Spin Tracking for EDM Searches in Storage Rings - Special Emphasis on Intricacies in Electrostatic Elements

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Transfer Map Method and Differential Algebras

- The transfer map $\mathcal{M}$ is the flow of the system ODE.

$$\vec{z}_f = \mathcal{M}(\vec{z}_i, \vec{\delta}),$$

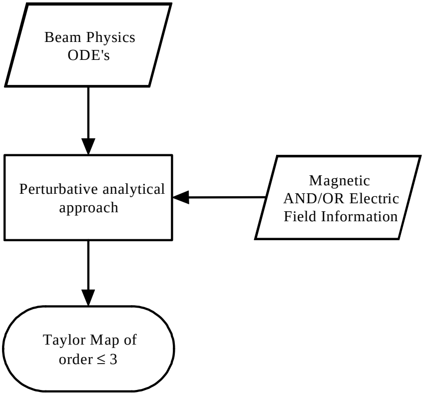
  where $\vec{z}_i$ and $\vec{z}_f$ are the initial and the final condition, $\vec{\delta}$ is system parameters.

- For a repetitive system, only one cell transfer map has to be computed. Thus, it is much faster than ray tracing codes (i.e. tracing each individual particle through the system).
- The Differential Algebraic method allows a very efficient computation of high order Taylor transfer maps.
- The Normal Form method can be used for analysis of nonlinear behavior.

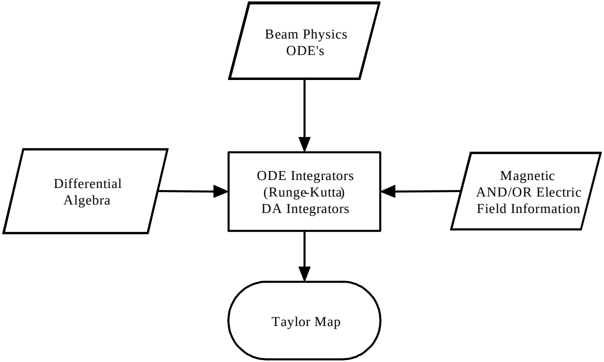
Differential Algebras (DA)

- it works to arbitrary order, and can keep system parameters in maps.
- very transparent algorithms; effort independent of computation order.

The code **COSY Infinity** has many tools and algorithms necessary.



- Method can be used to compute transfer map of order ≤ 3
- Analytic or local Taylor expansion (multipole decomposition) of the magnetic field should be specified
- Present/future accelerators require much higher order description



- DA methods were introduced in 1988 to compute maps to in principle arbitrary order
- Analytic formula or local expansion of the field should be specified

COSY INFINITY

- Arbitrary order
- Maps depending on parameters (mass dependence!)
- No approximations in motion or field description
- Large library of elements
- Arbitrary Elements (you specify fields)
- Very flexible input language
- Powerful interactive graphics
- Errors: position, tilt, rotation
- Tracking through maps
- Normal Form Methods
- Spin dynamics
- Fast fringe field models using SYSCA approach
- Reference manual (80 pages) and Programming manual (90 pages)
- As of December 2004, more than 1000 registered users

Codes Using DA methods

Since we introduced it, many/most modern codes use DA:

- PowerTrack (Berz) *)
- TPot (Talman) *)
- TLie (van Zeijts)
- ZLib (Yan)
- MXYZTPLK (Michelotti, ...)
- DACYC (Davies, ...)
- Classic (Iselin, ...)
- PTC (Forest, ...) *)
- MAD-X, SixTrack (Schmidt, ...) *)
- TPSALib (Yang)
- COSY Infinity *)

*) Using modern or earlier versions of our DA package

The Use of Schauder's Theorem

Re-write differential equation as integral equation

$$\vec{r}(t) = \vec{r}_0 + \int_{t_0}^{t} \vec{F}(\vec{r}(t'), t')\, dt'.$$

Now introduce the operator

$$A : \vec{C}^0[t_0, t_1] \to \vec{C}^0[t_0, t_1]$$

on space of continuous functions via

$$A\left(\vec{f}\right)(t) = \vec{r}_0 + \int_{t_0}^{t} \vec{F}\left(\vec{f}(t'), t'\right)\, dt'.$$

Then the solution of ODE is transformed to a fixed-point problem on space of continuous functions

$$\vec{r} = A(\vec{r}).$$

Theorem (Schauder): *Let A be a continuous operator on the Banach Space X. Let $M \subset X$ be compact and convex, and let $A(M) \subset M$. Then A has a fixed point in M, i.e. there is an $\vec{r} \in M$ such that $A(\vec{r}) = \vec{r}$.*

Field Description in Differential Algebra

There are various DA algorithms to treat the fields of beam optics efficiently. For example, **DA PDE Solver**

- requires to supply only
  - the midplane field for a midplane symmetric element.
  - the on-axis potential for straight elements like solenoids, quadrupoles, and higher multipoles.
- treats arbitrary fields straightforwardly.
  - Magnet (or, Electrostatic) fringe fields:
    The Enge function fall-off model

    $$F(s) = \frac{1}{1 + \exp(a_1 + a_2 \cdot (s/D) + ... + a_6 \cdot (s/D)^5)}$$

    where D is the full aperture.
    Or, any arbitrary model including the measured data representation.
  - Solenoid fields including the fringe fields.
  - Measured fields: E.g. Use Gaussian wevelet representation.
  - Etc. etc.

DA Fixed Point PDE Solvers

The **DA fixed point theorem** allows to **solve PDEs iteratively** in **finitely many steps** by rephrasing them in terms of a fixed point problem.

Consider the rather general PDE

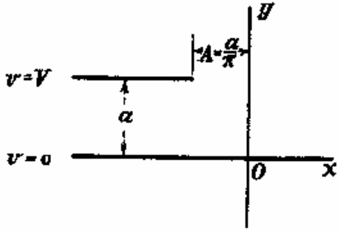
$$a_1\frac{\partial}{\partial x}\left(a_2\frac{\partial}{\partial x}V\right)+b_1\frac{\partial}{\partial y}\left(b_2\frac{\partial}{\partial y}V\right)+c_1\frac{\partial}{\partial z}\left(c_2\frac{\partial}{\partial z}V\right)=0,$$

where a_i, b_i, c_i are functions of x, y, z.

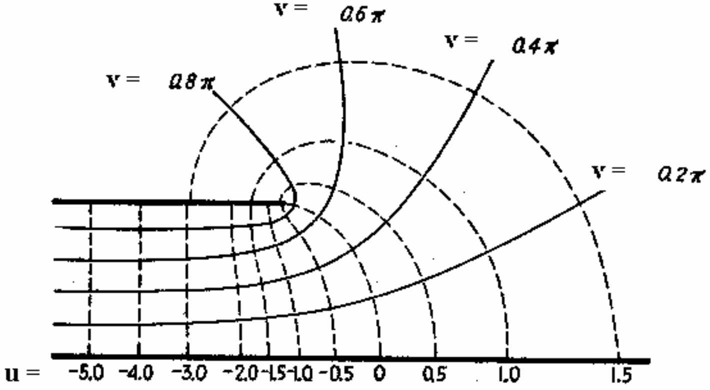
The PDE is re-written in **fixed point form** as

$$V = V|_{y=0} + \int_0^y \frac{1}{b_2}\left(b_2\frac{\partial V}{\partial y}\right)\bigg|_{y=0} - \int_0^y \frac{1}{b_2}\int_0^y \left(\frac{a_1}{b_1}\frac{\partial}{\partial x}\left(a_2\frac{\partial V}{\partial x}\right)+\frac{c_1}{b_1}\frac{\partial}{\partial z}\left(c_2\frac{\partial V}{\partial z}\right)\right) dydy.$$

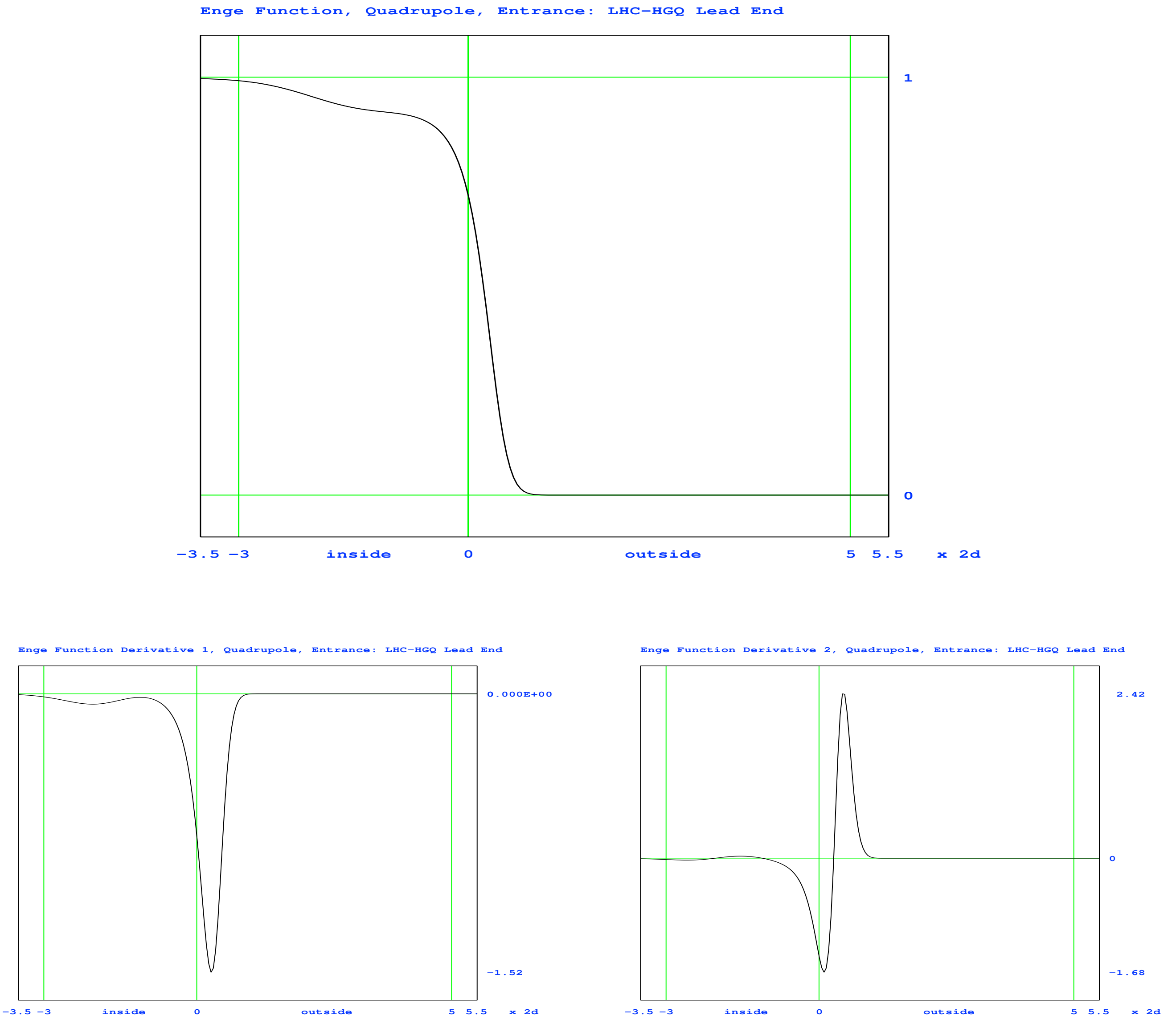
Assume the derivatives of V and $\partial V/\partial y$ with respect to x and z are **known in the plane** $y = 0$. Then the right hand side is **contracting** with respect to y (which is necessary for the DA fixed point theorem), and the various orders in y can be **iteratively** calculated by mere iteration.



(a)



LHC-HGQ Lead End Enge Function



Fringe Field Effects in Small Rings

Fringe fields are the regions of the element where the field falls off from being nearly constant with respect to position on reference axis to being nearly zero

- Fringe fields represent an important part of the ring
  - In small rings, the fringe field region can be about half of the total "length" of the optical element
  - (In LHC or Tevatron, elements are much longer compared to their aperture)
  - In small rings, the change of dynamics (phase advance or linear transfer matrix) in each element is large
  - (In LHC or Tevatron, the effect of each element is much less)
  - In small rings, even the reference orbit is affected
  - (In LHC or Tevatron, carefully finding the "Effective Field Boundary" avoids this problem

Fringe Fields and Their Nonlinearities

- Fringe fields are often the main source of (non-deliberate) nonlinearities
  - In main fields, one of course attempts very carefully to keep the field constant in direction of reference orbit, and imposes specific axial dependencies
  - In fringe fields, there is natural nonlinearity due to unavoidable curvature of electric or magnetic field lines.
  - These curvatures of fields affect particles at different distances from reference orbit differently, and because of curvature, they do so nonlinearly.
  - All these things are unavoidable; they are a direct consequence of Maxwell's equations.

Elements in COSY

- Magnetic and electric multipoles
- Superimposed multipoles
- Combined function bending magnets with curved edges
- Electrostatic deflectors
- Wien filters
- Wigglers
- Solenoids, various field configurations
- 3 tube electrostatic round lens, various configurations
- Exact fringe fields to all of the above
- Fast fringe fields (SYSCA)
- General electromagnetic element (measured data)
- Glass lenses, mirrors, prisms with arbitrary surfaces
- Misalignments: position, angle, rotation

All can be computed to arbitrary order, and the dependence on any of their parameters can be computed.

Normal Form Theory

Goal: perform a nonlinear change of variables such that the motion in the new variable pairs is rotationally invariant:

$\mathcal{M} \circ \mathcal{R} = \mathcal{R} \circ \mathcal{M}$

If the map is symplectic, this means circles. If the map is damped, we obtain logarithmic spirals.

Advantage: Tune with amplitude is trivial to compute, since each iteration of the map corresponds to the same angle advance.

Other Advantages: - Provides pseudo invariants the quality of which allows conclusions about the map; - sensitive to resonances, allows efficient study of resonances

Decoupling of Planes in Linear Map

Goal: Coordinate Transformation such that linear Map that has only two-by-two blocks along diagonal.

If Linear Map is not in block form, it can be brought into block form if it has n disctinct Eigenvalues. (Other cases are not of interest because they imply resonances)

Eigenvalues are either

- a complex conjugate pair $\rightarrow$ choose the real and imaginary parts of the eigenvector
- a real pair $\rightarrow$ choose the two eigenvectors

The resulting map is purely real. When it is fully diagonlized it becomes complex.

The DA Normal Form Algorithm

Assume linear part of map has been diagonalized by a linear change of basis:

$$\mathcal{M} = \mathcal{R} + \mathcal{S}$$

where $\mathcal{R}$ has on its diagonal the values $r_j \cdot e^{\pm i\nu_j}$ (pair structure).

Now attempt to simplify the map by a nonlinear transformation. Choose transformation

$$\mathcal{A}_m = \mathcal{E} + \mathcal{T}_m$$

Up to order m, the inverse is $\mathcal{A}_m^{-1} =_m \mathcal{E} - \mathcal{T}_m$, and we obtain

$$\begin{aligned}
&\mathcal{A} \circ \mathcal{M} \circ \mathcal{A}^{-1} \\
&=_m (\mathcal{E} + \mathcal{T}_m) \circ (\mathcal{R} + \mathcal{S}_{m-1}) \circ (\mathcal{E} - \mathcal{T}_m) \\
&=_m (\mathcal{E} + \mathcal{T}_m) \circ (\mathcal{R} + \mathcal{S}_{m-1}) \circ (\mathcal{E} - \mathcal{T}_m) \\
&=_m \mathcal{R} + \mathcal{S}_{m-1} + (\mathcal{T}_m \circ \mathcal{R} - \mathcal{R} \circ \mathcal{T}_m)
\end{aligned}$$

Removing Terms in the Normal Form Step

We can use the commutator $\mathcal{C}$ of $\mathcal{T}_m$ and $\mathcal{R}$ to remove terms from $\mathcal{S}_{m-1}$. We write

$$\mathcal{T}^{\pm}_{mj} = \sum (\mathcal{T}^{\pm}_{mj}|k_1^+, k_1^-, ..., k_n^+, k_n^-) \cdot (v_1^+)^{k_1^+}(v_1^-)^{k_1^-} \cdot ... \cdot (v_n^+)^{k_n^+}(v_n^-)^{k_n^-}$$

$$\mathcal{C}^{\pm}_{mj} = \sum (\mathcal{C}^{\pm}_{mj}|k_1^+, k_1^-, ..., k_n^+, k_n^-) \cdot (v_1^+)^{k_1^+}(v_1^-)^{k_1^-} \cdot ... \cdot (v_n^+)^{k_n^+}(v_n^-)^{k_n^-}$$

Because of the simple form of $\mathcal{R}$, we obtain

$$(\mathcal{C}^{\pm}_j|k_1, k_1^-, ..., k_n^+, k_n^-)$$
$$= -C^{\pm}_j(\vec{k}^+, \vec{k}^-) \cdot (\mathcal{T}^{\pm}_j|k_1^+, k_1^-, ..., k_n^+, k_n^-)$$

where

$$C^{\pm}_j(\vec{k}^+, \vec{k}^-) = r_j \cdot e^{\pm i\nu_j} - \left(\prod_{j=1}^{n}(r_j)^{k_j^+ + k_j^-}\right) \cdot e^{i\vec{\nu}\cdot(\vec{k}^+ - \vec{k}^-)}$$

So we can remove every term for which $C^{\pm}_j(\vec{k}^+, \vec{k}^-)$ is nonzero!

Removable Terms in the Symplectic Case

In the symplectic case, all r_j are one (no damping). Then everything is removable except if

$$\vec{\nu} \cdot (\vec{k}^{+} - \vec{k}^{-}) = l \cdot 2\pi \pm \nu_j \quad \forall l$$

This can occur in the following cases:

1. $\vec{n} \cdot \vec{\nu} = l \cdot 2\pi$ has nontrivial solutions (we are on a resonance; physics case)
2. $k_l^{+} = k_l^{-} \;\; \forall\, l \neq j$, and $k_j^{+} = k_j^{-} \pm 1$ (unavoidable; mathematics case)

Removable Terms under Damping

In case there is damping (or blow up), some of the r_j are not 1. In this case, additional terms can be removed.

Of particular interest is the case of total damping in which all r_j are less than one. Then everything can be removed except

1. $k_l^+ = k_l^- = 0 \;\; \forall\, l \neq j$, and $k_j^+ = k_j^- \pm 1$ (unavoidable; mathematics case)

But this is the identity!

This has important consequences:

- Damped systems are not susceptible to resonances
- There are no amplitude dependent tune shifts in damped systems

Resonance Correction and Tune Shifts

Amplitude dependent tune shifts obtained with DA normal form theory.Before and after resonance correction with sextupoles

1	0.4499999999999998	0	0	0	0	0
2	30.71450631162792	2	2	0	0	0
3	-39734.01363530685	2	0	0	2	0
4	3077.595867175395	4	4	0	0	0
5	-315212453990.7433	4	2	0	2	0
6	-294628537556.6385	4	0	0	4	0

1	0.4499999999999998	0	0	0	0	0
2	8.691395574893371	2	2	0	0	0
3	241.0982265780670	2	0	0	2	0
4	1247.729293865072	4	4	0	0	0
5	757332.8310454757	4	2	0	2	0
6	3727722.890469429	4	0	0	4	0

The BMT Equation

Classical equation of Motion for Spin:

$$\frac{d\vec{s}}{dt} = \vec{w} \times \bar{s}, \text{where}$$

$$\vec{w} = k \left(-(1+G\gamma)\vec{B} + \frac{G}{1+\gamma}(\vec{P}\cdot\vec{B})\ \vec{P} + (G + \frac{1}{1+\gamma})\ \vec{P} \times \frac{\vec{E}}{c} \right),$$

and $k = e/\gamma m_o c$, $G = (g-2)/g$, $\vec{P} = \vec{p}/m_0 c$.

In particle optical relative coordinates:

$$\frac{d\vec{s}}{ds} = t' \cdot \vec{w} \times \bar{s} + \vec{h} \times \vec{s}, \text{where } s\text{: arclength, } \vec{h}\text{: curvature}$$

Solution is a *linear orthogonal transformation* depending on orbital variables. Thus

$$\vec{s}_f = \hat{A}(\vec{z}) \cdot \vec{s}_i, \text{ where}$$

$$\hat{A}(\vec{z}) \in SO(3)$$

Motion Of Spin Matrix

Nine dimensional motion of particle with spin, neglecting spin-orbit coupling

$$\begin{pmatrix} \vec{z} \\ \vec{s} \end{pmatrix}' = \vec{F}(\vec{z}, \vec{s}, s) = \begin{pmatrix} \vec{f}(\vec{z}, s) \\ \hat{W}(\vec{z}, s) \cdot \vec{s} \end{pmatrix}$$

$$\begin{pmatrix} \vec{z}_f \\ \vec{s}_f \end{pmatrix} = \vec{M}(\vec{z}_i, \vec{s}_i, s) = \begin{pmatrix} \mathcal{M}(\vec{z}_i, s) \\ \hat{A}(\vec{z}_i, s) \cdot \vec{s} \end{pmatrix}$$

To *reduce dimensionality* and *utilize linearity*, it is advantageous to set up EOM for $\hat{A}$. Insertion yields EOM for 3×3 spin matrix depending on only the 6 orbital variables:

$$\hat{A}'(\vec{z}, s) = \hat{W}(\vec{z}, s) \cdot \hat{A}(\vec{z}, s)$$

Spin Dynamics in SU(2) Representation

Describe spin vector $\vec{s}$ by matrix

$$\hat{L} = \begin{pmatrix} s_3 & s_1 + is_2 \\ s_1 - is_2 & -s_3 \end{pmatrix}.$$

Then dynamics is described by

$$\frac{d\hat{L}}{dt} = \hat{B} \cdot \hat{L} - \hat{L} \cdot \hat{B}, \text{ where}$$

$$\hat{B} = \frac{i}{2} \cdot \begin{pmatrix} w_3 & w_1 + iw_2 \\ w_1 - iw_2 & -w_3 \end{pmatrix}$$

The propagation of the matrix $\hat{L}$ is described by a unitary transformation

$$\hat{L}_f = \hat{U}(\vec{x}_i) \cdot \hat{L}_i \cdot \hat{U}^*(\vec{x}_i) \text{ where}$$

$$\hat{U} \in SU(2), \text{ i.e. } \det(\hat{U}) = 1 \text{ and } \hat{U}^* \cdot \hat{U} = \hat{I}$$

Transformation Between SO(3) and SU(2)

Write $\hat{U} \in SU(2)$ as

$$\hat{U} = \begin{pmatrix} a & b \\ -b^* & a^* \end{pmatrix}, \text{ with } |a|^2 + |b|^2 = 1.$$

Then from $\hat{L}_f = \hat{U}(\vec{x}_i) \cdot \hat{L}_i \cdot \hat{U}^*(\vec{x}_i)$, we get

$$\hat{A} = \begin{pmatrix} \mathrm{Re}(a^2 - b^2) & -\mathrm{Im}(a^2 + b^2) & -2\mathrm{Re}(ab) \\ \mathrm{Im}(a^2 - b^2) & \mathrm{Re}(a^2 + b^2) & -2\mathrm{Im}(ab) \\ 2\mathrm{Re}(b^* a) & 2\mathrm{Im}(a^* b) & aa^* - bb^* \end{pmatrix}$$

Conversely, if $\hat{A} = (A_{i,j})$ is given, then from above

$$a^2 = \tfrac{1}{2}(A_{11} + iA_{21} + A_{22} - iA_{12})$$
$$b^2 = -\tfrac{1}{2}(A_{11} + iA_{21} - A_{22} + iA_{12})$$
$$ab = -\tfrac{1}{2}(A_{13} + iA_{23})$$

which can be solved uniquely using $|a|^2 + |b|^2 = 1$.

DA Computation of Spin Motion

Instead of motion in nine variables, it is sufficient to use six dimensional DA to determine Taylor expansion of $\hat{A}(\vec{z})$:

Autonomous Case (Main Fields): Utilize existence of two invariant subspaces of mixed symplectic-orthogonal propagator $\exp(sL_{\vec{F}})$ with directional derivative

$$L_{\vec{F}} = \vec{f}^t \cdot \vec{\nabla}_{\vec{z}} + (\hat{W} \cdot \vec{s})^t \cdot \vec{\nabla}_{\vec{s}}$$

and exploit

$$\vec{M}(\vec{z}_i, \vec{s}_i, s) = \exp(sL_{\vec{F}})\ \vec{I}.$$

Non-Autonomous Case (Fringe Fields, Wigglers, Measured Fields etc.): Integrate equations of motion for orbit and spin matrix in DA.

Composition of Maps: Let $(\vec{M}_{1,2}, \hat{A}_{1,2})$ and $(\vec{M}_{2,3}, \hat{A}_{2,3})$ be given. Then we get

$$\vec{M}_{1,3} = \vec{M}_{2,3} \circ \vec{M}_{1,2}$$
$$\hat{A}_{1,3}(\vec{z}) = \hat{A}_{2,3}(\vec{M}_{1,2}) \cdot \hat{A}_{1,2}(\vec{z})$$

Invariant Subspaces of $L_{\vec{F}}$

Define two spaces of functions $g(\vec{z}, \vec{s})$ on spin-orbit phase space as follows:

Z: space of functions depending on $\vec{z}$

S:space of linear forms in $\vec{s}$ with coefficients in Z

Then we have for $g \in Z$:

$$L_{\vec{F}}\ g = (\vec{f}^t \cdot \vec{\nabla}_{\vec{z}} + (\hat{W} \cdot \vec{s})^t \cdot \vec{\nabla}_{\vec{s}})\ g = \vec{f}^t \cdot \vec{\nabla}_{\vec{z}}\ g = L_{\vec{f}}\ g$$

Similarly, we have for $g =< a_j >= \sum_j a_j \cdot s_j \in S$:

$$L_{\vec{F}}\ < a_j >= (\vec{f}^t \cdot \vec{\nabla}_{\vec{z}} + (\hat{W} \cdot \vec{s})^t \cdot \vec{\nabla}_{\vec{s}})\ (\textstyle\sum_j a_j \cdot s_j)$$
$$= \textstyle\sum_j (\vec{f}^t \cdot \vec{\nabla}_{\vec{z}}) a_j \cdot s_j + \sum_{j,k} s_j W_{kj} a_k =< L_{\vec{f}} a_j + \sum_k W_{kj} a_k > .$$

Thus Z and S are invariant subspaces of $L_{\vec{F}}$. Further, the components of $\hat{I}$ are in Z and S, and hence so is $\exp(sL_{\vec{F}})$.

Since elements in either space are charecterized by just six dimensional functions, $\exp(sL_{\vec{F}})$ can be computed in a six dimensional differential algebra.

Spin Tracking

Spin tracking is performed iteratively with the following steps:

1. Evaluate orbit map on current orbit coordinates $\vec{z}_n$ to get new coordinates $\vec{z}_{n+1}$:
$$\vec{z}_{n+1} = \vec{M}(\vec{z}_n),$$
symplectify if needed or desired.

2. Insert new orbit coordinates into spin matrix to get
$$\hat{A}^* = \hat{A}(z_{n+1}),$$
orthogonalize $\hat{A}^*$ if needed or desired; simply requires renormalization of $\vec{s}_{n+1}$.

3. Multiply current spin matrix $\hat{A}^*$ with current spin coordianates $\vec{s}_n$ to get new spin coordinates $\vec{s}_{n+1}$

4. Display coordinates of $\vec{s}_{n+1}$

The Spin Tune Shifts

The determination of spin amplitude tune shifts requires the following steps:

1. Bring orbital motion to normal form N describing amplitude dependent rotations.
2. Express spin matrix $\hat{A}$ in terms of orbital normal form
3. Solve 3×3 eigenvalue problem for $\hat{A}$
4. Linear part of conjugate non-unity eigenvalues give spin tune
5. Nonlinear part of non-unity eigenvalues give spin tune dependence on orbital amplitudes

The Invariant Axis $\bar{n}$

One of the quantities of prime interest: Invariant polarization axis $\bar{n}(\vec{z})$. It depends on orbit quantities and satisfies

$$\hat{A}(\vec{z}) \cdot \bar{n}(\vec{z}) = \bar{n}(\vec{\mathcal{M}}(\vec{z}))$$

We develop a method to obtain $\bar{n}(\vec{z})$ with coupled $SO(3)$ and symplectic normal form methods.

To begin, assume orbital map is already in normal form and linear spin map is diagonalized.

For zeroth order, observe that $\vec{\mathcal{M}}(\vec{z}) =_0 0$. Denote $\bar{n}_0 = \bar{n}(0)$.The equation reduces to

$$\hat{A}_0 \cdot \bar{n}_0 = \bar{n}_0$$

Thus, $\bar{n}_0$ is the eigenvector to unit eigenvalue of the constant part of $\hat{A}$.

The Invariant Axis $\bar{n}$ - Higher Orders

Now proceed iteratively; assume we know $\bar{n}$ to order $m-1$ and want to determine it to order m. Assume $\vec{\mathcal{M}}(\vec{z})$ is in normal form, i.e. $\vec{\mathcal{M}}(\vec{z}) = \mathcal{R} + \mathcal{N}$. Write $\hat{A} = \hat{A}_0 + \hat{A}_{\geq 1}$, $\bar{n} = \bar{n}_{<m} + \bar{n}_m$, and obtain

$$(\hat{A}_0 + \hat{A}_{\geq 1}) \cdot (\bar{n}_{<m} + \bar{n}_m) = (\bar{n}_{<m} + \bar{n}_m) \circ (\mathcal{R} + \mathcal{N})$$

To order m, this can be rewritten

$$\hat{A} \cdot \bar{n}_{<m} + \hat{A}_0 \cdot \vec{n}_m =_m \bar{n}_{<m} \circ (\mathcal{R} + \mathcal{N}) + \bar{n}_m \circ R, \text{ or}$$
$$\hat{A}_0 \bar{n}_m - \bar{n}_m \circ \mathcal{R} =_m \bar{n}_{<m} \circ (\mathcal{R} + \mathcal{N}) - \hat{A} \cdot \vec{n}_{<m}.$$

The right hand side has to be balanced by choosing $\bar{n}_m$ appropriately. But like in orbit case, the coefficients of $\hat{A}_0 \vec{n}_m - \vec{n}_m \circ \mathcal{R}$ differ from those of $\bar{n}_m$ only by resonance denominators.

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The Pain with Electrostatic Elements

- Unless one is very careful, there will be various undesirable effects:
  - The motion from before to after the element satisfies energy conservation, but the integrator does not know this
  - Repeated small violations of energy conservation can lead to either oscillations, or big long term effects
  - Particular problem: due to offset of reference orbit, it is very useful to re-align elements. This is normally done after each part:
  - After entrance fringe field, after main field, after exit fringe field
  - Each re-alignment causes small change in geometry, and hence small change in potential!

Testing Electrostatic Elements

- The Spherical Capacitor is an excellent test case for studying the performance of codes:
  - Observe that just in the gravitational case, the motion follows Kepler orbits
  - Set up a “lattice” consisting of n spherical capacitors of angle $2\pi/n$
  - Compute dynamics for one revolution, it should be the identity
  - In particular, in map picture, this gives a nice effect because ALL transfer matrix elements have to vanish
  - Very important: need non-relativistic motion for testing! (Remember precession of perihelion of Mercury)

MS <radius> <angle> <aperture> $< n_1 >< n_2 >< n_3 >< n_4 >< n_5 >$;

ES <radius> <angle> <aperture> $< n_1 >< n_2 >< n_3 >< n_4 >< n_5 >$;

The radius is measured in meters, the angle in degrees, and the aperture is in meters and corresponds to half of the gap width. The indices n_i describe the midplane radial field dependence which is given by

$$F(x) = F_0 \cdot \left[1 - \sum_{i=1}^{5} n_i \cdot (\frac{x}{r_0})^i \right]$$

where r_0 is the bending radius. Note that an electric cylindrical condenser has $n_1 = 1$, $n_2 = -1$, $n_3 = 1$, $n_4 = -1$, $n_5 = 1$, etc, and an electric spherical condenser has $n_1 = 2$, $n_2 = -3$, $n_3 = 4$, $n_4 = -5$, $n_5 = 6$, etc. Homogeneous dipole magnets have $n_i = 0$.

Since an electric cylindrical condenser is invariant under translation along the y axis, the y motion is like a drift. An offset in the y direction does not alter the motion, so a map produced by **ES**, $\mathcal{M}_{ES}$, and a y offset map $\mathcal{M}_{\triangle y}$, produced by "**SA** 0 DA(5) ;" for example, commute. A similar consistency test can be performed for an electric spherical condenser. Consider an offset $\triangle\theta$ along the radius R sphere toward the positive y direction, and call the map $\mathcal{M}_{\triangle}$. For example, such a map $\mathcal{M}_{\triangle}$ can be produced by "**SA** -R 0 ; **RA** DA(5) ; **SA** R 0 ;". The map of a spherical condenser for a 180° travel, represented by $\mathcal{M}_{ES180}$, agrees with the map of a $\triangle$ offset, a 180° travel in the condenser and a $\triangle$ offset again. Another test is based on the observation that the motion is that of a Kepler problem. The motion should return to the original state after one cycle travel, thus $\mathcal{M}_{ES360}$ should become an identity map.

The element

unity map test (for ES Spherical)

```
  1.000000     0.6745274E-08  0.000000      0.000000      0.000000     1000000
-0.6745265E-08  1.000000      0.000000      0.000000      0.000000     0100000
  0.000000      0.000000      1.000000     0.7076424E-14  0.000000     0010000
  0.000000      0.000000     0.1078906E-14  1.000000      0.000000     0001000
 0.1960073E-13-0.6745267E-08  0.000000      0.000000      0.000000     0000001
 0.1250518E-11 0.4073090E-14  0.000000      0.000000      0.000000     2000000
-0.1349320E-07-0.5151423E-14  0.000000      0.000000      0.000000     1100000
-0.2007760E-11 0.6745271E-08  0.000000      0.000000      0.000000     0200000
  0.000000      0.000000    -0.3080058E-12 0.6745268E-08  0.000000     1010000
  0.000000      0.000000    -0.6748311E-08-0.1367794E-13  0.000000     0110000
 0.5851285E-12 0.3371156E-08  0.000000      0.000000      0.000000     0020000
  0.000000      0.000000    -0.7270661E-12 0.2327415E-13  0.000000     1001000
  0.000000      0.000000    -0.9280394E-12 0.6745264E-08  0.000000     0101000
-0.6744571E-08-0.3180281E-11  0.000000      0.000000      0.000000     0011000
-0.1771120E-11-0.1011791E-07  0.000000      0.000000      0.000000     1000001
 0.3368683E-08-0.9681441E-14  0.000000      0.000000      0.000000     0100001
  0.000000      0.000000    -0.3136933E-11-0.6745272E-08  0.000000     0010001
-0.6305797E-12 0.2567664E-12  0.000000      0.000000      0.000000     0002000
  0.000000      0.000000     0.3741119E-11-0.4766519E-14  0.000000     0001001
 0.1817235E-12 0.3372632E-08  0.000000      0.000000      0.000000     0000002
----------------------------------------------------------------------------
```

unity map test (for ES Spherical)

1.000000	0.6745274E-08	0.000000	0.000000	0.000000	1000000
-0.6745265E-08	1.000000	0.000000	0.000000	0.000000	0100000
0.000000	0.000000	1.000000	0.7076424E-14	0.000000	0010000
0.000000	0.000000	0.1078906E-14	1.000000	0.000000	0001000
0.1960073E-13	-0.6745267E-08	0.000000	0.000000	0.000000	0000001
0.1250518E-11	0.4073090E-14	0.000000	0.000000	0.000000	2000000
-0.1349320E-07	-0.5151423E-14	0.000000	0.000000	0.000000	1100000
-0.2007760E-11	0.6745271E-08	0.000000	0.000000	0.000000	0200000
0.000000	0.000000	-0.3080058E-12	0.6745268E-08	0.000000	1010000
0.000000	0.000000	-0.6748311E-08	-0.1367794E-13	0.000000	0110000
0.5851285E-12	0.3371156E-08	0.000000	0.000000	0.000000	0020000
0.000000	0.000000	-0.7270661E-12	0.2327415E-13	0.000000	1001000
0.000000	0.000000	-0.9280394E-12	0.6745264E-08	0.000000	0101000
-0.6744571E-08	-0.3180281E-11	0.000000	0.000000	0.000000	0011000
-0.1771120E-11	-0.1011791E-07	0.000000	0.000000	0.000000	1000001
0.3368683E-08	-0.9681441E-14	0.000000	0.000000	0.000000	0100001
0.000000	0.000000	-0.3136933E-11	-0.6745272E-08	0.000000	0010001
-0.6305797E-12	0.2567664E-12	0.000000	0.000000	0.000000	0002000
0.000000	0.000000	0.3741119E-11	-0.4766519E-14	0.000000	0001001
0.1817235E-12	0.3372632E-08	0.000000	0.000000	0.000000	0000002
0.5533440E-09	0.1011790E-07	0.000000	0.000000	0.000000	3000000
-0.1702689E-07	0.1478432E-13	0.000000	0.000000	0.000000	2100000
-0.1169976E-10	0.1686319E-07	0.000000	0.000000	0.000000	1200000
-0.9656353E-08	0.7932441E-14	0.000000	0.000000	0.000000	0300000
0.000000	0.000000	-0.1081623E-08	-0.6745259E-08	0.000000	2010000
0.000000	0.000000	-0.4412272E-08	0.2145719E-13	0.000000	1110000
0.000000	0.000000	0.8653551E-09	0.5243032E-15	0.000000	0210000
-0.6909305E-09	-0.6741664E-08	0.000000	0.000000	0.000000	1020000
0.7546713E-09	0.1426317E-11	0.000000	0.000000	0.000000	0120000
0.000000	0.000000	-0.2840469E-09	0.3608056E-08	0.000000	0030000
0.000000	0.000000	0.1412215E-08	0.2174629E-13	0.000000	2001000
0.000000	0.000000	-0.1534852E-09	0.6745295E-08	0.000000	1101000
0.000000	0.000000	-0.5394932E-10	0.3867638E-13	0.000000	0201000
-0.7391106E-08	0.2988533E-11	0.000000	0.000000	0.000000	1011000
0.6034106E-09	0.6752251E-08	0.000000	0.000000	0.000000	0111000
0.000000	0.000000	-0.7147032E-08	-0.5312460E-09	0.000000	0021000
0.1768529E-08	-0.3035372E-07	0.000000	0.000000	0.000000	2000001
0.2325481E-07	-0.2513020E-13	0.000000	0.000000	0.000000	1100001
0.1241695E-08	-0.1349054E-07	0.000000	0.000000	0.000000	0200001
0.000000	0.000000	0.2481365E-08	-0.3372599E-08	0.000000	1010001
0.000000	0.000000	0.2107295E-08	0.2355462E-14	0.000000	0110001
0.2328773E-09	-0.1689794E-08	0.000000	0.000000	0.000000	0020001
-0.7399307E-10	-0.8518364E-11	0.000000	0.000000	0.000000	1002000
0.3400791E-09	-0.4805750E-11	0.000000	0.000000	0.000000	0102000
0.000000	0.000000	0.3079728E-09	0.6477583E-08	0.000000	0012000
0.000000	0.000000	-0.6836197E-09	-0.1427027E-13	0.000000	1001001
0.000000	0.000000	0.8183832E-09	-0.3372569E-08	0.000000	0101001
0.2055210E-08	0.5635014E-11	0.000000	0.000000	0.000000	0011001
0.1545877E-08	0.3288320E-07	0.000000	0.000000	0.000000	1000002
-0.9206106E-08	-0.2608718E-13	0.000000	0.000000	0.000000	0100002
0.000000	0.000000	-0.2041249E-09	0.3372596E-08	0.000000	0010002
0.000000	0.000000	-0.4555206E-09	0.2368727E-09	0.000000	0003000

-0.1692201E-09	0.2279837E-11	0.000000	0.000000	0.000000	0002001
0.000000	0.000000	0.2838449E-09	0.2809226E-13	0.000000	0001002
0.6944120E-09	-0.9274779E-08	0.000000	0.000000	0.000000	0000003
0.1301495E-06	0.6451537E-14	0.000000	0.000000	0.000000	4000000
-0.3730385E-06	-0.5967129E-14	0.000000	0.000000	0.000000	3100000
0.1254413E-05	0.3035372E-07	0.000000	0.000000	0.000000	2200000
0.9394927E-06	0.1594603E-13	0.000000	0.000000	0.000000	1300000
0.1273916E-06	0.1011790E-07	0.000000	0.000000	0.000000	0400000
0.000000	0.000000	-0.3336485E-06	0.1686316E-07	0.000000	3010000
0.000000	0.000000	0.8664680E-06	-0.1861908E-13	0.000000	2110000
0.000000	0.000000	0.1066022E-05	0.1011791E-07	0.000000	1210000
0.000000	0.000000	0.1221455E-06	-0.2007998E-13	0.000000	0310000
0.5675642E-06	0.2530136E-07	0.000000	0.000000	0.000000	2020000
-0.1840534E-06	-0.3818392E-12	0.000000	0.000000	0.000000	1120000
-0.5580093E-06	0.5060438E-08	0.000000	0.000000	0.000000	0220000
0.000000	0.000000	-0.9677183E-07	-0.9546061E-08	0.000000	1030000
0.000000	0.000000	-0.5415099E-07	-0.2962033E-09	0.000000	0130000
0.1466944E-07	-0.3660510E-07	0.000000	0.000000	0.000000	0040000
0.000000	0.000000	-0.8748181E-07	0.4514130E-14	0.000000	3001000
0.000000	0.000000	0.7278789E-06	0.1011791E-07	0.000000	2101000
0.000000	0.000000	-0.6844266E-06	0.3843864E-13	0.000000	1201000
0.000000	0.000000	-0.2207211E-06	0.1011790E-07	0.000000	0301000
0.4805153E-07	-0.6085563E-11	0.000000	0.000000	0.000000	2011000
-0.7295426E-06	0.2025166E-07	0.000000	0.000000	0.000000	1111000
-0.1384967E-08	0.4243910E-11	0.000000	0.000000	0.000000	0211000
0.000000	0.000000	-0.4431538E-06	0.2012506E-08	0.000000	1021000
0.000000	0.000000	-0.2975732E-06	-0.1701831E-09	0.000000	0121000
-0.5849968E-07	-0.4233868E-06	0.000000	0.000000	0.000000	0031000
0.8664604E-07	-0.3878524E-07	0.000000	0.000000	0.000000	3000001
-0.6658891E-06	-0.4517800E-13	0.000000	0.000000	0.000000	2100001
0.7211017E-06	-0.7251163E-07	0.000000	0.000000	0.000000	1200001
0.1102797E-05	-0.4479199E-13	0.000000	0.000000	0.000000	0300001
0.000000	0.000000	-0.9274011E-06	-0.2866742E-07	0.000000	2010001
0.000000	0.000000	0.6697516E-06	0.2756410E-13	0.000000	1110001
0.000000	0.000000	0.9280774E-06	-0.8431581E-08	0.000000	0210001
0.2428273E-07	-0.2697255E-07	0.000000	0.000000	0.000000	1020001
-0.3642728E-06	0.4439068E-11	0.000000	0.000000	0.000000	0120001
0.000000	0.000000	0.1272174E-06	-0.1895547E-08	0.000000	0030001
0.5569780E-06	-0.2869571E-11	0.000000	0.000000	0.000000	2002000
-0.5201408E-06	-0.1219200E-10	0.000000	0.000000	0.000000	1102000
0.2391525E-06	0.1129476E-11	0.000000	0.000000	0.000000	0202000
0.000000	0.000000	0.2445912E-06	0.1980308E-08	0.000000	1012000
0.000000	0.000000	0.2473107E-06	-0.4403271E-09	0.000000	0112000
0.9141858E-08	0.7199813E-07	0.000000	0.000000	0.000000	0022000
0.000000	0.000000	0.2781381E-05	-0.4347771E-13	0.000000	2001001
0.000000	0.000000	-0.2107168E-05	-0.2698094E-07	0.000000	1101001
0.000000	0.000000	0.1591568E-07	-0.1208460E-13	0.000000	0201001
-0.9074227E-06	0.6708081E-11	0.000000	0.000000	0.000000	1011001
0.7191120E-06	-0.2360661E-07	0.000000	0.000000	0.000000	0111001
0.000000	0.000000	-0.2180719E-07	-0.9986139E-09	0.000000	0021001
-0.5992411E-06	0.8600213E-07	0.000000	0.000000	0.000000	2000002
-0.3438104E-06	-0.2412441E-13	0.000000	0.000000	0.000000	1100002
0.7345605E-06	0.2782412E-07	0.000000	0.000000	0.000000	0200002
0.000000	0.000000	0.5468222E-07	0.3288308E-07	0.000000	1010002

0.000000	0.000000	0.5541707E-07	-0.4032768E-13	0.000000	0110002
0.3058835E-07	0.1476017E-07	0.000000	0.000000	0.000000	0020002
0.000000	0.000000	0.2428966E-06	0.1290941E-08	0.000000	1003000
0.000000	0.000000	0.1981435E-06	0.5869586E-09	0.000000	0103000
0.3736079E-07	0.4244065E-06	0.000000	0.000000	0.000000	0013000
-0.8859546E-07	0.1520113E-10	0.000000	0.000000	0.000000	1002001
0.8159316E-07	-0.2189287E-11	0.000000	0.000000	0.000000	0102001
0.000000	0.000000	-0.2906939E-07	-0.2860252E-08	0.000000	0012001
0.000000	0.000000	0.2162195E-06	0.1277710E-12	0.000000	1001002
0.000000	0.000000	0.6788655E-06	0.1264737E-07	0.000000	0101002
0.3219852E-07	0.6836154E-11	0.000000	0.000000	0.000000	0011002
0.2006784E-06	-0.5522681E-07	0.000000	0.000000	0.000000	1000003
-0.1097666E-05	-0.1593299E-12	0.000000	0.000000	0.000000	0100003
0.000000	0.000000	-0.7096167E-07	-0.1096104E-07	0.000000	0010003
-0.6923941E-07	0.2830617E-07	0.000000	0.000000	0.000000	0004000
0.000000	0.000000	-0.8367360E-07	-0.3861988E-09	0.000000	0003001
-0.8964278E-06	-0.1053800E-10	0.000000	0.000000	0.000000	0002002
0.000000	0.000000	0.3915240E-06	-0.4008137E-13	0.000000	0001003
-0.3475201E-06	0.8853058E-08	0.000000	0.000000	0.000000	0000004
-0.4357295E-04	0.1264737E-07	0.000000	0.000000	0.000000	5000000
0.3736813E-03	0.2071608E-13	0.000000	0.000000	0.000000	4100000
-0.5650136E-03	0.4553050E-07	0.000000	0.000000	0.000000	3200000
0.1188522E-03	0.6443823E-13	0.000000	0.000000	0.000000	2300000
-0.2531612E-03	0.4300113E-07	0.000000	0.000000	0.000000	1400000
-0.2392416E-05	0.1490456E-13	0.000000	0.000000	0.000000	0500000
0.000000	0.000000	0.2206317E-05	-0.1686321E-07	0.000000	4010000
0.000000	0.000000	-0.1493999E-03	0.1094826E-13	0.000000	3110000
0.000000	0.000000	0.5704915E-03	0.1011787E-07	0.000000	2210000
0.000000	0.000000	-0.3019527E-03	-0.2654466E-14	0.000000	1310000
0.000000	0.000000	-0.2966571E-04	0.5628562E-14	0.000000	0410000
0.3112016E-04	-0.3375058E-07	0.000000	0.000000	0.000000	3020000
0.7055110E-04	-0.1578224E-10	0.000000	0.000000	0.000000	2120000
-0.3831207E-03	0.1011504E-07	0.000000	0.000000	0.000000	1220000
-0.6626683E-04	0.1386364E-11	0.000000	0.000000	0.000000	0320000
0.000000	0.000000	0.2841034E-04	0.3655323E-07	0.000000	2030000
0.000000	0.000000	-0.1954221E-04	0.1548137E-08	0.000000	1130000
0.000000	0.000000	0.2249254E-04	0.4467481E-08	0.000000	0230000
-0.1238939E-03	-0.2818974E-06	0.000000	0.000000	0.000000	1040000
0.7304833E-04	-0.5286833E-06	0.000000	0.000000	0.000000	0140000
0.000000	0.000000	-0.4657691E-04	-0.2024594E-04	0.000000	0050000
0.000000	0.000000	0.6623421E-04	-0.1670159E-14	0.000000	4001000
0.000000	0.000000	0.1189982E-03	0.1011789E-07	0.000000	3101000
0.000000	0.000000	-0.2932981E-03	-0.5077753E-13	0.000000	2201000
0.000000	0.000000	-0.1209702E-03	0.3035371E-07	0.000000	1301000
0.000000	0.000000	0.1884318E-04	0.3931602E-13	0.000000	0401000
-0.2310160E-03	-0.6154666E-11	0.000000	0.000000	0.000000	3011000
0.1756188E-02	0.3033636E-07	0.000000	0.000000	0.000000	2111000
-0.2122528E-03	-0.2325885E-10	0.000000	0.000000	0.000000	1211000
0.2297538E-03	0.3036302E-07	0.000000	0.000000	0.000000	0311000
0.000000	0.000000	-0.1071908E-04	0.4871924E-08	0.000000	2021000
0.000000	0.000000	-0.1284719E-03	0.3675568E-07	0.000000	1121000
0.000000	0.000000	-0.8238659E-05	0.1504311E-08	0.000000	0221000
0.2048355E-03	0.9939052E-07	0.000000	0.000000	0.000000	1031000
-0.2209440E-04	0.2672614E-06	0.000000	0.000000	0.000000	0131000

0.000000	0.000000	0.9651164E-04	-0.3715032E-04	0.000000	0041000
0.8340064E-03	-0.6323683E-07	0.000000	0.000000	0.000000	4000001
-0.3390547E-02	-0.7109447E-13	0.000000	0.000000	0.000000	3100001
0.3571336E-02	-0.1922404E-06	0.000000	0.000000	0.000000	2200001
0.7399221E-03	-0.1026198E-13	0.000000	0.000000	0.000000	1300001
0.3078823E-03	-0.3119670E-07	0.000000	0.000000	0.000000	0400001
0.000000	0.000000	0.3572171E-04	-0.8431284E-08	0.000000	3010001
0.000000	0.000000	0.1349220E-02	-0.2145255E-12	0.000000	2110001
0.000000	0.000000	-0.7446124E-03	-0.3878517E-07	0.000000	1210001
0.000000	0.000000	0.1172583E-03	-0.1195878E-12	0.000000	0310001
0.6394920E-04	-0.1603635E-07	0.000000	0.000000	0.000000	2020001
-0.8658305E-04	0.2374340E-11	0.000000	0.000000	0.000000	1120001
-0.2065831E-03	-0.2105290E-07	0.000000	0.000000	0.000000	0220001
0.000000	0.000000	0.6162355E-04	-0.2615678E-07	0.000000	1030001
0.000000	0.000000	0.6939270E-04	-0.1283447E-08	0.000000	0130001
-0.5511355E-04	-0.2475439E-06	0.000000	0.000000	0.000000	0040001
-0.2268112E-03	-0.1008890E-11	0.000000	0.000000	0.000000	3002000
0.2938630E-03	0.8180927E-11	0.000000	0.000000	0.000000	2102000
0.3787458E-03	0.3049136E-11	0.000000	0.000000	0.000000	1202000
-0.1517359E-04	-0.1085920E-10	0.000000	0.000000	0.000000	0302000
0.000000	0.000000	0.3414099E-03	0.1292740E-07	0.000000	2012000
0.000000	0.000000	-0.4144445E-03	0.4079816E-09	0.000000	1112000
0.000000	0.000000	0.4462021E-03	0.3092001E-07	0.000000	0212000
0.2686284E-03	0.5943656E-06	0.000000	0.000000	0.000000	1022000
0.3170400E-04	-0.4434056E-08	0.000000	0.000000	0.000000	0122000
0.000000	0.000000	0.1228696E-04	0.9894014E-04	0.000000	0032000
0.000000	0.000000	-0.1016611E-03	-0.9575522E-13	0.000000	3001001
0.000000	0.000000	-0.9568894E-03	-0.5058906E-07	0.000000	2101001
0.000000	0.000000	0.6737691E-03	-0.4509300E-14	0.000000	1201001
0.000000	0.000000	0.1848436E-03	-0.2023554E-07	0.000000	0301001
0.1365188E-02	0.7631074E-11	0.000000	0.000000	0.000000	2011001
-0.5492038E-03	-0.9787997E-07	0.000000	0.000000	0.000000	1111001
-0.7978308E-04	0.5783246E-11	0.000000	0.000000	0.000000	0211001
0.000000	0.000000	-0.2758545E-03	-0.2717260E-08	0.000000	1021001
0.000000	0.000000	-0.9989758E-05	-0.3096046E-07	0.000000	0121001
0.6211599E-04	0.3413042E-06	0.000000	0.000000	0.000000	0031001
-0.9525994E-03	0.1825440E-06	0.000000	0.000000	0.000000	3000002
0.1674260E-02	-0.1020928E-12	0.000000	0.000000	0.000000	2100002
-0.8444202E-03	0.1867590E-06	0.000000	0.000000	0.000000	1200002
-0.6423596E-03	0.1005905E-12	0.000000	0.000000	0.000000	0300002
0.000000	0.000000	0.2474151E-03	0.5227530E-07	0.000000	2010002
0.000000	0.000000	-0.5198361E-03	0.1208598E-13	0.000000	1110002
0.000000	0.000000	-0.8811385E-03	0.1770578E-07	0.000000	0210002
-0.6869196E-05	0.5310556E-07	0.000000	0.000000	0.000000	1020002
-0.9236970E-03	0.1048265E-10	0.000000	0.000000	0.000000	0120002
0.000000	0.000000	0.7530706E-04	0.1647242E-07	0.000000	0030002
0.000000	0.000000	0.2089722E-03	0.5357028E-09	0.000000	2003000
0.000000	0.000000	0.2058528E-03	0.3445231E-08	0.000000	1103000
0.000000	0.000000	0.1174229E-04	0.1736581E-08	0.000000	0203000
-0.1018205E-04	0.3895462E-06	0.000000	0.000000	0.000000	1013000
-0.1703426E-03	0.4663838E-06	0.000000	0.000000	0.000000	0113000
0.000000	0.000000	-0.1292773E-03	0.1724617E-03	0.000000	0023000
-0.2959914E-04	0.1727596E-08	0.000000	0.000000	0.000000	2002001
0.2763638E-03	-0.3134279E-10	0.000000	0.000000	0.000000	1102001

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 0.3661618E-03-0.1707484E-08  0.000000      0.000000      0.000000     0202001
  0.000000      0.000000     0.7662359E-03-0.2194845E-07  0.000000     1012001
  0.000000      0.000000    -0.6488619E-03-0.3581858E-08  0.000000     0112001
-0.4835207E-03 0.1994027E-07  0.000000      0.000000      0.000000     0022001
  0.000000      0.000000     0.1494336E-03 0.1015586E-12  0.000000     2001002
  0.000000      0.000000     0.2281940E-03 0.5143277E-07  0.000000     1101002
  0.000000      0.000000    -0.5831523E-03 0.4309075E-12  0.000000     0201002
-0.3915750E-03 0.2464082E-10  0.000000      0.000000      0.000000     1011002
-0.9667518E-03 0.4642991E-07  0.000000      0.000000      0.000000     0111002
  0.000000      0.000000     0.1705244E-03 0.2448350E-09  0.000000     0021002
-0.1624803E-03-0.2086817E-06  0.000000      0.000000      0.000000     2000003
-0.1060660E-02-0.6621711E-12  0.000000      0.000000      0.000000     1100003
-0.2591765E-03-0.4300154E-07  0.000000      0.000000      0.000000     0200003
  0.000000      0.000000     0.2825920E-03-0.4426537E-07  0.000000     1010003
  0.000000      0.000000    -0.2309635E-03-0.3811017E-12  0.000000     0110003
-0.1588611E-03-0.2128057E-07  0.000000      0.000000      0.000000     0020003
-0.9345362E-04-0.1703433E-06  0.000000      0.000000      0.000000     1004000
-0.1763581E-04-0.3705832E-07  0.000000      0.000000      0.000000     0104000
  0.000000      0.000000    -0.1721734E-04-0.5052216E-04  0.000000     0014000
  0.000000      0.000000     0.4354669E-03 0.1546786E-08  0.000000     1003001
  0.000000      0.000000    -0.6749205E-05-0.5807740E-09  0.000000     0103001
-0.3855160E-03 0.5475406E-06  0.000000      0.000000      0.000000     0013001
-0.2215067E-04-0.3428119E-08  0.000000      0.000000      0.000000     1002002
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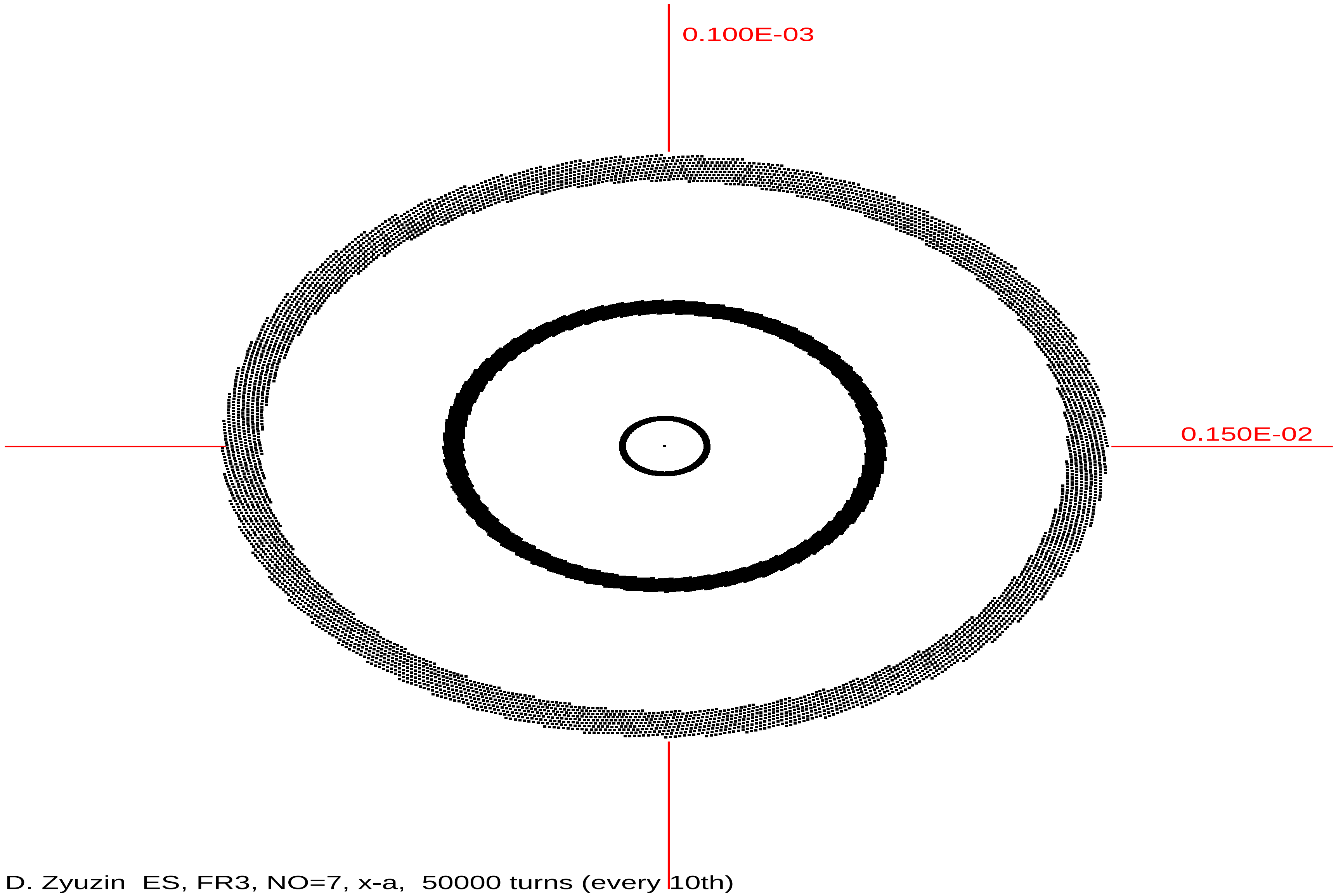
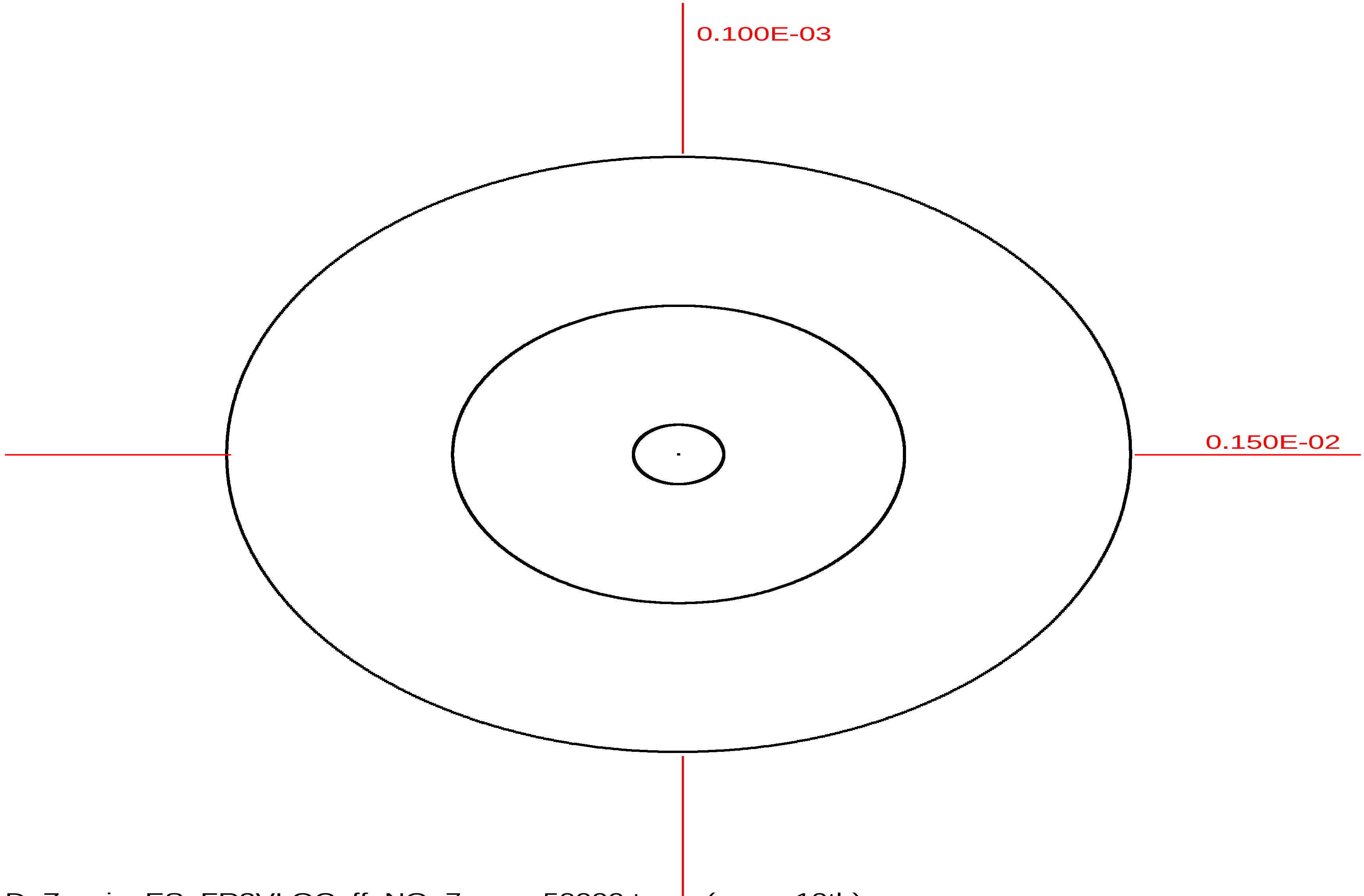
  0.000000      0.000000     0.2936376E-04-0.4636029E-13  0.000000     1001003
  0.000000      0.000000    -0.4288197E-03-0.8853119E-08  0.000000     0101003
-0.1870915E-03 0.1902511E-10  0.000000      0.000000      0.000000     0011003
-0.4417931E-03 0.8995486E-07  0.000000      0.000000      0.000000     1000004
-0.3436738E-04-0.8123444E-12  0.000000      0.000000      0.000000     0100004
  0.000000      0.000000    -0.1481154E-03 0.8009784E-08  0.000000     0010004
  0.000000      0.000000     0.5465326E-05-0.2912335E-04  0.000000     0005000
-0.1852932E-04-0.7348978E-08  0.000000      0.000000      0.000000     0004001
  0.000000      0.000000     0.2584482E-03 0.9979594E-09  0.000000     0003002
-0.1560408E-03 0.1685662E-08  0.000000      0.000000      0.000000     0002003
  0.000000      0.000000    -0.1663985E-03-0.2632633E-13  0.000000     0001004
-0.2456936E-04-0.1280640E-07  0.000000      0.000000      0.000000     0000005
--------------------------------------------------------------------------------
360 degree test (for ES Cylindrical)
-0.8582162    -0.7258994      0.000000      0.000000      0.000000     1000000
 0.3629497    -0.8582162      0.000000      0.000000      0.000000     0100000
  0.000000      0.000000      1.000000      0.000000      0.000000     0010000
  0.000000      0.000000      6.283185      1.000000      0.000000     0001000
 0.9291081     0.3629497      0.000000      0.000000      0.000000     0000001
-0.6170785     0.8667631E-01  0.000000      0.000000      0.000000     2000000
-0.8477916     0.4437621      0.000000      0.000000      0.000000     1100000
 -1.085123    -0.5877627      0.000000      0.000000      0.000000     0200000
  0.000000      0.000000     0.7258994      0.000000      0.000000     1001000
  0.000000      0.000000      1.858216      0.000000      0.000000     0101000
 0.6170785     0.2762734      0.000000      0.000000      0.000000     1000001
 0.6053707    -0.2218810      0.000000      0.000000      0.000000     0100001
-0.9291081    -0.3629497      0.000000      0.000000      0.000000     0002000
  0.000000      0.000000      2.778643      0.000000      0.000000     0001001
 0.7800739E-01-0.6906835E-01  0.000000      0.000000      0.000000     0000002
```

The Pain with Electrostatic Elements

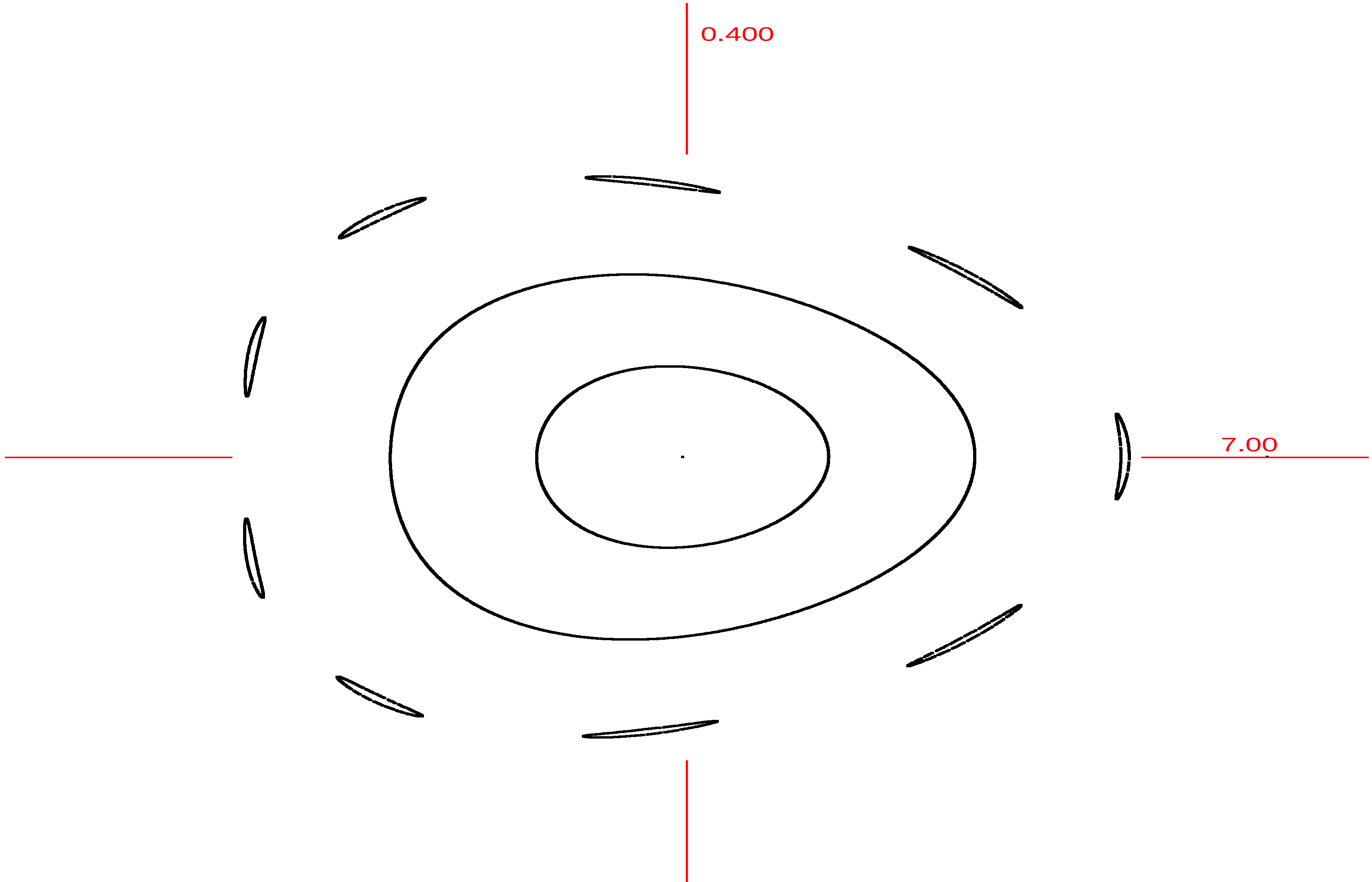
- Unless one is very careful, there will be various undesirable effects:
  - The motion from before to after the element satisfies energy conservation, but the integrator does not know this
  - Repeated small violations of energy conservation can lead to either oscillations, or big long term effects
  - Particular problem: due to offset of reference orbit, it is very useful to re-align elements. This is normally done after each part:
  - After entrance fringe field, after main field, after exit fringe field
  - Each re-alignment causes small change in geometry, and hence small change in potential!



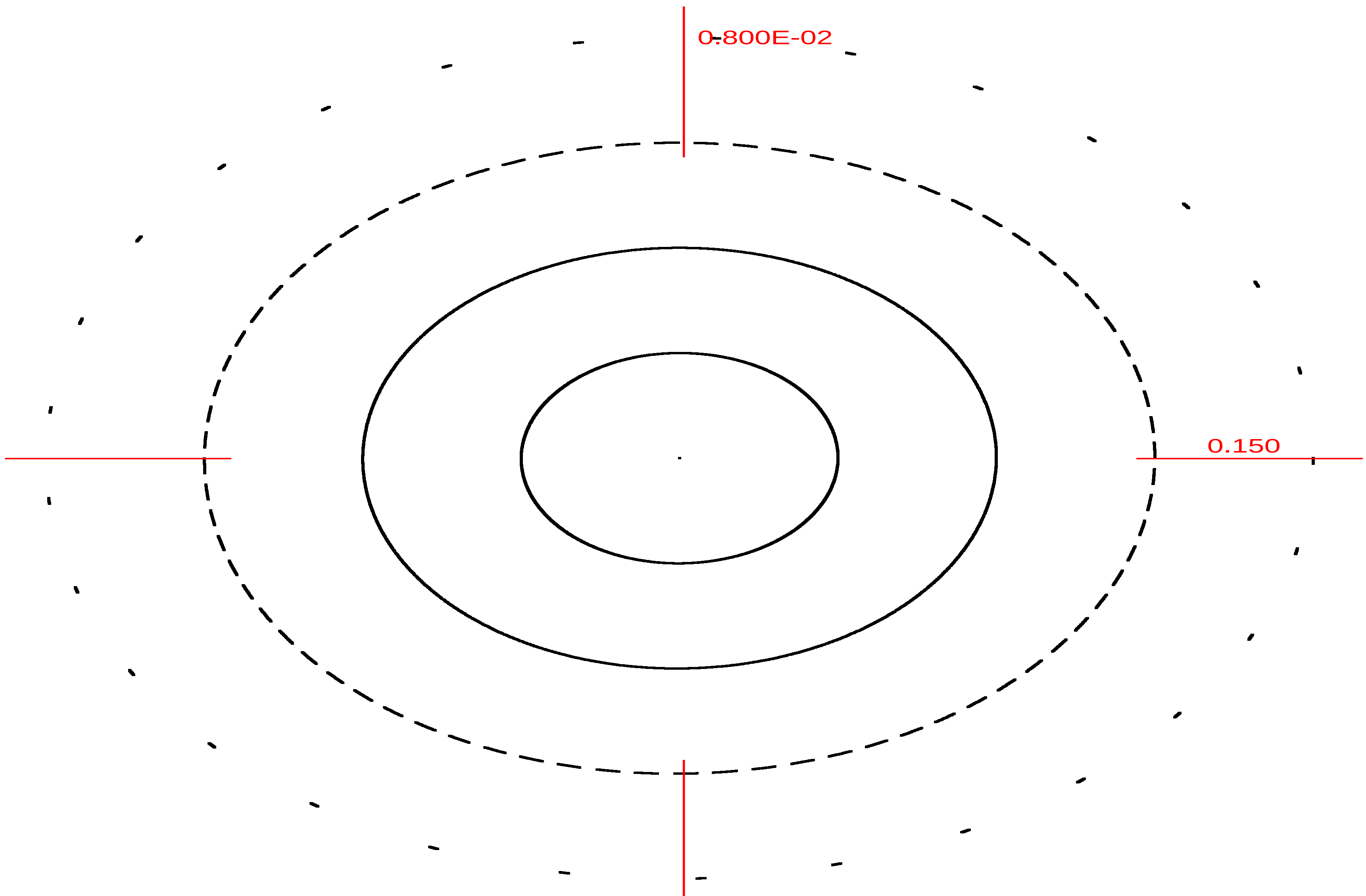
D. Zyuzin ES, FR3, NO=7, x-a, 50000 turns (every 10th)



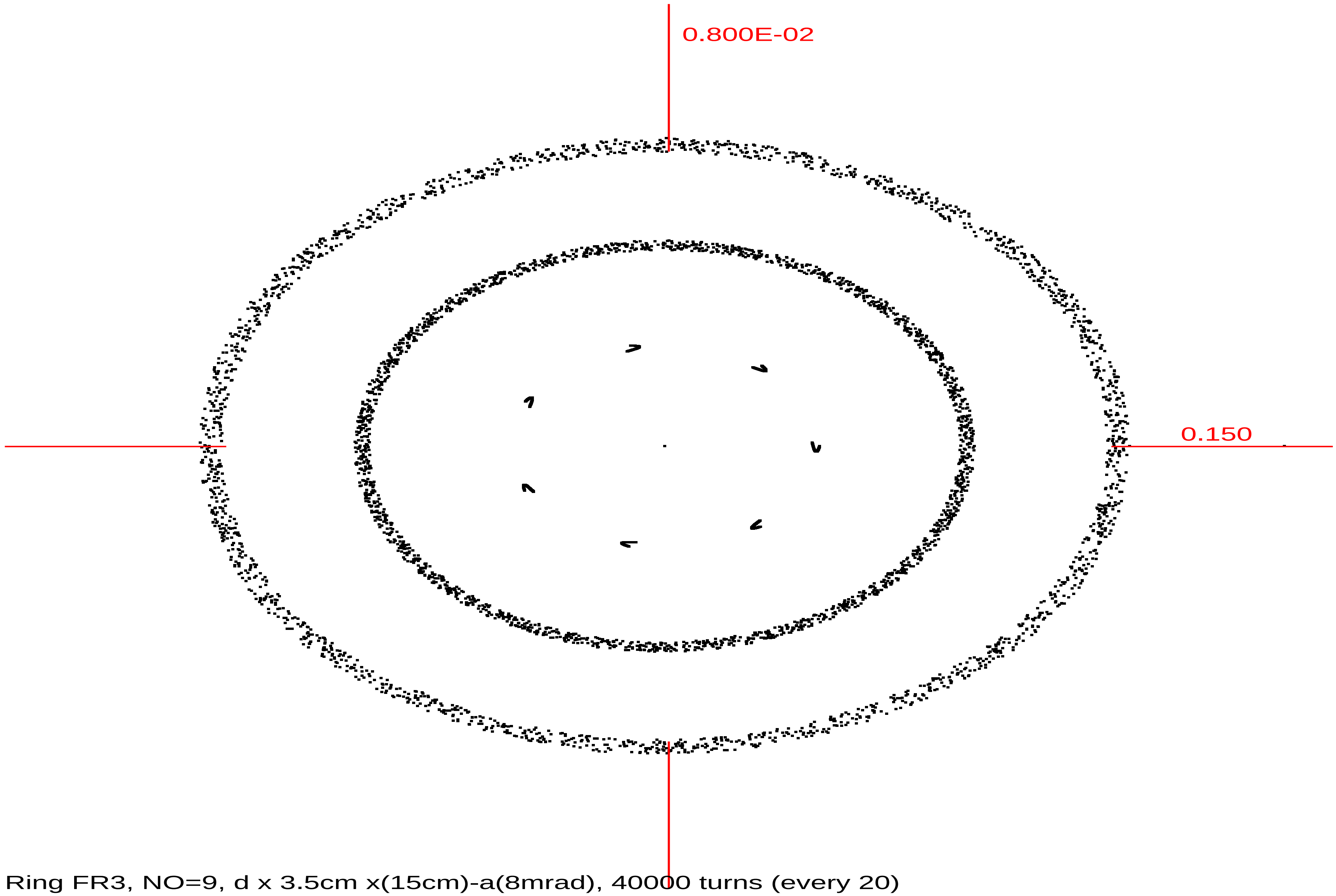
D. Zyuzin ES, FR3VLOCoff, NO=7, x-a, 50000 turns (every 10th)



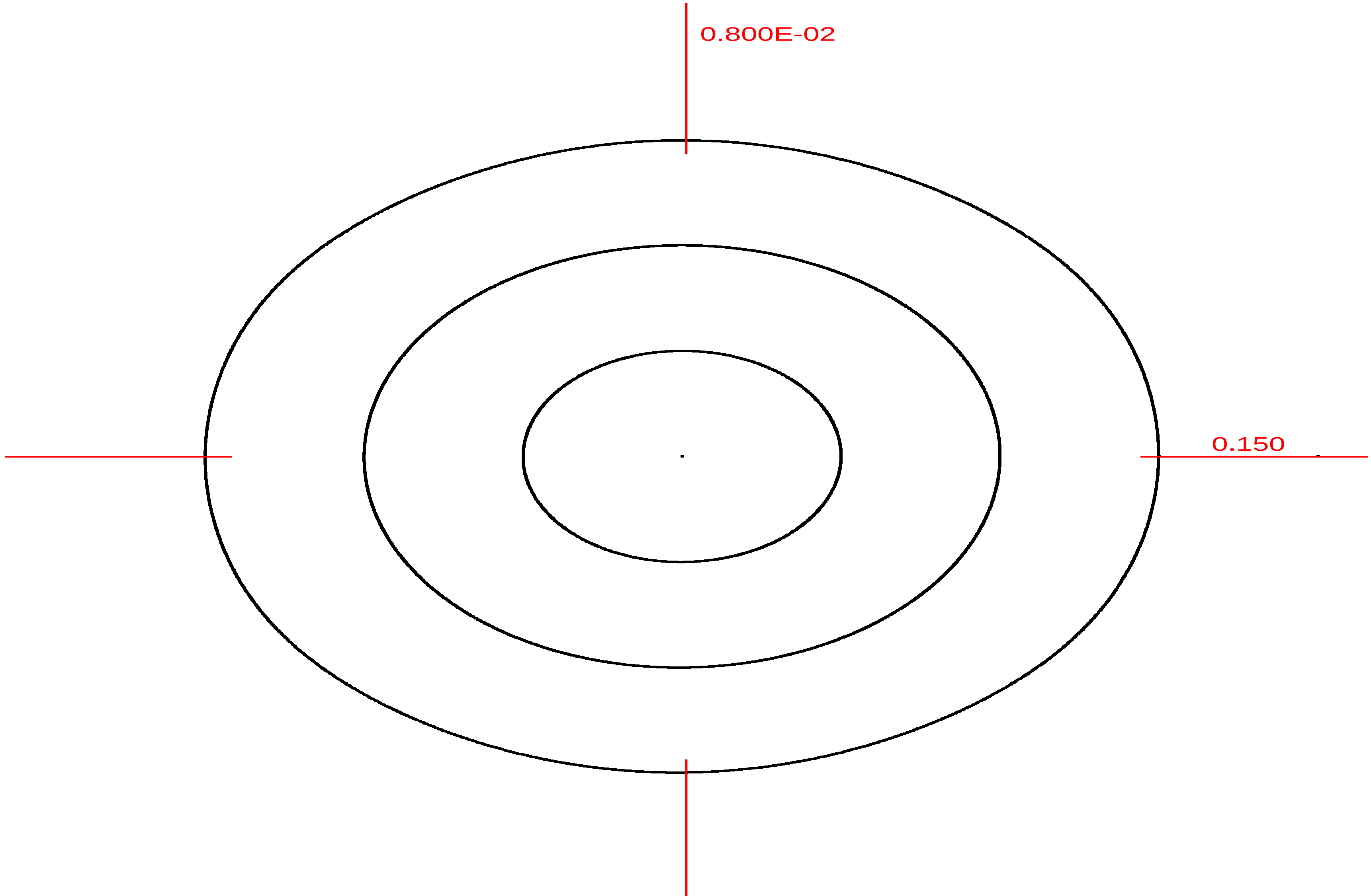
Ring FR0, NO=9, d x 1.5m x(7m)-a(400mrad), 40000 turns (every 20)



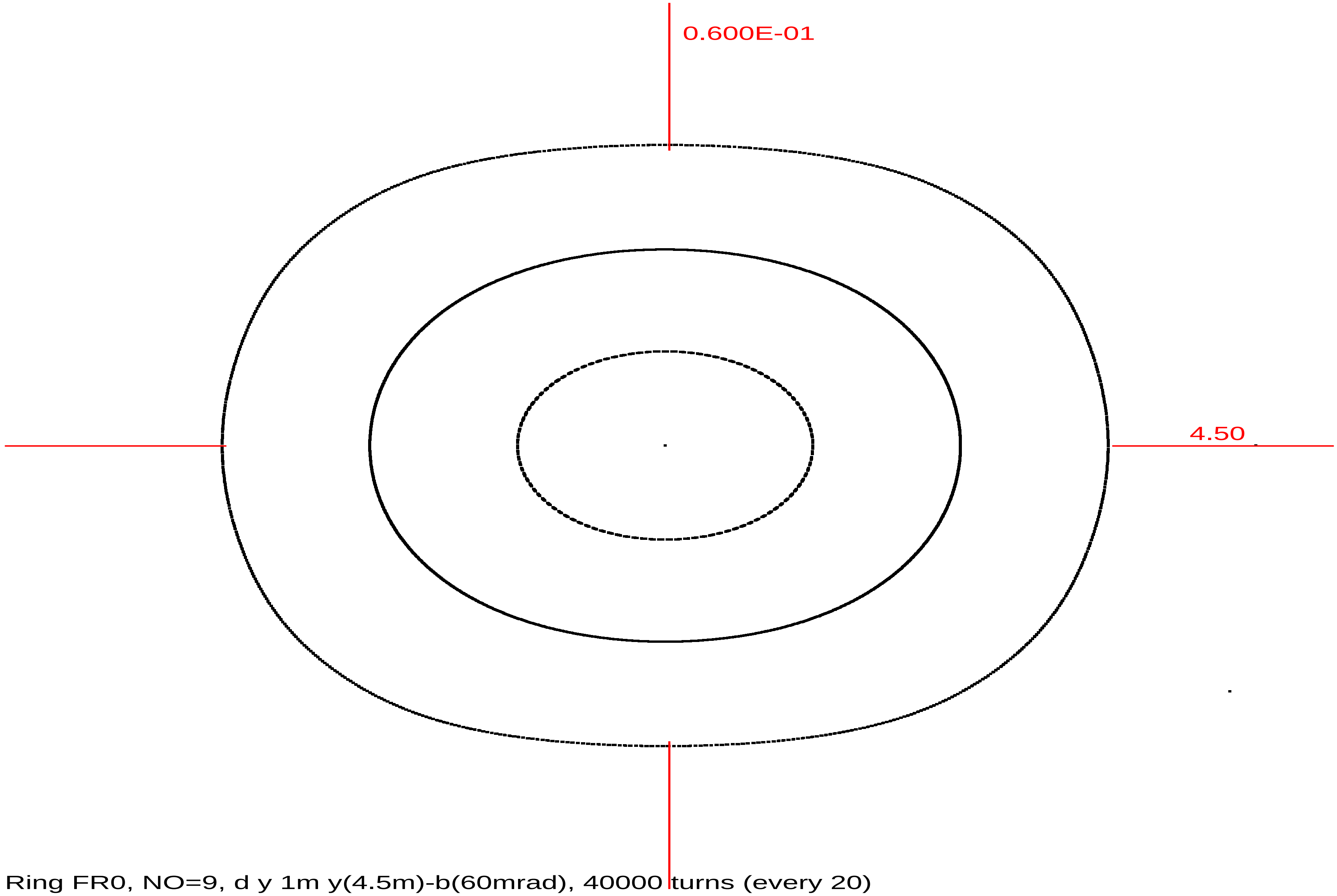
Ring FR0, NO=9, d x 3.5cm x(15cm)-a(8mrad), 40000 turns (every 20)



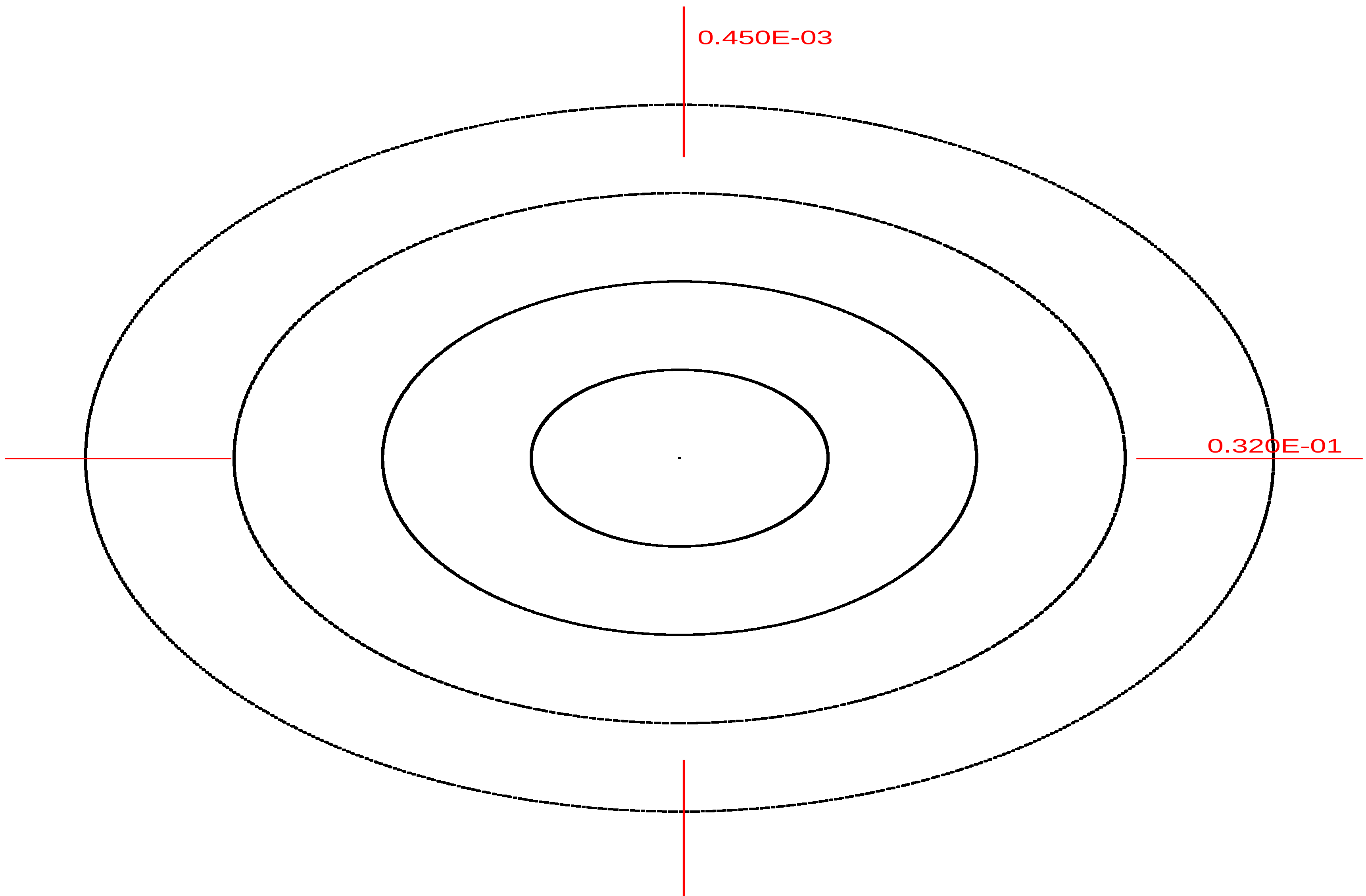
Ring FR3, NO=9, d x 3.5cm x(15cm)-a(8mrad), 40000 turns (every 20)



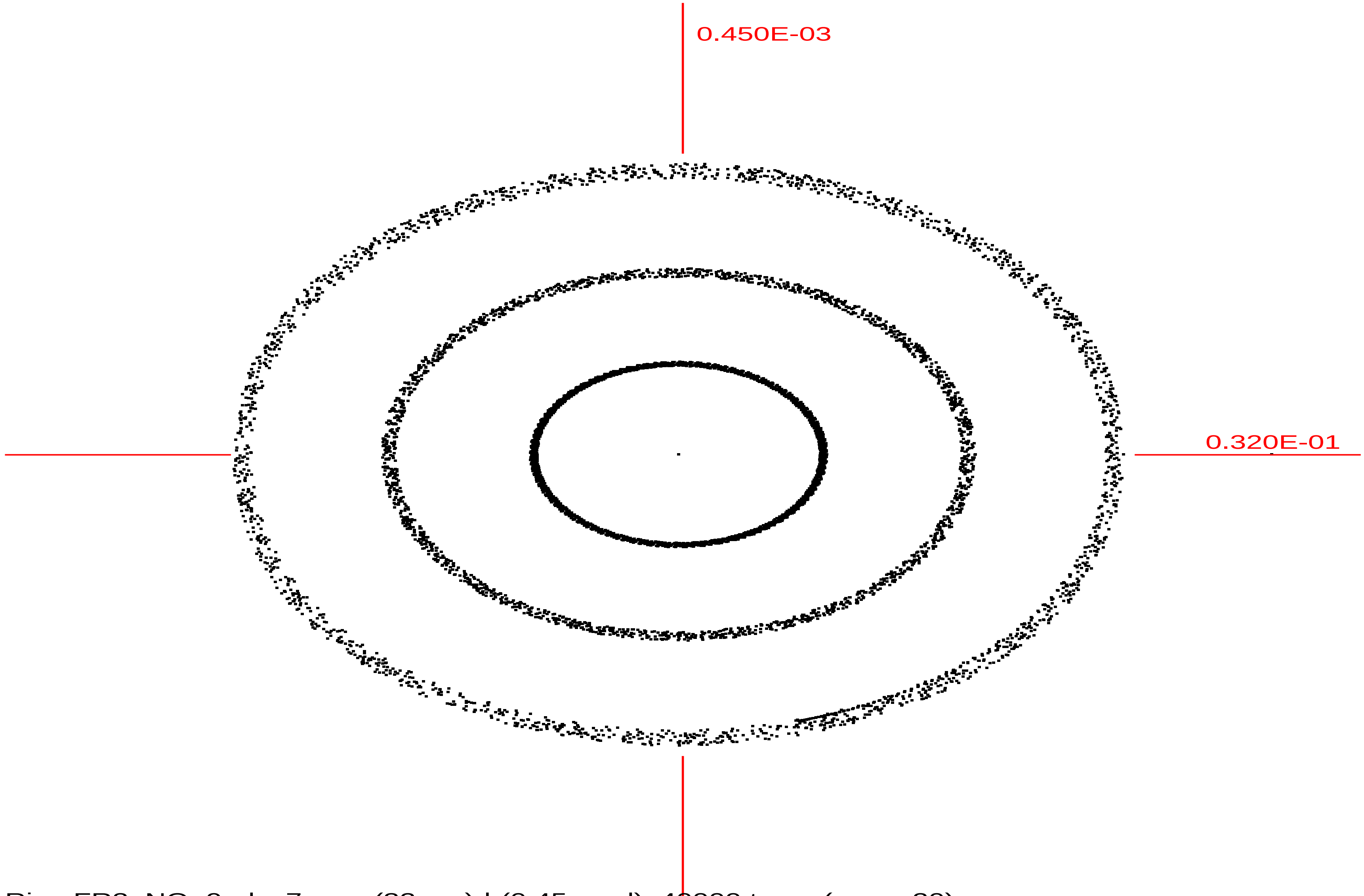
Ring FR3VLOCoff, NO=9, d x 3.5cm x(15cm)-a(8mrad), 40000 turns (every 20)



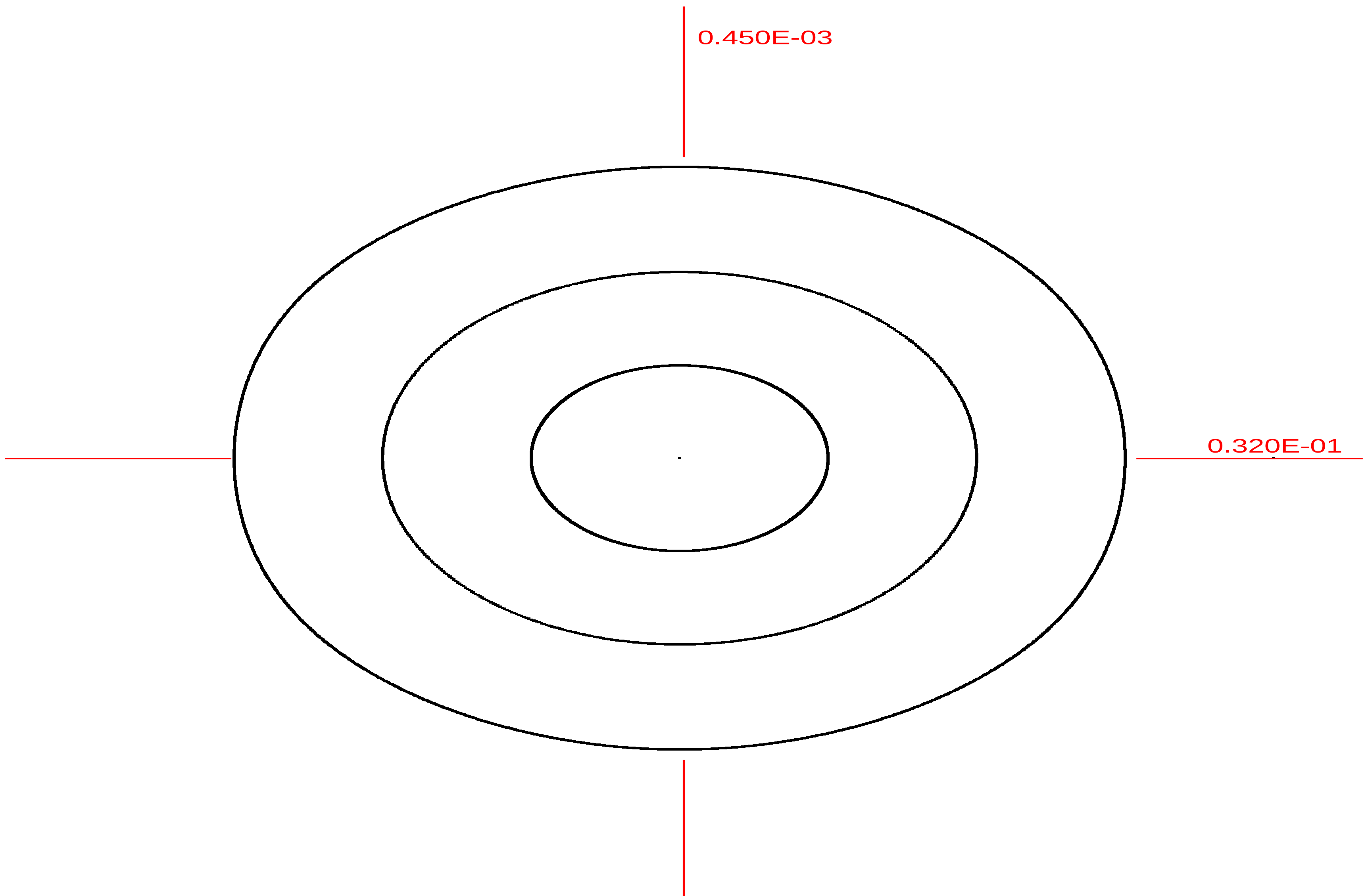
Ring FR0, NO=9, d y 1m y(4.5m)-b(60mrad), 40000 turns (every 20)



Ring FR0, NO=9, d y 7mm y(32mm)-b(0.45mrad), 40000 turns (every 20)



Ring FR3, NO=9, d y 7mm y(32mm)-b(0.45mrad), 40000 turns (every 20)



Ring FR3VLOCoff, NO=9, d y 7mm y(32mm)-b(0.45mrad), 40000 turns (every 20)