Transverse Stern-Gerlach Deflection Polarimetry

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2 Outline

3 Introduction

- A century ago Bohr and Pauli stated that the Stern-Gerlach (SG) experiment cannot be performed using electrons.
- We propose an experiment which, if successful, will "refute" this claim—the quotation marks here acknowledge that the B-P claim was predicated on technical capabilities of their day—they had no way of anticipating future technical advances. Furthermore, the most essential aspect of their claim is not disputed.
- An apparatus capable of detecting and measuring Stern-Gerlach deflection of polarized electrons at Jefferson Lab using only well understood inexpensive apparatus is to be described.

 Ancient and modern Stern-Gerlach apparatus

- The historic SG apparatus used
  - a dipole magnetic field (to orient the spins)
  - with quadrupole magnetic field superimposed (to deflect opposite spins oppositely)
  - and a neutral, somewhat mono-energetic, unpolarized, neutral atomic beam of spin 1/2 silver atoms.
- The proposed SG apparatus uses
  - highly-monochromatic, already-polarized beams produced from an electron gun cathode, the uniform magnetic field has become superfluous, and a quadrupole produces polarization dependent SG deflection.,
  - modern quadrupoles, electro- or permanent- magnet, produce peak gradient comparable to SG, but with good field quality over far greater area than the original (1923) SG magnets.
- Neutral silver atoms of velocity 500 m/s in the original experiment, produced $\Delta\theta^{Ag} \approx 0.002\,\mathrm{r}$ angular deflections. The SG displacements of 500 KeV electrons downstream of a single modern quadrupole will be in the Å ngstrom range.

- The defining relations, in the electron rest frame, of electric and Stern-Gerlach forces are

$$\mathbf{F}^{\mathrm{elec}} = -e\mathbf{E},$$
$$\mathbf{F}^{SG} = \mu_e \nabla(\hat{\mathbf{s}} \cdot \mathbf{B}).$$

- $\hat{\mathbf{s}}$ is a unit spatial 3-vector specifying the orientation of the spin angular momentum in the rest frame.
- Experimentally, transverse $\hat{\mathbf{s}}$ is expected to produce a Stern-Gerlach signal that will be absent with unpolarized beam, with $\hat{\mathbf{s}}$ longitudinal—this dependence on the beam polarization state will provide the experimental evidence for SG deflection.

- If it is "natural" for E and cB magnitudes to be comparable, the ratio of Stern-Gerlach to electromagnetic force is determined by the ratio

$$\frac{\mu_B/c}{e} \equiv \frac{1}{4\pi}\,\frac{hc}{m_e c^2} = 1.930796 \times 10^{-13}\,\mathrm{m},$$

  where, except for sign and fractionally-small anomalous magnetic moment, Bohr magneton μ_B is the electron magnetic moment.
- It is curious that the charge force has been confirmed to high accuracy, while the SG force has never been confirmed directly, to much better than the accuracy of the original Stern-Gerlach experiment.
- The poor quality (even after a century) of experimental checks of Stern-Gerlach deflection can be ascribed to the smallness of this ratio.
- The proposed experiment is motivated partly by the desire to improve this experimental determination.

- This paper is concerned with just a single aspect of the Stern-Gerlach phenomenon; namely spin-dependent particle deflection.
- The proposed experiment *starts with a pre-polarized beam*, prepared upstream, which passes on-axis through a quadrupole representing the non-uniform magnetic field needed to cause SG deflection.
- Unlike the original SG experiment, existence of this deflection does not imply any ability to separate electrons based on their quantum mechanical spin
- In fact our analysis supports the claim that any SG orbit shift (e.g. several nm) will always be several orders of magnitude less than achievable electron beam sizes at the same location.

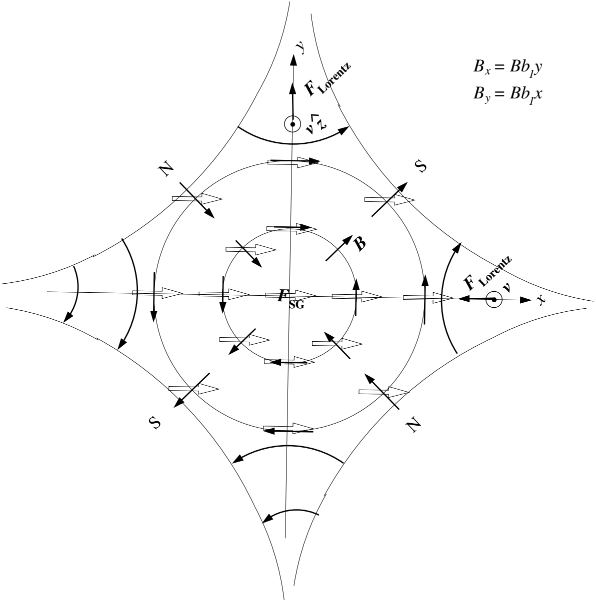
- The Bohr-Pauli claim proving the impossibility of replicating the original Stern-Gerlach experiment with electrons is that the Lorentz force deflection of off-axis electrons overwhelms the Stern-Gerlach deflection of on-axis electrons.
- The argument applies to the deflection of single electrons, for their eventual downstream separation.
- It does not apply directly to the downstream centroid shift of an intense beam that has been pre-polarized upstream of the SG deflecting magnet. Modern equipment can measure a beam centroid shift small compared to beam size.

9 SG deflection of a relativistic particle by a quadrupole

- We are primarily interested in the Stern-Gerlach deflection caused by the on-axis passage of an electron with velocity $v\hat{\mathbf{z}}$ and rest frame, transversely-polarized magnetic dipole moment vector $\mu_x\hat{\mathbf{x}}$ or $\mu_y\hat{\mathbf{y}}$, through a DC quadrupole, of (short) length L_q.
- Stern-Gerlach deflections in a quadrupole are strictly proportional to the (equal in magnitude, opposite in sign) inverse focal lengths q_x and q_y of the quadrupole;

$$\Delta\theta_x^{SG} = \frac{\mu_x}{ec\beta} q_x, \quad \text{and} \quad \Delta\theta_y^{SG} = \frac{\mu_y}{ec\beta} q_y.$$

10 Stern-Gerlach orbit deflection in a quadrupole



$$s \circ B = Bb_1 x$$

$$s = \hat{y}$$

$$
\begin{aligned}
F_{SG} &= \mu \nabla (s \circ B) \\
&= \mu \hat{x} \frac{\partial}{\partial x} Bb_1 x \\
&= \mu Bb_1 \hat{x}
\end{aligned}
$$

$$s \circ B = Bb_1 y$$

$$s = \hat{x}$$

$$
\begin{aligned}
F_{SG} &= \mu \nabla (s \circ B) \\
&= \mu \hat{y} \frac{\partial}{\partial y} Bb_1 y \\
&= \mu Bb_1 \hat{y}
\end{aligned}
$$

11 Magnifying the SG orbit deflection

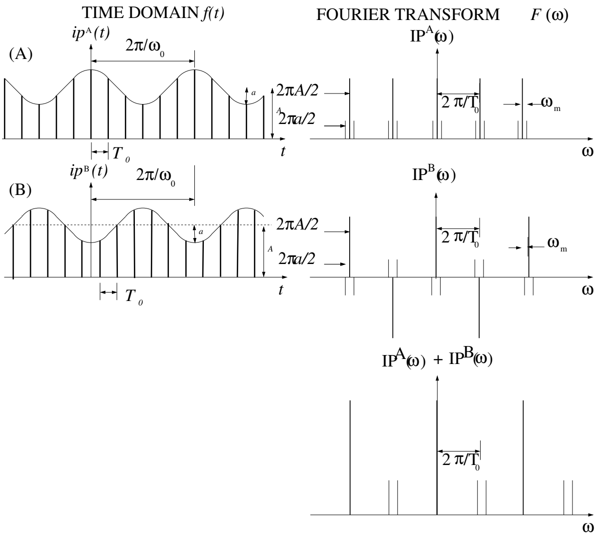
- The electron beam is to be conveyed through N_c identical cells,
- Each cell has one focusing and one defocusing permanent magnet quadrupole, with length l_Q and magnetic field gradient $\partial B_x/\partial x$.
- Spin-dependent SG deflections occur in each of the quadrupoles, and the betatron phase advances are arranged so that the deflections all add constructively.
- The total SG angular deflection is therefore greater than each individual factor by a factor $2N_c$.

12 CEBAF polarized beam preparation

- Dual CEBAF electron sources produce oppositely polarized A and B beams having bunch separation 4 ns. Interleaved, the resulting A & B beam has bunch separation 2 ns.
- The effect of this beam preparation is to produce a bunch charge repetition frequency of 0.5 GHz different from the bunch polarization frequency of 0.25 GHz. This difference will make it possible to distinguish Stern-Gerlach-induced bunch deflections from spurious charge-induced deflections.
- Transverse bunch displacements produce narrow band BPM signals proportional to the f_r Fourier frequency components of transverse beam centroid displacement. Because linac bunches are short there is substantial resonator response at numerous strong low order harmonics of the 0.25 GHz bunch polarization frequency. The proposed SG responses are centered at odd harmonics, $f_r = 0.25, 0.75, 1.25, \dots$ GHz.

- The absence of beam-induced detector response at these frequencies greatly improves the rejection of spurious "background" bunch displacement correlated with bunch charge.
- For further background rejection the polarization amplitudes are modulated at a low, kHz, frequency, which shifts the SG response to sidebands of the central SG frequencies.
- Exactly the same beam preparation will be optimal for for the test of resonant longitudinal polarimetry to be described later.
- Current and polarization time domain amplitudes are plotted on the left in the following figure; their frequency domain signals are plotted on the right.

14 Time domain beam structure and frequency domain spectra



15 FODO quadrupole line to produce Stern-Gerlach electron deflection

A Jefferson Lab test is proposed to detect Stern-Gerlach (SG) electron deflection in a polarimeter consisting of 8 small bore permanent magnet quadrupoles like this.

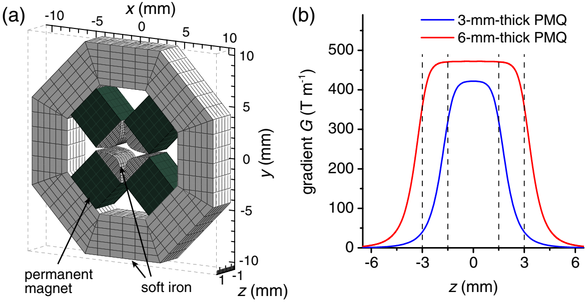
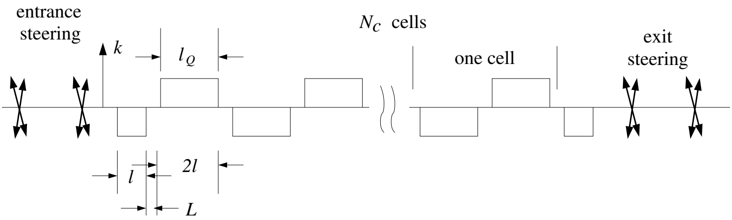


FIG. 5. (a) RADIA model of a 3-mm-thick PMQ magnet and (b) the calculated on-axis focusing gradient of a 3-mm and a 6-mm PMQ. Dashed lines indicate the physical boundaries of the 3- and 6-mm-thick PMQs.



- Parameter values for numerical calculations in this talk:
  quadrupole length $l_Q = 2l = 0.02\,\text{m}$
  quadrupole separation $L = 0.005\,\text{m}$
  number of FODO cells $N_c = 4$
- Entrance and exit steering is needed to correct for quadrupole misalignment steering.
- Positive detection would "refute" the Bohr-Pauli assertion that the SG effect is undetectable for electrons

- But not really!
- The quotation marks on "refute" acknowledge that Bohr and Pauli had no knowledge of modern technical capabilities
- More important, the most essential aspect of their claim —that electrons cannot be "separated" by spin states with an SG apparatus—is not disputed.
- polarization-independent deflection dwarfs any achievable separation into a spin-up and a spin-down beam
- It should, however, be possible to measure the polarization state of an electron beam by measuring its bunch-magnetization centroid deflection
- This is what needs to be demonstrated
- If and when it is demonstrated, a high analysing power, non-destructive form of (transverse) polarimetry will have been demonstrated

- For the initial test described in this talk I choose $N_c = 4$ but, for an eventual apparatus, N_c could be several times greater, depending on tolerance issues to be discussed.
- Since the design uses permanent magnets, any realization of the design is static, specific to a particular electron beam energy.
- But the design scales easly to other energies and parameter choices.
- The assumed quadrupoles are patterned after permanent magnet quadrupole papers by Li, Musumeci, Maxson and others

Calculated SG deflection

- During passage through a short quadrupole, the bend radius is determined by the centripetal force equation,

$$\frac{pv}{r} = evB = ev\frac{\partial B_x}{\partial x}\,x$$

- Re-arranging this equation, the integrated particle deflection angle during passage is

$$\theta = \frac{l_Q}{r} = \frac{cl_Q\langle\partial B_x/\partial x\rangle x}{pc/e},$$

- For a quadrupole of strength (i.e. inverse focal length) $q = 1/f$, the deflection angle is $\pm qx$ where

$$q = \pm\frac{\theta}{x} = \pm\mathcal{C}_\gamma(3\times10^8)/(0.511\times10^6)\left[\frac{l_Q\langle\partial B_x/\partial x\rangle}{\gamma_e}\right]$$

$$\approx \pm587\mathrm{T}^{-1}\mathrm{m}^{-1}\left[\frac{l_Q\langle\partial B_x/\partial x\rangle}{\gamma_e}\right].$$

- The γ_e factor inside the square bracket "cancels" the momentum dependence, allowing the lens strength to be expressed as an inverse focal length.
- The lens can be treated as purely geometric (i.e. independent of momentum) by varying $\partial B_x/\partial x$ proportional to γ_e,
- but only until the gradient cannot be increased further !
- For this talk I take $l_q = 0.02\,\mathrm{m}$ and (already achievable) field gradient $\partial B_x/\partial x = 500\,\mathrm{T/m}$ as nominal values.
- Higher field gradient, $\partial B_x/\partial x = 1000\,\mathrm{T/m}$, at shorter length, $l_Q = 0.01\,\mathrm{m}$ is expected to be achievable.
- This would yield the same length-strength product of $10\,\mathrm{T}$, but be more useful in the (important) sense of allowing a lens of the same strength to be shorter relative to its focal length.

- Limited only by the maximum achievable permanent magnetic field gradient, even with careful element alignment and coherent multiplication of the displacement by the number of quadrupoles in the beamline, the Stern-Gerlach deflection can be expected to be only comparable in magnitude with deflection caused by misaligned quadrupoles.
- This spurious excitation will be suppressed by the interleaving of opposite-polarization A and B beams.
- This shifts the spectral frequency of the SG deflection to one half the spectral frequency of the spurious deflection,
- This will allow the SG contribution to be isolated in a frequency-sensitive BPM.

Beamline optics

Optical properties of the proposed beamline are shown in the following figures.

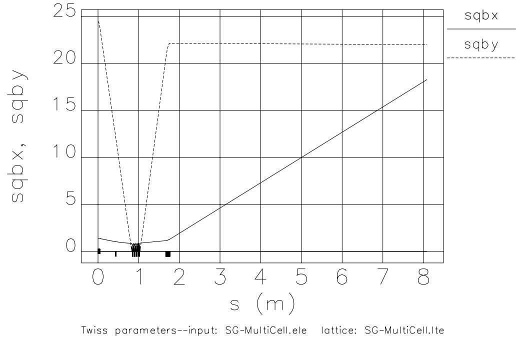


Figure: Beta functions for the Stern-Gerlach detection beamline. The length of the beamline is as long as possible consistent with the requirement that the rms beam size is conservatively smaller than the vacuum chamber radius. An SG-detecting BPM is located as far along the beam line as possible.

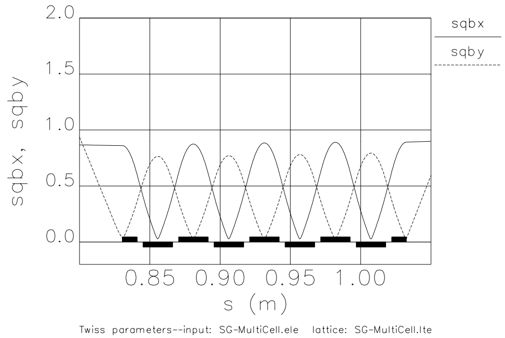


Figure: Optics in the periodic, SG deflection, multiple cell FODO lattice. The full quadrupole lengths are $l_Q = 2l = 0.02\,\text{m}$ and the quad separation distances are $L = 0.005\,\text{m}$. So the full cell length is $L_{\text{cell}} = 0.05\,\text{m}$.

- Only $N_c = 4$ cells for the FODO section are shown.
- but N_c could be increased with little effect on the matching.
- N_c is limited, however, by the fact that the same optics that magnifies the SG deflection also magnifies the sensitivity to transverse beam displacement injection error.

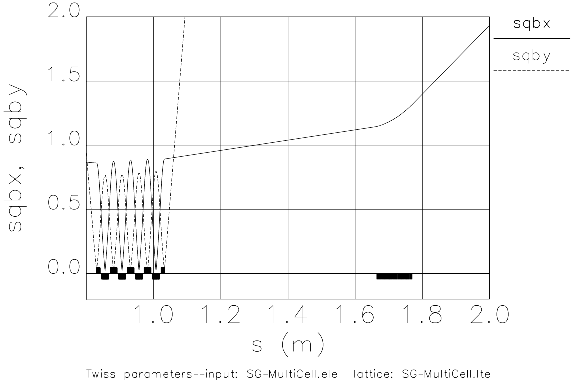


Figure: The quadrupole at $s = 1.72\,\mathrm{m}$ (at a distance $L_{\mathrm{coll.}} = 0.8\,\mathrm{m}$ from the center of the FODO lattice) is needed to restrict the growth of the defocussed transverse coordinate. But it also has the beneficial effect of magnifying the SG deflection. At low electron energy the beam emittance may limit the exit drift length to be shorter than shown to prevent beam loss before the beam passes through the BPM's.

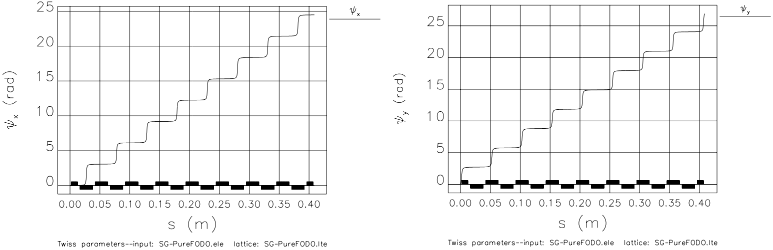


Figure: Phase advances ψ_x and ψ_y through a lattice with $N_c = 8$ cells. Since $25/8 = 3.125$, one sees that the phase advances per half cell are quite close to the value of 180 degrees, the maximum value that could be stable for arbitrarily large value of N_c. It is also the value for which all SG deflections superimpose constructively.

26 Dependence on electron energy

- Adiabatic dampling causes the beam emittances to shrink proportional to γ_e
- For fixed q, this produces a $\gamma_e^{1/2}$ SG enhancement factor with increasing γ_e.
- This capability "saturates" when the quadrupole strength required to produce the necessary focal length is no longer physically achievable.

- Using the ELEGANT program, the focal lengths of the individual quadrupoles in the FODO line tuned for π phase advance per half cell are
$$q = kl_Q = 68.1\,\mathrm{m}^{-1}.$$
- The corresponding focal length is $f = 0.0147\,\mathrm{m}$ —about 0.3 times the full cell length, as seems about right.
- Substitution of this q value and $l_Q = 0.02$ and rearranging produces

$$l_Q \langle \partial B_x / \partial x \rangle = \frac{68.1\,\mathrm{m}^{-1}}{587\mathrm{T}^{-1}\mathrm{m}^{-1}}\,\gamma_e, \quad \text{or} \quad \left\langle \frac{\partial B_x}{\partial x} \right\rangle = \frac{68.1\,\mathrm{m}^{-1}}{0.02\,\mathrm{m} \times 587\mathrm{T}^{-1}\mathrm{m}^{-1}}$$

If the practical limit for $\partial B_x / \partial x$ is $500\,\mathrm{T/m}$, then the apparatus being described could act as a Stern-Gerlach polarimeter up to $\gamma_e = 86$, or electron energy of 43 MeV.

Stern-Gerlach displacement

- The Stern-Gerlach deflection in a quadrupole is strictly proportional to the inverse focal lengths of the quadrupole;

$$\Delta\theta_x^{SG} = \frac{\mu_x^*}{ec\beta} q_x, \quad \text{and} \quad \Delta\theta_y^{SG} = \frac{\mu_y^*}{ec\beta} q_y.$$

- The magnetic moments μ_x^* and μ_y^* differ from the Bohr magnetron μ_B only by $\sin\theta$ and $\cos\theta$ factors respectively
- For a single quadrupole, the Stern-Gerlach-induced angular deflection is

$$\Delta\theta^{SG} = (1.93 \times 10^{-13}\,\mathrm{m})\, q.$$

To determine the downstream dispacement, one can use linear transfer matrix evolution;

$$\begin{pmatrix} \Delta x_{SG} \\ \cdot \end{pmatrix} = \begin{pmatrix} 1 & L_{\mathrm{drift}} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1.49 & 1 \end{pmatrix} \begin{pmatrix} 1 & L_{\mathrm{coll}} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ \Delta\theta_{SG} \end{pmatrix},$$

- The collimating quadrupole strength is 1.49 /m. Completing the matrix multiplication yields

$$\Delta x_{SG} = (0.8 + 2.19 L_{\rm drift}) \Delta\theta_{SG}.$$

- The horizontal SG displacement is then given by

$$\begin{aligned}\Delta x_{\rm SG} &= \pm 2 N_c \, (1.93 \times 10^{-13}\,{\rm m}) \, \times 68.1\,{\rm m}^{-1} (0.8\,{\rm m} + 2.19 L_{\rm drift}) \\ &= 1.59 \times 10^{-9}\,{\rm m}.\end{aligned}$$

- The $\pm$ factor doubles the SG displacement to 3.2 nm; because the BPM is tuned to half the bunch passage frequency, it responds constructively to the oppositely polarized A and B beam bunches.

30 Energy dependence of transverse polarimetry

- Expressing the quadrupole strength as an inverse focal length, as we have done, has had the effect of making the SG deflection independent of γ.
- Transverse beam size adiabatic damping enhances the energy dependence by a factor $\sqrt{\gamma}$.
- Even with the magnetic field gradient limited, the SG quadrupole lengths can be increased to preserve the optics described in this note, though with a longer FODO section.
- So the actual scaling with energy is such that the maximum achievable Stern-Gerlach deflection increases as $\sqrt{\gamma}$ until the gradient can no longer be increased, and falls as $1/\sqrt{\gamma}$ as the electron energy is increased from there.
- As for the test at CEBAF, the most convenient energy remains to be determined. Discussions so far have assumed 500 KeV electron kinetic energy, but this is for reasons of economy and accessibility, not because the SG signal is strongest at low energy. For the geometric parameters assumed in this note, the magnetic field gradient for $\gamma_e = 2$ would be 12 T/m, far less than the maximum possible.

31 Signal levels and noise suppression

- The resonant BPM relies on precise, on-axis, alignment of a cavity tuned to have an anti-symmetric mode at the bunch charge passage frequency.
- Extreme selectivity is needed to separate the beam polarization signal from the spurious direct beam charge signal (and misaligned equipment).
- Also the signal power induced in the position-sensitive cavity by SG-induced displacement has to exceed the inherent thermal noise “floor”. This noise floor could, if necessary, be lowered by using liquid Helium temperature apparatus, but our estimates indicate that such an extreme measure is unnecessary.

- Pusch et al. report BPM measurement at the 0.1 mm level for beam currents greater than 250 pA. The J-Lab current is a million times greater. The off-axis shunt impedance of a resonant cavity is proportional to the square of the (beam-current $\times$ beam-displacement) product. By this estimate, the resonator excitation of 1 Å will be at the noise floor. The SG displacement predicted for our beamline is approximately 30 Å.
- International Linear Collider motivated BPM performance design studies have shown that the $\pm$20 Å beam position pulse-to-pulse reproduceability planned for effective ILC operation will be achievable.
- A CEBAF beam is CW, with average current about five orders of magnitude higher than for the BPM test at the KEK, ATF Test Facility. Averaging over longer times can reduce some noise sources. For these, the increased average beam current can improve the signal to noise by the square root of the current ratio.
- Also the ILC cavity discharging time is far shorter than the ATF repetition period, which makes it necessary for them to treat their BPM resonant response on a pulse-by-pulse basis.
- Our high bunch frequency permits phase-sensitive CW signal treatment.

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