Potential for a discovery of new spin-dependent effects in the framework of the EDM experiments

Alexander J. Silenko

Research Institute for Nuclear Problems, BSU, Minsk, Belarus

Joint Institute for Nuclear Research, Dubna, Russia

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OUTLINE

- Violation of the Lorentz and CPT symmetry
- Cartan torsion of spacetime
- Is the spin tune independent of the beam polarization?
  - Analytical description of spin dynamics in oscillating fields rotating the spin about horizontal axes
- Spin dragging
- Summary

Violation of the Lorentz and CPT symmetry

CPT violation results in a difference between particles and antiparticles. Violation of the Lorentz symmetry makes different frames (e.g., being at rest and moving relative to the galactic center) to be nonequivalent.

Laboratory tests of Lorentz and CPT symmetry with muons

André H. Gomes,[1] V. Alan Kostelecký,[2] and Arnaldo J. Vargas[2]
[1] *Departamento de Física, Universidade Federal de Viçosa, 36570-000 Viçosa, MG, Brazil*
[2] *Physics Department, Indiana University, Bloomington, Indiana 47405, USA*

The prospects are explored for testing Lorentz and CPT symmetry in the muon sector via the spectroscopy of muonium and various muonic atoms, and via measurements of the anomalous magnetic moments of the muon and antimuon. The effects of Lorentz-violating operators of both renormalizable and nonrenormalizable dimensions are included. We derive observable signals, extract first constraints from existing data on a variety of coefficients for Lorentz and CPT violation, and estimate sensitivities attainable in forthcoming experiments. The potential of Lorentz violation to resolve the proton radius puzzle and the muon anomaly discrepancy is discussed.

Phys. Rev. D **90** (2014) 076009

The rate of change of the spin expectation value for the μ^- due to Lorentz violation is given by

$$\frac{d\langle \boldsymbol{S} \rangle}{dt} \approx 2(\boldsymbol{h}_g + \boldsymbol{h}_H) \times \langle \boldsymbol{S} \rangle.$$

The corrections to the $\mu^\pm$ anomaly frequencies are given by

$$\delta\omega_a^\pm = \pm 2h_g + 2h_H.$$

Comment: storage ring experiments with the $e^\pm$ beams can be more sensitive

Atomic experiments with comagnitometers

PRL **112**, 110801 (2014) PHYSICAL REVIEW LETTERS week ending 21 MARCH 2014

New Limit on Lorentz-Invariance- and *CPT*-Violating Neutron Spin Interactions Using a Free-Spin-Precession ^{3}He-^{129}Xe Comagnetometer

F. Allmendinger,[1,*] W. Heil,[2] S. Karpuk,[2] W. Kilian,[3] A. Scharth,[2] U. Schmidt,[1]
A. Schnabel,[3] Yu. Sobolev,[2] and K. Tullney[2]
[1]*Physikalisches Institut, Ruprecht-Karls-Universität, 69120 Heidelberg, Germany*
[2]*Institut für Physik, Johannes Gutenberg-Universität, 55099 Mainz, Germany*
[3]*Physikalisch-Technische Bundesanstalt Berlin, 10587 Berlin, Germany*
(Received 12 December 2013; revised manuscript received 4 February 2014; published 17 March 2014)

We report on the search for a *CPT*- and Lorentz-invariance-violating coupling of the ^{3}He and ^{129}Xe nuclear spins (each largely determined by a valence neutron) to posited background tensor fields that permeate the Universe. Our experimental approach is to measure the free precession of nuclear spin polarized ^{3}He and ^{129}Xe atoms in a homogeneous magnetic guiding field of about 400 nT using LT$_C$ SQUIDs as low-noise magnetic flux detectors. As the laboratory reference frame rotates with respect to distant stars, we look for a sidereal modulation of the Larmor frequencies of the colocated spin samples. As a result we obtain an upper limit on the equatorial component of the background field interacting with the spin of the bound neutron $\tilde{b}^n_\perp < 8.4 \times 10^{-34}$ GeV (68% C.L.). Our result improves our previous limit (data measured in 2009) by a factor of 30 and the world's best limit by a factor of 4.

Spin Physics (SPIN2014)
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While atomic experiments with comagnitometers are the most sensitive, storage ring experiments may be performed with other objects

PHYSICAL REVIEW C **94**, 025502 (2016)

Tests of Lorentz and *CPT* symmetry with hadrons and nuclei

J. P. Noordmans,[1,2] J. de Vries,[3] and R. G. E. Timmermans[1]

[1] *Van Swinderen Institute for Particle Physics and Gravity, University of Groningen, 9747 AG Groningen, The Netherlands*

[2] *CENTRA, Departamento de Física, Universidade do Algarve, 8005-139 Faro, Portugal*

[3] *Nikhef, Theory Group, Science Park 105, 1098 XG, Amsterdam, The Netherlands*

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We explore the breaking of Lorentz and *CPT* invariance in strong interactions at low energy in the framework of chiral perturbation theory. Starting from the set of Lorentz-violating operators of mass-dimension five with quark and gluon fields, we construct the effective chiral Lagrangian with hadronic and electromagnetic interactions induced by these operators. We develop the power-counting scheme and discuss loop diagrams and the one-pion-exchange nucleon-nucleon potential. The effective chiral Lagrangian is the basis for calculations of low-energy observables with hadronic degrees of freedom. As examples, we consider clock-comparison experiments with nuclei and spin-precession experiments with nucleons in storage rings. We derive strict limits on the dimension-five tensors that quantify Lorentz and *CPT* violation.

PHYSICAL REVIEW D **95**, 075030 (2017)

Gluonic Lorentz violation and chiral perturbation theory

J. P. Noordmans

CENTRA, Departamento de Física, Universidade do Algarve, 8005-139 Faro, Portugal

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By applying chiral-perturbation-theory methods to the QCD sector of the Lorentz-violating Standard-Model Extension, we investigate Lorentz violation in the strong interactions. In particular, we consider the *CPT*-even pure-gluon operator of the minimal Standard-Model Extension. We construct the lowest-order chiral effective Lagrangian for three as well as two light quark flavors. We develop the power-counting rules and construct the heavy-baryon chiral-perturbation-theory Lagrangian, which we use to calculate Lorentz-violating contributions to the nucleon self-energy. Using the constructed effective operators, we derive the first stringent limits on many of the components of the relevant Lorentz-violating parameter. We also obtain the Lorentz-violating nucleon-nucleon potential. We suggest that this potential may be used to obtain new limits from atomic-clock or deuteron storage-ring experiments.

Parameters of the Lorentz-violating nucleon-nucleon potential can be bounded

Spin precession

Nuclear matrix elements:

$\langle I M_I | V_{\rm LV} | I' M_I' \rangle$ ⟶ Comagnetometer experiments
But matrix elements of two-body operator are hard

Easier for the deuteron:

$\langle D | V_{\rm LV} | D \rangle$ ⟶ Spin precession and storage-ring experiments

spin-precession for spin-1/2 particle: $\dfrac{d\boldsymbol{\sigma}}{dt} = i\,[H, \boldsymbol{\sigma}]$ (3 parameters to specify polarization)

spin-precession for spin-1 particle: $\dfrac{d\varrho}{dt} = i\,[H, \varrho]$ (8 parameters to specify polarization)

spin-density matrix: $\varrho = \dfrac{1}{3}\left[1 + \dfrac{3}{2}\mathbf{P}\cdot\mathbf{S} + \sqrt{\dfrac{3}{2}}T_{ij}(S_i S_j + S_j S_i)\right]$

polarization ($\mathbf{P}$); rank 2 polarization tensor (T_{ij})

Noordmans' talk

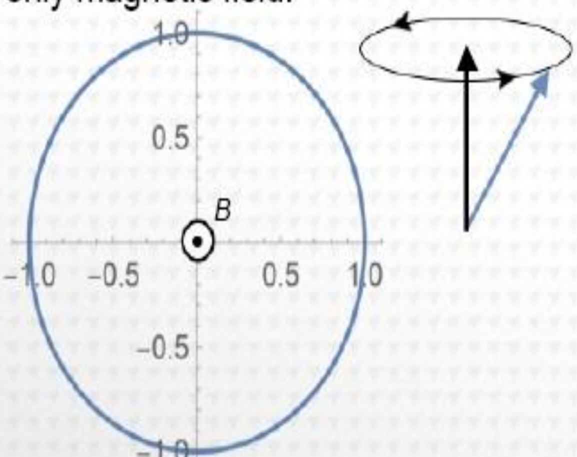
Spin precession

$\frac{d\varrho}{dt} = i\,[H, \varrho]$ ⟶ (coupled) differential equations for polarization and alignment in terms of Lorentz-violating couplings

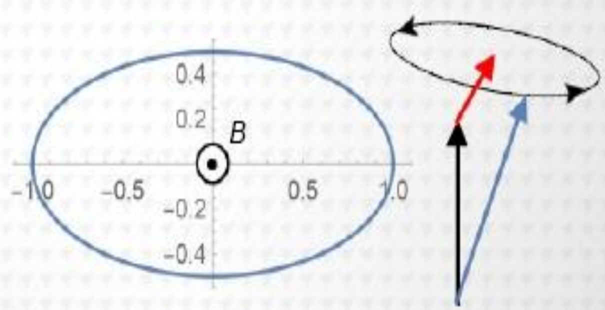
$H = \mathbf{\Omega} \cdot \mathbf{S} + Q_{ij} S_i S_j$ ⟶ Without LV, effects of Q_{ij} are usually negligable, but the LV parts of $\mathbf{\Omega}$ and Q_{ij} are of the same size.

Unpolarized sample: $\mathbf{P} = 0$, $T_{ij} = 0$ ⟶ Remains unpolarized

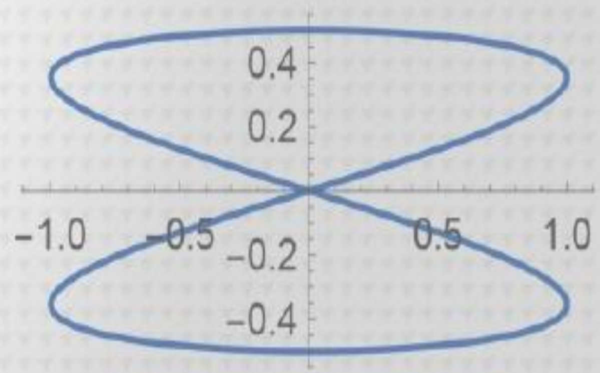
only magnetic field:



nonzero LV contribution to $\mathbf{\Omega}$:



nonzero LV contribution to Q_{ij}:



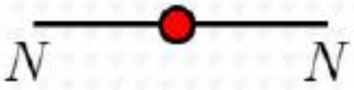
$\mathbf{P}$ precesses around magnetic field (+ LV background field) with frequency ω_s (+ LV contribution). $|\mathbf{P}|$ remains constant. Vanishing (small) tensor polarization remains zero (small)

The size of the polarization changes, because differential equations for $\mathbf{P}$ and T_{ij} are coupled.

Noordmans' talk

Limits on free nucleon terms

Nonrelativistic limit:



$$\propto (\tilde{C}^{w}_{0ij}\epsilon^{ijk} - \tilde{H}^{k00})\bar{N}\sigma_k N$$

Looks like spin coupling to magnetic field

➡ Spin precession gets component around the LV direction

Dedicated experiment for 'other' LV coefficient: $b_\mu \bar{N}\gamma^5\gamma^\mu N \rightarrow b_k \bar{N}\sigma^k N$

PRL **112,** 110801 (2014) PHYSICAL REVIEW LETTERS week ending 21 MARCH 2014

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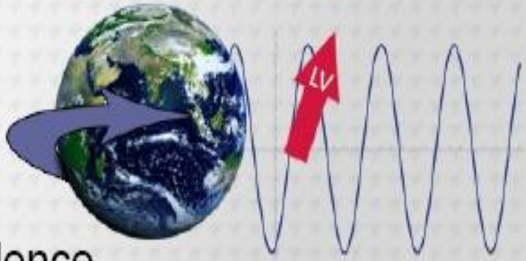
[1]*Physikalisches Institut, Ruprecht-Karls-Universität, 69120 Heidelberg, Germany*
[2]*Institut für Physik, Johannes Gutenberg-Universität, 55099 Mainz, Germany*
[3]*Physikalisch-Technische Bundesanstalt Berlin, 10587 Berlin, Germany*
(Received 12 December 2013; revised manuscript received 4 February 2014; published 17 March 2014)

Noordmans' talk

Spin precession



Simplest case: assume no contribution to Q_{ij}

measure $\frac{\omega_s}{\omega_c}$ (around a certain direction) and analyse sidereal dependence

$$\frac{\langle D\rangle \cdot \mathrm{LV} \cdot f(\chi)}{(2\pi)(750\mathrm{kHz})\hbar} \lesssim 10^{-10} \longrightarrow \mathrm{LV} \lesssim 10^{-26}\ \mathrm{GeV}^{-1} - 10^{-28}\mathrm{GeV}^{-1}$$

However, in general Q_{ij} will not vanish, e.g.: $D^{\pm}_{\mu\nu\rho}$, $W_{\mu\nu\rho\sigma}$

Even variation of polarization in one direction will have contribution of higher harmonics

e.g.: $\mathbf{P}(t) = (0, A_y \sin(2\omega_{\mathrm{LV}} t), A_z \cos(\omega_{\mathrm{LV}} t))$

Also: what is measured gets (significant) contribution from tensor polarization?

Define smart asymmetry?

Noordmans' talk

PHYSICAL REVIEW C **94**, 025502 (2016)

Tests of Lorentz and *CPT* symmetry with hadrons and nuclei

J. P. Noordmans,[1,2] J. de Vries,[3] and R. G. E. Timmermans[1]

Definite plans [54,55] exist to search for electric dipole moments of the proton and the deuteron in this way. Such experiments can be adapted to search for Lorentz and *CPT* violation as well.

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IOP Conf. Series: Journal of Physics: Conf. Series **873** (2017) 012009 doi:10.1088/1742-6596/873/1/012009

Tests of Lorentz and CPT symmetry with hadrons and nuclei

J P Noordmans

nucleon potential in Eq. (23). The effects of the NN interaction in nuclei could provide much better bounds than the electromagnetic interactions. Especially the spin precession of the deuteron is promising in this respect and for example storage-ring experiments might be able to place stringent constraints [28].

Constraining Lorentz and CPT symmetry through deuteron spin-tune measurements

J. P. Noordmans[1]

$$\frac{d\mathcal{P}}{dt} = \boldsymbol{\Omega}_s \times \mathcal{P} = (\boldsymbol{\Omega}_{\text{MDM}} + \boldsymbol{\Omega}_{\text{LV}}) \times \mathcal{P} \ , \tag{15}$$

with the spin precession angular velocities defined in terms of restframe quantities by $\Omega^i_{\text{LV}} = 18\pi W_1 \epsilon^{ijk} X^{jk}(\mathring{v})$ and $\Omega^i_{\text{MDM}} = -\frac{qg}{2m} B^i$, where q, g, and m are the deuteron charge, g-factor, and mass, respectively. The magnetic field $\mathbf{B}$, is the field observed in the particle restframe. In the case of an asymmetric tensor X^{ij}, the LV thus forms an additive contribution to the spin-precession frequency of the deuteron.

Conclusion: results of the deuteron EDM experiment can be used for constraining Lorentz and CPT symmetry

The angular velocity of spin precession caused by the Lorentz violation can be discovered with clockwise and counterclockwise deuteron beams. However, there are also other possibilities caused by nontrivial polarization effects. The absolute value of the polarization vector oscillates. The Lorentz violation causes the rotation of tensor and vector polarizations with different frequencies *(J. Noordmans, private communication).*

The last property can be checked with the transition to the frame rotating with the angular velocity of the spin rotation. In any case, the polarization vector in this frame is not constant $\left(\frac{d\mathbf{S}'}{dt}\neq 0\right)$.

When there is no any spin-tensor interaction, **S'** is constant in the frame rotating with the angular velocity of the spin rotation. In this case, components of the polarization tensor rotate with a twice bigger frequency as compared with components of the polarization vector.

A.J. Silenko, General dynamics of tensor polarization of particles and nuclei in external fields, J. Phys. G. : Nucl. Part. Phys. 42, 075109 (2015).

The best conditions for an observation of spin-tensor interactions are provided by the use of an initially tensor-polarized beam because such a beam acquires a final vector polarization.

Cartan torsion of spacetime

Riemann-Cartan spacetimes

Cartan spacetime torsion tensor **Christoffel symbols**

$$S^{\lambda}_{\mu\nu} = \Gamma^{\lambda}_{\mu\nu} - \Gamma^{\lambda}_{\nu\mu}.$$

The Cartan spacetime torsion does not influence a particle motion but affects the spin

The classical limit of the quantum-mechanical Hamiltonian obtained by Y. N. Obukhov, A. J. Silenko, and O. V. Teryaev, Phys. Rev. D **90**, 124068 (2014) **for a Dirac particle in a magnetic field is given by**

$$H = -g_N \frac{\mu_N}{\hbar} \mathbf{B} \cdot \mathbf{S} - \boldsymbol{\omega} \cdot \mathbf{S} - \frac{c}{2} \breve{\mathbf{T}} \cdot \mathbf{S}, \qquad \breve{T}^{\alpha} = -\frac{1}{2} \eta^{\alpha\mu\nu\lambda} T_{\mu\nu\lambda},$$

where g_N is the nuclear g factor and μ_N is the nuclear magneton. $\eta^{\alpha\mu\nu\lambda}$ is the totally antisymmetric Levi-Civita tensor.

$\boldsymbol{\omega}$ is the angular velocity of the Earth rotation.

The strongest restriction extracted from the experimental data presented by C. Gemmel et al., Eur. Phys. J. D **57**, 303 (2010) **is**

$$|\breve{T}| \cdot |\cos\Theta| < 2.4 \times 10^{-15}\ \mathrm{m}^{-1}.$$

Θ is the angle between $\boldsymbol{B}$ and the torsion $\breve{T}$

About 1% of the angular velocity of the Earth rotation ($|\boldsymbol{\omega}| = 7.29 \times 10^{-5}$ rad/s).

Eur. Phys. J. D **57**, 303–320 (2010)
DOI: 10.1140/epjd/e2010-00044-5

THE EUROPEAN PHYSICAL JOURNAL D

Regular Article

Ultra-sensitive magnetometry based on free precession of nuclear spins

C. Gemmel[1], W. Heil[1,a], S. Karpuk[1], K. Lenz[1], Ch. Ludwig[1], Yu. Sobolev[1,b], K. Tullney[1], M. Burghoff[2], W. Kilian[2], S. Knappe-Grüneberg[2], W. Müller[2], A. Schnabel[2], F. Seifert[2], L. Trahms[2], and St. Baeßler[3,c]

[1] Institut für Physik, 55099 Mainz, Germany
[2] Physikalisch-Technische Bundesanstalt, 10587 Berlin, Germany
[3] University of Virginia, Charlottesville, VA 22904, USA

Conclusion 1: **The effect of the Cartan torsion of spacetime on the spin does not depend on the matter structure. In any case, it appears as a correction to the measured frequency of the Earth rotation.**

Conclusion 2: **An *absolute* precision of the spin rotation frequency measured in storage ring EDM experiments cannot be so high as the corresponding precision in atomic experiments with comagnitometers. Therefore, it is not expedient to bound the Cartan torsion in the storage ring EDM experiments.**

Is the spin tune independent of the beam polarization?

- **We consider the case in which there is the main magnetic (or quasimagnetic) field rotating the spin about the vertical axis** z **and the oscillating magnetic (or quasimagnetic) field rotating the spin about one or two horizontal axes** x, y (**or** $\mathbf{e}_\rho$, $\mathbf{e}_\phi$).

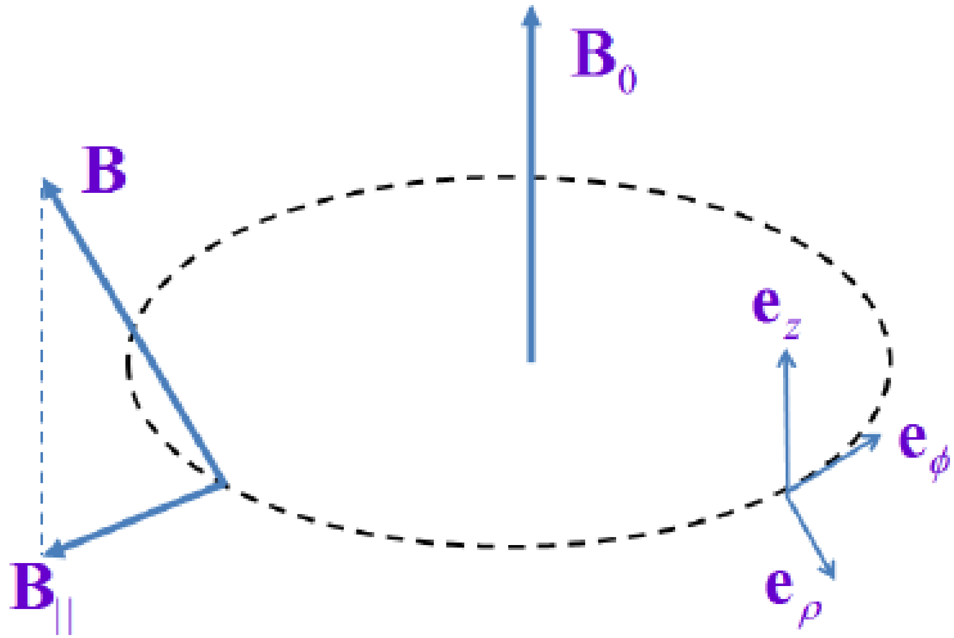


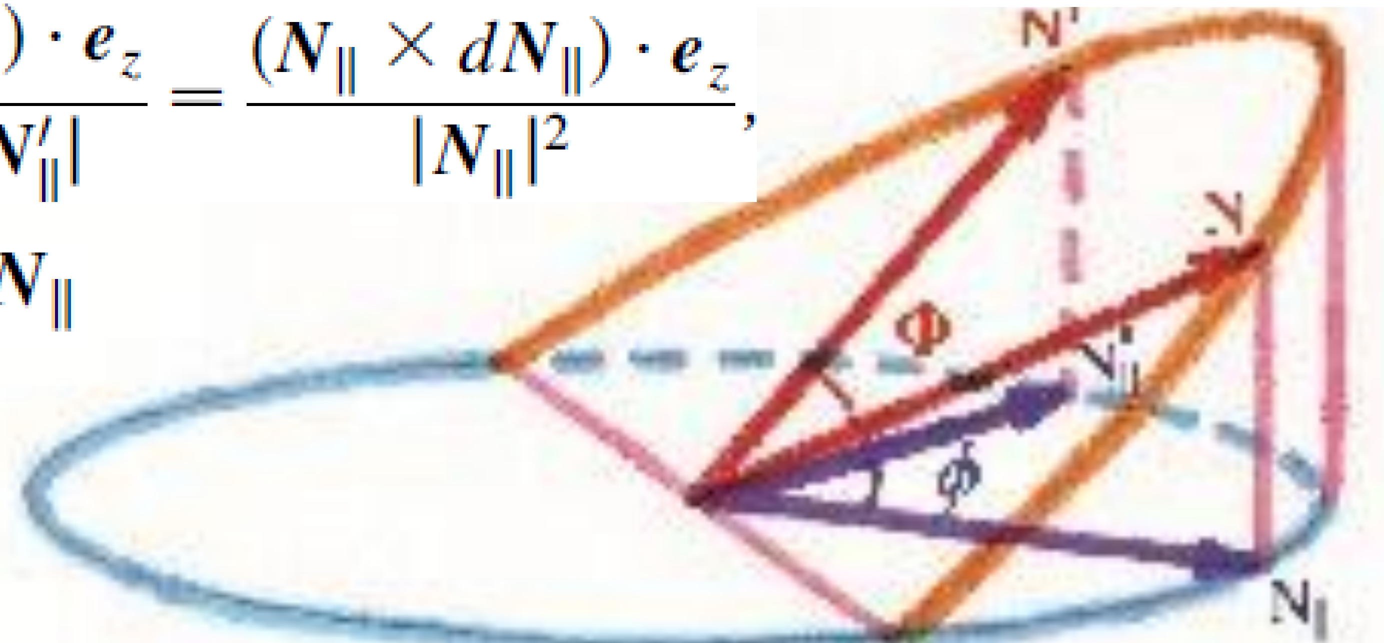
FIG. 1: The storage ring geometry when the cylindrical coordinate system is used. The figure relates to the storage ring with the main magnetic field and magnetic focusing.

Analytical description of spin dynamics in oscillating fields rotating the spin about horizontal axes

The instantaneous plane of particle motion does not coincide with the horizontal plane, and the instantaneous plane of rotation of vector $\boldsymbol{N}=\boldsymbol{p}/p$ *is not* horizontal. The angle Φ between two positions of the rotating vector $\boldsymbol{N}$ in the tilted plane is not equal to the angle ϕ between two corresponding horizontal projections. Therefore, the instantaneous angular velocity of particle motion is changed. The infinitesimal angle of particle rotation in the xy *plane,* $d\phi$*,* is given by

$$d\phi = \frac{(\boldsymbol{N}_{\|} \times \boldsymbol{N}'_{\|}) \cdot \boldsymbol{e}_z}{|\boldsymbol{N}_{\|}| \cdot |\boldsymbol{N}'_{\|}|} = \frac{(\boldsymbol{N}_{\|} \times d\boldsymbol{N}_{\|}) \cdot \boldsymbol{e}_z}{|\boldsymbol{N}_{\|}|^2},$$

$$d\boldsymbol{N}_{\|} = \boldsymbol{N}'_{\|} - \boldsymbol{N}_{\|}$$



A. J. Silenko, Equation of spin motion in storage rings in the cylindrical coordinate system, Phys. Rev. ST Accel. Beams **9**, 034003 (2006).

$$\dot{\phi} \equiv \frac{d\phi}{dt} = \frac{(\boldsymbol{N}_{\|} \times \dot{\boldsymbol{N}}_{\|}) \cdot \boldsymbol{e}_z}{|\boldsymbol{N}_{\|}|^2} = \omega_z - o,$$

$$o = \frac{(\omega_x N_x + \omega_y N_y) N_z}{1 - N_z^2} = \frac{(\omega_\rho N_\rho + \omega_\phi N_\phi) N_z}{1 - N_z^2}.$$

As a rule, o is a rather small correction. These equations are exact.

$$\dot{\phi} = \omega_z = -\frac{e}{\gamma m}\left(B_z - \frac{(\boldsymbol{N} \times \boldsymbol{E})_z}{\beta}\right).$$

The instantaneous angular velocity of spin rotation in the horizontal plane, $\dot{\psi}$, is characterized by the change of angle ψ determining the spin orientation in this plane.

$$\dot{\psi} \equiv \frac{d\psi}{dt} = \frac{(\xi_{\|} \times \dot{\xi}_{\|}) \cdot e_z}{|\xi_{\|}|^2} = (\omega_a)_z - O, \qquad \xi = s/s$$

$$O = \frac{[(\omega_a)_x \xi_x + (\omega_a)_y \xi_y]\xi_z}{1 - \xi_z^2} = \frac{[(\omega_a)_\rho \xi_\rho + (\omega_a)_\phi \xi_\phi]\xi_z}{1 - \xi_z^2}$$

As a rule, *O* is a rather small correction. These equations are exact.

The equations of the spin motion needed for next calculations are either exact (for a circular horizontal field, clockwise or counterclockwise) or obtained in the approximation of weak horizontal fields. The exact case takes usually place at resonance conditions. For storage ring experiments, the above-mentioned approximation is very good.

Eur. Phys. J. C (2017) 77:341
DOI 10.1140/epjc/s10052-017-4845-2

THE EUROPEAN PHYSICAL JOURNAL C

CrossMark

Regular Article - Theoretical Physics

General classical and quantum-mechanical description of magnetic resonance: an application to electric-dipole-moment experiments

Alexander J. Silenko[1,2,a]

We have the oscillating horizontal magnetic field

$$\mathcal{B}\cos(\omega t+\chi)$$

$$\omega_0=-\frac{g_N\mu_N}{\hbar}B_0,$$

$$\mathfrak{C}=-\frac{g_N\mu_N}{2\hbar}\mathcal{B},$$

For particles in a storage ring, we should take into account the Thomas-Bargmann-Mishel-Telegdi equation

the final result is given by

$$
\begin{aligned}
P_x(t) = {} & \cos\Omega t \sin\theta \cos(\omega t + \psi) \\
& + \frac{\mathfrak{E}^2}{\Omega^2}(1 - \cos\Omega t)\sin\theta\cos(\psi - \chi)\cos(\omega t + \chi) \\
& - \frac{\omega_0 - \omega}{\Omega}\sin\Omega t \sin\theta \sin(\omega t + \psi) \\
& + \frac{\mathfrak{E}}{\Omega}\left[\frac{\omega_0 - \omega}{\Omega}(1 - \cos\Omega t)\cos(\omega t + \chi)\right. \\
& \left. + \sin\Omega t \sin(\omega t + \chi)\right]\cos\theta ,
\end{aligned}
$$

$$
\begin{aligned}
P_y(t) = {} & \frac{\omega_0 - \omega}{\Omega} \sin \Omega t \sin \theta \cos(\omega t + \psi) \\
& + \cos \Omega t \sin \theta \sin(\omega t + \psi) \\
& + \frac{\mathfrak{E}^2}{\Omega^2}(1 - \cos \Omega t) \sin \theta \cos(\psi - \chi) \sin(\omega t + \chi) \\
& + \frac{\mathfrak{E}}{\Omega}\left[\frac{\omega_0 - \omega}{\Omega}(1 - \cos \Omega t) \sin(\omega t + \chi) \right. \\
& \left. - \sin \Omega t \cos(\omega t + \chi)\right] \cos \theta, \\
P_z(t) = {} & \frac{(\omega_0 - \omega)\mathfrak{E}}{\Omega^2}(1 - \cos \Omega t) \sin \theta \cos(\psi - \chi) \\
& + \frac{\mathfrak{E}}{\Omega} \sin \Omega t \sin \theta \sin(\psi - \chi) \\
& + \left[1 - \frac{\mathfrak{E}^2}{\Omega^2}(1 - \cos \Omega t)\right] \cos \theta.
\end{aligned}
$$

Spin evolution at frequencies far from the resonance (clockwise and counterclockwise beams are taken into account)

$$P_x(t) = \sin\theta \cos(\omega_0 t + \psi)$$
$$+ \left\{ \frac{\mathfrak{E}}{\omega_0 + \omega} \left[\cos(\omega t + \chi) - \cos(\omega_0 t - \chi) \right] \right.$$
$$\left. + \frac{\mathfrak{E}}{\omega_0 - \omega} \left[\cos(\omega t + \chi) - \cos(\omega_0 t + \chi) \right] \right\} \cos\theta,$$
$$P_y(t) = \sin\theta \sin(\omega_0 t + \psi)$$
$$+ \left\{ -\frac{\mathfrak{E}}{\omega_0 + \omega} \left[\sin(\omega t + \chi) + \sin(\omega_0 t - \chi) \right] \right.$$
$$\left. + \frac{\mathfrak{E}}{\omega_0 - \omega} \left[\sin(\omega t + \chi) - \sin(\omega_0 t + \chi) \right] \right\} \cos\theta,$$

$$P_x(t) = \sin\theta \cos(\omega_0 t + \psi) + \left\{ \frac{\mathfrak{E}}{\omega_0 + \omega} \left[\cos(\omega t + \chi) - \cos(\omega_0 t - \chi) \right] + \frac{\mathfrak{E}}{\omega_0 - \omega} \left[\cos(\omega t + \chi) - \cos(\omega_0 t + \chi) \right] \right\} \cos\theta,$$

$$P_y(t) = \sin\theta \sin(\omega_0 t + \psi) + \left\{ -\frac{\mathfrak{E}}{\omega_0 + \omega} \left[\sin(\omega t + \chi) + \sin(\omega_0 t - \chi) \right] + \frac{\mathfrak{E}}{\omega_0 - \omega} \left[\sin(\omega t + \chi) - \sin(\omega_0 t + \chi) \right] \right\} \cos\theta,$$

$$P_z(t) = \cos\theta + \left(\frac{\mathfrak{E}}{\omega_0 + \omega} \left\{ \cos(\psi + \chi) - \cos\left[(\omega_0 + \omega)t + \psi + \chi\right] \right\} + \frac{\mathfrak{E}}{\omega_0 - \omega} \times \left\{ \cos(\psi - \chi) - \cos\left[(\omega_0 - \omega)t + \psi - \chi\right] \right\} \right) \sin\theta.$$

Next step is the consideration of the case in which the vector $\mathbf{B}_{\|}$ (or $(\mathbf{v} \times \mathbf{E})_{\|}$) circumscribes an ellipse. It is important for taking into account the vertical BOs.

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General description of spin motion in storage rings in the presence of oscillating horizontal fields

Alexander J. Silenko

$$P_x(t) = \sin\theta\cos(\omega_0 t + \psi)$$
$$+\frac{1}{2}\left\{\frac{a_1 - a_2}{\omega_0 + \omega}\left[\cos(\omega t + \chi) - \cos(\omega_0 t - \chi)\right]\right.$$
$$\left.+\frac{a_1 + a_2}{\omega_0 - \omega}\left[\cos(\omega t + \chi) - \cos(\omega_0 t + \chi)\right]\right\}\cos\theta,$$
$$P_y(t) = \sin\theta\sin(\omega_0 t + \psi)$$
$$+\frac{1}{2}\left\{-\frac{a_1 - a_2}{\omega_0 + \omega}\left[\sin(\omega t + \chi) + \sin(\omega_0 t - \chi)\right]\right.$$
$$\left.+\frac{a_1 + a_2}{\omega_0 - \omega}\left[\sin(\omega t + \chi) - \sin(\omega_0 t + \chi)\right]\right\}\cos\theta,$$
$$P_z(t) = \cos\theta + \frac{1}{2}\left(\frac{a_1 - a_2}{\omega_0 + \omega}\left\{\cos(\psi + \chi)\right.\right.$$
$$\left. - \cos\left[(\omega_0 + \omega)t + \psi + \chi\right]\right\} + \frac{a_1 + a_2}{\omega_0 - \omega}\left\{\cos(\psi - \chi)\right.$$
$$\left.\left. - \cos\left[(\omega_0 - \omega)t + \psi - \chi\right]\right\}\right)\sin\theta.$$

We can now use the general theory developed in A. J. Silenko, Equation of spin motion in storage rings in the cylindrical coordinate system, Phys. Rev. ST Accel. Beams **9**, 034003 (2006) **and can calculate the spin tune**

$$\boldsymbol{\omega}_a = \omega_0 \mathbf{e}_z + a_1 \cos(\omega t + \chi)\mathbf{e}_x + a_2 \sin(\omega t + \chi)\mathbf{e}_y .$$

$$\Omega = \omega_0 - \left\langle \frac{a_1 \cos(\omega t + \chi) P_\rho(t) P_z(t) + a_2 \cos(\omega t + \chi) P_\phi(t) P_z(t)}{1 - P_z^2(t)} \right\rangle .$$

We present the following relations in order to demonstrate a nontriviality of the final result:

$$\left\langle \cos(\omega t + \chi) P_\rho(t) P_z(t) \right\rangle = \frac{1}{4}\left(3\cos^2\theta - 1\right)\frac{a_1\omega_0 + a_2\omega}{\omega_0^2 - \omega^2},$$

$$\left\langle \sin(\omega t + \chi) P_\phi(t) P_z(t) \right\rangle = \frac{1}{4}\left(3\cos^2\theta - 1\right)\frac{a_1\omega + a_2\omega_0}{\omega_0^2 - \omega^2}.$$

Taking into account the denominator results in

$$\left\langle \frac{a_1 \cos\left(\omega t + \chi\right) P_\rho(t) P_z(t) + a_2 \cos\left(\omega t + \chi\right) P_\phi(t) P_z(t)}{1 - P_z^2(t)} \right\rangle$$

$$= -\frac{\left(a_1^2 + a_2^2\right)\omega_0 + 2a_1 a_2 \omega}{4\left(\omega_0^2 - \omega^2\right)}.$$

$$\Omega = \omega_0 + \frac{\left(a_1^2 + a_2^2\right)\omega_0 + 2a_1 a_2 \omega}{4\left(\omega_0^2 - \omega^2\right)}.$$

Finally, we can check that this result *obtained for an arbitrary initial polarization* agrees with the Farley formula for the vertical BO (pitch).

F. J. M. Farley, Pitch correction in (g-2) experiments, Phys. Lett. B **42**, 66 (1972).

We can conclude that in the nonresonance case the angular frequency of the spin rotation about the vertical axis does not depend on the initial beam polarization. When a_1^2 and a_2^2 are small as compared with $\omega_0^2-\omega^2$, the correction is small.

However, this result is obtained on the assumption that the action of the perturbing fields is much weaker than that of the main field. This assumption means the adiabatic approximation, wherein the time scale over which a time-dependent Hamiltonian varies is long compared to typical quantum-mechanical oscillation periods. In this case, a quantum-mechanical system remains in the same eigenstate but develops a dynamical phase factor. This is the geometric (Berry) phase. What happens when this assumption is not satisfied?

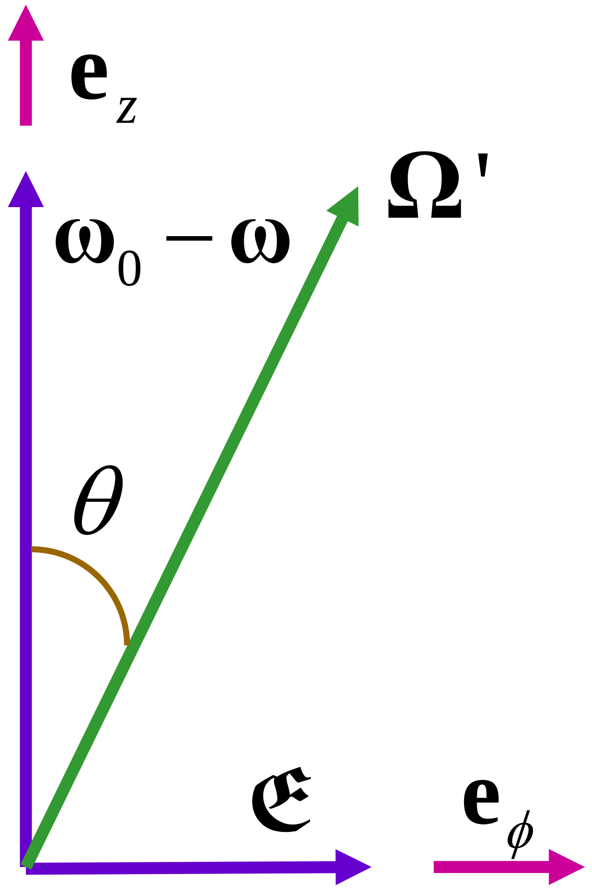
Spin dragging

The adiabatic approximation is violated in the very simple case which is well known in the theory of magnetic resonance.

It is generally accepted to consider the spin motion in the frame rotating with the angular velocity of the oscillating field, $\boldsymbol{\omega}$. If we have a rotating horizontal magnetic field or can neglect one of two horizontal magnetic fields rotating in opposite directions, the angular velocity of the spin rotation in the above-mentioned rotating frame is given by

$$\boldsymbol{\Omega}' = \boldsymbol{\omega}_0 - \boldsymbol{\omega} + \boldsymbol{\mathfrak{E}} = \left(\omega_0 - \omega\right)\mathbf{e}_z' + \mathfrak{E}\mathbf{e}_\phi'.$$

Motional (unstable) spin axis



If the spin direction is collinear to $\boldsymbol{\Omega}'$, it remains unchanged in the frame, rotating with the angular velocity $\boldsymbol{\omega}$. *For this spin direction*, the angular velocity of the spin rotation in the above-mentioned rotating frame is, therefore, equal to $\boldsymbol{\omega}$. The adiabatic approximation is violated because torques caused by $\boldsymbol{\omega}_0 - \boldsymbol{\omega}$ and $\mathfrak{E}$ are mutually balanced.

The spin precession frequency remains the same in narrow intervals of spin directions close to Ω' and −Ω'.

Conclusion 1: The spin precession frequency has a resonance-like dependence on the initial beam polarization. When adiabatic approximation is valid and the initial spin direction is far from the directions (θ,π/2) and (-θ,-π/2), the spin precession frequency does not depend on the initial beam polarization and is close to ω_0 . However, the spin precession frequency becomes equal to ω in the narrow intervals of spin directions close to Ω' and −Ω'. ***This is a new resonance-like spin effect. In the narrow intervals of directions, the spin is dragged by the rotating horizontal field.***

Conclusion 2: The new resonance-like spin effect can be observed in the framework of a preparation of storage ring EDM experiments. It is sufficient to take $|\omega_0 - \omega| / |\omega_0 + \omega| \leq 0.1$. In this case, a weaker horizontal field rotating either clockwise or counterclockwise can be neglected. Then, experiments for initial beam polarizations far from the resonance directions, $(\theta, \pi/2)$ and $(-\theta, -\pi/2)$, and close to these directions *give different spin precession frequencies.* To confirm this conclusion, we need to monitor an evolution of the spin defected from the vertical at an angle of about 0.1 rad.

Summary

- We can constrain Lorentz and CPT symmetry in the framework of the storage ring EDM experiments. While atomic experiments with nuclear spins performed by the Berlin group is potentially more sensitive, this group does not work with deuterons.
- It is not expedient to bound the Cartan torsion in the storage ring EDM experiments because the *absolute* precision of the measured spin rotation frequency cannot be so high as the corresponding precision in atomic experiments performed by the Berlin group.
- We have a possibility to discover the new spin effect of a resonance-like dependence of the spin rotation frequency on a direction of the initial beam polarization.


Thank you for your attention