Theory IV: Electric Dipole Moments
Hadron Physics Summer School 2012

August 29, 2012 | Andreas Wirzba
Motivation: Matter Excess in the Early Universe

- matter : antimatter
- as
- \[ \frac{1,000,000,001}{1,000,000,000} \]

Problem

Standard Model falls short by several orders of magnitude!

What is missing?

\[ T \geq m_B \approx \mathcal{O}(1 \text{ GeV}) : \frac{n_B - n_{\bar{B}}}{n_B + n_{\bar{B}}} \approx 10^{-9} \] [A.D. Dolgov (1997)]
Motivation: Matter Excess in the Present Universe

Matter-Antimatter Asymmetry

- Sakharov Conditions:
  1. baryon number $B$ violation
  2. C- and CP(T)-symmetry violation
  3. no thermal equilibrium

- observed asymmetry
  
  \[
  \frac{\langle n_B - n_{\bar{B}} \rangle}{n_\gamma} = 6 \cdot 10^{-10}
  \]

  vs. Standard Cosmological Model:
  
  \[
  \frac{\langle n_B - n_{\bar{B}} \rangle}{n_\gamma} = 10^{-18}
  \]

- Investigation of CP induced by SM extensions required

- Complementary approaches:
  
  high energy ↔ high precision

* WMAP + COBE (2003): $n_B/n_\gamma = \left(6.1^{+0.3}_{-0.2}\right) \times 10^{-10}$, $n_\gamma \approx 411 \text{cm}^{-3} \approx 2.736 \text{K}$

JETP Lett. 5 (1967) 24
Outline:

1. Motivation: Matter–Antimatter Asymmetry
2. The Permanent EDM and its Features
3. CP-Violating Sources beyond the Standard Model
4. CP-Violating Sources at the Hadronic Level
5. Electric Dipole Moments of Neutron and Proton
6. Nuclear Electric Dipole Moments: the EDM of the Deuteron
7. Outlook
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1. Motivation: Matter–Antimatter Asymmetry
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Come to the dark side of the force!
The Electric Dipole Moment (EDM)

EDM: \( \vec{d} = \sum_i \vec{r}_i e_i \) (polar) \( \rightarrow d \cdot \vec{\sigma} \) (axial)

\[
H = -\vec{\mu} \cdot \vec{B} - d \vec{\sigma} \cdot \vec{E}
\]

T: \( H = -\vec{\mu} \cdot \vec{B} + d \vec{\sigma} \cdot \vec{E} \)

P: \( H = -\vec{\mu} \cdot \vec{B} + d \vec{\sigma} \cdot \vec{E} \)

A non-vanishing permanent EDM of a stable subatomic particle is P- and T-violating

- Assuming CPT to hold, CP is violated as well
- Strongly suppressed in SM (CKM-matrix): \( d_n \sim 10^{-31} \text{ e cm} \)
- Current bounds: \( d_n < 3 \cdot 10^{-26} \text{ e cm}, \, d_p < 8 \cdot 10^{-25} \text{ e cm} \)


* calculation with input from Hg atom measurement: Griffith et al. (2009)
A *naive* estimate of the *natural* EDM scale

Khriplovich & Lamoreaux (1997)

- **CP & P** conserving magnetic moment \( \sim \) nuclear magneton \( \mu_N \)

\[
\mu_N = \frac{e}{2m_p} \sim 10^{-14} \text{ e} \cdot \text{cm}
\]

- EDM demands for parity \( P \) violation *:

\[
\Rightarrow \text{ pay the price } \sim 10^{-7}.
\]

- EDM demands for \( \text{CP} \) violation :

Scale for \( \text{CP} \) from K-decays*\(^\dagger\) is \( \sim 10^{-3} \Rightarrow \) Pay this price.

- In summary:

\[
d_N \sim 10^{-7} \times 10^{-3} \times \mu_N \sim 10^{-24} \text{ e} \cdot \text{cm}
\]

* \( G_F \cdot m^2_\pi \sim 10^{-7} \) with \( G_F \approx 1.166 \cdot 10^{-5} \text{ GeV}^{-2} \),

\[
|\eta_{+ -}| = \frac{|A(K_L^0 \rightarrow \pi^+ \pi^-)|}{|A(K_S^0 \rightarrow \pi^+ \pi^-)|} = (2.232 \pm 0.011) \cdot 10^{-3}
\]

\( \dagger \) divides.alt0 /divides.alt0 \'= \frac{A(K^0_L \rightarrow \pi^+ \pi^-)}{A(K^0_S \rightarrow \pi^+ \pi^-)}$
Upper bounds on $d_n$ in the course of time

Upper bounds on the neutron EDM

Smith, Purcell, Ramsey (1957) ......................... Baker et al. (2006)
(Permanent) EDMs of (stable) non-selfconjugate\(^*\) particles with spin

EDM operator \(\vec{d} = \int d^3x \bar{x}\rho(\bar{x})\) in stationary state \(|j\rangle\) of definite parity

\[
\langle j|\vec{d}|j\rangle = d\langle j|\vec{J}|j\rangle :
\text{time reversal: } \vec{d} \rightarrow \bar{\vec{d}}, \quad \vec{J} \rightarrow -\bar{\vec{J}},
\]
\[
\text{space reflection: } \vec{d} \rightarrow -\bar{\vec{d}}, \quad \vec{J} \rightarrow \bar{\vec{J}}.
\]

\(\text{If } d \neq 0 \text{ and state } |j\rangle \text{ has no degeneracy (besides rotations), then } \mathcal{P} \& \mathcal{T}.\)

- \(|j\rangle\) can be ‘elementary’ particle (quark, charged lepton, \(W^\pm\) boson, Dirac neutrino, \ldots) or a ‘composite’ neutron, proton, nuclei, atom, molecule
- Do not confuse with huge EDM of \(H_2O\) or \(NH_3\) molecules — no \(\mathcal{P} \& \mathcal{T}\): Ground state of these molecules at non-zero temperatures is mixture of 2 opposite parity states: above theorem does not apply
- If the interactions are described by a local, Lorentz-invariant, hermitian Lagrangian, then CPT invariance holds: then \(\mathcal{T} \iff \mathcal{CP}\)
- Thus, under very mild assumptions: EDM \(d \neq 0 \Rightarrow \mathcal{P}\) and \(\mathcal{CP}\)

\(^*\) self-conjugate particle \(\equiv\) particle equal to its antiparticle: all its “charges” are zero
Permanent EDMs and Form Factors

- Consider here \( s = \frac{1}{2} \) fermions: \( f = \text{quark, lepton or nucleon} \)

\[
\langle f(p') | J_{\text{em}}^\mu | f(p) \rangle = \bar{u}_f(p') \Gamma^\mu (q^2) u_f(p)
\]

\[
\Gamma^\mu (q^2) = \gamma^\mu F_1(q^2) - i\sigma^{\mu\nu} q_\nu \frac{F_2(q^2)}{2m_f} + \sigma^{\mu\nu} q_\nu \gamma_5 \frac{F_3(q^2)}{2m_f} + (\not{q} q^\mu - q^2 \gamma^\mu) \gamma_5 F_a(q^2)/m_f
\]

Dirac \( F_1(q^2) \), Pauli \( F_2(q^2) \), electric dipole \( F_3(q^2) \) and anapole \( F_a(q^2) \) FFs

- quark, lepton or nucleon EDM \( d_f := F_{3,f}(0)/(2m_f) \)

\[
\mathcal{H}_{\text{eff}} = \frac{i}{2} \bar{f} \sigma^{\mu\nu} \gamma_5 f F_{\mu\nu} \longrightarrow -d_f \sigma \cdot E \longrightarrow \text{linear Stark effect}
\]

- Likewise \( C\mathcal{P} \text{- chromo (color quark)} \) EDM in quark-gluon vertex

\[
i \frac{d_{cq}}{2} \bar{q} \sigma^{\mu\nu} \gamma_5 T^a q G^{a\mu\nu} \text{ etc.}
\]

or weak dipole moment (WDM) in \( ffZ\)-boson vertex \( i \frac{d_f^2}{2} \bar{f} \sigma^{\mu\nu} \gamma_5 f Z_{\mu\nu} \).
**Generic features of EDM, chromo EDM or WDM**

\[
\mathcal{L}_{\text{EDM}} = -i \frac{d_f}{2} \bar{f} \sigma^{\mu\nu} \gamma_5 f F_{\mu\nu} = -i \frac{d_f}{2} \bar{f}_L \sigma^{\mu\nu} f_R F_{\mu\nu} + i \frac{d_f}{2} \bar{f}_R \sigma^{\mu\nu} f_L F_{\mu\nu}
\]

- has mass dimension 5 (\(i.e.\) \(\text{dim}(d_f) = e \times \text{length} = e \times \text{mass}^{-1}\))
  \(\Rightarrow\) non-renormalizable effective interaction
- In renormalizable theories, \(\mathcal{L}_{\text{EDM}}\) must be induced by quantum corrections, \(i.e.\) at 1-loop order or higher
- If EDM or chromo EDM or WDM non-zero, then \(\mathcal{CP}\) in flavor-diagonal amplitudes
  - Note: \(\mathcal{CP}\) in SM model via CKM matrix is flavor changing. Thus extra \(\mathcal{P} \approx 10^{-7}\) factor multiplying naive estimate \(d \approx 10^{-24}\) e cm.
- \(\mathcal{L}_{\text{EDM}}\) flips fermion chirality \(\frac{1}{2} (\bar{1}1 - \gamma_5) f = f_L \leftrightarrow f_R = \frac{1}{2} (\bar{1}1 + \gamma_5) f\)
  \(\Rightarrow\) fermion mass \(m_F\) term (\(e.g.\) via Higgs) must be involved:
  \(d_F \propto m_f^n, \quad n = 1, 2, 3\)
  \(i.e.\) mass scaling depends on model of \(\mathcal{CP}\)
CP violation in the Standard Model (SM)

3 generations of ‘up/down’ quarks (& ‘el./neutrino’ leptons)

with interactions with gluons, the photon, $W^\pm$, $Z$, and Higgs boson

Empirical facts:
- $0 < m_u < m_d < m_s < m_c < m_b < m_t$ and $m_e < m_\mu < m_\tau$
- quarks & leptons in mass basis ≠ quarks & leptons in weak-interaction basis
- $\mathcal{L}_{SM} = \mathcal{L}_{gauge} + \mathcal{L}_{gauge-fermion} + \mathcal{L}_{gauge-Higgs} + \mathcal{L}_{Higgs-fermion}$ CP invariant,
  - except $\theta$ term of QCD (see below)
  - and charged-weak-current interaction $\mathcal{L}_{cc} \subset \mathcal{L}_{gauge-fermion}$
    \[
    \mathcal{L}_{cc} = -\frac{g_w}{\sqrt{2}} \sum_{ij=1}^{3} \bar{d}_i \gamma^\mu V_{ij} u_{Lj} W^-_{\mu} - \frac{g_w}{\sqrt{2}} \sum_{ij=1}^{3} \bar{\ell}_i \gamma^\mu U_{ij} \nu_{Lj} W^-_{\mu} + \text{h.c.}
    \]
- $V$: CKM (Cabibbo-Kobayashi-Maskawa matrix), $U$: lepton-mixing (Maki-Nakagawa-Sakata m.)
  - 3 angles + 1 $CP$ phase $\delta_{KM}$
  - 3 angles +1(3) $CP$ phase(s) for Dirac (Majorana) $\nu_i$'s
EDMs in the SM

- \( \mathcal{CP} \) effects \( \propto \prod_{i>j} \left( \frac{m_{ui}^2 - m_{uj}^2}{M_{EW}^2} \right) \prod_{k>l} \left( \frac{m_{dk}^2 - m_{dl}^2}{M_{EW}^2} \right) \cdot J_{KM} \approx 10^{-15} J_{KM} \)

with \( J_{KM} = \text{Im}(V_{ud}^* V_{ub} V_{cb}^* V_{cd}) = -\text{Im}(V_{cb}^* V_{cd} V_{td}^* V_{tb}) \approx 3 \cdot 10^{-5} \)

and, especially, flavor-changing (！)

- EDM flavor-neutral \( \Rightarrow \) predictions of KM mechanism tiny (\( \propto G_F^2 \))

- 1-loop

\[ O(g_w^2) \]

\( CP \) phase \( \delta_{KM} \) cancels
\( \Rightarrow \) prefactor real \( \Rightarrow d_q^{1\text{loop}} = 0 \)

- 2-loop: \( d_q^{2\text{loop}} = d_{cq}^{2\text{loop}} = 0 \)

Shabalin (1978)

- 3-loop: \( O(g_W^4 g_s^2) \), \( \delta_{KM} \) induces \( d_q \) only at \( \geq 3 \) loops: \( d_n^{KM} \approx 10^{-34} \ldots 10^{-31} \) e cm

Shabalin (1978); Khriplovich & Zhitnitsky (1982); Czarnecki & Krause (1997)
Effective Lagrangian on the quark/gluon level

Construction of an effective (low-energy) Lagrangian that incorporates the relevant $\mathcal{CP}$ (light) quark, gluon & electromagnetic interactions

- **SM** (light quarks w. Higgs integrated out) with at most marginal (dim=4) terms:
  
  $$
  \mathcal{L}_{SM} = \mathcal{L}_{EW} + \mathcal{L}_{QCD} \equiv \mathcal{L}_{CP}^{SM} + \mathcal{L}_{cc}
  $$

  $$
  \quad = \mathcal{L}_{EW} + \sum_{f=u,d,s} \bar{q}^c_f \left( i \gamma^\mu D^c_{\mu} - \delta^c_{cc'} m_f \right) q^c_{f'} - \frac{1}{4} G^{a}_{\mu\nu} G^{a}_{\mu\nu}
  $$

  - Note: because of $d_q$ and $d_{cq} \propto m_f$ we have effectively $\text{dim}=6$!
Effective Lagrangian on the quark/gluon level

Construction of an effective (low-energy) Lagrangian that incorporates the relevant $\mathbb{C P}$ (light) quark, gluon & electromagnetic interactions

- SM (light quarks w. Higgs integrated out) with at most marginal (dim=4) terms:

$$
\mathcal{L}_{SM} = \mathcal{L}_{EW} + \mathcal{L}_{QCD} + \mathcal{L}_{QCD}^\theta 
\equiv \mathcal{L}_{SM}^{CP} + \mathcal{L}_{cc} + \mathcal{L}_{\text{dim}=4}^{CP}
$$

$$
= \mathcal{L}_{EW} + \sum_{f=u,d,s} \bar{q}_f^c \left( i \gamma^\mu D_{\mu}^{cc'} - \delta^{cc'} m_f \right) q_{f'}^c - \frac{1}{4} G_{\mu\nu}^a G^{a\mu\nu} + \frac{\theta}{32\pi^2} G_{\mu\nu}^a \tilde{G}^{a\mu\nu}
$$
Effective Lagrangian on the quark/gluon level

Construction of an effective (low-energy) Lagrangian that incorporates the relevant \(\mathbb{C}\mathbb{P}\) (light) quark, gluon & electromagnetic interactions

- SM (light quarks w. Higgs integrated out) with at most marginal (dim=4) terms:

\[
L_{SM} = L_{EW} + L_{QCD} + L_{\theta QCD} = L_{SM}^{\mathbb{C}\mathbb{P}} + L_{cc} + L_{\mathbb{C}\mathbb{P}}^{dim=4}
\]

\[
= L_{EW} + \sum_{f=u,d,s} \bar{q}_f^c \left( i \gamma^\mu D_{\mu}^{cc'} - \delta^{cc'} m_f \right) q_f^{c'} - \frac{1}{4} G_{\mu\nu}^a G^{a\mu\nu} + \frac{\theta g_s^2}{32\pi^2} G_{\mu\nu}^a \tilde{G}^{a\mu\nu}
\]

- Add \(\mathbb{C}\mathbb{P}\) non-relevant (dim=5 and higher) effective terms to \(L_{SM}\):

\[
L_{SM} \rightarrow L_{SM} + L_{\mathbb{C}\mathbb{P}}^{dim=5} + L_{\mathbb{C}\mathbb{P}}^{dim=6} + \cdots \equiv L_{SM}^{\mathbb{C}\mathbb{P}} + L_{cc} + L_{\mathbb{C}\mathbb{P}}^{\mathbb{C}\mathbb{P}}
\]

with

\[
L_{\mathbb{C}\mathbb{P}} = L_{\mathbb{C}\mathbb{P}}^{dim=4} + L_{\mathbb{C}\mathbb{P}}^{dim=5} + L_{\mathbb{C}\mathbb{P}}^{dim=6} + \cdots
\]

\[
= \frac{\theta g_s^2}{32\pi^2} G_{\mu\nu}^a \tilde{G}^{a\mu\nu} - i \sum_{f=u,d,s} \frac{d_q}{2} \bar{q}_f \sigma^{\mu\nu} \gamma_5 q_f F_{\mu\nu} - i \sum_{f=u,d,s} \frac{d_{cq}}{2} \bar{q}_f \sigma^{\mu\nu} \gamma_5 T^a q_f G_{\mu\nu}^a
\]

\[
+ \frac{w_{3G}}{3} f^{abc} G_{\mu\nu}^a \tilde{G}^{b\nu\rho} G_{\rho}^{c\mu} + \sum_{f,g=u,d,s} C_{fg} \left( \bar{q}_f q_f \right) \left( \bar{q}_g i \gamma_5 q_g \right)
\]
Effective Lagrangian on the quark/gluon level

Construction of an effective (low-energy) Lagrangian that incorporates the relevant CP (light) quark, gluon & electromagnetic interactions

- SM (light quarks w. Higgs integrated out) with at most marginal (dim=4) terms:

\[ \mathcal{L}_{SM} = \mathcal{L}_{EW} + \mathcal{L}_{QCD} + \mathcal{L}_{\theta}^{QCD} \equiv \mathcal{L}_{SM}^{CP} + \mathcal{L}_{cc} + \mathcal{L}_{\dim=4}^{CP} \]

\[ = \mathcal{L}_{EW} + \sum_{f=u,d,s} \bar{q}_f^c \left( i \gamma^\mu D_{\mu}^{cc'} - \delta^{cc'} m_f \right) q_f^{c'} - \frac{1}{4} G_{\mu\nu}^a G^{a\mu\nu} + \frac{\theta g_s^2}{32\pi^2} G_{\mu\nu}^a \tilde{G}^{a\mu\nu} \]

- Add CP non-relevant (dim=5 and higher) effective terms to \( \mathcal{L}_{SM} \):

\[ \mathcal{L}_{SM} \rightarrow \mathcal{L}_{SM} + \mathcal{L}_{\dim=5}^{CP} + \mathcal{L}_{\dim=6}^{CP} + \cdots \equiv \mathcal{L}_{SM}^{CP} + \mathcal{L}_{cc} + \mathcal{L}_{\dim=5}^{CP} \text{ with } \]

\[ \mathcal{L}_{\dim=5}^{CP} = \mathcal{L}_{\dim=4}^{CP} + \mathcal{L}_{\dim=5}^{CP} + \mathcal{L}_{\dim=6}^{CP} + \cdots \]

\[ = \frac{\theta g_s^2}{32\pi^2} G_{\mu\nu}^a \tilde{G}^{a\mu\nu} - i \sum_{f=u,d,s} \frac{d_q}{2} \bar{q}_f \sigma^{\mu\nu} \gamma_5 q_f F_{\mu\nu} - i \sum_{f=u,d,s} \frac{d_{cq}}{2} \bar{q}_f \sigma^{\mu\nu} \gamma_5 T^a q_f G_{\mu\nu}^a \]

\[ + \frac{w_{3G}}{3} f^{abc} G_{\mu\nu}^a \tilde{G}^{b\nu\rho} G_{\rho}^{c\mu} + \sum_{f,g=u,d,s} C_{fg} (\bar{q}_f q_f)(\bar{q}_g i\gamma_5 q_g) \]

- Note: because of \( d_q \) and \( d_{cq} \propto m_f \rightarrow \mathcal{L}_{\dim=5}^{CP} \text{ effectively dim=6} ! \]
Strong CP violation: dimension 4

- QCD has non-trivial vacua with topological quantum number $|n\rangle$:
  - Cluster decomposition theorem $\rightarrow$ QCD vacuum characterized by the parameter $\theta$ value: $|\theta\rangle = \sum_{n=-\infty}^{\infty} e^{-i n \theta} |n\rangle$ with $\Delta n = \frac{g_s^2}{32\pi^2} \int d^4 x E_\mu \tilde{G}_\mu^a G^{a \mu \nu} \in \mathbb{Z}$

- $\mathcal{L}_{QCD} = \mathcal{L}_{QCD}^{CP} + \theta \frac{g_s^2}{32\pi^2} \tilde{G}_\mu^a G^{a,\mu \nu} = \mathcal{L}_{QCD}^{CP} + \theta \frac{g_s^2}{32\pi^2} \frac{1}{2} \epsilon^{\mu \nu \rho \sigma} G_{\mu \nu}^a G_{\alpha \beta}^a$

- $\mathcal{L}_{QCD}^{CP}$ term in QCD Lagrangian
  - (remember $\epsilon^{\mu \nu \alpha \beta} \neq 0$ only if $\mu \nu \rho \sigma = 0123$ modulo permutations)
Strong CP violation: dimension 4

- QCD has non-trivial vacua with topological quantum number \( |n\rangle \):
  - Cluster decomposition theorem \( \rightarrow \) QCD vacuum characterized by the parameter \( \theta \) value: 
    \[
    |\theta\rangle = \sum_{n=-\infty}^{\infty} e^{-i\theta n} |n\rangle \quad \text{with} \quad \Delta n = \frac{g_s^2}{32\pi^2} \int d^4x E \tilde{G}_\mu^a \tilde{G}^{a\mu\nu} \in \mathbb{Z}
    \]

- \( \mathcal{P} \) & \( \mathcal{T} \) (\( = \mathcal{CP} \)) term in QCD Lagrangian
  (remember \( \epsilon_{\mu\nu\alpha\beta} \neq 0 \) only if \( \mu\nu\rho\sigma = 0123 \) modulo permutations)

\[
\mathcal{L}_{QCD} = \mathcal{L}^{\mathcal{CP}}_{QCD} + \theta \frac{g_s^2}{32\pi^2} \tilde{G}_\mu^a G^{a,\mu\nu} = \mathcal{L}^{\mathcal{CP}}_{QCD} + \theta \frac{g_s^2}{32\pi^2} \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} G^{a}_{\mu\nu} G^{a}_{\rho\sigma}
\]

- Under \( U_A(1) \) rotation of the quark fields \( q_f \rightarrow e^{i\alpha \gamma_5/2} q_f \approx (1 + i\frac{1}{2} \alpha \gamma_5) q_f \):

\[
\mathcal{L}_{QCD} \rightarrow \mathcal{L}^{\mathcal{CP}}_{QCD} - \alpha \sum_f m_f \bar{q}_i \gamma_5 q + (\theta - N_f \alpha) \frac{g_s^2}{32\pi^2} \tilde{G}_\mu^a G^{a,\mu\nu}
\]

\[\mathcal{L}^{\text{str CP}}_{\text{SM}} = \mathcal{L}^{\mathcal{CP}}_{\text{SM}} - \bar{\theta} m^* \sum_f \bar{q}_f i \gamma_5 q_f \quad \text{with} \quad \bar{\theta} = \theta + \text{arg det } \mathcal{M} \quad \text{and} \quad m^* = \frac{m_u m_d}{m_u + m_d}\]
Strong CP violation: dimension 4

- QCD has non-trivial vacua with topological quantum number $|n\rangle$:
- Cluster decomposition theorem $\rightarrow$ QCD vacuum characterized by the parameter $\theta$ value:

$$|\theta\rangle = \sum_{n=-\infty}^{\infty} e^{-in\theta} |n\rangle$$

with $\Delta n = \frac{g_s^2}{32\pi^2} \int d^4x \tilde{G}_{\mu\nu}^a G^{a\mu\nu} \in \mathbb{Z}$

- $\mathcal{F}$ & $\mathcal{F}' (=\mathcal{CP})$ term in QCD Lagrangian
  (remember $\epsilon_{\mu\nu\alpha\beta} \neq 0$ only if $\mu\nu\rho\sigma = 0123$ modulo permutations)

$$\mathcal{L}_{QCD} = \mathcal{L}^{\mathcal{CP}}_{QCD} + \theta \frac{g_s^2}{32\pi^2} \tilde{G}_{\mu\nu}^a G^{a,\mu\nu} = \mathcal{L}^{\mathcal{CP}}_{QCD} + \theta \frac{g_s^2}{32\pi^2} \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} G_{\mu\nu}^a G_{\alpha\beta}^a$$

- Under $U_A(1)$ rotation of the quark fields $q_f \rightarrow e^{i\alpha \gamma_5/2} q_f \approx (1 + i\frac{1}{2} \alpha \gamma_5) q_f$:

$$\mathcal{L}_{QCD} \rightarrow \mathcal{L}^{\mathcal{CP}}_{QCD} - \alpha \sum_f m_f \bar{q}_i \gamma_5 q + (\theta - N_f \alpha) \frac{g_s^2}{32\pi^2} \tilde{G}_{\mu\nu}^a G^{a,\mu\nu}$$

$$\rightarrow \mathcal{L}^{\mathcal{strCP}}_{SM} = \mathcal{L}^{\mathcal{CP}}_{SM} - \bar{\theta} m^* \sum_f \bar{q}_i \gamma_5 q_f \quad \text{with} \quad \bar{\theta} = \theta + \text{arg det } \mathcal{M} \quad \text{and} \quad m^* = \frac{m_u m_d}{m_u + m_d}$$

- Estimate of natural $\theta$ EDM size (note $\bar{\theta} \rightarrow 0$ if one $m_f \rightarrow 0$):

$$d_{N}^{\theta} \sim \bar{\theta} \cdot \frac{e}{2m_N} \cdot \frac{m_q}{m_N} \sim \bar{\theta} \cdot 10^{-14} \text{ e} \cdot \text{cm} \cdot 10^{-2} \sim \bar{\theta} \cdot 10^{-16} \text{ e} \cdot \text{cm}$$
CP-violating sources beyond SM: dimension 6

- $CP$-violation from extensions of the standard model
  $\Rightarrow$ SUSY, multi-Higgs, Left-Right Symmetric Models, . . .

- Treatment of SM as an effective field theory:
  - All higher d.o.f. than those of a given scale are integrated out:
  - Theory contains only relevant d.o.f. and non-relevant contact terms governed by symmetry: Lorentz + SM gauge symmetries
  - All information about the physics beyond this scale are collected in the values of the low-energy constants (LECS)
CP-violating sources beyond SM: dim. 6

Add to SM all possible T- and P-odd contact interactions

100 GeV
\[ q, G, \gamma, H \]

10 GeV
\[ q, G, \gamma, H \]

\[ \ll 1 \text{ GeV} \]
\[ N, \pi, \gamma, \ldots \]

\[ q, G, \gamma, H \]

\[ qEDM \]
\[ qCEDM \]
\[ gCEDM \]
\[ 4q \]

\[ N \]
EDM-Translator from “quarkish” to “hadronic" language?
EDM-Translator from “quarkish” to “hadronic" language?

Symmetries, esp. Chiral Symmetry and Goldstone Theorem
Low-Energy Effective Field Theory with External Sources
Effective CP-violating sources on the hadronic level:

Non perturbative techniques required: e.g. 2-flavor-ChPT  
J. DeVries et.al. (2010, 2011)  

- Symmetries of QCD preserved by the effective field theory
- Association of terms by chiral transformation properties  
  (each source transforms \textit{differently} under chiral symmetry)

\[
\begin{array}{|c|c|c|c|c|}
\hline
\text{\(P\) \& \(T\) hadronic operator} & \theta & qEDM & qCEDM & gCEDM \\
\hline
N^\dagger \pi \cdot \pi N & \checkmark & - & - & \checkmark & - \\
N^\dagger \pi_3 N & - & - & - & \checkmark & \checkmark \\
N^\dagger S^\mu \nu N_{F_{\mu\nu}} & \checkmark & \checkmark & - & \checkmark & \checkmark & \checkmark \\
N^\dagger S^\mu \nu \pi_3 N_{F_{\mu\nu}} & \checkmark & - & \checkmark & \checkmark & \checkmark & \checkmark \\
\hline
\end{array}
\]

(\checkmark): Suppressed by Goldstone theorem

All sources contribute to nucleon EDMs  
Measurement of nuclear EDMs required for disentanglement!
\( \theta \)-term at the hadronic level:

\( \theta \)-term related to charge symmetry breaking (CSB) mass term:

Crewther et al. (1979); Ottnad et al. (2010); Mereghetti et al. (2011); de Vries et al. (2011)

\( \epsilon = \frac{m_u - m_d}{m_u + m_d} \), \hspace{1cm} m^* = \frac{m_u m_d}{m_u + m_d} = \frac{m_u + m_d}{4} \left(1 - \epsilon^2\right) \)

\[
\begin{align*}
\frac{m_u - m_d}{2} \frac{\bar{q} \tau_3 q}{2} & \quad \text{Non-perturbative part of } \pi NN\text{-vertex fixed by CSB studies:} \\
\delta M_{str} & = \frac{(2.0 \pm 0.3)}{MeV} \quad \text{[Gasser, Leutwyler (1982)]}
\end{align*}
\]
**θ-term induced Nucleon EDM:** complete 1-loop calculation

chiral $\mathcal{L}$: Crewther, di Vechia, Veneziano & Witten (1979/80)

complete 1-loop: Ottnad, Kubis, Meißner & Guo (2010)

\[ d_n = d_n^{\text{tree}} + d_n^{\text{loop}} = ([2.9 \pm 1.1] + [(-3 \cdots - 5) \pm 2]) \times 10^{-16} \tilde{\theta} \text{ e cm} \]

\[ \Rightarrow \quad \tilde{\theta} < 3 \times 10^{-10} \text{ (using } d_n < 3 \times 10^{-26} \text{ e cm)} \]

- Counter term at LO: estimate is model dependent
  \[ \Rightarrow \quad \text{Lattice calculation required for reliable prediction} \]
- EDM of p and n not related by isovector symmetry

Nuclear EDMs might be larger and ‘simpler’

Sushkov, Flambaum, Khriplovichv (1984)
Loop contributions to neutron EDM $d_n$

Ottnad, Kubis, Meißner & Guo, PLB 687 (2010) 42

- Note that the participating baryons and mesons in the loops are both charged and the baryons and mesons are the same in both loop topologies.
Loop contributions to the proton EDM $d_p$

Ottnad, Kubis, Meißner & Guo, PLB 687 (2010) 42

- In contradistinction to $d_n$, the graphs relevant for $d_p$ are not symmetric with respect to the photon coupling
- Note the additional neutral mesons in the second loop topology
Why $d_n \neq -d_p$?

- $d_p$ has additional (counter) terms due to
  - the contributions of additional neutral loop baryons
  - and contributions of additional neutral loop mesons
  - and the electric charge of the proton: additional LECs since $[Q, B_p] \neq 0$ while $[Q, B_n] = 0$.

- In the lattice calculation, additional noise because the charge form factor $F_1(q^2)$ and the electric dipole form factor $F_3(q^2)/2m_N$ mix in the electromagnetic current of the proton.
First lattice results for the nucleon EDM

R. Horsley et al., arXiv:0808.1428v2

- simulation details: $V = 16^3 \times 32$, $a \approx 0.11$ fm, $m_\pi/m_\rho \approx 0.8 (!)$
- proton and neutron dipole form factors:
  (here $\bar{\theta}' = 0.2$ with $\bar{\theta} = -i \bar{\theta}'$)

- extract:
  $$d_n = -0.049(5) \bar{\theta} \text{ e fm}, \quad d_p = 0.080(10) \bar{\theta} \text{ e fm} \rightarrow |\bar{\theta}| < 6 \cdot 10^{-12}$$
- Note: no isovector symmetry, $d_p \neq -d_n$!
- $d_p$ noisy since $F_1(0)$ has to be subtracted from $\langle p', s' | J^\mu | p, s \rangle$
Nuclear EDM — Two-Nucleon Contribution

for gCEDM and 4q; changes for $\theta$-term and qCEDM in [... ];

\[
\begin{align*}
\frac{e\delta q}{f} \frac{1}{m_\pi^2} \frac{1}{E} \frac{m^2_\pi}{\Lambda f_\pi} \left[ \times 1 \right] & \sim 1 \left[ 1 \right] \\
\frac{e\delta q}{\Lambda} \frac{1}{E} \frac{1}{f_\pi^2} \left[ \times \frac{m^2_\pi}{\Lambda^2} \right] & \sim 1 \left[ \ln\left( \frac{m^2_\pi}{\mu^2} \right) \frac{m^2_\pi}{\Lambda^2} + C \right] \\
\frac{e\delta q}{\Lambda} \frac{1}{E} \frac{1}{f_\pi^2} \left[ \times \frac{m^2_\pi}{\Lambda^2} \right] & \sim 1 \left[ \frac{m^2_\pi}{\Lambda^2} \right] \\
\frac{e\delta q}{f_\pi M} \frac{1}{m_\pi^2} \frac{m^2_\pi}{\Lambda f_\pi} \left[ \times 1 \right] & \sim \frac{E}{\Lambda} \left[ \frac{E}{\Lambda} \right]
\end{align*}
\]

pion ranged operators enhanced for $\theta$-term and qCEDM

Sushkov, Flambaum, Khriplovich (1984)
Two-Nucleon Contribution to the Deuteron EDM at LO

\[ N^\dagger \pi \cdot \tau N: \quad 3S_1 \xrightarrow{\gamma} 1P_1 \xrightarrow{\gamma} 3S_1 \]
\[ N^\dagger \pi_3 N : \quad 3S_1 \xrightarrow{\gamma} 3P_1 \xrightarrow{\gamma} 3S_1 \]

⇒ No $\theta$EDM contribution to $^2H$ at LO
$\theta$EDM contribution to $^3He$ at LO

in $G_{\pi}^{(1)} \times \text{e fm}$

<table>
<thead>
<tr>
<th>State</th>
<th>CDBonn [U] (3)</th>
<th>AV18 [U] (3)</th>
<th>Reid93 [U] (2)</th>
<th>ZRA [U] (1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^3S_1$</td>
<td>$-1.46 \cdot 10^{-2}$</td>
<td>$-1.41 \cdot 10^{-2}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^3D_1$-adm.</td>
<td>$-0.48 \cdot 10^{-2}$</td>
<td>$-0.49 \cdot 10^{-2}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>$-1.94 \cdot 10^{-2}$</td>
<td>$-1.91 \cdot 10^{-2}$</td>
<td>$-1.92 \cdot 10^{-2}$</td>
<td>$-1.8 \cdot 10^{-2}$</td>
</tr>
</tbody>
</table>

(2): Afnan, Gibson (2010)
(3): Bsaisou, Hanhart, Meißner, Wirzba, ... (2012, in prep.)
Two-Nucleon Contribution to the Deuteron EDM at LO

\[ \sum_{3S_1} = ie(1 + \tau_3) \]

\[ N^\dagger \pi \cdot \vec{\tau} N: \quad 3S_1 \rightarrow 1P_1 \rightarrow 3S_1 \]

\[ N^\dagger \pi_3 N: \quad 3S_1 \rightarrow 3P_1 \rightarrow 3S_1 \]

\[ \Rightarrow \text{No } \theta \text{EDM contribution to } ^2H \text{ at LO} \]

\[ \theta \text{EDM contribution to } ^3He \text{ at LO} \]

in \( G^{(1)}_\pi \times \text{e fm} \)

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<td>(-1.92 \times 10^{-2})</td>
<td>(-1.8 \times 10^{-2})</td>
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<tr>
<td>(^3P_1)-int.</td>
<td>0.43 \times 10^{-2}</td>
<td></td>
<td>0.39 \times 10^{-2}</td>
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</tr>
<tr>
<td>Total</td>
<td>(-1.51 \times 10^{-2})</td>
<td></td>
<td>(-1.52 \times 10^{-2})</td>
<td></td>
</tr>
</tbody>
</table>


(2): Afnan, Gibson (2010)

(3): Bsaisou, Hanhart, Meißner, Wirzba, ... (2012, in prep.)
# Two-Nucleon Contribution to the Deuteron EDM at NLO

\[ = N^\dagger \pi_3 N \text{ (qCEDM)} \]

Bsaisou, Hanhart, Meißner, Wirzba ... (2012, in prep.)

<table>
<thead>
<tr>
<th>LO</th>
<th>( N^2 \text{LO} \sim 1% )</th>
<th>( N^4 \text{LO} )</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="LO Diagram" /></td>
<td><img src="image2" alt="N^2 LO Diagram" /></td>
<td><img src="image3" alt="N^4 LO Diagram" /></td>
</tr>
</tbody>
</table>

- **Finite:**
  - ![Finite Diagram](image4)
  - ![Finite Diagram](image5)

- **Ren.:**
  - ![Renormalization Diagram](image6)
  - ![Renormalization Diagram](image7)

- All other topologies yield no EDM contribution, blue: also \( \theta \text{EDM contr.} \)
- Consistent power counting scheme
Outlook:

<table>
<thead>
<tr>
<th>source</th>
<th>$\theta$</th>
<th>$qEDM$</th>
<th>$qCEDM$</th>
<th>$gCEDM$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
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<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>$n$</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>$^2H$</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>$^3He$</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

- EDMs are ideal probes for CP violation in the hadronic sector
- EDMs of light nuclei provide independent information to $p$ and $n$
- EDMs of light nuclei may be larger & simpler than nucleon EDMs
- qEDM dominates if nuclear EDM is sum of nucleon EDMs
- Nuclear calculation possible up to accuracy of a few %
- Deuteron is a filter for the isospin-dependent qCEDM
- $\theta$EDM: $d_{^3He} - 2d_p - d_n \iff \bar{\theta} \iff p-,n$-EDM
Outlook: *May the force be with us!*

<table>
<thead>
<tr>
<th>source</th>
<th>$\theta$</th>
<th>$q\text{EDM}$</th>
<th>$q\text{CEDM}$</th>
<th>$g\text{CEDM}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
<td>$\checkmark$</td>
<td>$\checkmark$</td>
<td>$\checkmark$</td>
<td>$\checkmark$</td>
</tr>
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<td>$\checkmark$</td>
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</tr>
<tr>
<td>$^2H$</td>
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<td></td>
<td>$\checkmark$</td>
</tr>
<tr>
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<td></td>
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- EDMs are **ideal probes** for CP violation in the hadronic sector
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- Deuteron is a filter for the isospin-dependent $q\text{CEDM}$
- $\theta\text{EDM}$: $d_{^3\text{He}} - 2d_p - d_n \iff \bar{\theta} \iff p-,n-$EDM

A measurement of $p$, $n$, $d$, and $^3\text{He}$ EDM is necessary to learn about the underlying physics