Spin Tune Analysis for Longer Cycles

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Overview

Electric Dipole Moment (EDM)

- The electric dipole moment (EDM) is a fundamental property of a particle
- EDMs violate time and parity symmetry, therefore, under the CPT theorem, violate CP symmetry
- EDMs can be used investigate the matter-antimatter asymmetry in the universe, axions etc.

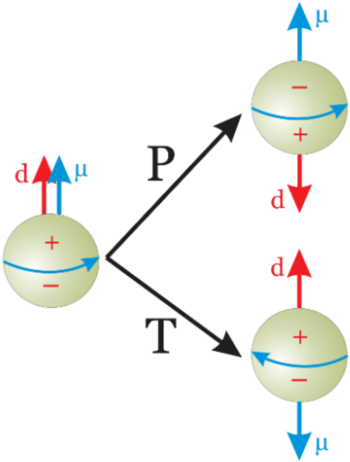


Figure: Behaviour of an EDM under P and T transformations

Spin Precession in a Storage Ring

- The motion of the spin vector $\overrightarrow{S}$ in an accelerator is given by the Thomas-BMT equation, which is written in a simplified form as

$$\frac{d\overrightarrow{S}}{dt} = (\overrightarrow{\Omega}_{MDM} + \overrightarrow{\Omega}_{EDM}) \times \overrightarrow{S} \tag{1}$$

- In COSY, vertically polarized particles are injected and the polarization is flipped onto the horizontal plane using an RF solenoid, causing the spin to precess
- The spin tune is defined as the number of times the spin precesses about its axis per particle turn in the ring, i.e

$$\nu_s = \frac{f_{spin}}{f_{beam}} \tag{2}$$

Detectors and Data Acquisition

- If a beam is polarized in the horizontal plane, the count rates in the up and down detectors, due to spin precession, is $N_{up/dn} \propto \sigma_0 \left(1 \pm PA \sin(\omega t + \phi)\right)$
- σ_0 is the unpolarized cross section, P is the polarization, A is the analyzing power, $\omega = 2\pi f_{spin} = 2\pi \nu_s f_{beam}$
- In order to get the three parameters P, ω and ϕ, an asymmetry is formed

$$\epsilon(t) = \frac{N_{up} - N_{dn}}{N_{up} + N_{dn}} = PA \sin(\omega t + \phi) \tag{3}$$

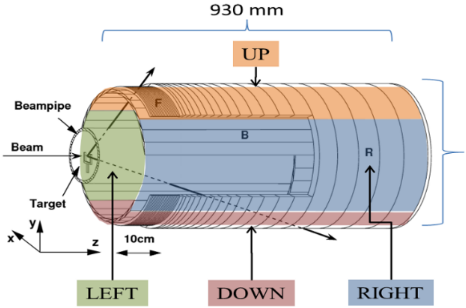
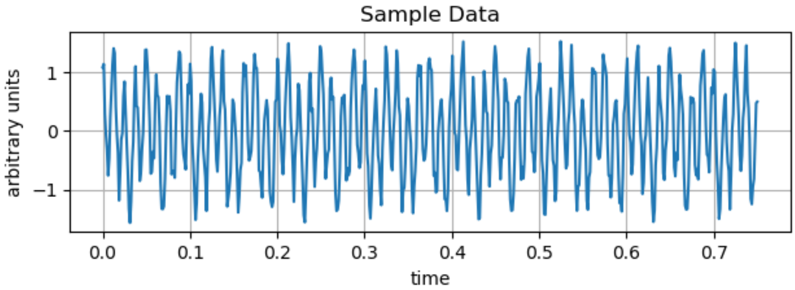
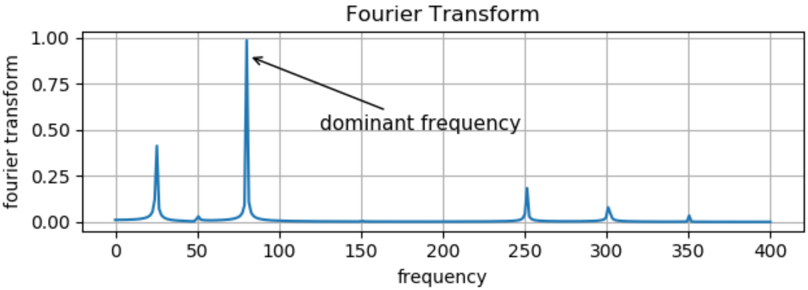


Figure: EDDA detector showing the separate detector quadrants

Fourier Transform Review





Analysis Using Fourier Transform

- A simple parameterization of the recorded data, as a function of turn number n, is given by

$$p_s(n) = 1 \pm \epsilon \sin\left(2\pi\nu_s n + \phi_s\right) \tag{4}$$

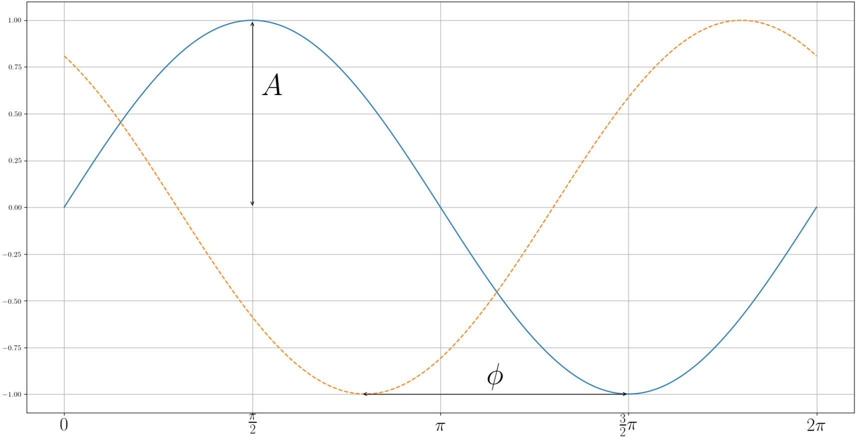
where ν_s is the spin tune and ϕ_s is the phase

- Performing a Fourier transformation on this data allows us to decompose it into a frequency spectrum and easily locate the spin tune frequency
- Since the data is recorded in discrete points in time, a discrete Fourier transform is performed, given by

$$\hat{f}_{\nu_s} = \sum_{n_{ev}=1}^{N} f(n_{ev}) \left(\cos(2\pi\nu_s n_{ev}(n)) - \imath \sin(2\pi\nu_s n_{ev}(n)\right) \tag{5}$$

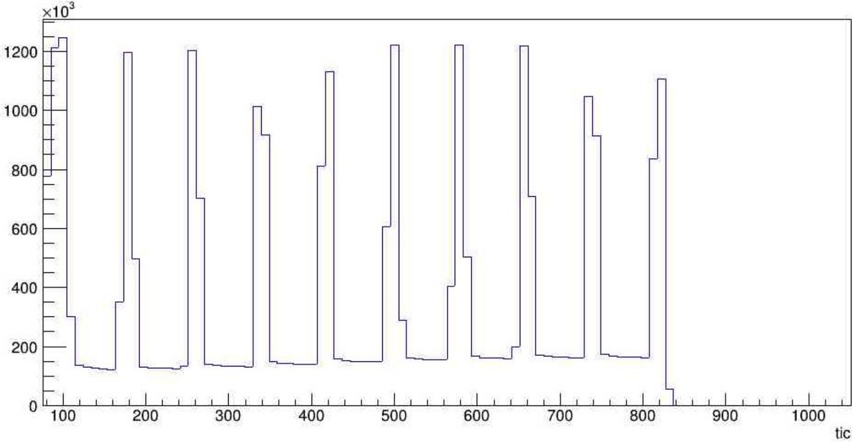
Phase & Amplitude

- The Fourier coefficients are then given by
  $a_{\nu_s} = \frac{2}{N}\sum_{n_{ev}=1}^{N} \cos(2\pi\nu_s n_{ev}(n))$, $b_{\nu_s} = \frac{2}{N}\sum_{n_{ev}=1}^{N} -\sin(2\pi\nu_s n_{ev}(n))$
- Therefore, the phase ϕ_{ν_s} and amplitude A_{ν_s} are given by $\phi_{\nu_s} = \text{atan2}\left(b_{\nu_s}, a_{\nu_s}\right)$ and $A_{\nu_s} = \sqrt{a_{\nu_s}^2 + b_{\nu_s}^2}$



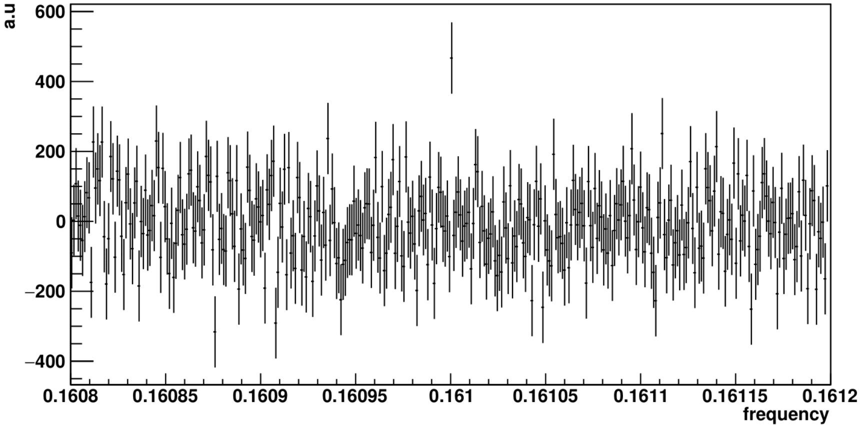
Features of the Run

- Analyzing Run 51180 - May 2019
- Polarized deuterons at 970 MeV/c
- 2 polarized cycles of 900 s $\sim 675 \times 10^6$ turns, 4 bunches
- Extraction occurs at 10 intervals during the cycle



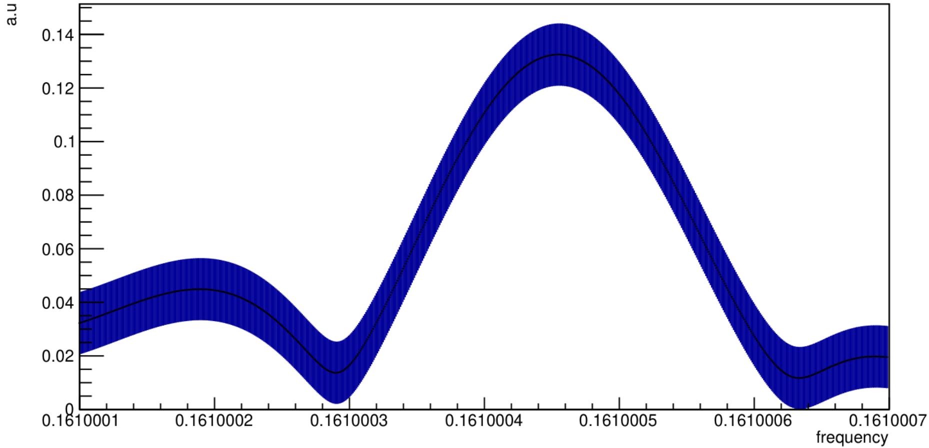
Fourier Spectrum

A clear peak at the spin tune frequency $\nu_s \approx 0.161$ is observed in the Fourier spectrum



Amplitude of the Fourier Spectrum

Analysing a much smaller frequency range, the Fourier spectrum shows clear peaks around the spin tune $\nu_s = 0.16100045$



Relation between Phase and Spin Tune

- The phase of the spin tune is given by $\phi_{\nu_s}(n) = \text{atan2}(b_{\nu_s}, a_{\nu_s})$
- The turn dependent spin tune $\nu_s(n)$ is derived using the angular frequency

$$\delta\nu_s(n) = \frac{1}{2\pi}\delta\omega_{\nu_s}(n) = \frac{1}{2\pi}\frac{d\delta\phi_{\nu_s}(n)}{dn} \quad (6)$$

- The spin tune is defined as $\nu_s = \frac{f_{spin}}{f_{beam}}$, therefore

$$\begin{aligned}\frac{\Delta\nu_s}{\nu_s} &= \frac{\Delta f_{spin}}{f_{spin}} - \frac{\Delta f_{beam}}{f_{beam}} \\ &= \frac{1}{2\pi\nu_s}\frac{\partial\phi_{\nu_s}(n)}{\partial n}\end{aligned} \quad (7)$$

- The turn dependent spin tune is therefore given by

$$\nu_s(n) = \nu_s^0 + \delta\nu_s = \nu_s^0 + \frac{1}{2\pi}\frac{\partial\phi_{\nu_s}(n)}{\partial n} \tag{8}$$

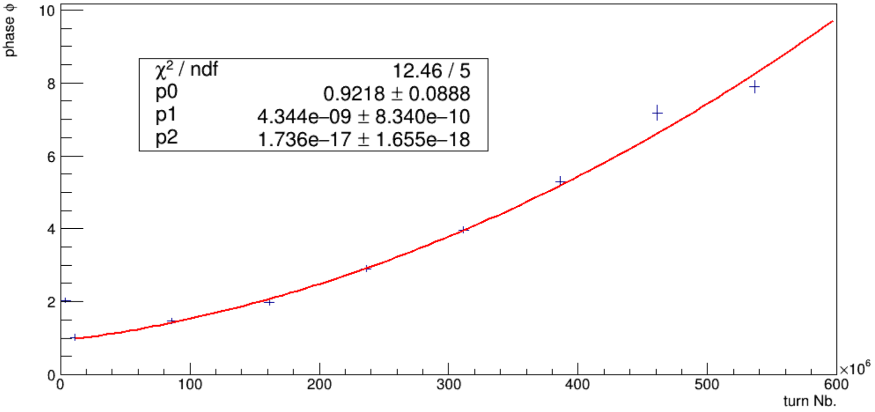
  where ν_s^0 is the initial spin tune
- Therefore, the phase $\phi_{\nu_s}(n)$, which is calculated using the Fourier spectrum, is plotted against the turn number and fitted with a polynomial (order 2)
- From this fit, we get the slope of the phase with respect to turn number, i.e $\frac{\partial\phi_{\nu_s}(n)}{\partial n}$
- Using this, we calculate the spin tune $\nu_s(n)$

Phase Calculation

- The turn dependent phase is calculated by varying the frequency, hence Fourier coefficients, and plotting them at different turn numbers $\phi(n) = \text{atan2}(b_{\nu_s}, a_{\nu_s})$
- The peak frequency from the Fourier spectrum is taken as the centre and a frequency range is created for frequencies lesser than and greater than the peak
- Varying the frequency changes the overall slope of the phase evolution
- If the frequency at which the phase is calculated and the spin tune frequency match then the phase remains constant

Phase of the Fourier Spectrum

- The figure shows the phase distribution for a frequency of $\nu = 0.161000509$ for one bunch in one cycle, which is then fitted with a second order polynomial



Phase in Different Bunches

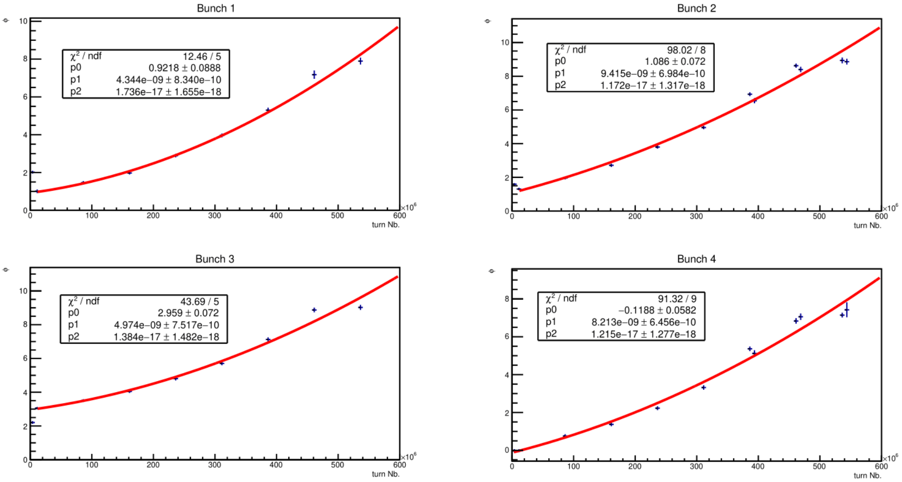


Figure: Phase plots and fits in 4 bunches

Spin Tune

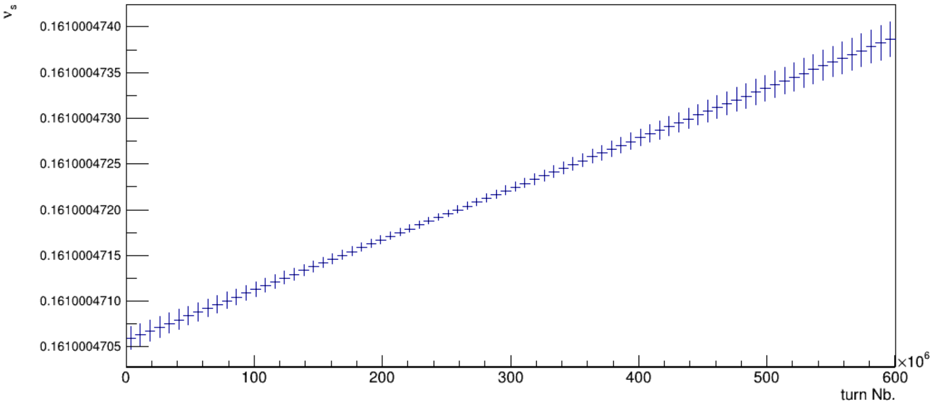


Figure: Spin tune drift during the cycle in one bunch

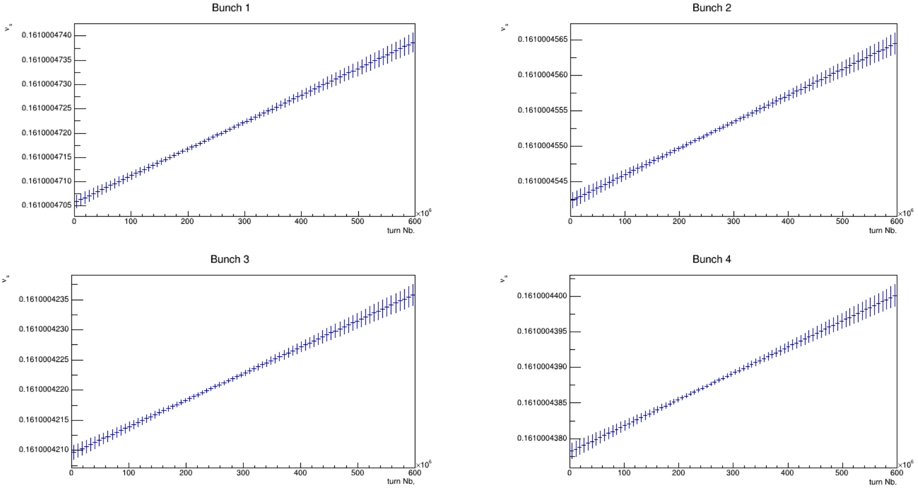


Figure: Spin tune evolution in different bunches in one polarized cycle

Error Analysis

- Fitting a polynomial of order 2 to the phase $\phi(n) = c_0 + c_1 n + c_2 n^2$
  $\Rightarrow \frac{\partial \phi(n)}{\partial n} = c_1 + 2c_2 n$
- Therefore, the spin tune is given by $\nu_s(n) = \nu_s^0 + \frac{1}{2\pi}(c_1 + 2c_2 n)$
- The error on the spin tune is given by

$$\sigma_{\nu_s}^2 = \frac{1}{4\pi^2}\left(\sigma_{c_1}^2 + 4\sigma_{c_2}^2 n^2 + 4n\mathrm{cov}(c_1, c_2)\right) \tag{9}$$

- Earlier spin tune measurements with cycles $\sim$ 100 s had achieved an uncertainty in spin tune of the order of 10^{-10}
- The minimum uncertainty observed for this cycle is in the order of $\sim 10^{-11}$

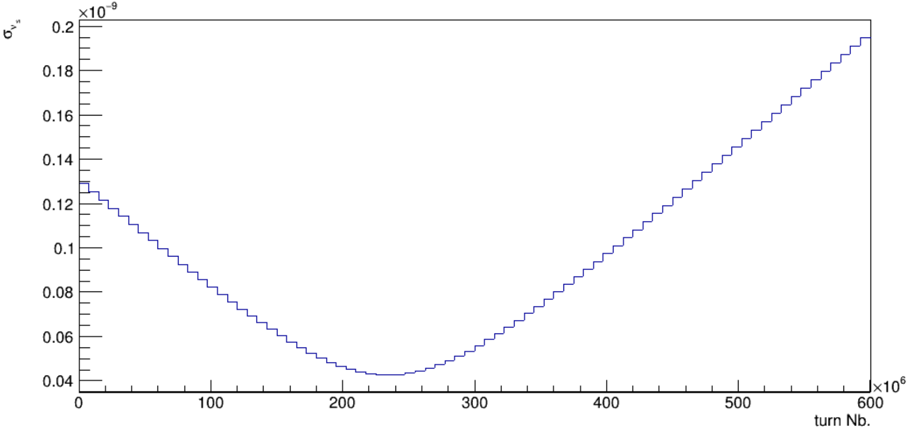


Figure: Uncertainty in the spin tune during the cycle in one bunch

Uncertainty in Spin Tune in Bunches

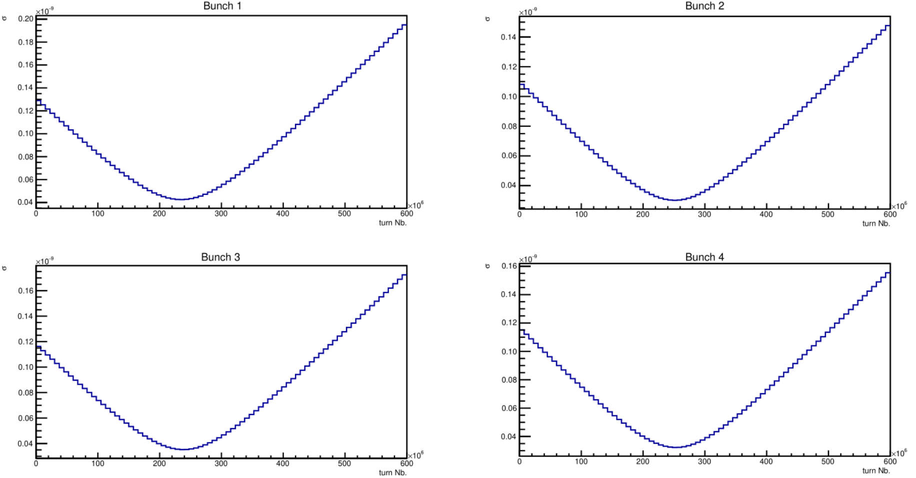


Figure: Uncertainty in spin tune in different bunches

Comparing to Expected Uncertainty

- The uncertainty in the phase is given by $\sigma_\phi^2 = \frac{2}{NP^2}$ where N is the number of events and P is the polarization
- If the variance in the time values when the phase measurements took place is $V(t)$, the uncertainty in the spin tune is given by

$$\sigma_{\nu_s}^2 = \frac{\sigma_\phi^2}{N_{bin} V(t)} = \frac{\frac{2}{NP^2}}{N_{bin} V(t)} \tag{10}$$

- $V(t) \propto T_{cycle}^2$, $N_{bin} \cdot N = N_{tot}$, implies

$$\sigma_{\nu_s}^2 \propto \frac{2}{P^2 T_{cycle}^2 N_{tot}} \implies \sigma_{\nu_s} \propto \frac{1}{T_{cycle}} \tag{11}$$

- Therefore, if T_{cycle} is ten times greater (1000 s vs. 100 s), σ_{ν_s} is expected to drop by an order of 10, which is what is observed

Summary

- The spin tune is an important quantity in the study of EDMs
- Longer cycles are expected to help make more precise calculations of the spin tune
- 1000 s cycles were analyzed using the Fourier method
- The uncertainty in the spin tune was observed to have gone down to 10^{-11}, compared to 10^{-10} for 100 s cycles