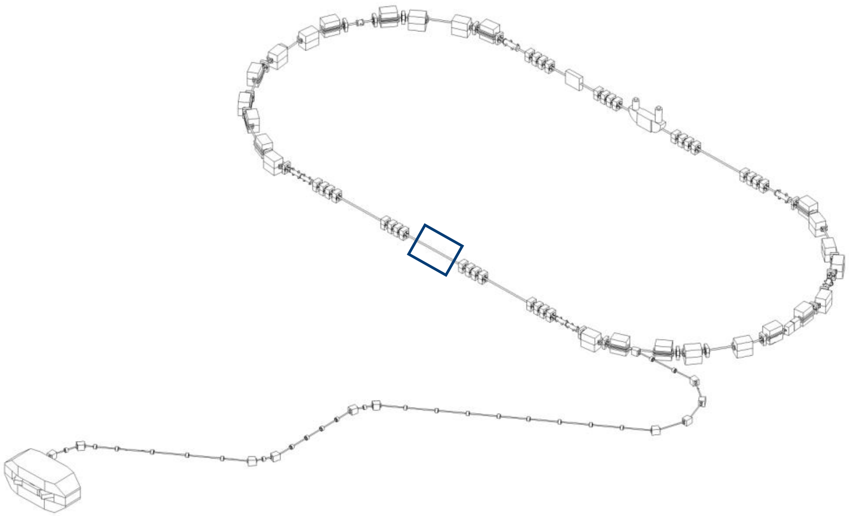
- Reconstruction of the wire positions

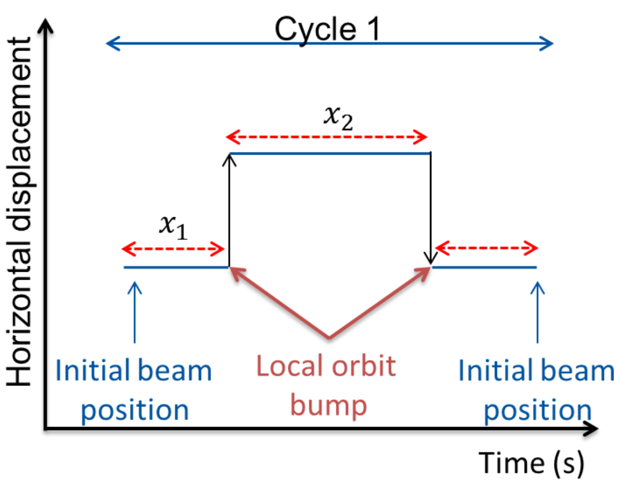


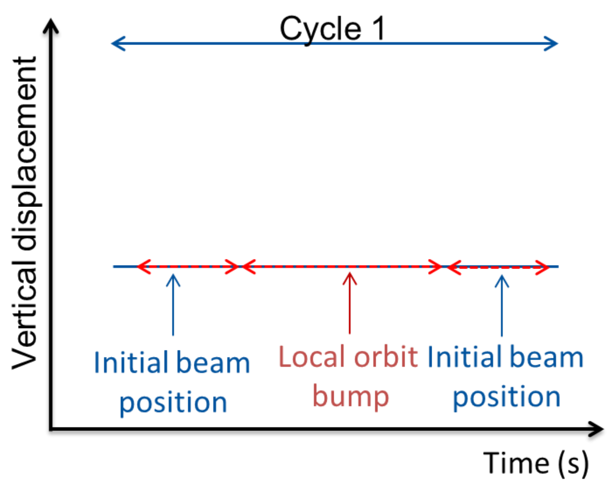
- Accuracy: ≈ 150 μm
- Caused by the asymmetry of the coil
- This effect is not considered in the calibration algorithm
- Resolution: ≈ 1.25 μm
- The theoretical resolution limit is reached

Beam position measurements with a Rogowski coil BPM at COSY









Horizontal orbit bumps:

- Values: -2% to 2%
- Step size: 0.2%
- Frequency: 750 kHz
- Number of particles: 10^{10}

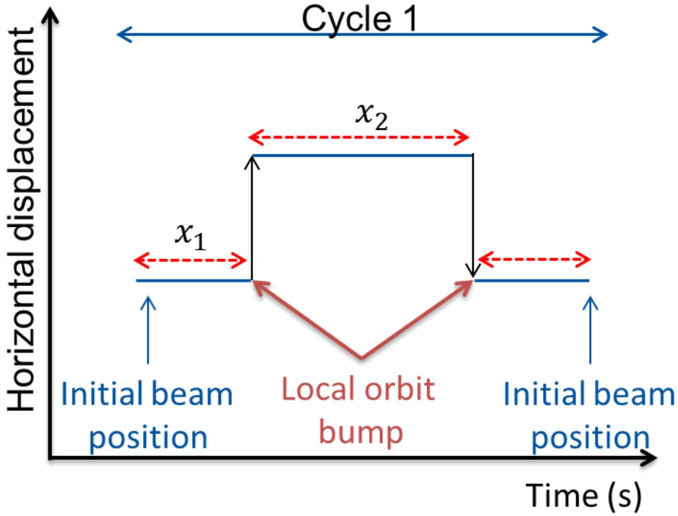
Beam position measurements with a Rogowski coil BPM in COSY

Horizontal ratio:

$$\Delta\frac{\Delta \mathrm{U_{hor}}}{\Sigma \mathrm{U_i}} = \frac{\Delta \mathrm{U_{hor,bump}}}{\Sigma \mathrm{U_i}} - \frac{\Delta \mathrm{U_{hor,initial}}}{\Sigma \mathrm{U_i}}$$

Horizontal model ratio expectation:

$$\Delta\frac{\Delta \mathrm{U_{hor}}}{\Sigma \mathrm{U_i}} = c_1 \cdot \underbrace{(x_2 - x_1)}_{\mathrm{const}\cdot\Delta I} = a_1 \cdot \Delta I$$

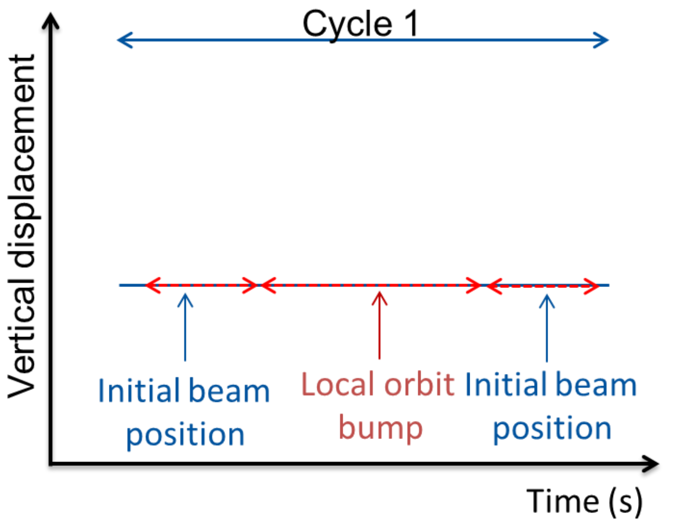


Vertical ratio:

$$\Delta\frac{\Delta \mathrm{U_{ver}}}{\Sigma \mathrm{U_i}} = \frac{\Delta \mathrm{U_{ver,bump}}}{\Sigma \mathrm{U_i}} - \frac{\Delta \mathrm{U_{ver,initial}}}{\Sigma \mathrm{U_i}}$$

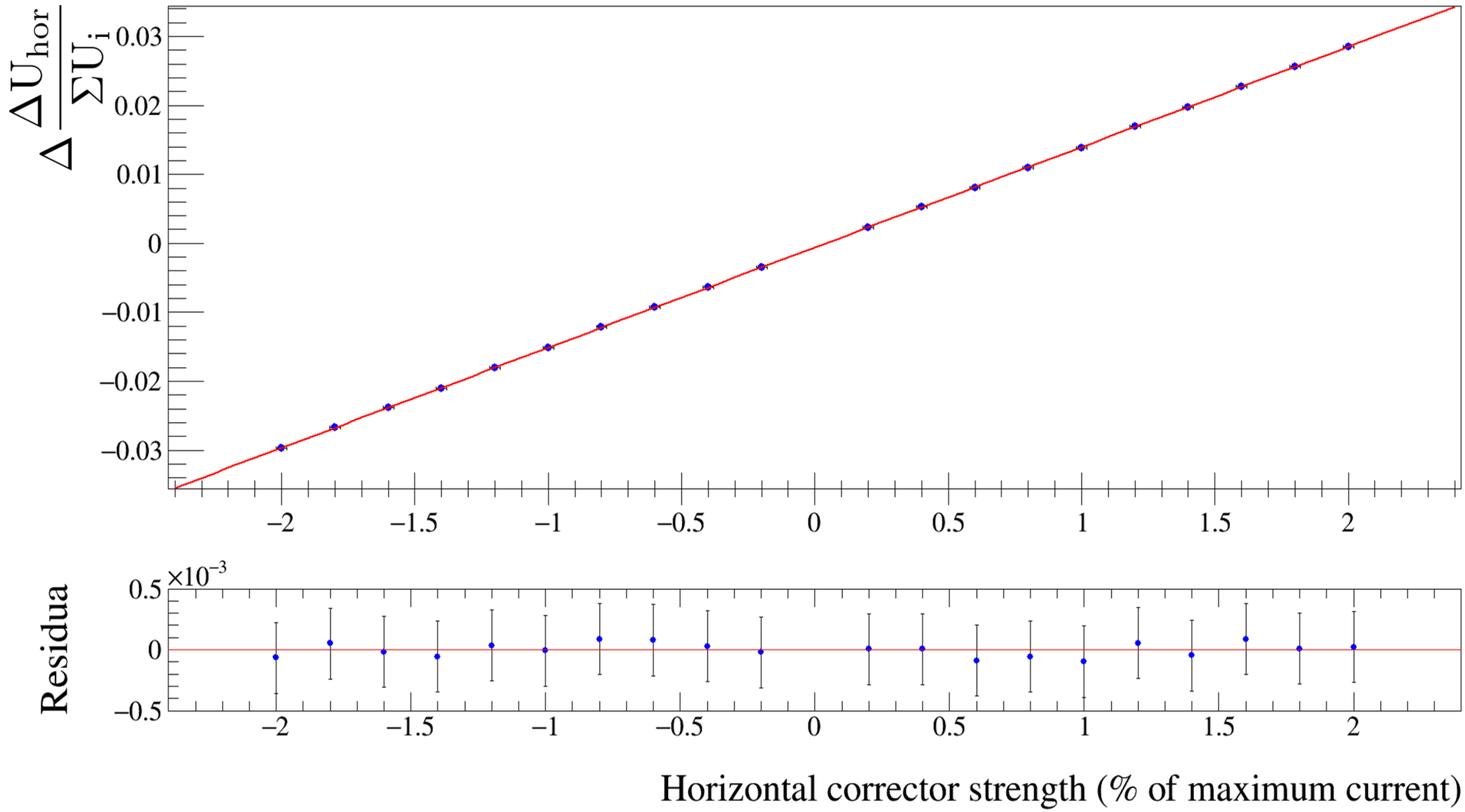
Vertical model ratio expectation:

$$\Delta\frac{\Delta \mathrm{U_{ver}}}{\Sigma \mathrm{U_i}} = b_1 + b_2 \cdot \Delta I + b_3 \cdot \Delta I^2$$



Beam position measurements with a Rogowski coil BPM in COSY

$$\Delta \frac{\Delta U_{hor}}{\Sigma U_i} = a_1 \cdot \Delta I$$



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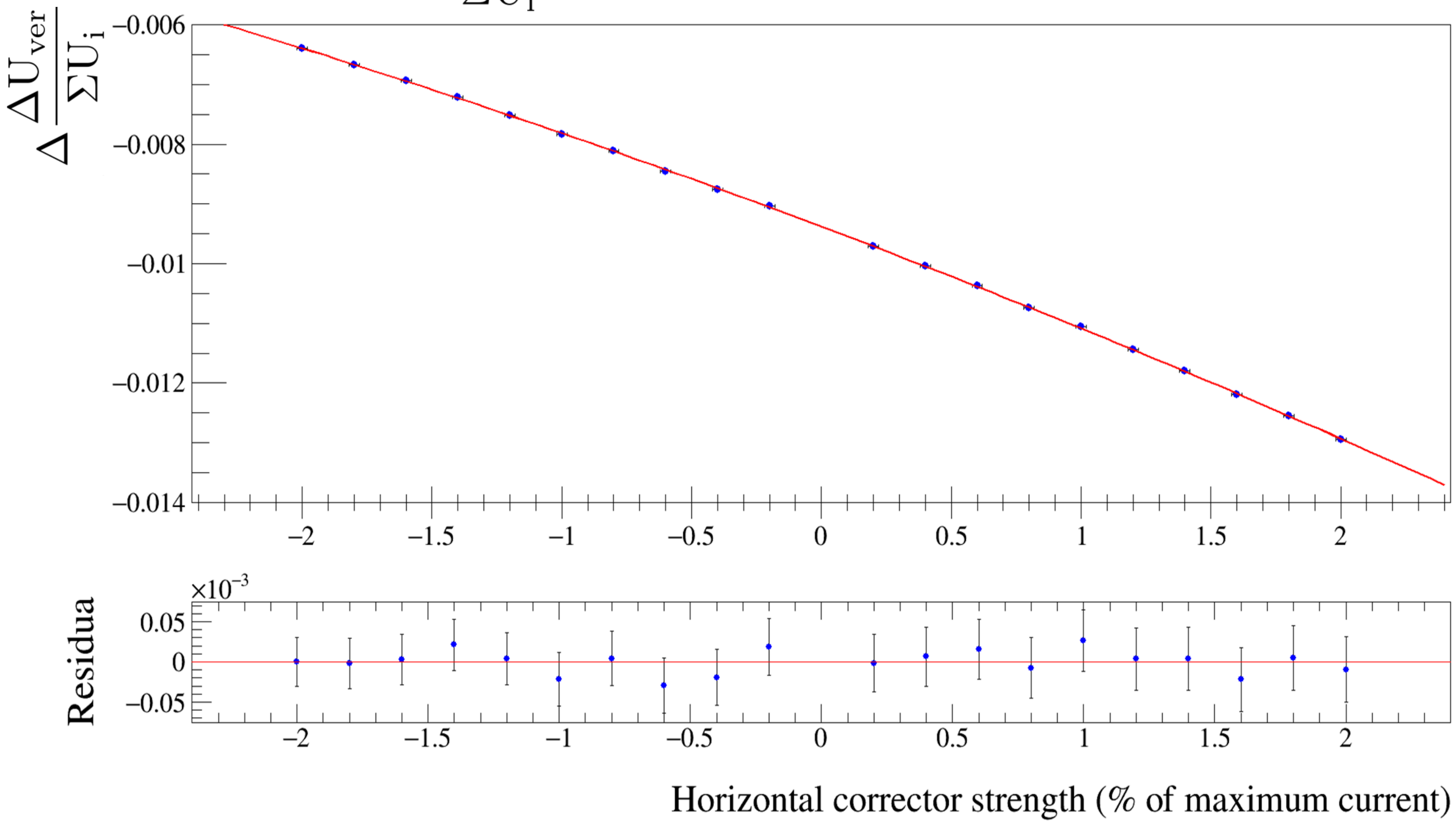
Beam position measurements with a Rogowski coil BPM in COSY

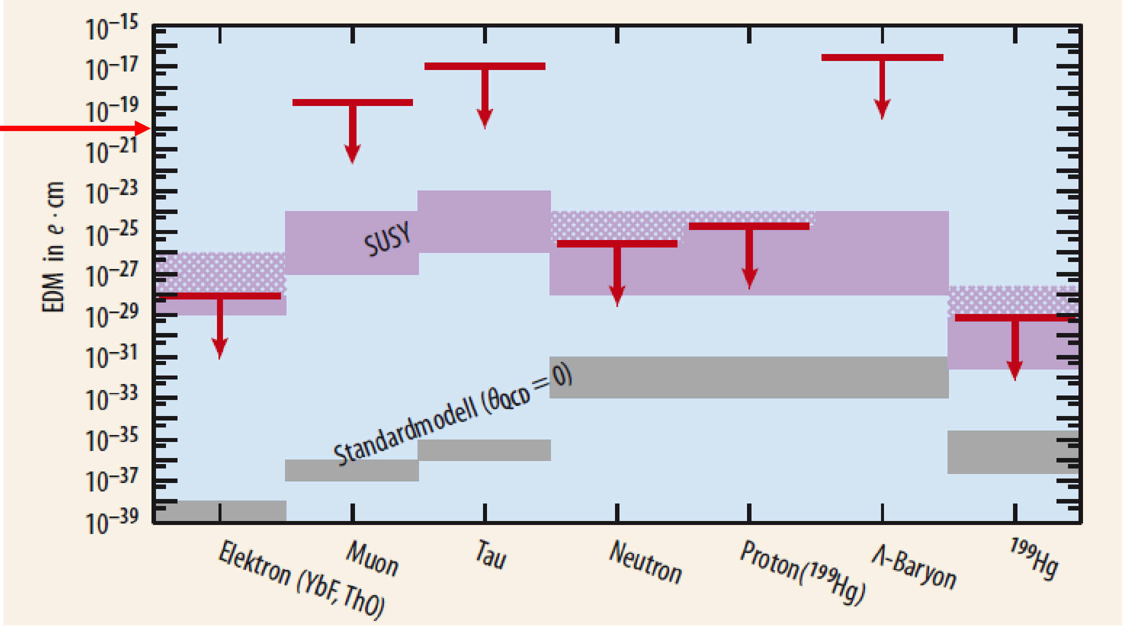
$$\Delta \frac{\Delta \mathrm{U_{ver}}}{\Sigma \mathrm{U_i}} = b_1 + b_2 \cdot \Delta I + b_3 \cdot \Delta I^2$$



Deuteron EDM limits for the precursor experiments at COSY

Measurement method		Common BPM (μm)	Rogowski coil BPM (μm)	Orbit RMS (μm)	$\sigma_{\mathrm{EDM,sys,orbit}}$ $(e \cdot cm)$
Absolute beam positions	Accuracy	100.00	150.00	≈ 100	$\approx 5 \cdot 10^{-20}$

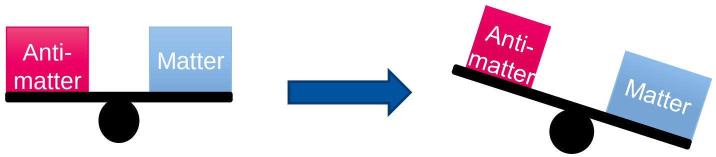
First deuteron EDM limit measured with COSY

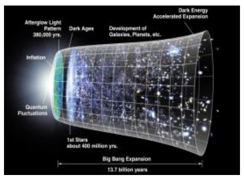


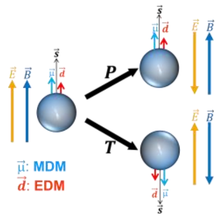
- A relative beam position measurement would reduce the $\sigma_{\mathrm{EDM,sys,orbit}}$ in magnitudes

Summary

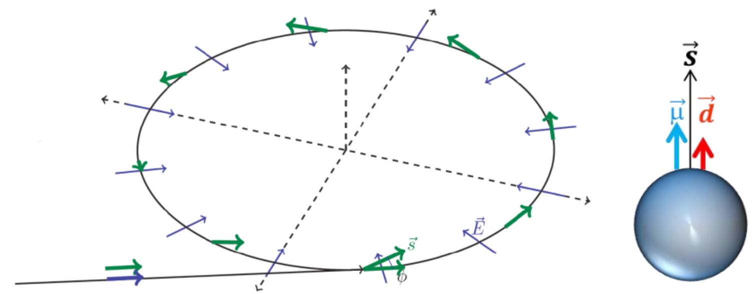
CP-violating process (EDMs) for matter over antimatter dominance





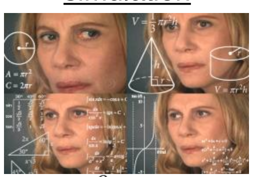


Hardware development to supress systematic effects (orbit control)



Development of a Rogowski coil-based BPM

Theoretical calculations & simulation

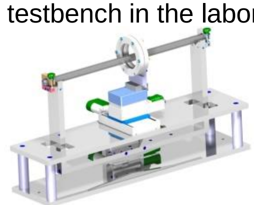


$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0} \qquad \vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{j} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial B}{\partial t}$$

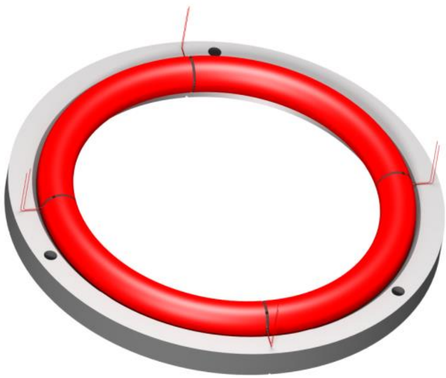
Construction, manufacturing & test measurements with a testbench in the laboratory

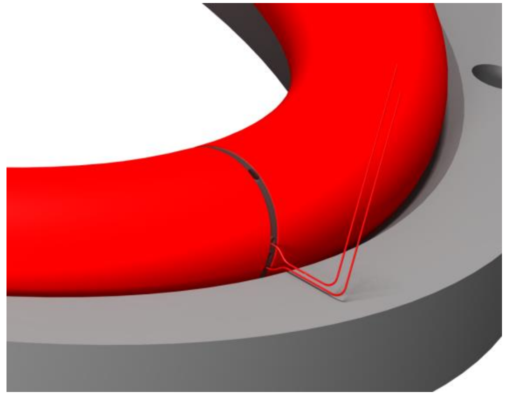


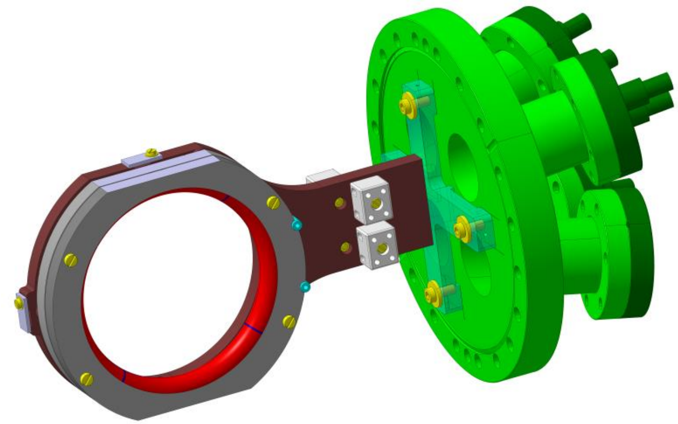
Test measurements in the accelerator COSY

Future Rogowski coil developments

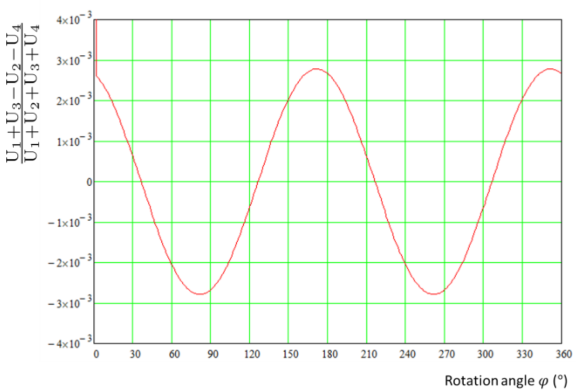
Coil design and flange improvements

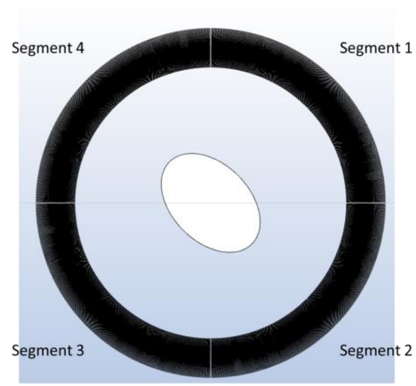




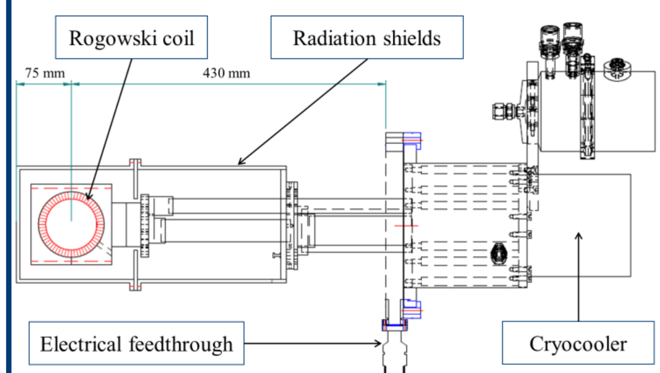


Investigation as a non-invasive beam profile monitor





SQUID development



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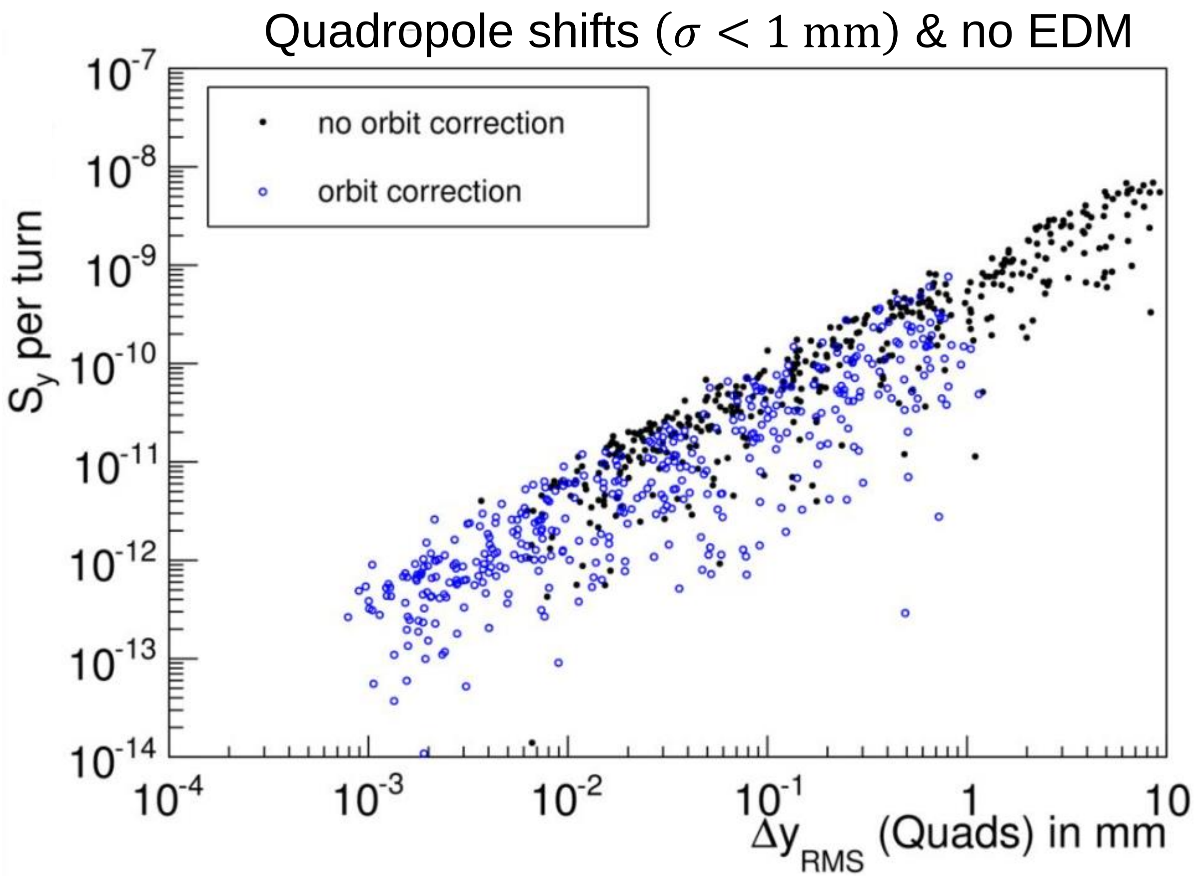
Backup

Systematic Effects I

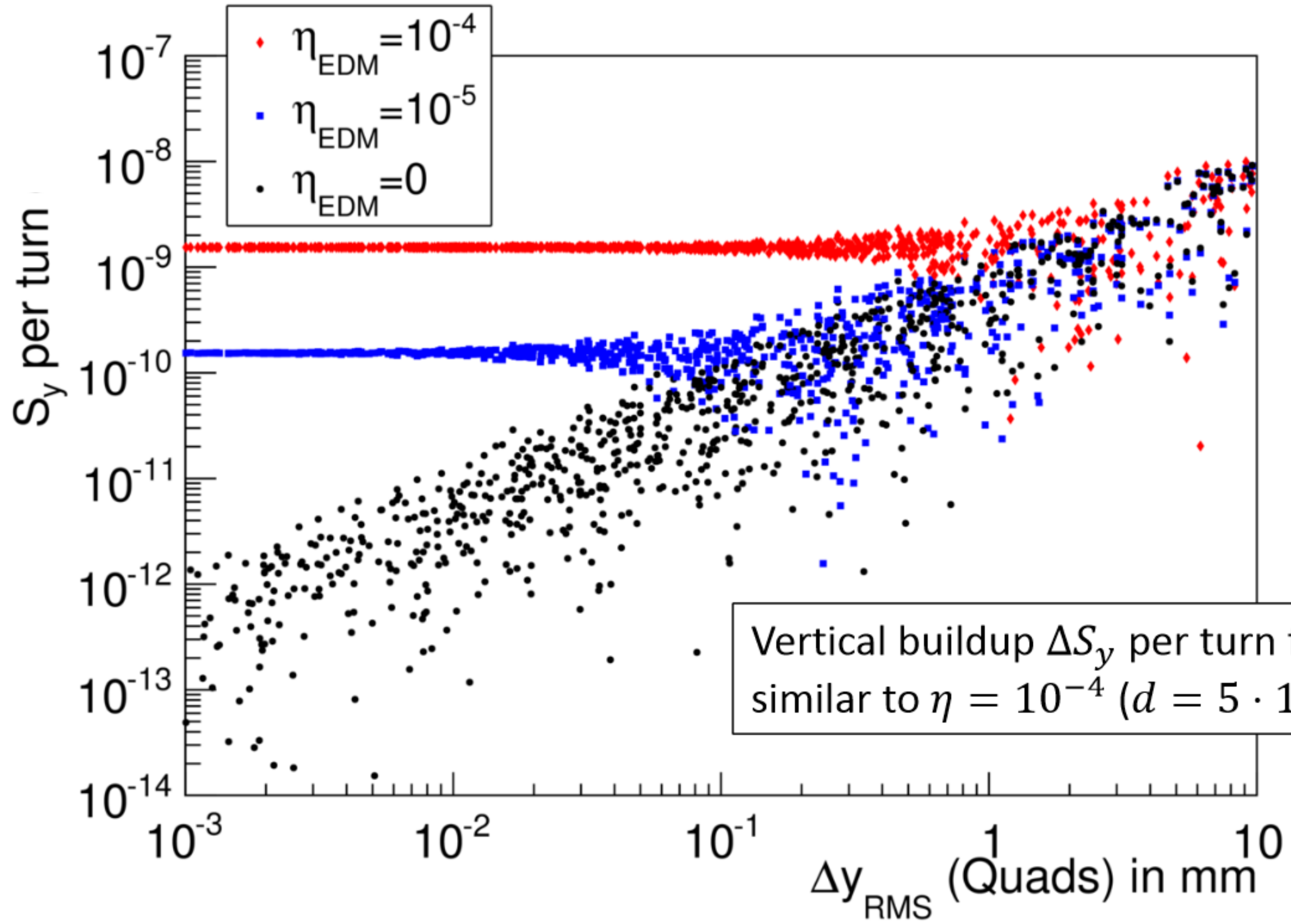
- Misaligned magnets lead to
- polarization build up
- orbit distortion

➢ Correct orbit to minimize polarization build up



Courtesy: Marcel Rosenthal (m.rosenthal@fz-juelich.de)

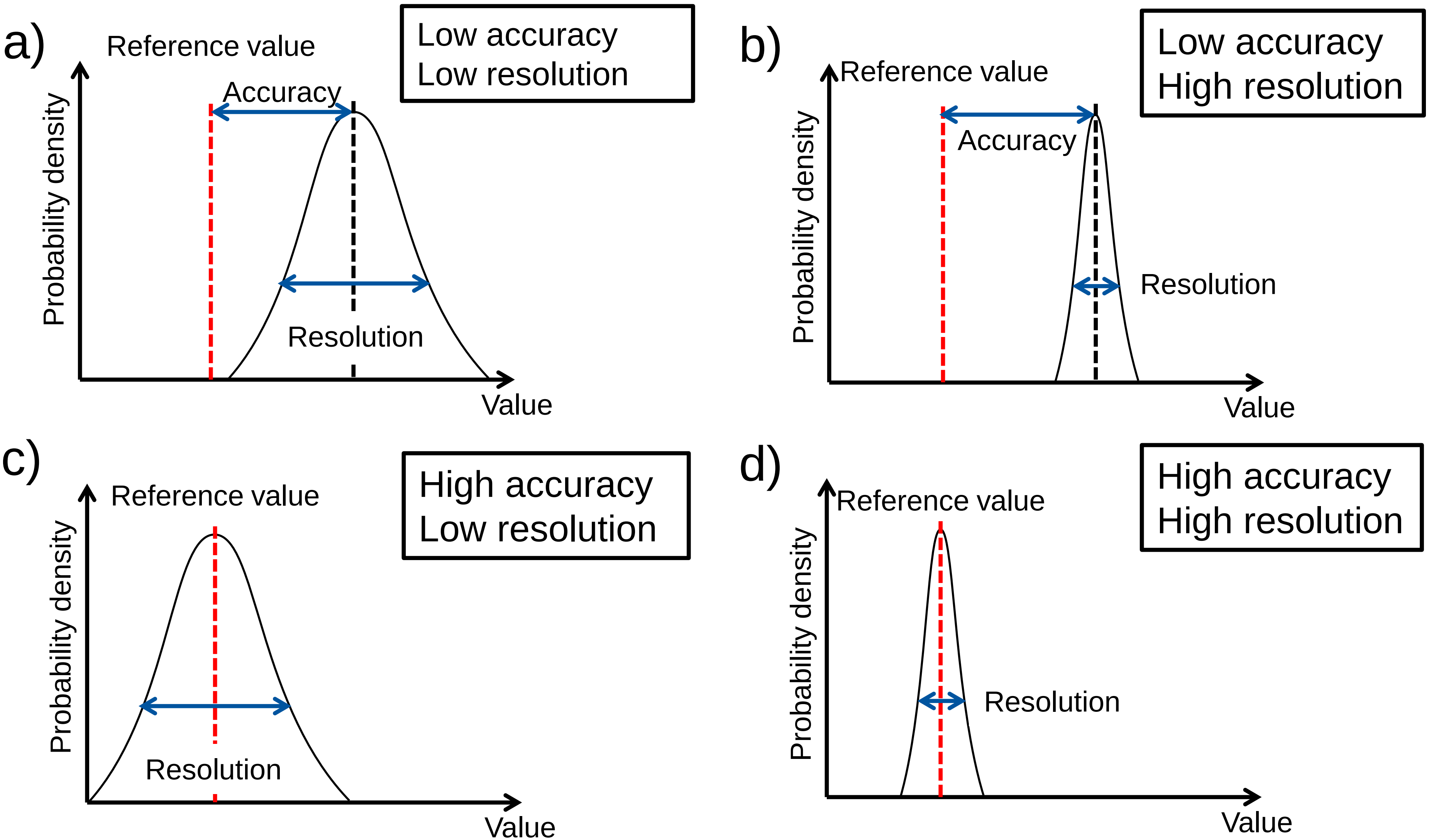
Systematic Effects II



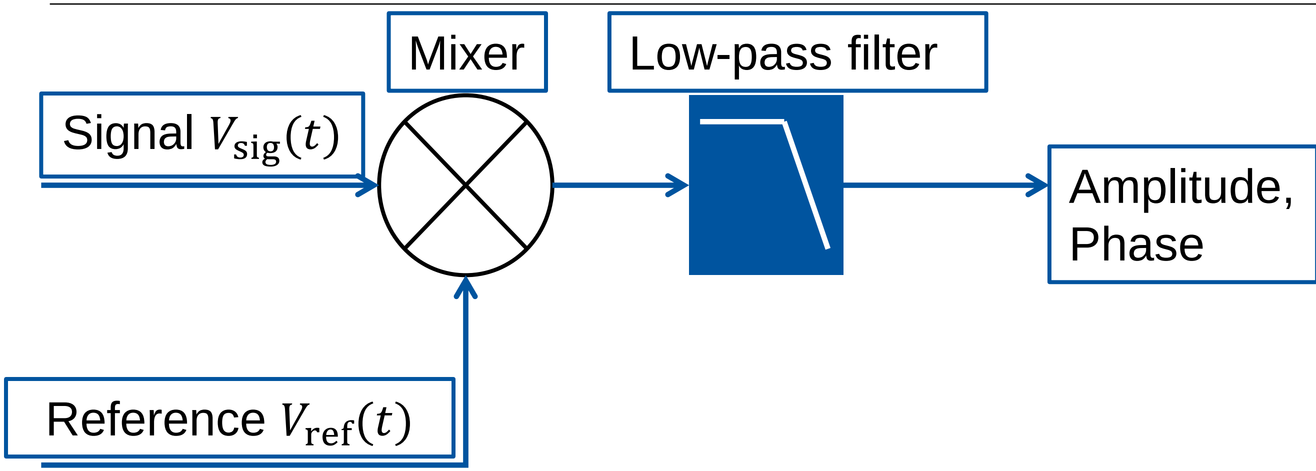
Vertical buildup ΔS_y per turn for RMS of 2 mm similar to $\eta = 10^{-4}$ $(d = 5 \cdot 10^{-19}\text{e cm})$

Courtesy: Marcel Rosenthal (m.rosenthal@fz-juelich.de)

Definition of accuracy and resolution



Voltage measurement principle with a lock-in amplifier



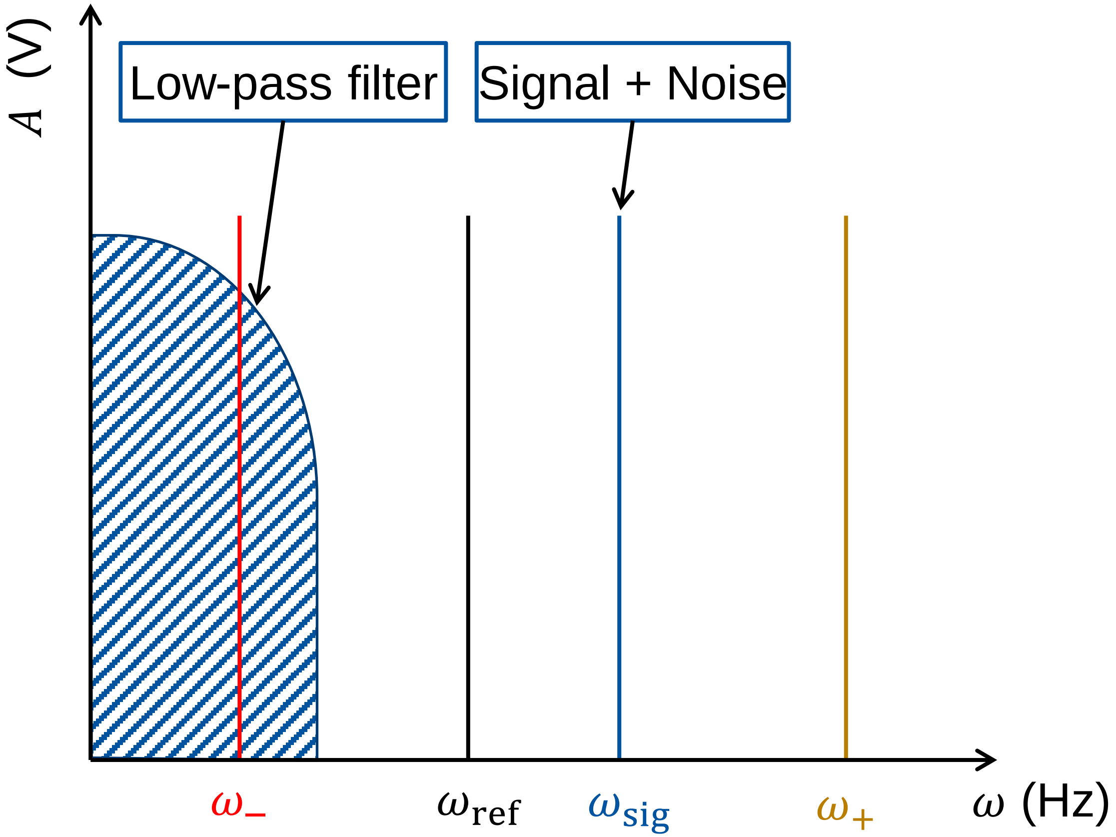
Zurich Instruments HF2LI Lock-in amplifier

$$V_{\text{sig}}(t) = A_{sig}\cos\left(\omega_{\text{sig}}t\right) \qquad V_{\text{ref}}(t) = A_{\text{ref}}\cos\left(\omega_{\text{ref}}t\right)$$

$$\begin{aligned} V_{\text{mod}}(t) &= V_{\text{sig}}(t)\cdot V_{\text{ref}}(t) \\ &= A_{\text{sig}}\cos\left(\omega_{\text{sig}}t\right)\cdot A_{\text{ref}}\cos\left(\omega_{\text{ref}}t\right) \\ &= \underbrace{A_{\text{mod}}}_{\frac{1}{2}A_{\text{sig}}A_{\text{ref}}}\left[\cos(t(\underbrace{\omega_{\text{sig}}-\omega_{\text{ref}}}_{\omega_-})) + \cos(t(\underbrace{\omega_{\text{sig}}+\omega_{\text{ref}}}_{\omega_+}))\right] \qquad \omega_{\text{sig}} \approx \omega_{\text{ref}} \end{aligned}$$

$$\boxed{V_{\text{mod}}(t) = \frac{1}{2}A_{sig}A_{ref}}$$

Voltage measurement principle with a lock-in amplifier



$\omega_{\mathrm{sig}} \approx \omega_{\mathrm{ref}}$

$$V_{\mathrm{mod}}(t) = \frac{1}{2} A_{sig} A_{ref}$$

Theoretical calculations for the Rogowski coil BPM

Magnetic field:

$$\mathrm{B}_{\mathrm{e}_\varphi} = \frac{\mu_0 \mathrm{I}_0}{2\pi}\frac{1}{\mathrm{r}} \underbrace{\frac{1-\frac{\mathrm{r}_0}{\mathrm{r}}\cos(\varphi-\varphi_0)}{1+\left(\frac{\mathrm{r}_0}{\mathrm{r}}\right)^2 - 2\frac{\mathrm{r}_0}{\mathrm{r}}\cos(\varphi-\varphi_0)}}_{:=\mathrm{A} \text{ with } u=\frac{\mathrm{r}_0}{\mathrm{r}} \text{ and } \Delta\varphi=\varphi-\varphi_0}$$

Taylor expansion series:

$$\mathrm{B}_{\mathrm{e}_\varphi} = \frac{\mu_0 \mathrm{I}_0}{2\pi}\frac{1}{\mathrm{r}}\left[\left.\frac{\mathrm{d}A}{\mathrm{du}^0}\right|_{\mathrm{u}=0} + \left.\frac{\mathrm{d}A}{\mathrm{du}^1}\right|_{\mathrm{u}=0}(u) + \frac{1}{2}(u)^2\left.\frac{\mathrm{d}^2A}{\mathrm{du}^2}\right|_{\mathrm{u}=0} + \frac{1}{6}(u)^3\left.\frac{\mathrm{d}^3A}{\mathrm{du}^3}\right|_{\mathrm{u}=0} \right.$$
$$\left. + \frac{1}{24}(u)^4\left.\frac{\mathrm{d}^4A}{\mathrm{du}^4}\right|_{\mathrm{u}=0} + \frac{1}{120}(u)^5\left.\frac{\mathrm{d}^5A}{\mathrm{du}^5}\right|_{\mathrm{u}=0} + \mathcal{O}\left((u)^6\right)\right]$$

Induced voltage for N windings:

$$\mathrm{U}_{\mathrm{ind}} = -N\frac{d\Phi}{dt}$$
$$= \frac{-N}{(\varphi_2-\varphi_1)}\frac{d\mathrm{I}_0}{dt}\int_{\varphi_1}^{\varphi_2}\int_{-\mathrm{a}}^{\mathrm{a}}\int_{R-\sqrt{a^2-z^2}}^{R+\sqrt{a^2-z^2}} \mathrm{B}(r,\varphi)\,dr\,dz\,d\varphi.$$

Theoretical calculations for the Rogowski coil BPM

Induced voltage for segment 1:

$$\mathrm{U}_1 = \frac{N_{1/4,1}\mu_0}{\pi}\frac{dI}{dt}\Bigg[\pi\left(R-\sqrt{R^2-a^2}\right)+(x_0+y_0)\frac{2\left(R-\sqrt{R^2-a^2}\right)}{\sqrt{R^2-a^2}}$$

$$+2x_0y_0\frac{a^2}{(R^2-a^2)^{3/2}}+\left(-x_0^3-y_0^3+3y_0x_0^2+3x_0y_0^2\right)\frac{a^2R}{3(R^2-a^2)^{5/2}}$$

$$+\left(x_0^5+y_0^5-10x_0^3y_0^2-10y_0^3x_0^2+5y_0^4x_0+5x_0^4y_0\right)\frac{a^2R\left(4R^2+3a^2\right)}{20(R^2-a^2)^{9/2}}\Bigg]$$

$$\frac{\Delta\mathrm{U}_{\mathrm{hor}}}{\Sigma\mathrm{U}_{\mathrm{i}}}=\frac{(\mathrm{U}_1+\mathrm{U}_2)-(\mathrm{U}_3+\mathrm{U}_4)}{\mathrm{U}_1+\mathrm{U}_2+\mathrm{U}_3+\mathrm{U}_4} \qquad \frac{\Delta\mathrm{U}_{\mathrm{ver}}}{\Sigma\mathrm{U}_{\mathrm{i}}}=\frac{(\mathrm{U}_1+\mathrm{U}_4)-(\mathrm{U}_2+\mathrm{U}_3)}{\mathrm{U}_1+\mathrm{U}_2+\mathrm{U}_3+\mathrm{U}_4}$$

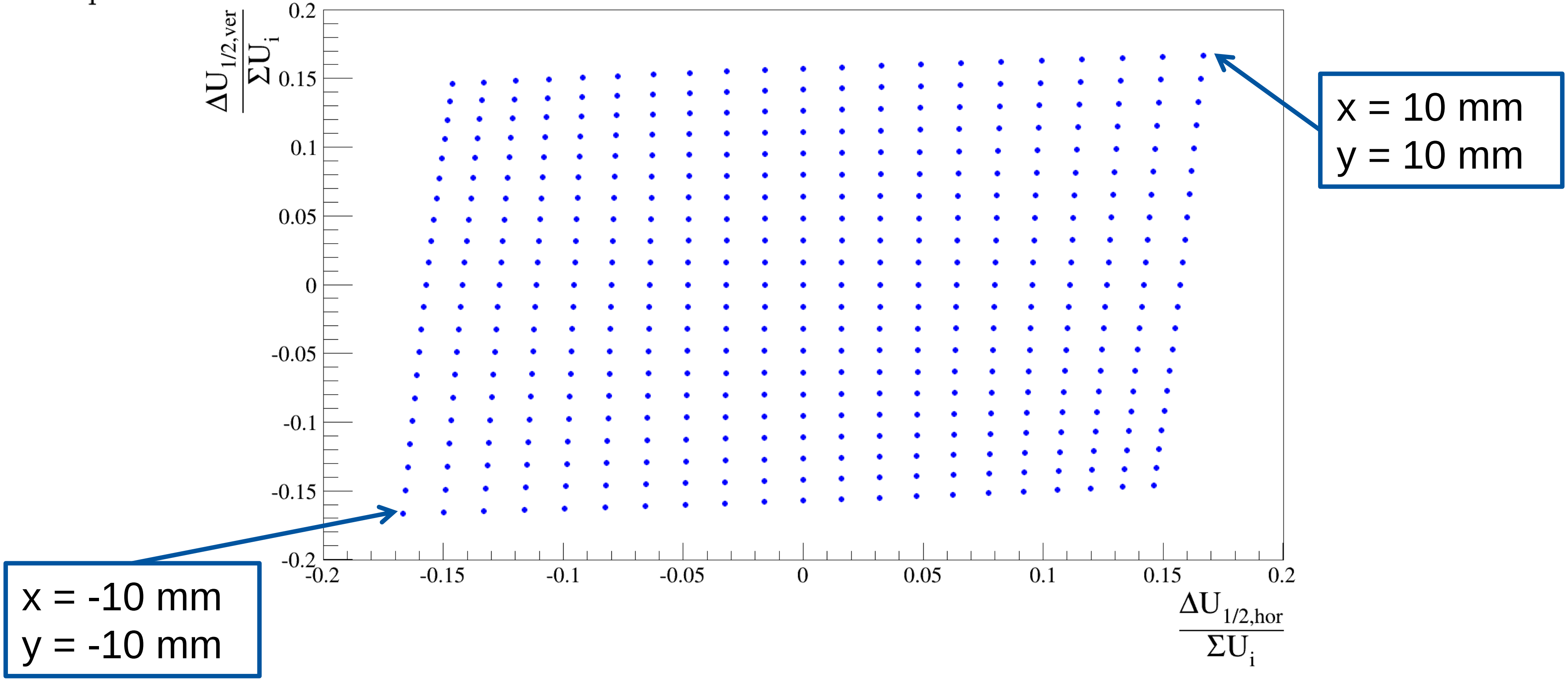
$$\frac{\Delta\mathrm{U}_{\mathrm{hor}}}{\Sigma\mathrm{U}_{\mathrm{i}}}=c_1x_0-c_2\left(x_0^3-3y_0^2x_0\right)+c_3\left(x_0^5-10y_0^2x_0^3+5y_0^4x_0\right)+\mathcal{O}\left(x_0^n\cdot y_0^m\right)_{n,m\geq 6}$$

$$\frac{\Delta\mathrm{U}_{\mathrm{ver}}}{\Sigma\mathrm{U}_{\mathrm{i}}}=c_1y_0-c_2\left(y_0^3-3x_0^2y_0\right)+c_3\left(y_0^5-10x_0^2y_0^3+5x_0^4y_0\right)+\mathcal{O}\left(x_0^n\cdot y_0^m\right)_{n,m\geq 6}$$

Theoretical calculations for the Rogowski coil BPM

$$\frac{\Delta \mathrm{U_{hor}}}{\Sigma \mathrm{U_i}} = c_1 x_0 - c_2 \left(x_0^3 - 3y_0^2 x_0\right) + c_3 \left(x_0^5 - 10 y_0^2 x_0^3 + 5 y_0^4 x_0\right) + \mathcal{O}\left(x_0^n \cdot y_0^m\right)_{n,m\geq 6}$$

$$\frac{\Delta \mathrm{U_{ver}}}{\Sigma \mathrm{U_i}} = c_1 y_0 - c_2 \left(y_0^3 - 3x_0^2 y_0\right) + c_3 \left(y_0^5 - 10 x_0^2 y_0^3 + 5 x_0^4 y_0\right) + \mathcal{O}\left(x_0^n \cdot y_0^m\right)_{n,m\geq 6}$$



Calculation of the voltage ratios for the orbit bump measurements

Horizontal:

$$\Delta \frac{\Delta \mathrm{U_{hor}}}{\Sigma \mathrm{U_i}} = \frac{\Delta \mathrm{U_{hor,bump}}}{\Sigma \mathrm{U_i}} - \frac{\Delta \mathrm{U_{hor,initial}}}{\Sigma \mathrm{U_i}} = c_1 \cdot \underbrace{(x_2 - x_1)}_{\mathrm{const} \cdot \Delta I} = a_1 \cdot \Delta I$$

Theoretical:

$$\frac{\Delta \mathrm{U_{hor}}}{\Sigma \mathrm{U_i}} = c_1 x_0$$

Takes the first order of x_0 for the horizontal voltage ratio into account

Vertical:

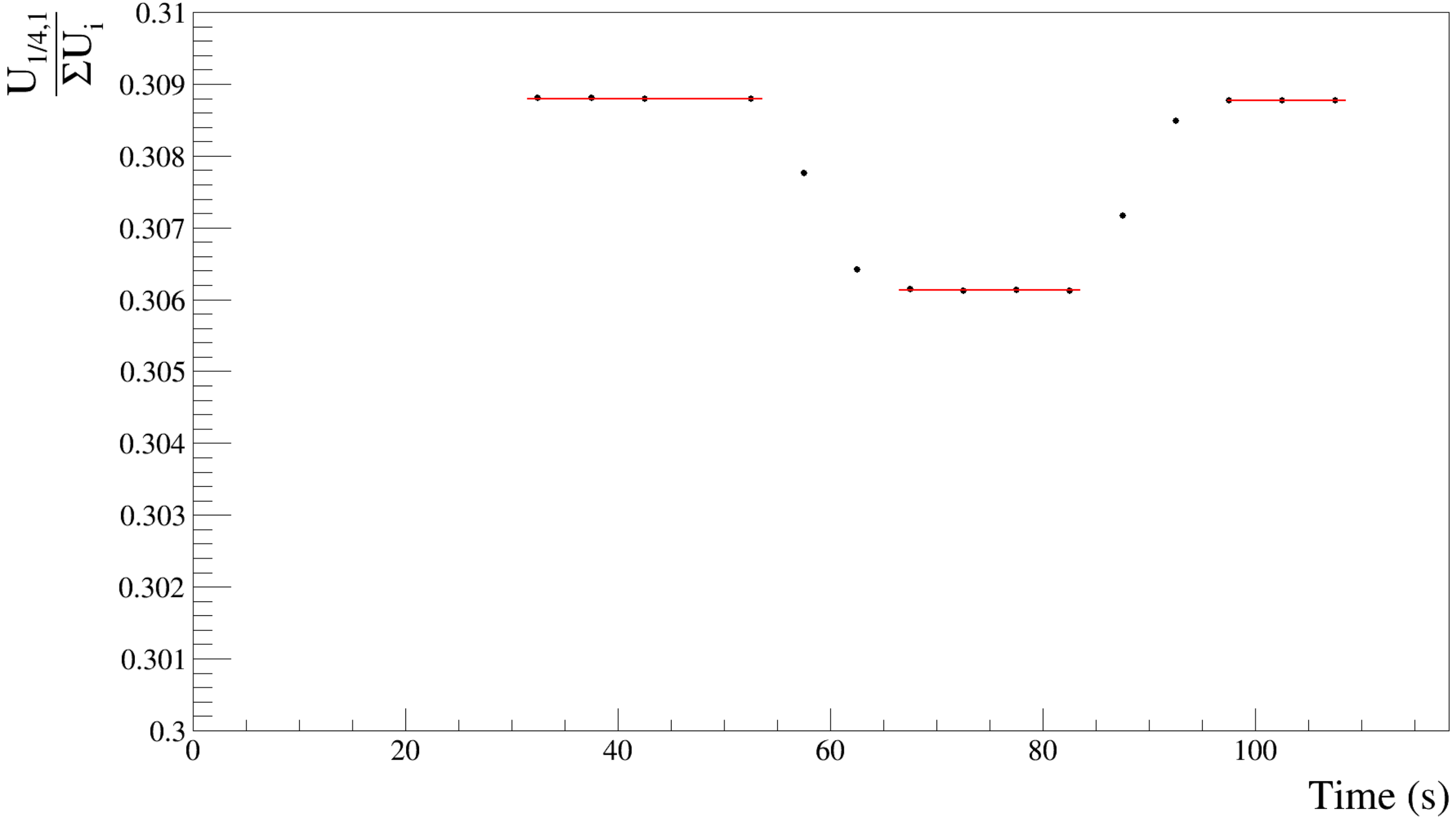
$$\Delta \frac{\Delta \mathrm{U_{ver}}}{\Sigma \mathrm{U_i}} = \frac{\Delta \mathrm{U_{ver,bump}}}{\Sigma \mathrm{U_i}} - \frac{\Delta \mathrm{U_{ver,initial}}}{\Sigma \mathrm{U_i}} = \underbrace{a_{1,2} - a_{1,1}}_{b_1} + a_2 \underbrace{(x_2 - x_1)}_{\Delta I} \underbrace{(x_2 + x_1)}_{I_0 + \Delta I} = b_1 + b_2 \cdot \Delta I + b_3 \cdot \Delta I^2$$

$$\frac{\Delta \mathrm{U_{ver}}}{\Sigma \mathrm{U_i}} = c_1 y_0 - c_3 (y_0^3 - 3 x_0^2 y_0) = a_1 + a_2 x_0^2$$

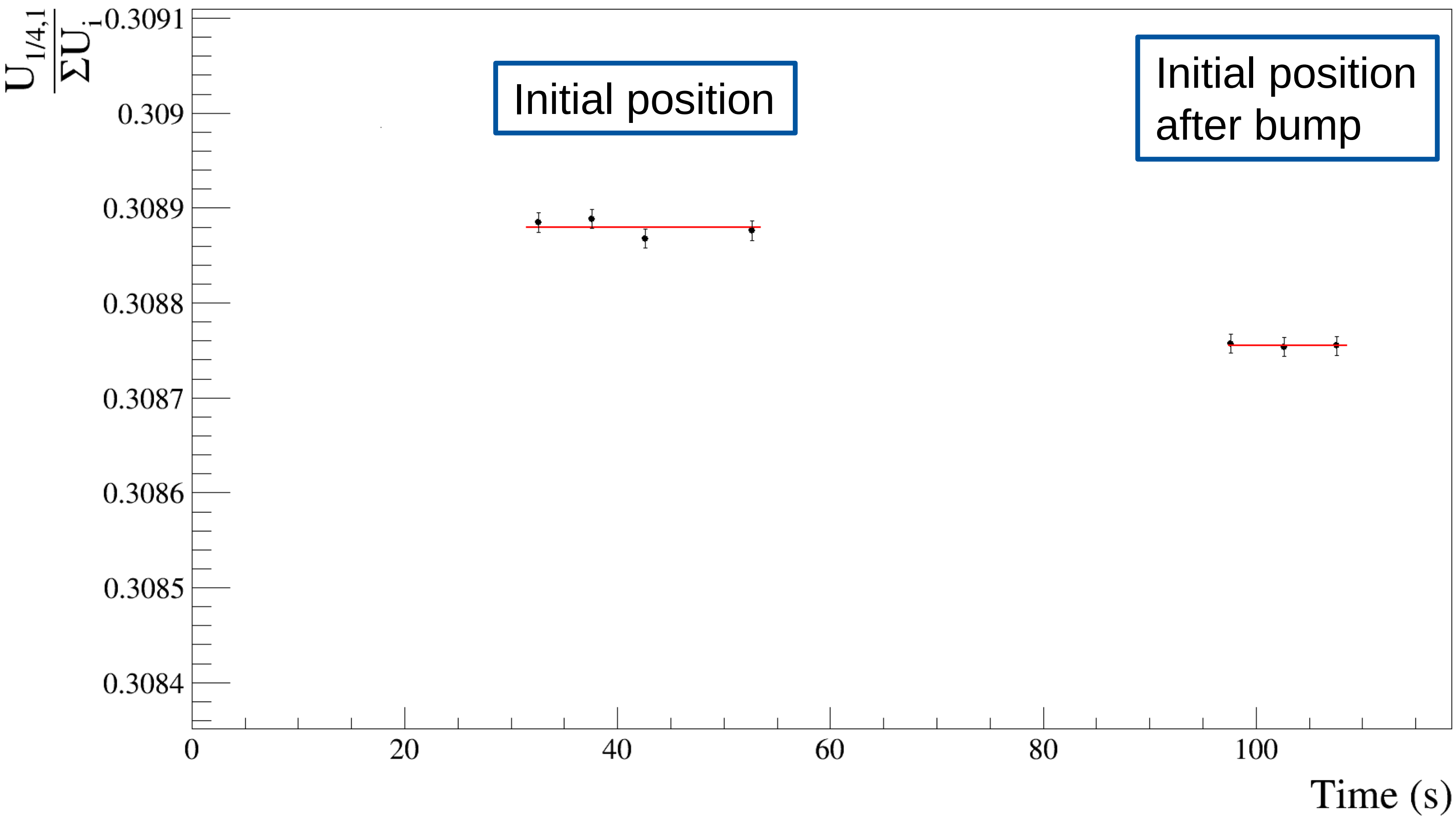
Takes the first x_0 for the vertical voltage ratio into account

Orbit bump

Horizontal orbit bumps with steerer value of 0%



Orbit bump: Comparison of initial and final orbit



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Voltage noise calculations

Thermal noise:

$$U = \sqrt{4k_{\mathrm{B}}TR\Delta f}$$

Calculation of the thermal noise for the different devices with a bandwidth of Δf = 6.81 Hz

Device	T (K)	σ_U (nV)
Lock-in amplifier	293.15	13.05
Low-noise preamplifier	293.15	2.00
Quartered segment	293.15	1.14
Cooled quartered segment	77.15	0.22
Quartered segment amplified	293.15	10.34
Cooled quartered segment amplified	77.15	2.07

Voltage noise calculations

Two different signal chains:

1. quartered segment
2. quartered segment + low-noise preamplifier

Readout of the signal chain is an uncooled lock-in amplifier

$$\sigma_{\mathrm{U_{total}}} = \sqrt{\sum_{\mathrm{i}} \sigma_{\mathrm{U_i}}^2}$$

Signal chain	T (K) for quartered segment	$\sigma_{\mathrm{U,total}}$ (nV)	σ_x (μm)
1	293.15	13.10	1.07
2	293.15	16.77	1.35
1	77.15	13.05	1.05
2	77.15	13.36	1.08

- Cooling only the signal chain lead not to a major increase of resolution because the dominant noise source is the lock-in amplifier itself