Comparison of Different Numerical Modelling Methods for Beam Dynamics in Electrostatic Rings

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Abstract

To search the electric dipole moment was proposed to use polarized protons at the so-called ”magic” momentum of 0.7 GeV/c in an electric storage ring. For studying beam dynamics in electrostatic rings different computational methods can be used. We used differential algebra methods realized in COSY Infinity and integrating program with symplectic Runge-Kutta methods. These methods were observed and compared for orbital and spin motion.

INTRODUCTION

The results of numerical modeling and comparative analysis of long-term evolution of the particle dynamics in electrostatic fields are presented. The goal of the research is to study how the spin tracking results, obtained with COSY Infinity program, coincide with the results of direct integrating of motion and spin equations. Effective in sense of performance algorithm that used in COSY Infinity, can be tested with less efficient, but accurate traditional algorithm of numerical simulation based on the Runge-Kutta scheme. In both cases the symplectic version of the algorithms are used, and in step-by-step integration additional conditions are taken into account for corresponding to energy conservation.

COSY INFINITY

To confirm analytical calculations we need to do long term (hundreds of millions turns or even more) tracking of bunch of particles. Full spin-tracking simulations of the entire experiment are absolutely crucial to explore in a systematic way the feasibility of the planned experiments.

One of the program that can simulate particle evolution both in phase and spin spaces is COSY Infinity. COSY Infinity is a code for the simulation, analysis and design of particle optical systems, based on differential algebraic methods. It is planned to use the COSY Infinity program and to include higher-order nonlinearities, normal form analysis, symplectic tracking and especially spin tracking upon incorporation of RF-E and RF-B flippers into the code. In order to study subtle effects and simulate the particle and especially spin dynamics during the storage and build-up of the EDM signal, one needs custom-tailored fast trackers capable of following up to 10–100 billions turns for samples of up to 10^4–10^6 particles.

COSY Infinity operates with phase coordinates:

$$\begin{aligned} r_1 &= x, & r_2 &= a = \frac{p_x}{p_0} \\ r_3 &= y, & r_4 &= b = \frac{p_y}{p_0} \\ r_5 &= l = -\tfrac{(t-t_0)v_0\gamma}{1+\gamma}, & r_6 &= \delta_K = \frac{K-K_0}{K_0}, \end{aligned} \quad (1)$$

and finds the solution in form

$$\mathbf{X} = \mathbb{M}^1\mathbf{X_0} + \mathbb{M}^2\mathbf{X_0}^{[2]} + \mathbb{M}^3\mathbf{X_0}^{[3]} + \ldots + \mathbb{M}^N\mathbf{X_0}^{[N]}. \quad (2)$$

Here x and y are the horizontal and vertical distances to the optic axis, respectively. The quantities p_0, v_0, K_0 denote momentum, velocity and kinetic energy of the reference particle, and p, v, K stand for the same quantities of the particle under consideration. $\mathbf{X}^{[N]} = \underbrace{\mathbf{X} \otimes \ldots \otimes \mathbf{X}}_{N times}$ — Kronecker power of $\mathbf{X}$. $\mathbf{X}^k$ is a vector with C^k_{6+k-1} elements. Matrices $\mathbb{M}^k$ have dimensions $6 \times C^k_{6+k-1}$. Transfer maps $\mathbb{M}^k$ can be generated up to any order.

And for spin motion we have initial spin coordinates:

$$\mathbf{S_0} = \begin{pmatrix} S_{x_0} \\ S_{y_0} \\ S_{z_0} \end{pmatrix}, \quad (3)$$

$$S^2_{x_0} + S^2_{y_0} + S^2_{z_0} = 1.$$

Spin coordinates after one revolution:

$$\mathbf{S} = \mathbb{M}_S \mathbf{S_0}, \quad (4)$$

where $\mathbb{M}_S$ — spin rotation matrix.

To simulate bunch of particles we used initial set of particles with random distribution. We took the values $-3mm < x < 3mm$, $-0.001 < p_x < 0.001$, $y = 0$, $p_y = 0$, $-2 \cdot 10^{-4} < \Delta E < 2 \cdot 10^{-4}$, $S_z = 1$ (spin of all particles oriented along the reference orbit, so $S_z = 1$, $S_x = S_y = 0$).

For this initial set we used COSY Infinity to track the evolution for two million of turns in different lattices. The final spin distribution (after evolution in optimized structure with customly shaped deflectors, see [1]) is represented on the figure (1). X-axis is a particle number and Y-axis shows S_x component of spin vector. There are S_x coordinates of 32768 particles with RMS less than 1 mrad which means that for one billion of turns RMS should be less than 1 radian.

Calculation of a million or turns takes less than one hour and one can use a compute cluster to calculate a long-term evolution of bunch of particles in reasonable time.

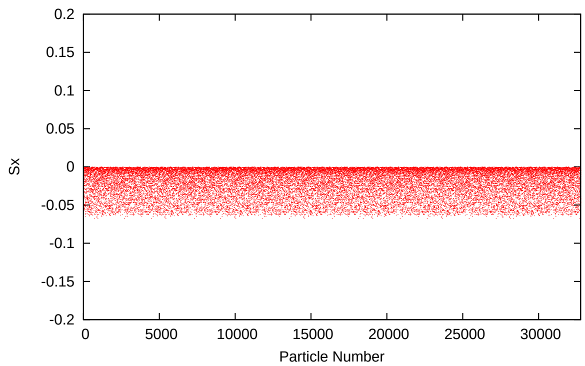

Figure 1: Spin distribution after 2 million turns.

TRACKING APPROACH

To simulate the beam dynamics in electrostatic we used a program for step-by-step symplectic integration of motion equations and spin equations. Dynamics of charge particle is described in generalized coordinates along a design orbit. This section presents the basic equations of the motion and spin dynamics. Also integration scheme is briefly discussed.

Orbital motion

Derivation of the trajectory equations that describes the orbital motion uses generalized coordinates [2]. In the research we use (x, y, s) space, where s is independed variable that equals to the length along the design orbit.

The design orbit is choosed in accordance to symmetry of field distribution. For example, in quadrupole lenses it is a straight line, in cylindrical or spherical deflectors it is an arc of a circle.

In case of straight design orbit the motion is presented as Newton-Lorentz equation in Cartesian space. When the design orbit is a circle arc the equations are following:

$$
\begin{aligned}
x'' + \frac{1}{\gamma}\frac{HG}{v}x' - (1 + \frac{x}{R})\frac{1}{R} = \frac{QH}{m_0 v}\frac{1}{\gamma}HE_x/v, \\
y'' + \frac{1}{\gamma}\frac{HG}{v}y' = \frac{QH}{m_0 v}\frac{1}{\gamma}HE_y/v,
\end{aligned}
\tag{5}
$$

where H, G are functions of variable x, x', y, y', R, and R is a curvature radius of the design orbit.

Spin dynamics

Spin dynamics is described by the T–BMT equation. Along the circle arc we have

$$
\begin{aligned}
S_x' &= S_s/R + \frac{Q}{m_0 c^2}\left(G + \frac{1}{1+\gamma}\right)\Big((h_s E_x - x' E_s)S_s - \\
&\quad -(x' E_y - y' E_x)S_y\Big), \\
S_y' &= \frac{Q}{m_0 c^2}\left(G + \frac{1}{1+\gamma}\right)\Big((x' E_y - y' E_x)S_x - \\
&\quad -(y' E_s - h_s E_y)S_s\Big), \\
S_s' &= -S_x/R + \frac{Q}{m_0 c^2}\left(G + \frac{1}{1+\gamma}\right)\Big((y' E_s - h_s E_y)S_y - \\
&\quad -(h_s E_x - x' E_s)S_x\Big),
\end{aligned}
\tag{6}
$$

where γ is Lorentz factor.

Symplectic Runge-Kutta scheme

The equations (5) and (6) can be written as

$$
\frac{d}{ds}\mathbf{X} = \mathbf{F}(s, \mathbf{X}), \tag{7}
$$

where $\mathbf{X} = (x, x', y, y', S_x, S_y, S_s)$.

It allows us to use classical step-by-step integration methods to solve this system. As basic method for the tracking program, a symplectic Runge-Kutta scheme was implemented [3].

$b_1 + \tilde{c_1}$	$b_1/2$	$b_1/2 + \tilde{c_1}$
$b_1 - \tilde{c_1}$	$b_1/2 - \tilde{c_1}$	$b_1/2$
	$b_1 = 1/2$, $2b_1\tilde{c_1}^2 = 1/12$	

Table 1: 2-stage 4-order implicit Runge-Kutta scheme.

According to this scheme (Table 1), the solution of the equations (7) can be presented in iterative form

$$
\begin{aligned}
\mathbf{X}_{n+1} &= \mathbf{X}_n + h\sum_{j=1}^{2} b_j \mathbf{F}(s + hc_j, \mathbf{X}^{(i)}), \\
\mathbf{X}^{(i)} &= \mathbf{X}_n + h\sum_{j=1}^{2} a_{ij} \mathbf{F}(s + hc_j, \mathbf{X}^{(i)}).
\end{aligned}
\tag{8}
$$

Note that symplectic scheme (8) imposes the condition of constant integration step. Moreover this scheme requires to solve implicit equations and appropriate numerical methods can be used.

MAIN GOALS

We have been studying the spin tracking problem in the accelerators, in paper [4] we considered various causes leading to aberrations of spin motion, we estimated their values using simple analytic techniques and compared them with numerical results obtained with the simulation program COSY Infinity. Based on these results, we considered different methods to reduce spin aberrations — desirable spin coherence time (time when RMS spin orientation of the bunch particles reaches one radian) is more than 1000 seconds. Now it is clear that obtaining desired spin coherence time is possible [1] with custom shaped electrostatic plates, so we need to make calculations for several different condenser shapes.

COMPARISON

To compare the computation results of both codes we used the lattice with cylindrical deflectors, which was described in [4]. Comparing the results of tracking through

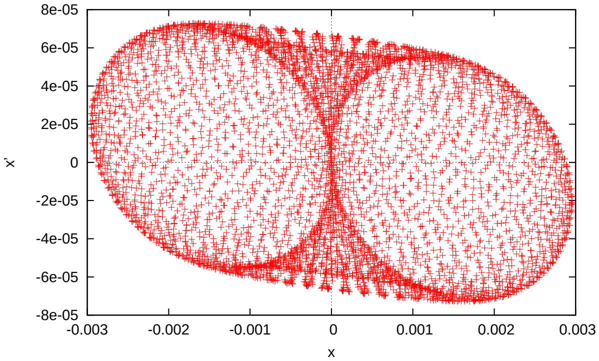

Figure 2: x–x' motion in COSY Infinity.

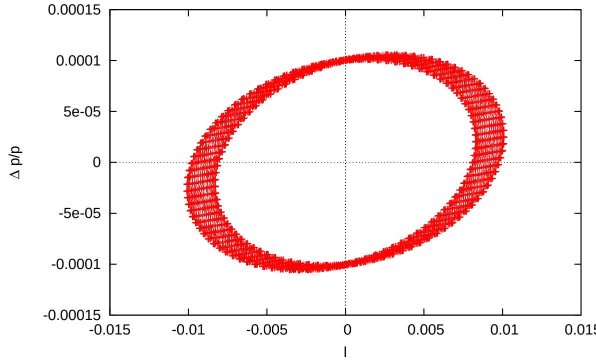

Figure 4: Longitudinal plane in COSY Infinity.

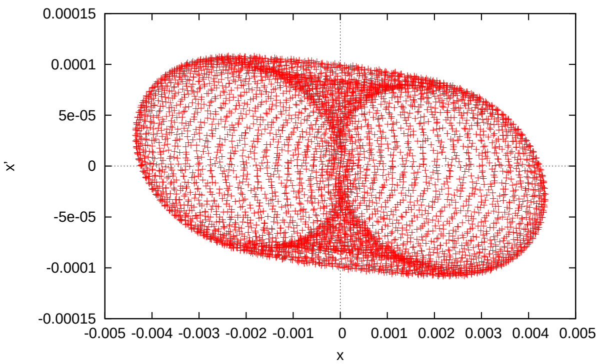

Figure 3: x–x' motion in the integrating program.

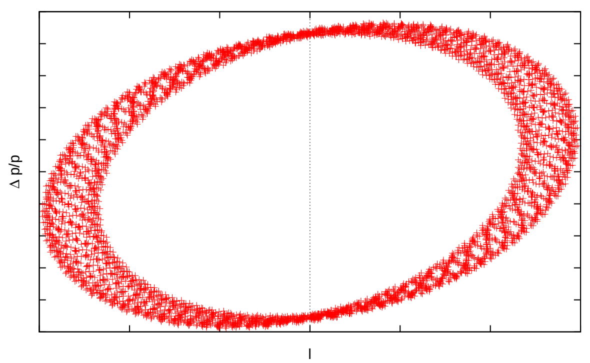

Figure 5: Longitudinal plane in the integrating program.

a single element we found good coincidence in computational models of COSY Infinity and integrating program. However the different choice of the reference orbits and different simplectification methods doesn't allow to obtain the same numerical results, but we can compare dynamics and behaviour of particles in both programs.

As initial values we used a particle without initial deviation in x–x' and y–y' spaces, $\Delta p/p = 10^{-4}$ and a particle with initial deviation $x = 3mm$, $\Delta p/p = 0$ and tracked for 10000 turns in the lattice without RF field. For these cases we got almost the same results for spin coherence time (for the first case 3292 seconds and 3658 seconds respectively, for the second particle — 323 and 349 seconds). After turning on RF field to average the motion in longitudinal plane for particle without initial deviation and with $\Delta p/p = 10^{-4}$ we got SCT in COSY Infinity about 7300 seconds and in the integrating program about 9800 seconds.

On figures 2, 3, 4, 5 one can see transverse and longitudinal planes of motion in the lattice with turned RF field turned on in both programs.

CONCLUSION

The results obtained in the both programs coincide with each other, but some disagreements have been identified. These disagreements come from the different choice of the reference orbits and the different simplectification methods, but we have good coincidence in qualitative behaviour of the orbital motion and SCT values coincide well. The possible approaches for further verification are based on alternative methods of integration that lead us to complex investigation of numerical simulation results. Also we need to study behaviour of orbital and spin motion in the lattice with fringe fields because the fringe fields can significantly affect spin motion.

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