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Frozen and Quasi-frozen spin lattices

on behalf of Collaboration "JülichElectricDipole moment Investigation"

10. März 2017

The EDM experiment problems

1. **Beam optics** (betatron tunes, sextupoles, DA, RF, straignt sections and so on)
2. **Spin coherence time** maximizing up to t_{coh} >**1000 sec** to provide the possible EDM signal observation
3. **Systematic errors** investigation to exclude "fake EDM signal"
4. Maximum **beam polarization** P~80%
5. **Beam intensity** $\sim 10^{10} \div 10^{11}$ particle per fill
6. Maximum **analyzing power** of polarimeter A~0.6
7. Maximum **efficiency of polarimeter** f> 10^{-3}
8. Total **running time** of accelerator ~5÷7 thousand hours
9. Minimum **radius of machine** with **E~10÷12 MV/m**

Most important problems in the ring for EDM search:

1. Frozen (or quasi-frozen) spin lattice
2. Spin decoherence
3. Systematic errors

Two concepts of lattices for deuteron EDM search have been investigated:

1. Frozen Spin (FS) method
2. Quasi-frozen spin (QFS) method

T-BMT and the basic measurement principle

$$\frac{d\vec{S}}{dt} = \vec{S}\times\vec{\Omega}_{MDM} + \vec{S}\times\vec{\Omega}_{EDM}$$

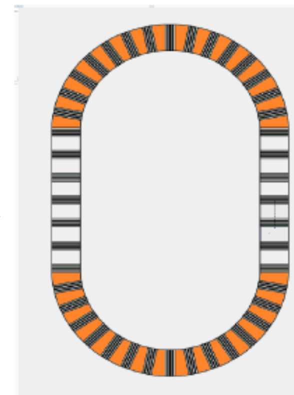
$$\vec{\Omega}_{MDM} = \frac{e}{m}\left[G\vec{B} - \left(G - \frac{1}{\gamma^2-1}\right)\frac{\vec{E}\times\vec{\beta}}{c}\right]$$

$$\vec{\Omega}_{EDM} = \frac{e}{m}\frac{\eta}{2}\left[\frac{\vec{E}}{c} + \vec{\beta}\times\vec{B}\right]$$

Frozen spin condition: $\left(\frac{1}{\gamma^2-1} - G\right)\left(\frac{\vec{\beta}\times\vec{E}}{c}\right) + G\vec{B} = 0$

$\vec{\Omega}_{MDM} = 0$

The spin stays parallel to the momentum



Quasi-frozen spin condition: $\gamma G \Phi_B = \left[\frac{1}{\gamma}(1-G) + \gamma G\right]\Phi_E$

Φ_E and Φ_B - the angles of the momentum rotation in the electric and the magnetic parts of the ring

$\langle\vec{\Omega}_{MDM}\rangle = 0$

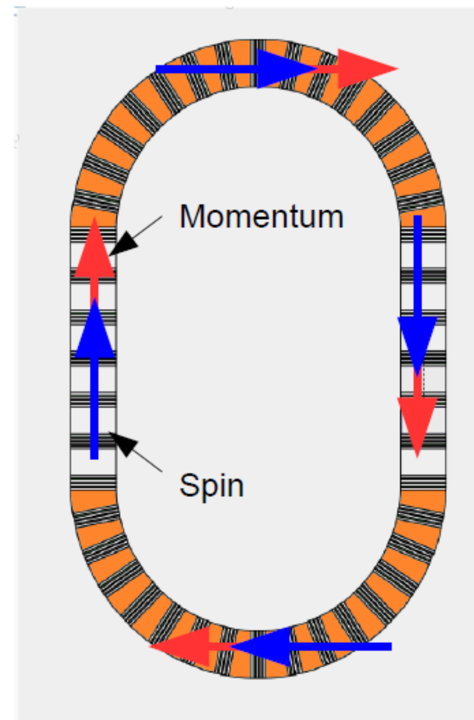
Frozen spin method

$$\left(\frac{1}{\gamma^2-1}-G\right)\left(\frac{\vec{\beta}\times\vec{E}}{c}\right)+G\vec{B}=0$$

- Combined E+B elements:
  radial E field and vertical B field
- The ring radius depends on deuteron G factor and energy
- For 270 MeV deuterons the ring radius and length are:

$$R=\frac{|G|}{(|G|-1)}\cdot\left[\frac{mc^2}{eE}\right]\gamma^3\beta^2\approx 9.2\,\mathrm{m}$$

Length = 145 m



Quasi-frozen spin method

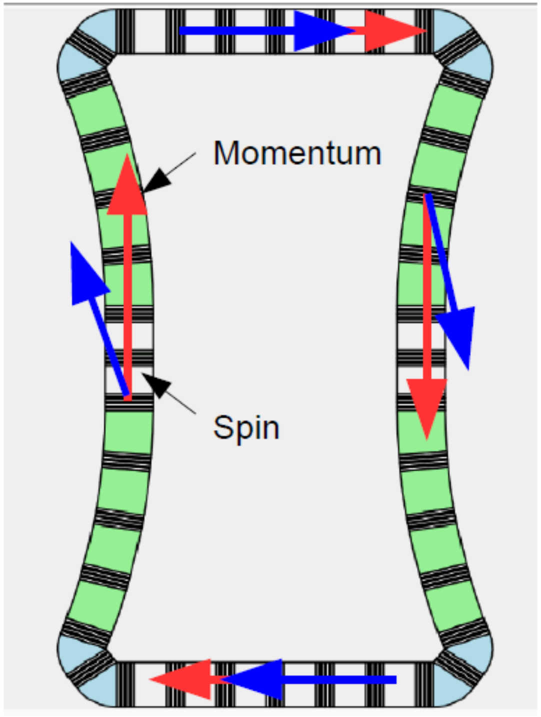
$$\gamma G \Phi_B = \left[\frac{1}{\gamma}(1-G) + \gamma G \right] \Phi_E$$

Φ_E and Φ_B - the angles of the momentum rotation in the electric and the magnetic parts of the ring

- Electric arcs with negative curvature and magnetic arcs
- The radii should fulfil the condition that keeps the spin vector parallel to the momentum after one revolution

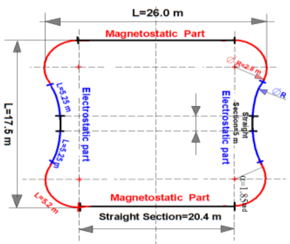
$R_{magnetic} \approx 2.3$ m
$R_{electric} \approx 42.1$ m

Length = 166 m



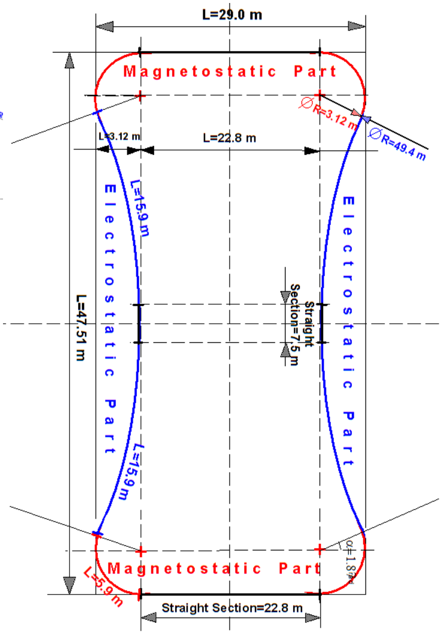
d-EDM ring for different energies

dEDM-75 MeV



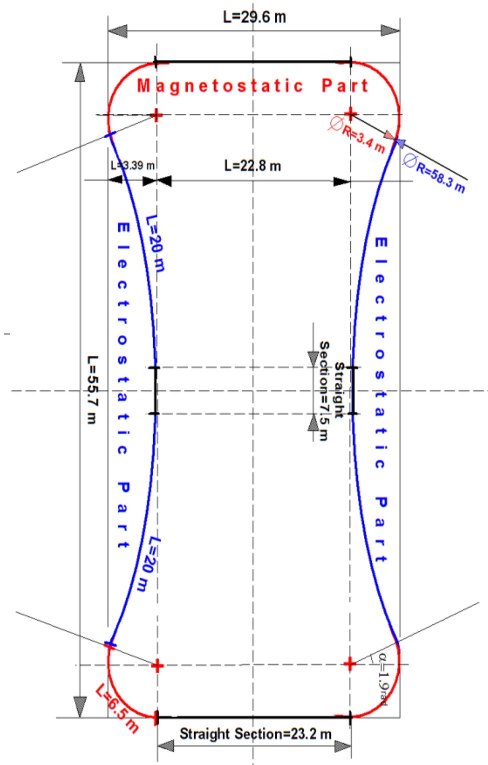
26 m x 17 m

dEDM-200 MeV



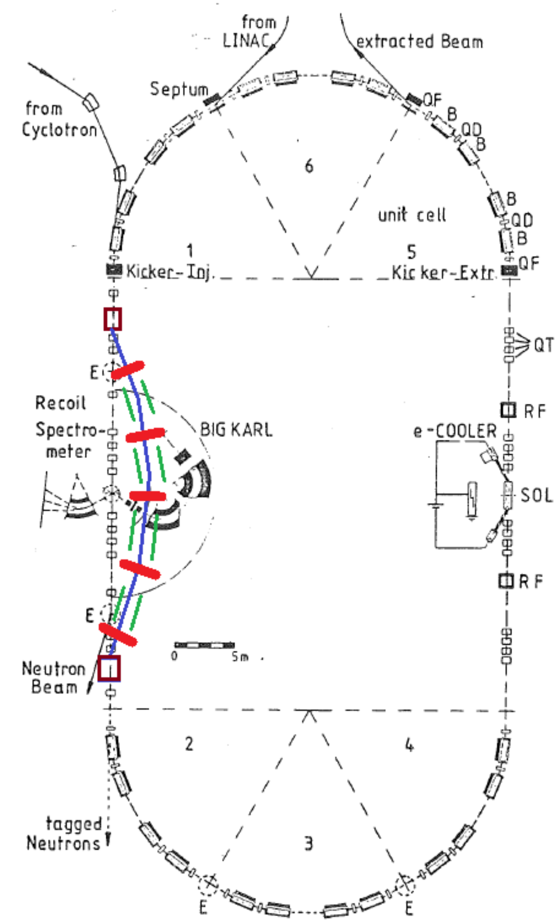
29 m x 47 m

dEDM-270 MeV



29.6 m x 55.7 m

COSY

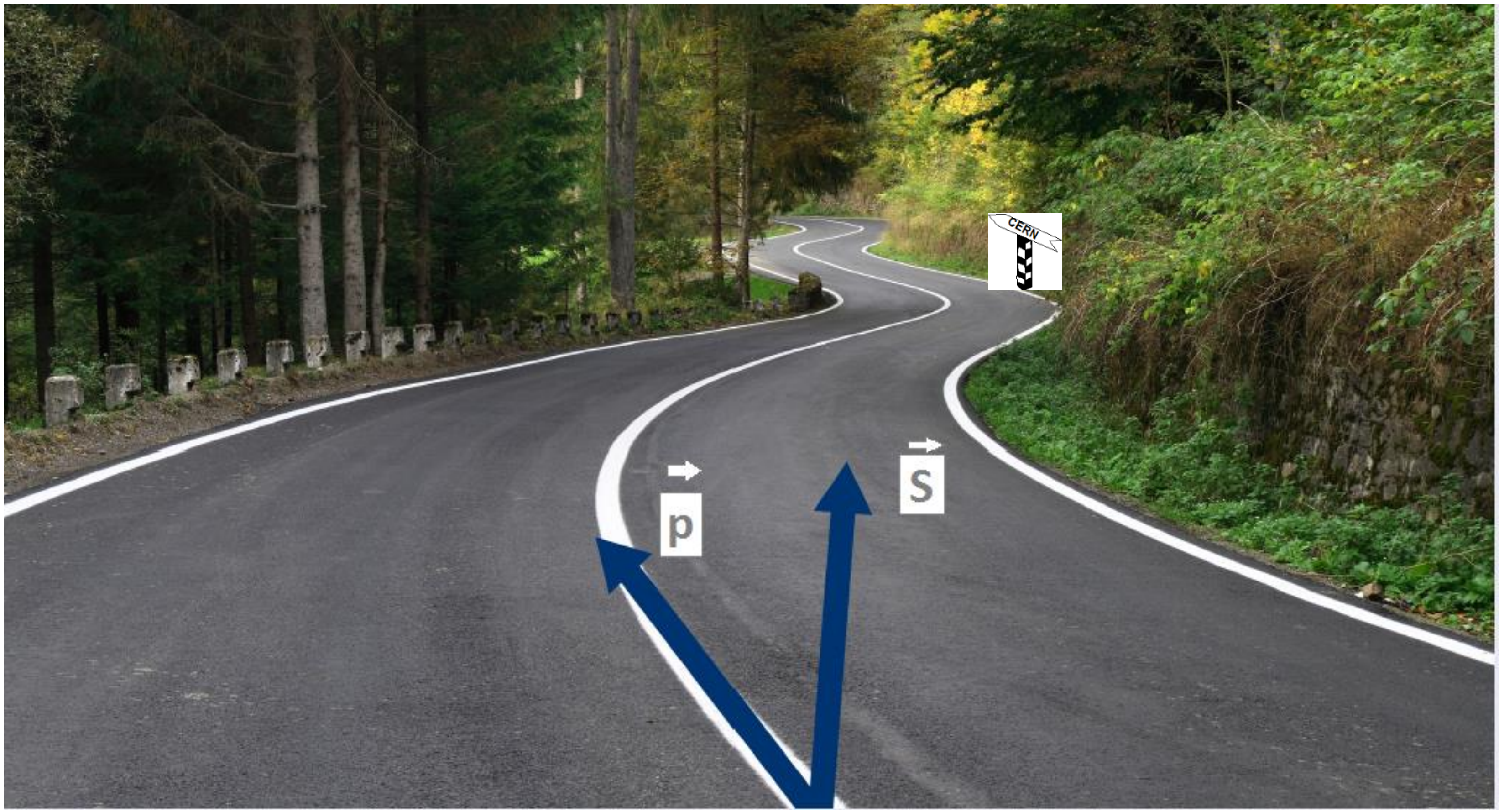


34 m x 71 m

Parameters of rings

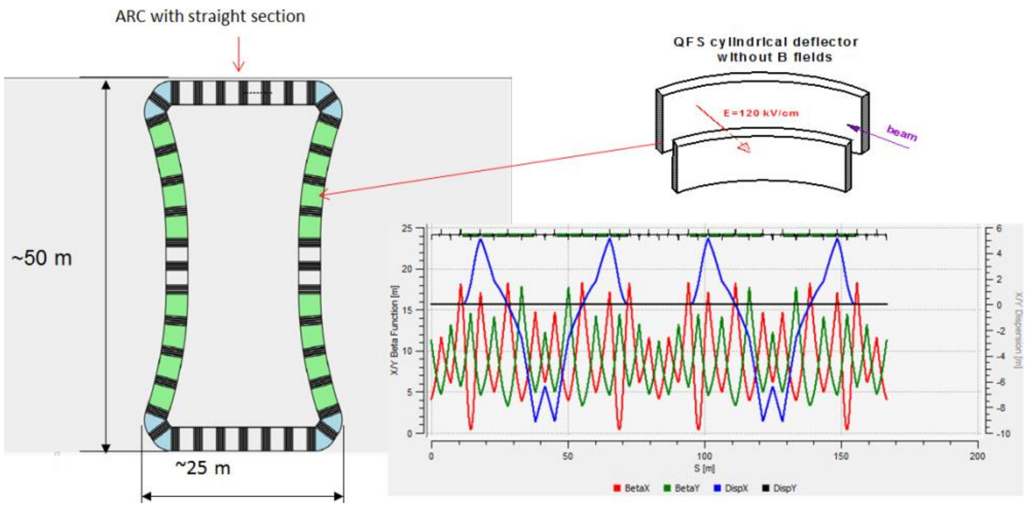
Energy	75 MeV	200 MeV	270 MeV
Number FODO cells	18 cells	20 cells	20 cells
Number of quadrupole magnets, effective length, gradient (T/m)	36; 0.2 m; 6÷1.0 T/m	40; 0.2 m; 6÷4 T/m	40; 0.2 m; 4.5÷4 T/m
Number of bend magnets, field (T), length (m), radius of curvature	8; 1.2 T; 1.4 m; 1.5 m	8; 1.5 T; 1.9 m; 2 m	8; 1.5 T; 2.2 m; 2.3 m
Number of electrostatic deflectors, field, length, radius of curvature	8; 120 kV/cm; 1.74 m; 12.3 m	16; 120 kV/cm; 2.54m; 31.7 m	16; 120 kV/cm; 3.6m; 42.6 m
Circumference, m	93 m	151 m	171 m
Momentum compaction factor	0.12	0.09	0.096
Maximum dispersion, m	2.5 m	7 m	9 m
Straight sections number with D≠0 and D=0 , length	2x5.0 m;(D≠0) **2x20.4 m; (D=0)**	2x7.5 m;(D≠0) **2x22.8 m; (D=0)**	2x7.5 m;(D≠0) **2x23.2 m; (D=0)**
Maximum beta-function X and Y planes	$ß_x$ 10 m; $ß_y$ changes in range 10÷500 m	$ß_x$ 14 m; $ß_y$ changes in range 14÷500 m	$ß_x$ 20 m; $ß_y$ changes in range 20÷500 m
Tune, X and Y	ν_x= 6.4; ν_y= 5.4÷0.4	ν_x= 4.8; ν_y= 4.3÷0.2	ν_x= 4.6; ν_y= 3.3÷0.1
Number of sextupoles, effective length, gradient	20; 0.15 m; S_x= 24 T/m^2; S_y=43 T/m^2	28; 0.15 m; S_x= 24 T/m^2; S_y=28 T/m^2	28; 0.15 m; S_x= 15 T/m^2; S_y=17 T/m^2

An analogue of QFS option



Jülich

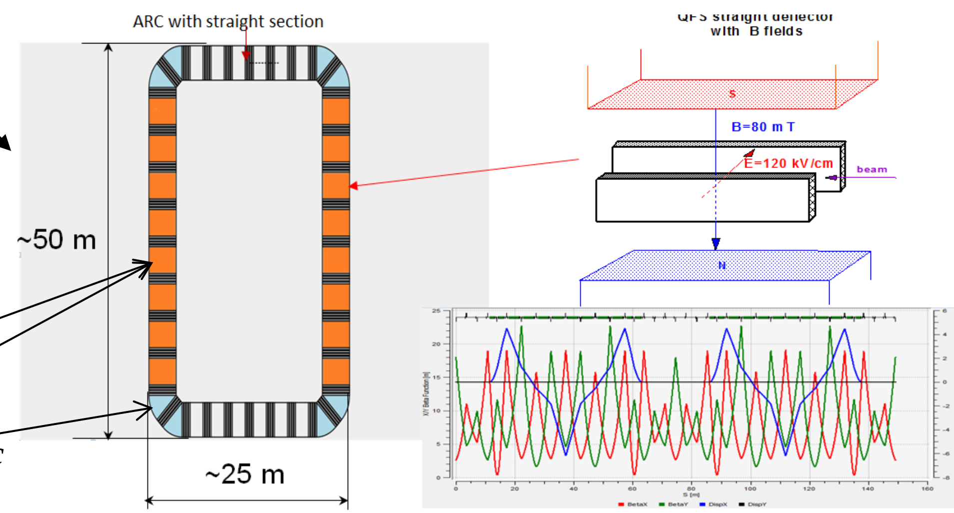
QFS lattice



$$r'' - \frac{1}{r} + \frac{1}{R_{eq}^2} r = 0$$

$$\frac{d^2x}{ds^2} = -\frac{2eU_0}{mv_\vartheta^2 d}$$

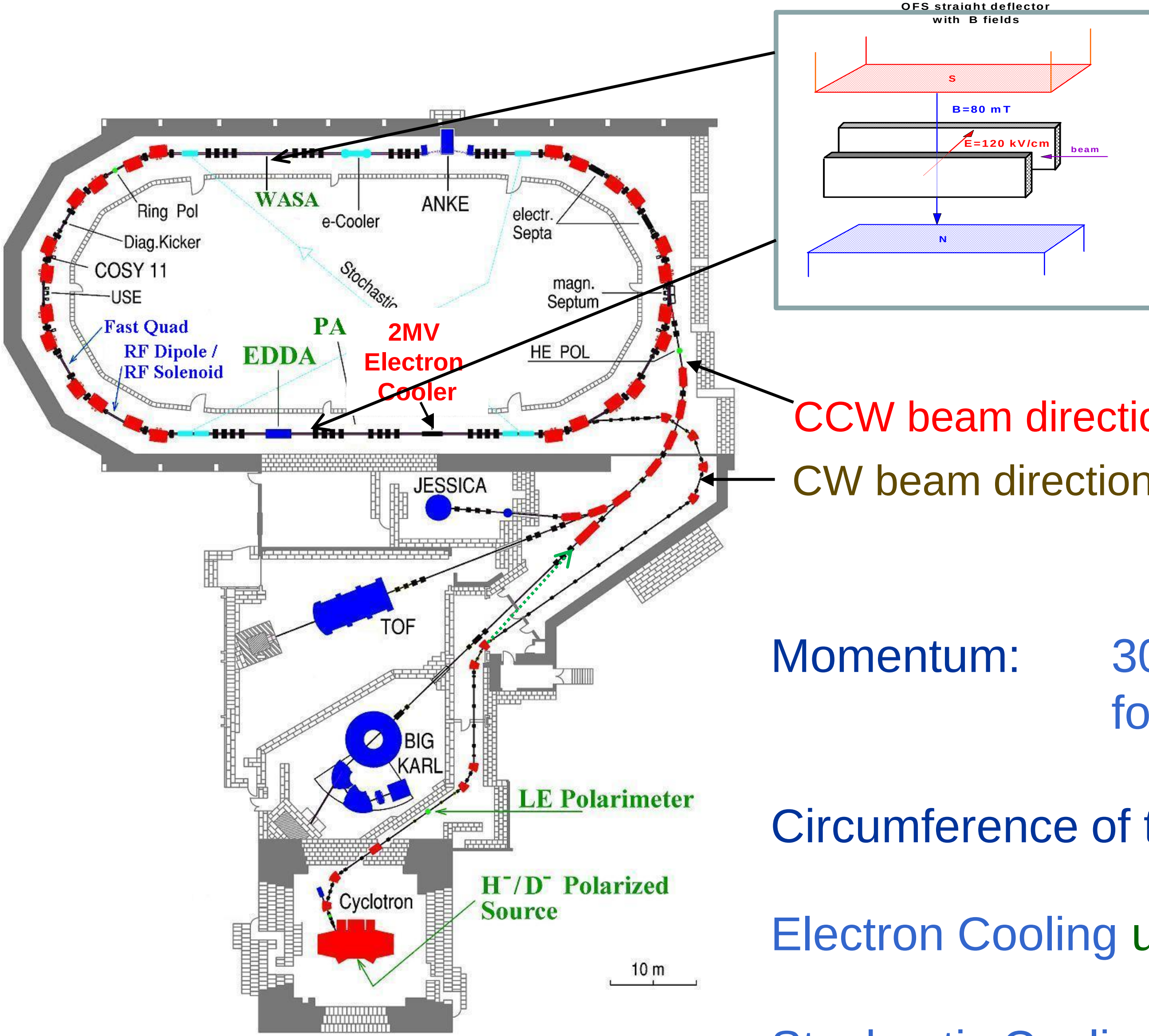
$$\frac{d^2y}{ds^2} = 0$$



$$\Phi_{spin}^{Bss} - \Phi_{spin}^{Ess} = \Phi_{spin}^{Barc}$$

B~100 mT

Cooler Synchrotron COSY in QFS mode



CCW beam direction

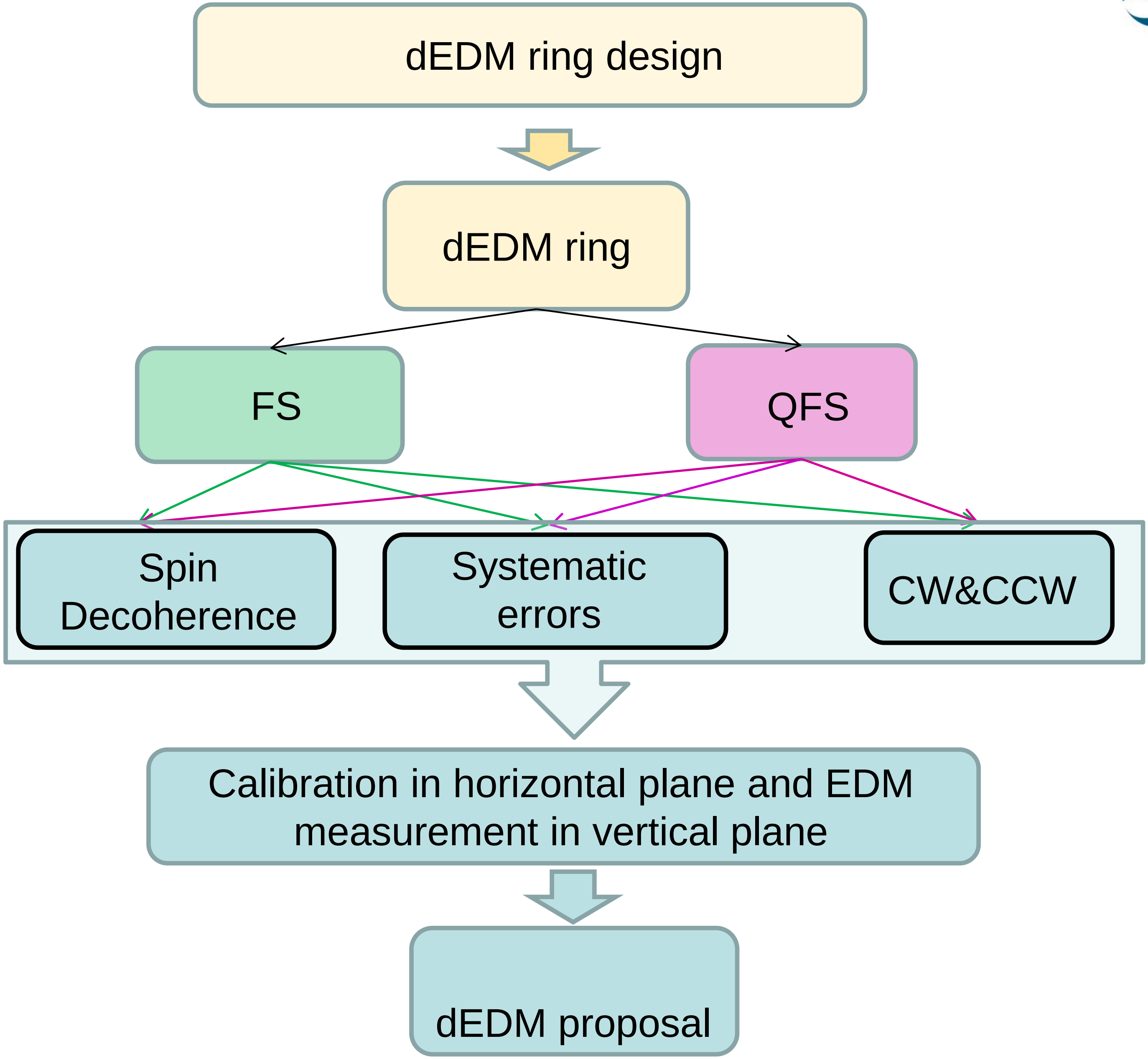
CW beam direction

Momentum: 300/600 to 3700 MeV/c for p/d, respectively

Circumference of the ring: 184 m

Electron Cooling up to 550 MeV/c

Stochastic Cooling above 1.5 GeV/c


dEDM ring design
dEDM ring
FS
QFS
Spin Decoherence
Systematic errors
CW&CCW
Calibration in horizontal plane and EDM measurement in vertical plane
dEDM proposal

- The proposed method of searching for EDM is based on measuring the sum and difference frequency of **spin precession in the vertical plane** due to EDM and MDM, respectively, for the CW and CCW cases.

- In order to **exclude the influence of spin precession frequencies** in other planes on the spin precession frequency in the vertical plane, certain relations are fulfilled between them, having the character of an approximate value.

- For the transition from CW to CCW, it is suggested to use **the calibration** of the equilibrium Lorentz factor in terms of the precession of the spin **in the horizontal plane**, which is then used for **EDM search in the vertical plane**.

Systematic errors:

In fact, for the EDM measurement at 10^{-29}e cm we have two possibilities: either we should strive to reduce the misalignments to the values for today unrealistic $\sim 10^{-11}$m, or to develop a procedure for EDM measurement in the presence of the actual error values.

And we have gone on the second way!!!

COSY Infinity and MODE codes

Spin-orbit dynamics of polarized beam investigated using:

- the code COSY Infinity

(M. Berz,Michigan State University, USA)

- the code MODE

(S. Andrianov and A. Ivanov St.Petersburg University).

The algorithms of COSY Infinity and MODE are different and it gives the possibility to check the results of calculation.

Conclusion

- We have formulated the basic requirements for the accelerator, in which it is possible measuring EDM at 10^{-29} e·cm
- We have developed and tested experimentally the method of how to achieve a long spin coherence time using sextupoles
- We learned how to measure the spin frequency with absolute accuracy 10^{-6}rad /sec in one fill
- We have developed the concept of quasi-frozen spin lattice and learned how to adapt the concept of QFS to COSY ring
- We have developed the methodology how to take into account the systematic errors

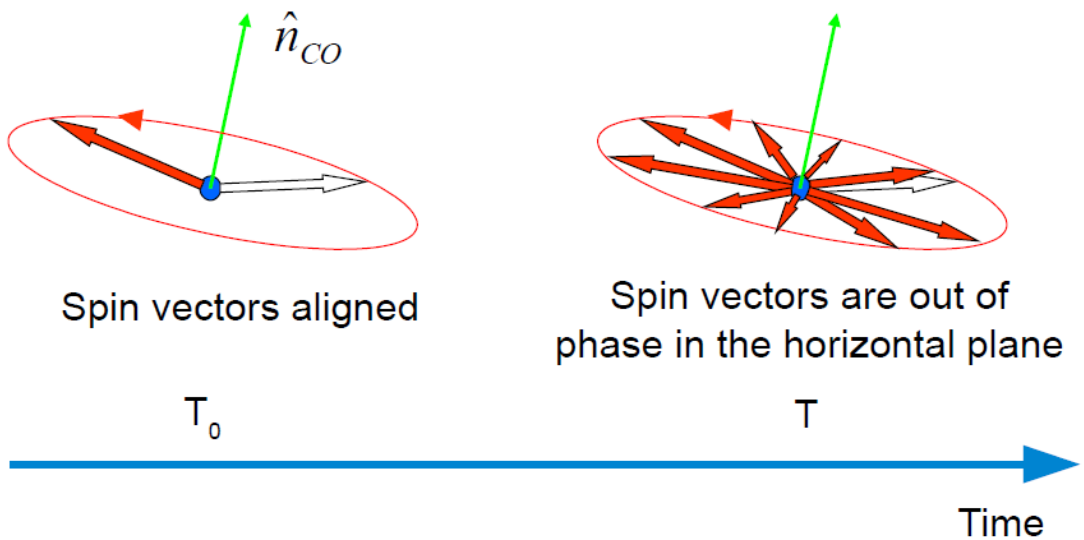

Thank you!

Spare slides in case of questions

Spin Coherence and Depolarisation

One of the main challenges is to have spin coherence time (SCT) of ~1000 seconds

What is SCT?



Loss of longitudinal polarisation causes loss of EDM effect

$$\vec{\Omega}_{MDM}=\frac{e}{m}\left[G\vec{B}-\left(G-\frac{1}{\gamma^2-1}\right)\frac{\vec{E}\times\vec{\beta}}{c}\right]$$

The averaged Lorentz factor over all particles will be called the generalized Lorentz factor

$$\Delta\delta_{equil}=\frac{\gamma_s^2}{\gamma_s^2\alpha_0-1}\left[\frac{\delta_m^2}{2}\left(\alpha_1-\frac{\alpha_0}{\gamma_s^2}+\frac{1}{\gamma_s^4}\right)+\left(\frac{\Delta L}{L}\right)_\beta\right]$$

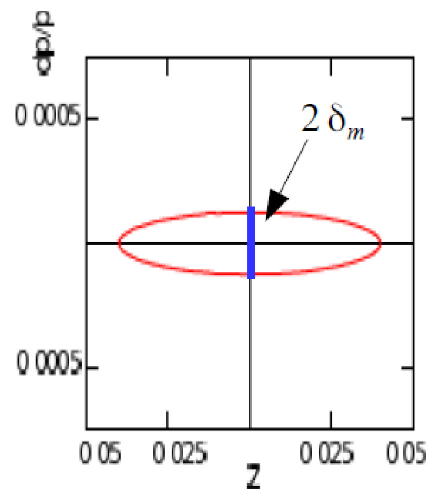
$$\left(\frac{\Delta L}{L}\right)_\beta=\frac{\pi}{2L}\left[\epsilon_x\nu_x+\epsilon_y\nu_y\right]$$

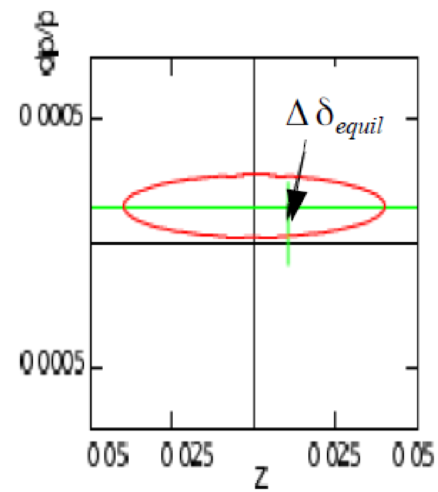
ν_x, ν_y are horizontal and vertical betatron tunes

$$\Delta\delta_{equil}=\Delta\left(\frac{\Delta p}{p}\right)_{equil}$$

ϵ_x, ϵ_y are horizontal and vertical emittances

- Different orbit lengths for different particles in the beam
- X and Y emittance sizes play the main role
- ΔP/P effects





Solutions:

- X and Y correction with sextupoles and beam cooling

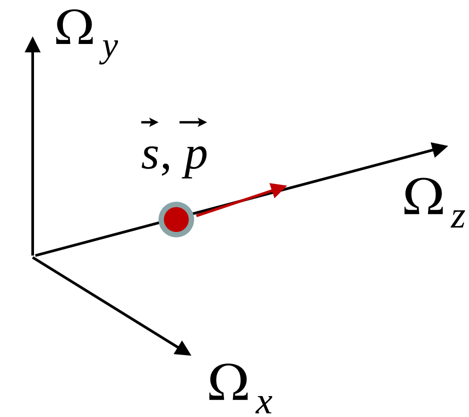
- ΔP/P correction with RF

General relation between EDM and fake MDM signals

In vertical plane: $\Omega_x = \Omega_{EDM} + \Omega_{Bx}$

In horizontal plane: $\Omega_y = \Omega_{decoh}$

In longitudinal direction: $\Omega_z = \Omega_{Bz}$



The total spin signal for case $\Omega_{Bz} = 0$ and $\Omega_{B_x} \neq 0$

$$S_y(t) = \frac{(\Omega_{EDM} + \Omega_{Bx}) \cdot \sin\left(\sqrt{(\Omega_{EDM} + \Omega_{Bx})^2 + \delta\Omega_{decoh}^2} \cdot t\right)}{\sqrt{(\Omega_{EDM} + \Omega_{Bx})^2 + \delta\Omega_{decoh}^2}}$$

Contribution of vertical oscillation into the horizontal plane should be negligibly small in comparison with EDM signal which we expect in vertical plane:

$$\delta\Omega_{decoh}{}^2 < 2\Omega_{EDM}\Omega_{B_x}$$

Similarly for longitudinal plane its contribution into the horizontal plane should be negligibly small in comparison with EDM signal:

$$\Omega_{B_z}{}^2 < 2\Omega_{EDM}\Omega_{B_x}$$

1. FS concept with no misalignments and no MDM decoherence

In the coordinate system with momentum

$$\frac{d\vec{S}}{dt} = \vec{S} \times \frac{e}{\gamma m}\left[\underbrace{G\gamma \vec{B}_{\perp} + \left(\frac{\gamma}{\gamma^2 - 1} - \gamma G\right)\frac{\vec{E}_{\perp} \times \vec{\beta}}{c}}_{\vec{\Omega}_{MDM}} + \underbrace{\frac{\eta}{2}\left(\vec{\beta} \times \vec{B}_{\perp} + \frac{\vec{E}_{\perp}}{c}\right)}_{\vec{\Omega}_{EDM}} \right] \quad \text{or} \quad \frac{d\vec{S}}{dt} = \vec{S} \times \left[\vec{\Omega}_{MDM} + \vec{\Omega}_{EDM}\right]$$

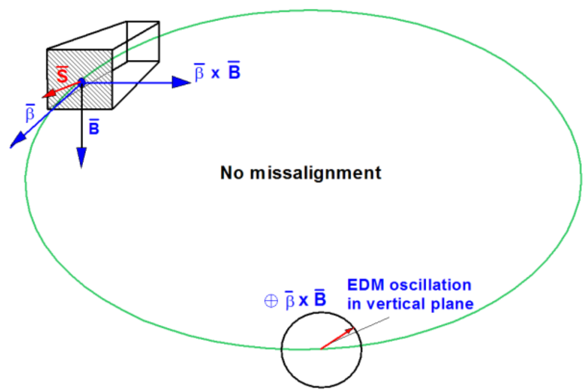
FS condition: Ω_{MDM}=0, that is $\Omega_x = \Omega_{EDM}$, $\Omega_y = 0$

$$\frac{dS_y}{dt} = -\Omega_{EDM} S_z$$

$$\frac{dS_z}{dt} = \Omega_{EDM} S_y$$

At $S_x = S_y = 0,\ S_z = 1$

EDM signal ~ $S_y(t) = \sin(\Omega_{EDM} t)$



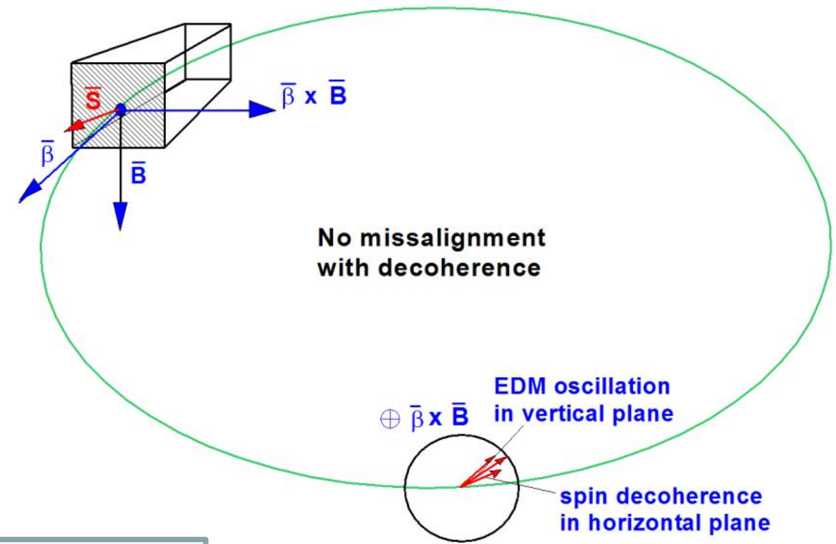
At presumable value of EDM ~ 10^{-29} e·cm after 10^9 turns (~1000 sec) S_y grows up 10^{-6} rad

2. FS concept with no misalignments and with MDM decoherence

$$\frac{d\vec{S}}{dt} = \vec{S} \times \left[\vec{\Omega}_{MDM} + \vec{\Omega}_{EDM}\right] \quad \text{and} \quad \begin{array}{l} \Omega_x = \Omega_{EDM} \\ \Omega_y = 0 + \delta\Omega_{decoh} \end{array}$$

$$S_x(t) = \frac{\Omega_{decoh} \cdot \sin(\sqrt{\Omega_{EDM}^2 + \Omega_{decoh}^2} \cdot t)}{\sqrt{\Omega_{EDM}^2 + \Omega_{decoh}^2}};$$

$$S_y(t) = -\frac{\Omega_{EDM} \cdot \sin(\sqrt{\Omega_{EDM}^2 + \Omega_{decoh}^2} \cdot t)}{\sqrt{\Omega_{EDM}^2 + \Omega_{decoh}^2}};$$



$\Omega_{decoh} >> \Omega_{EDM}$ → EDM signal is negligible in the horizontal plane but in vertical plane the EDM is decisive

$$S_x(t) \approx \sin\Omega_{decoh} t \quad \text{and} \quad \text{EDM signal} \sim \sum_i S_y^i(t) = \Omega_{EDM}\, t \cdot \sum_i \frac{\sin \Omega_{decoh}^i t}{\Omega_{decoh}^i t}$$

$$\max\{\Omega_{decoh} t_{\text{onefill}}\} < \pi \quad \text{or} \quad \langle \Omega_{decoh} \rangle \cdot t_{\text{onefill}} < 1$$

in case of it

$$\underbrace{\Omega_{decoh} \sim 10^{-3} rad/\sec}_{\text{in horizontal plane}} \quad \text{and}$$

$$\underbrace{\Omega_{EDM} \sim 10^{-9} rad/\sec}_{\text{in vertical plane}}$$

Methodology of EDM search

The proposed method of searching for EDM is based on:

- measuring the sum and difference frequency of spin precession in the vertical plane due to EDM and MDM, correspondingly, for the CW and CCW cases with absolute accuracy 10^{-7} rad per second in one fill;

- independence of the absolute error in determining the spin precession frequency from the frequency itself;

- Unchangeable position of the accelerator elements and as a consequence the constant ratio of the leading field By to the component of the field that determine the fake signal Bx;

- the non-influence of spin precession frequencies in other planes on the spin precession frequency in the vertical plane. Certain relations are fulfilled between them, having the character of an approximate value.

- For the transition from CW to CCW it is suggested to use the calibration of the equilibrium Lorentz factor in terms of the precession of the spin in the horizontal plane, which is then used in the vertical plane.

3. FS concept with misalignments $B_x \neq 0$ and MDM decoherence

$$\frac{d\vec{S}}{dt} = \vec{S} \times \left[\vec{\Omega}_{MDM} + \vec{\Omega}_{EDM}\right] \text{ and } \begin{array}{l} \Omega_x = \Omega_{EDM} + \Omega_{Bx} \\ \Omega_y = 0 + \delta\Omega_{decoh} \end{array}$$

At $S_x = S_y = 0,\ S_z = 1$

$$S_x(t) = \frac{\delta\Omega_{decoh} \cdot \sin\left(\sqrt{(\Omega_{EDM} + \Omega_{Bx})^2 + \delta\Omega_{decoh}^2} \cdot t\right)}{\sqrt{(\Omega_{EDM} + \Omega_{Bx})^2 + \delta\Omega_{decoh}^2}};$$

$$S_y(t) = \frac{(\Omega_{EDM} + \Omega_{Bx}) \cdot \sin\left(\sqrt{(\Omega_{EDM} + \Omega_{Bx})^2 + \delta\Omega_{decoh}^2} \cdot t\right)}{\sqrt{(\Omega_{EDM} + \Omega_{Bx})^2 + \delta\Omega_{decoh}^2}}$$

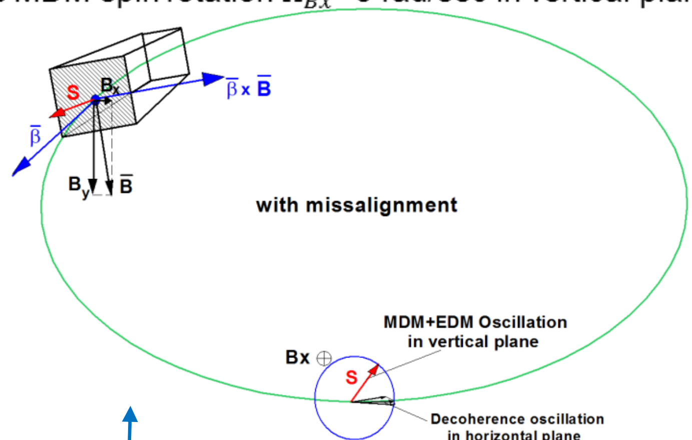
and taking into account
$\Omega_{EDM} = 10^{-9}$ rad/sec,
$\delta\Omega_{decoh} = 10^{-3}$ rad/sec
$\Omega_{Bx} = 3$ rad/sec

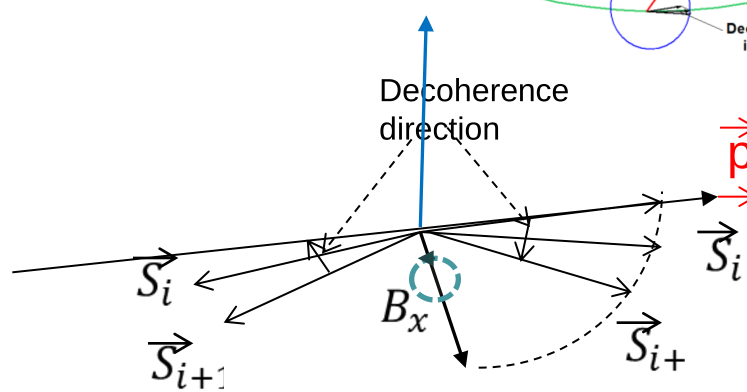
$$\Omega_{Bx} = \frac{e}{m\gamma} \cdot \gamma G B_x \gg \Omega_{decoh}$$

$$< S_x(t) > = \frac{< \delta\Omega_{decoh} >}{\Omega_{Bx}} \cdot \sin\Omega_{Bx} \cdot t;$$

$$S_y(t) \approx -\sin(\Omega_{Bx} + \Omega_{EDM}) \cdot t$$

Misalignment, Bx≠0:
Due to magnet rotation relative to the longitudinal axis the horizontal component of the magnetic field arises and causes the spin rotation $\Omega_x = \Omega_{Bx}$. In COSY simulation we took the magnets misalignment 10 μm, which corresponds to the rotation angle of magnet around the axis about $\alpha_{max} = \pm 10^{-5}$ rad and causes MDM spin rotation Ω_{Bx}=3 rad/sec in vertical plane



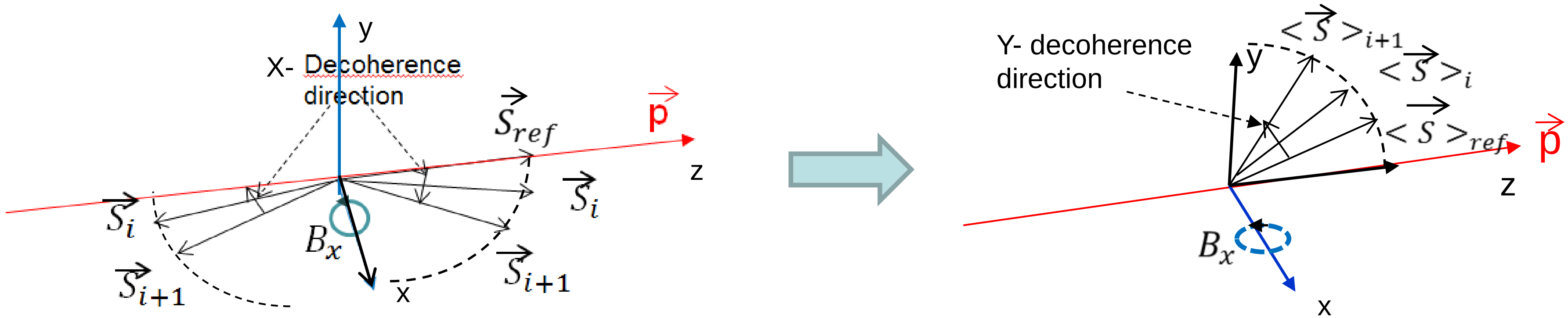


Due to rapid rotation in vertical plane and the preservation of growth direction of decoherence the growth of decoherence is restricted by one revolution in the Bx field

Compared with the previous case the EDM can be determined through measurement of frequency $\Omega_{Bx} + \Omega_{EDM}$ only

3a. FS concept: Conversion of decoherence from horizontal plane into vertical plane

Due to oscillation in Bx field $<S_x(t)>=\frac{<\delta\Omega_{decoh}>}{\Omega_{Bx}}\cdot\sin\Omega_{Bx}\cdot t$ the decoherence in horizontal plane remaines to be on the level $<S_x(t)>=\frac{<\delta\Omega_{decoh}>}{\Omega_{Bx}}\sim 0.0001$



Due to misalignments spin oscillates in a vertical plane with frequency Ω_x (in our case 3 rad/sec) and restricts the decoherence in the horizontal plane at the level of ~0.01 degrees. Simultaneously due to dependence $\Omega_x=\frac{e}{\gamma m}(1+G\gamma)\vec{B}_x$ on γ we have the spin decoherence already in vertical plane.

4. QFS concept: Effect of Coherent component of spin precession

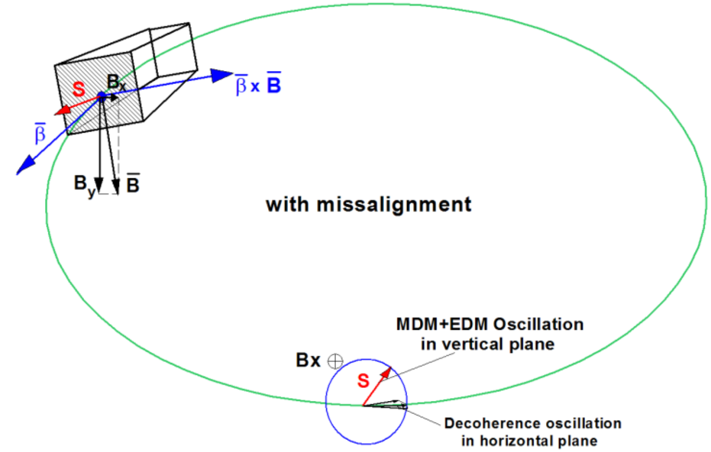
$$\frac{d\vec{S}}{dt} = \vec{S} \times \left[\vec{\Omega}_{MDM} + \vec{\Omega}_{EDM}\right] \text{ and }$$

$$\Omega_x = \Omega_{EDM} + \Omega_{Bx}$$
$$\Omega_y = 0 + \delta\Omega_{decoh} + \frac{\gamma G}{4} \cdot 2\pi \cdot$$

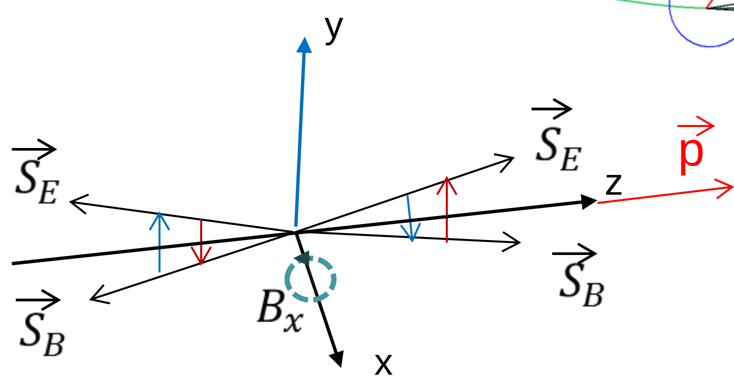
Bipolar Heaviside step function

$$\Omega_{E \leftrightarrow B} >> \Omega_{B_x}$$

During one revolution in the field Bx the spin jumps from position S_E to position S_B and back one million times. Averaging over time we now have to work with the averaged vector in a middle position.



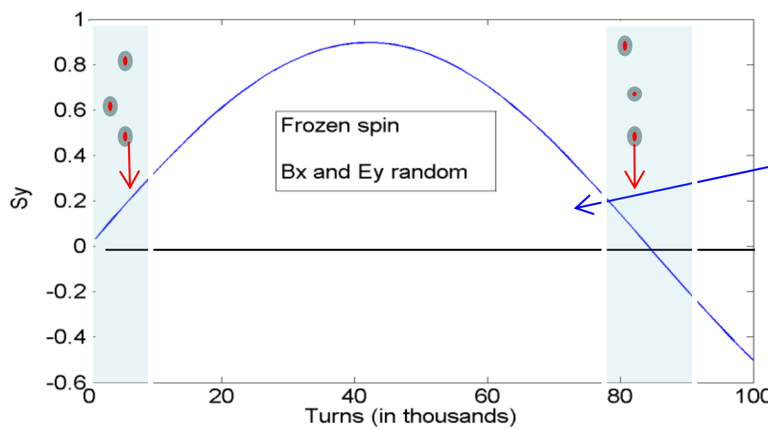
$$S_x(t) = \frac{\delta\Omega_{decoh}}{\Omega_{Bx}} \cdot \sin\Omega_{Bx} \cdot t;$$
$$S_y(t) \approx -\sin(\Omega_{Bx} + \Omega_{EDM}) \cdot t$$



So passing to the measurement of EDM in the vertical plane, we almost lose the distinction between FS and QFS method

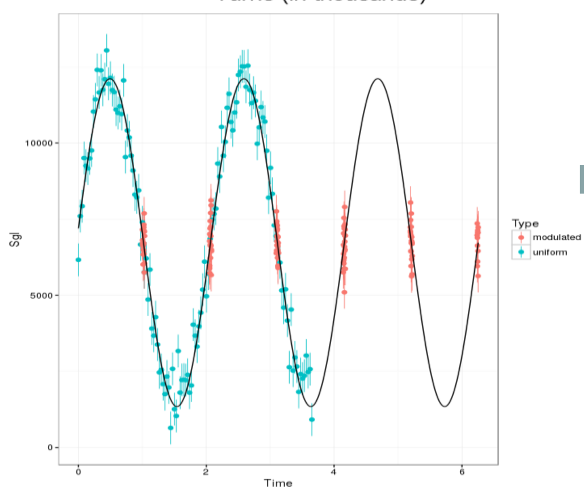
COSY Inf+MODE simulation of systematic errors due to magnet rotation around the longitudinal axis

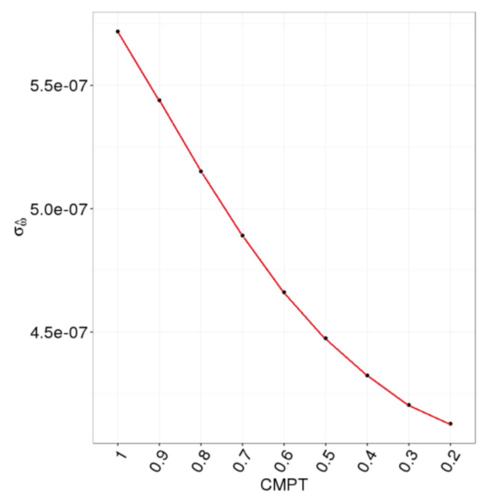
Coherent component

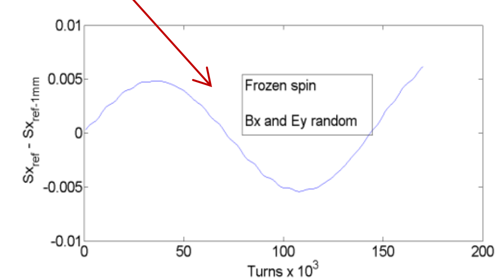


$$S_y(t) \approx -\sin(\Omega_{Bx} + \Omega_{EDM}) \cdot t$$

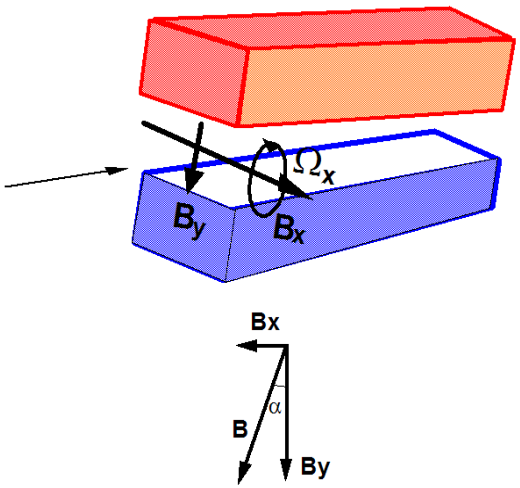
$$< S_x(t) >= \frac{\langle \delta\Omega_{decoh} \rangle}{\Omega_{Bx}} \cdot \sin\Omega_{Bx} \cdot t$$







Main steps of EDM search using CW+CCW procedure



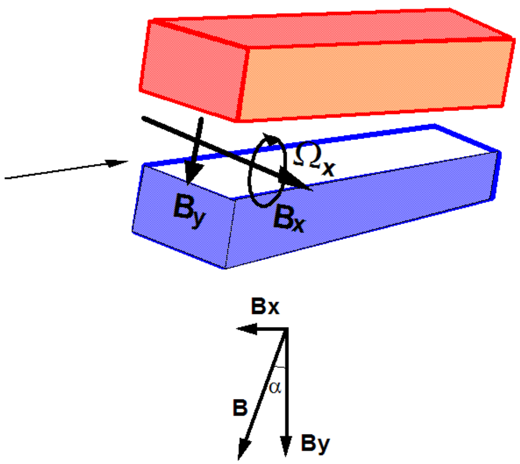
To split out the EDM signal from the sum signal we use **CW+CCW procedure:**

1. Calibration of **Bx** through **By**
2. *Measurement of the total spin frequency* in the experiment with a counter clock-wise (**CW**) direction of the beam $\Omega_{CW} = \Omega_{Bx}^{CW} + \Omega_{EDM}$
3. Calibration of **Bx** through **By** and installation of B field after the polarity change
4. Measurement of the total spin frequency in the experiment with a counter clock-wise (**CCW**) direction of the beam $\Omega_{CCW} = -\Omega_{Bx}^{CCW} + \Omega_{EDM}$
5. Compare **CCW** with clock-wise (**CW**) measurements

$$\Omega_{EDM} = (\Omega_{CW} + \Omega_{CCW})/2 + (\Omega_{Bx}^{CCW} - \Omega_{Bx}^{CW})/2$$

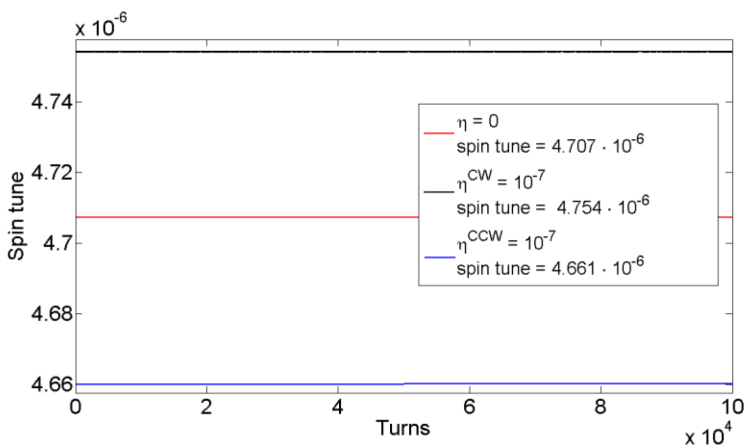
6. The difference $\Delta\Omega_{Bx} = \Omega_{Bx}^{CCW} - \Omega_{Bx}^{CW}$ determines the accuracy of the EDM measurement. Calibrating **Bx** we can minimize $\Delta\Omega_{Bx}$ up to value of calibration accuracy.

Bx and By calibration procedure



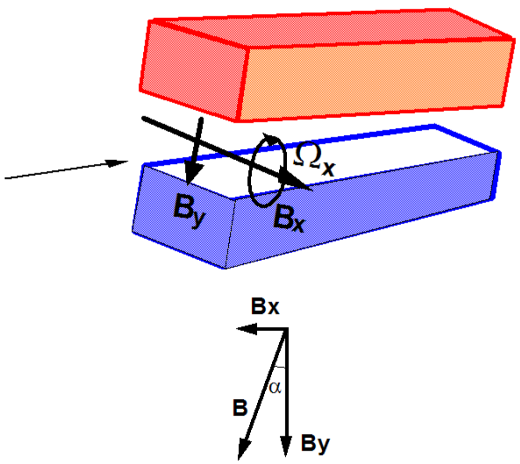
First, we suggest calibrating the field of the magnets using the relation between the beam gamma and the spin precession frequency in the horizontal plane, that is, determined by the vertical component By. Since the magnet orientation remains unchanged, and the magnets are fed from one power supply, the calibration of By will restore the component Bx with the same relative accuracy 10^(-9), that is the difference $\Omega_{Bx}^{CCW} - \Omega_{Bx}^{CW}$ as well. Such procedure does not involve EDM signal.

If we assume that we can measure the spin frequencies $\Omega_{CW}, \Omega_{CCW}$ with an accuracy of 10^(-9) and reach the calibration accuracy of **Bx** up to 10^(-9) we will be able to determine the EDM frequency up to 10^(-9) rad/sec, which corresponds to the EDM measurement on the level of **10^(-28)÷10^(-29) e·cm**



the results of a numerical simulation of the EDM measurement procedure, we took the EDM 10^(-21), that is $\Omega_{EDM} = 0.1 rad/\sec$

Bx coil



Nevertheless, an additional question of how to calibrate the field By using the spin tune measurement in a horizontal plane, if due to misalignments the spin rotates in the vertical plane with relatively high frequency ~3 rad/sec, remains. To solve this problem, we plan for the calibration time only to introduce the inhibitory vertical field, for example by means of a horizontal coil. Having inhibited rotation in the vertical plane to the reasonable value of ~0.1 rad/sec and calibrated, then we turn off the coil. In this case we do not need to know the value of the field in the coil

Nevertheless introducing the coil we can modify the integral value of the guiding magnetic field By. Let us estimate this value. We know that due to misalignment of magnets with an accuracy of 10 micrometer, we have in Bx/By = 10 ^(-)6. Obviously the coil can be installed with the same accuracy and By(coil)/Bx(coil)=10^(-6). Thus, the coil introduces in By of ring 10^(-12)

Sensitivity of EDM experiment

$$\sigma_{d\,p} \approx \frac{3\hbar}{PAE_R\sqrt{N_{Beam}fT_{Tot}\tau_{Spin}}}$$

P = 0.8	Beam polarization
A = 0.6	Analyzing power of polarimeter
E_R = 12 MV/m	Radial electric field strength
N_{Beam} = $2\cdot 10^{10}$ p/fill	Total number of stored particles per fill
f = 0.55%	Useful event rate fraction (polarimeter efficiency)
T_{Tot} = 10^7 s	Total running time per year
τ_{Spin} = 10^3 s	Polarization lifetime (Spin Coherence Time)

$$\sigma_{d\,p} \approx 3\cdot 10^{-29}\,\mathrm{e}\cdot\mathrm{cm}$$ for one year measurement