ALTERNATING SPIN ABERRATION ELECTROSTATIC LATTICE FOR EDM

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Abstract

The idea of the electric dipole moment search using the storage ring (SrEDM) with polarized beam is realized under condition of the long-time spin coherency of all particles, the time during which the rms spread of the orientation spin of all particles in the bunch reaches one radian [1]. Following the requirements of the planned EDM experiment the SCT should be more than 1000 seconds. During this time each particle performs about 10^9 turns in the storage ring moving on different trajectories through the optics elements. At such conditions the spin-rotation aberrations associated with various types of space and time dependent nonlinearities start to play a crucial role. In this paper we consider a new method based on the alternating spin rotating, thereby limiting the growth of aberrations at one order of magnitude lower. As a result, using this method we can achieve the SCT of the order of 5000-6000 seconds. The difficulties of these studies are still in the fact that the aberrations growth observed in the scale of a 10^9 turns. For the study we use the analytical method in composition with a numerical simulation by COSY-infinity [2].

INTRODUCTION

Time-dependent aberration is a spin tune aberration due to the different time-flight of particles in the focusing-deflecting fields. The space-dependent spin aberrations are associated with differences in the focusing-deflecting fields on the trajectory of particles. Besides the spin tune itself depends on the particle energy, which also introduces additional aberrations.

In [3] we studied the growth of aberrations in the presence of the momentum spread and the initial deviation from the equilibrium orbit in the horizontal plane. It was shown that at the momentum spread 10^{-4} the SCT is less than one millisecond. This disappointed fact coincides perfectly with the numerical simulation of COSY-infinity code. Then, following the previously proposed method [4,5], we have included an RF field to average the momentum deviation. Under the action of RF field the spin begins to oscillate with the longitudinal tune ν_z two orders of magnitude higher than the spin tune ν_s and therefore with very small amplitude $\Phi_{\max} \sim (\nu_s/\nu_z)^2$ relative to a central position, which in turn is drifting with very low frequency. Figure 1 shows the behavior of the horizontal projection of the spin S_x when you turn on RF. Since the oscillating component is always within $\Phi_{\max}$, hereinafter we will be interested only in the slow component drift. Just this averaged over time drift term does give non-zero contribution [3] and therefore SCT is limited at the momentum spread $\sim 5\cdot 10^{-5}$ by 180 sec. Thus the RF field increases the SCT by five orders, but it nevertheless remains unsatisfactory for EDM experiment.

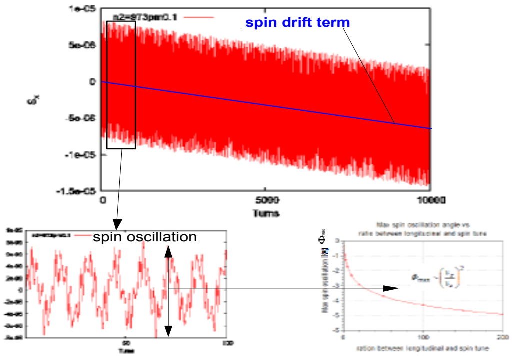

Figure 1: Oscillating and drift terms of spin behavior

Besides, it is unfortunately feasible only for particles with zero initial deviation from the axis. For non-zero deviation the particles receive a new equilibrium orbit energy with the momentum shift $\delta p/p$, which inevitably leads to a rapid increase of aberrations. In paper [3] studying this phenomenon, we found a method to introduce additional oscillations on the momentum that gave the averaging of the equilibrium orbit itself. As a result, we can achieve a longer SCT time up to 500 sec at $\Delta W_{kin}/W_{kin}=10^{-4}$, but it is still not enough.

TUNE OF SPIN AND ITS ABERRATION

The equation of the spin oscillations has the form:

$$\frac{d\vec{S}}{dt} = \vec{\omega}_G \times \vec{S}$$
$$\vec{\omega}_G = -\frac{e}{m_0\gamma c}\left(\frac{1}{\gamma^2-1}-G\right)\cdot(\vec{\beta}\times\vec{E}) \qquad (1)$$

We consider so-called “magic” [6] purely electrostatic ring for polarized protons, when for the reference particle $1/(\gamma_m^2-1)-G=0$. Now we should mention again that further the spin projections indications will be made in the following line: z is orientated along the momentum, x and y are horizontal and vertical directions correspondingly. Taking into account that the vertical and longitudinal electric field components are expected to be small and

$\beta_x, \beta_y << \beta_z$ we can get an expression for the number of spin oscillation per one turn that is spin tune:

$$\nu_s = \frac{e}{2\pi m_0 c^2} \cdot L_{orb} E_x \frac{1}{\gamma}\left(\frac{1}{\gamma^2 - 1} - G\right), \qquad (2)$$

where L_{orb} is orbit length. For the particle with energy different from the "magic" value $\gamma \neq \gamma_m$ the factor $G - 1/(\gamma_m^2 - 1) \neq 0$ does not equal to zero, and the spin rotates with a frequency dependent on particle energy, which leads to the spin tune aberrations.

Assuming "magic" condition we define a variation of the spin tune through the finite differences up to second order:

$$\delta\nu_s = \frac{e}{2\pi m_0 c^2} \cdot \delta\left(\frac{1}{\gamma^2 - 1} - G\right) \cdot L_{orb} E_x \frac{1}{\gamma}\left[1 + \frac{\delta L_{orb}}{L_{orb}} + \frac{\delta E_x}{E_x} + \gamma \cdot \delta\left(\frac{1}{\gamma}\right)\right] \qquad (3)$$

Representing each of them through the Taylor series expansion in powers of the finite difference $\Delta p / p$ up to second order:

$$\delta\left(\frac{1}{\gamma^2 - 1} - G\right) = -2G\frac{\Delta p}{p} + \frac{1 + 3\gamma^2}{\gamma^2} G \left(\frac{\Delta p}{p}\right)^2 + \ldots$$

$$\frac{\delta L_{orb}}{L_{orb}} = \alpha_1 \cdot \frac{\Delta p}{p} + \alpha_2 \cdot \left(\frac{\Delta p}{p}\right)^2 + \ldots \qquad (4)$$

$$\frac{\delta E_x}{E_x} = -k_1 \frac{x}{R} + k_2 \left(\frac{x}{R}\right)^2 + \ldots$$

$$\gamma\delta\left(\frac{1}{\gamma}\right) = -\frac{\gamma^2 - 1}{\gamma^3}\left(\frac{\Delta p}{p}\right) + \frac{(\gamma^2 - 1)^2}{2\gamma^5}\left(\frac{\Delta p}{p}\right)^2 + \ldots$$

where α_1 and α_2 are the momentum compaction factor in the first and second approach correspondingly; k_1 and k_2 are coefficients of the expansion of the field in the vicinity of the equilibrium orbit. As example for the cylindrical deflector the coefficients are $k_1 = 1$ and $k_2 = 1$.

ABERRATION OF SPIN OSCILLATION

After averaging over time the term $\Delta p / p$ gives zero contribution in to the spin tune. Substituting equations (4) in (9), and grouping the coefficients of $\Delta p / p$ powers up to second order, we obtain:

$$\delta\nu_s = \frac{e L_{orb} E_x G}{2\pi m_0 \gamma c^2} \cdot \left[F_2\left(\alpha_1, k_1, k_2, \frac{x}{R}\right) \cdot \left(\frac{\Delta p}{p}\right)^2 + 2F_1\left(k_1, k_2, \frac{x}{R}\right) \cdot \frac{\Delta p}{p}\right] \qquad (5)$$

$$F_2\left(\alpha_1, k_1, k_2, \frac{x}{R}\right) = \frac{1 + 3\gamma^2}{\gamma^2} k_2 \left(\frac{x}{R}\right)^2 - \frac{1 + 3\gamma^2}{\gamma^2} k_1 \frac{x}{R} + \frac{5\gamma^2 - 1}{\gamma^2} - 2\alpha_1$$

$$F_1\left(k_1, k_2, \frac{x}{R}\right) = -k_1 \frac{x}{R} + k_2 \left(\frac{x}{R}\right)^2$$

Thus, the aberration of the spin is determined by a parabolic equation.

We did not include in our consideration the coefficients k_n with $n > 2$ and α_2 because we consider the aberrations growth only up to second order of $(\Delta p / p)^2$ and $(x / R)^2$. Figure 2 shows the two dimensional parabolic dependence of spin tune aberration in 3D representation, where one axis is momentum spread in units of 10-4 and other axis is horizontal deviation in mm. The tune spin is normalized by a factor $N_F = \frac{e L_{orb} E_x G}{2\pi m_0 \gamma c^2}$. The coefficients k_1, k_2 depend on shape of deflector and the momentum compaction factor is defined by the lattice as whole.

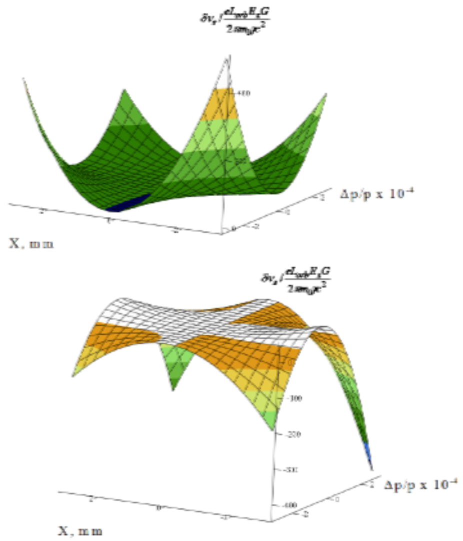

Figure 2: Spin tune aberration dependence on momentum spread and horizontal deviation at different k_1, k_2

Thus, these results show that it is impossible to exclude the growth aberrations of the tune spin for non-monochromatic beam with non-zero emittance, that is at $\Delta p / p \neq 0$ and/or $x \neq 0$.

SPIN ABERRATION MINIMIZING

However, from the above derived formula we can see there are two methods to minimize aberrations of spin. The first method is to choose the lattice with a compensation of the mutual influence of parameters k_1, k_2, α_1. In other words, we need to make a two-dimensional parabola maximally flat in the workspace of $(\Delta p / p)^2$ and $(x / R)^2$. To verify the analytic results we have done a full-scale simulation using the COSY-infinity code [2] symplectically calculating the spin-orbital motion in the pure electrostatic lattice consisting of the electrostatic deflectors and electrostatic quadrupoles only. Figure 3 shows the lattice in a format of OPTIM code [7]. Ring consists of two arcs, each arc has 4 FODO cells, one cell has 4 electrostatic deflectors in each gap between

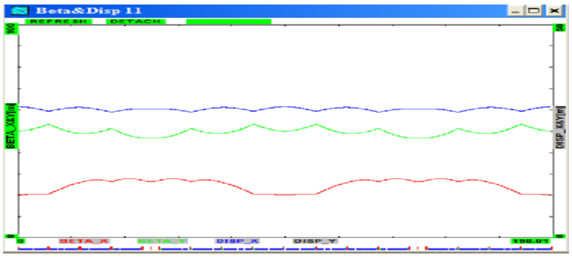

Figure 3: Twiss functions of electrostatic ring for ring and one cell

quadrupoles F and D. As example the straight section is designed of one FODO cell. Horizontal and vertical tune has 1.3 and 0.635 value correspondingly. The electric field between plates of deflector is taken 17 MV/m.
The maximum flatness is reached by choice of parameters of deflector k_1, k_2 and α_1 momentum compaction factor. The requirement for the momentum compaction factor to be as large as possible obviously follows from the expressions (5) in the ring with a cylindrical or similar geometry of deflector, when the electric field has the coefficients k_1 and k_2 close to unit.
Figure 4 shows the results of numerical simulation with the optimum parameters of the deflector k_1=0.94 and k_2=0.96 in the whole range of operating parameters of the beam. The red curve is described by a parabola $\Delta\nu_s/NF=0.012\cdot x^2$.

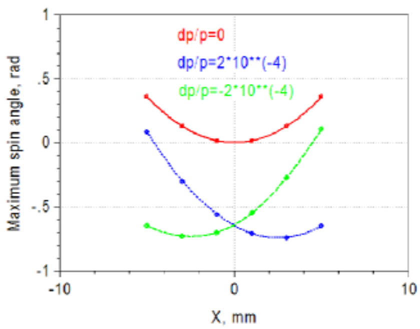

Figure 4: Maximum spin deflection angle after 10^9 turns versus x deviation at $\Delta p/p=2\cdot10^{-4}$, 0, $-2\cdot10^{-4}$

ALTERNATING SPIN OSCILLATION AS METHOD TO MINIMIZE ABERRATION

The second method is to alternately change the deflector parameters and thereby alternating the rotation of the spin. In mathematical terms, this means minimizing all the factors F_0, F_1, F_2 by averaging them in time. For this purpose, we suggest the alternating spin aberrations lattice rotating spin, for instance, in even deflectors in one direction and in odd deflectors in the other direction. That is, the ring is equipped with two types of deflector having k_1 = const, and $k_2 = k_{av} \pm \Delta k$ changing from a deflector to deflectors. Figure 5 shows the results of numerical simulation. We see that by choosing k_2=0.974±0.1 we can get practically zero aberration for particles with $\Delta p/p = 0$ and the function is described by a parabola $\Delta\nu_s/NF=0.004\cdot x^2$. Comparing it with previous one we can

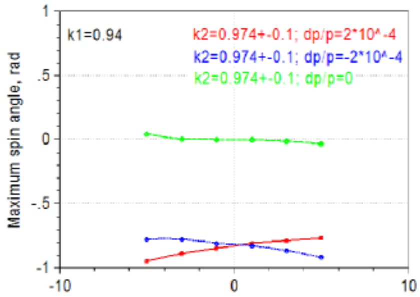

Figure 5: Maximum spin deflection angle after 10^9 turns versus x deviation in mm at $\Delta p/p=0$ (a) and $\pm2\cdot10^{-4}$, 0 (b)

see that the flatness in the workspace of the beam has improved nearly 90 times. However, the particle with nonzero momentum deviation has a finite value of the spin deflection getting parallel shift downward. This spread due to final $\Delta p/p$ it is impossible to remove using the correct k_1 and k_2. Nevertheless the total spread of spin deflection angle does not exceed ±0.5 rad after 10^9 turns, which one corresponds to a SCT about 5000 seconds.
Secondly, raising the field strength between the plates in even deflectors and reducing in the odd deflectors it effectively adjust the required coefficients k_1 and k_2. This allows adjusting the spin of aberration to a minimum.
Another possibility is the creation of the required potential distribution due to potential changes in stripline deposited on the surface of the ceramic plates.
In this work we have studied the behavior of spin aberrations in the structure and developed techniques to minimize them. One of the most effective methods is the alternating spin aberration. The analytical model allows finding the general solution of the retention of aberrations within the values allowed SCT to have about 5000 seconds. Authors would like to thank A. Lehrach for fruitfull discussion and K.Makino for help in COSY-infinity installation, adjusting it for the EDM problem.

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