Spin Tune Analysis for Longer Cycles

Abhiroop Sen

Forschungszentrum Jülich

March 31, 2020
Overview

1. Basics

2. Features & Fourier Spectra

3. Spin Tune Determination

4. Uncertainty in the Spin Tune

5. Summary
The electric dipole moment (EDM) is a fundamental property of a particle. EDMs violate time and parity symmetry, therefore, under the CPT theorem, violate CP symmetry. EDMs can be used to investigate the matter-antimatter asymmetry in the universe, axions etc.

Figure: Behaviour of an EDM under P and T transformations
The motion of the spin vector $\vec{S}$ in an accelerator is given by the Thomas-BMT equation, which is written in a simplified form as

$$\frac{d\vec{S}}{dt} = (\vec{\Omega}_{MDM} + \vec{\Omega}_{EDM}) \times \vec{S}$$  \hspace{1cm} (1)

In COSY, vertically polarized particles are injected and the polarization is flipped onto the horizontal plane using an RF solenoid, causing the spin to precess.

The spin tune is defined as the number of times the spin precesses about its axis per particle turn in the ring, i.e.

$$\nu_s = \frac{f_{\text{spin}}}{f_{\text{beam}}}$$  \hspace{1cm} (2)
If a beam is polarized in the horizontal plane, the count rates in the up and down detectors, due to spin precession, is

\[ N_{up/dn} \propto \sigma_0 (1 \pm PA \sin(\omega t + \phi)) \]

\( \sigma_0 \) is the unpolarized cross section, \( P \) is the polarization, \( A \) is the analyzing power, \( \omega = 2\pi f_{spin} = 2\pi \nu_s f_{beam} \).

In order to get the three parameters \( P, \omega \) and \( \phi \), an asymmetry is formed

\[ \epsilon(t) = \frac{N_{up} - N_{dn}}{N_{up} + N_{dn}} = PA \sin(\omega t + \phi) \]

(3)
Fourier Transform Review

Sample Data

Fourier Transform

dominant frequency
A simple parameterization of the recorded data, as a function of turn number \( n \), is given by

\[
p_s(n) = 1 \pm \epsilon \sin (2\pi \nu_s n + \phi_s)
\]  

(4)

where \( \nu_s \) is the spin tune and \( \phi_s \) is the phase.

Performing a Fourier transformation on this data allows us to decompose it into a frequency spectrum and easily locate the spin tune frequency.

Since the data is recorded in discrete points in time, a discrete Fourier transform is performed, given by

\[
\hat{f}_{\nu_s} = \sum_{n_{ev}=1}^{N} f(n_{ev}) \left( \cos(2\pi \nu_s n_{ev}(n)) - i \sin(2\pi \nu_s n_{ev}(n)) \right)
\]  

(5)
The Fourier coefficients are then given by
\[ a_{\nu s} = \frac{2}{N} \sum_{n_{ev}=1}^{N} \cos(2\pi \nu_s n_{ev}(n)), \quad b_{\nu s} = \frac{2}{N} \sum_{n_{ev}=1}^{N} -\sin(2\pi \nu_s n_{ev}(n)) \]

Therefore, the phase \( \phi_{\nu s} \) and amplitude \( A_{\nu s} \) are given by \( \phi_{\nu s} = \text{atan2} (b_{\nu s}, a_{\nu s}) \) and \( A_{\nu s} = \sqrt{a_{\nu s}^2 + b_{\nu s}^2} \).
Features of the Run

- Analyzing Run 51180 - May 2019
- Polarized deuterons at 970 MeV/c
- 2 polarized cycles of 900 s $\sim 675 \times 10^6$ turns, 4 bunches
- Extraction occurs at 10 intervals during the cycle
A clear peak at the spin tune frequency $\nu_s \approx 0.161$ is observed in the Fourier spectrum.
Analysing a much smaller frequency range, the Fourier spectrum shows clear peaks around the spin tune $\nu_s = 0.16100045$. 

![Amplitude of the Fourier Spectrum](chart.png)
The phase of the spin tune is given by \( \phi_{\nu_s}(n) = \text{atan}2(b_{\nu_s}, a_{\nu_s}) \).

The turn dependent spin tune \( \nu_s(n) \) is derived using the angular frequency

\[
\delta \nu_s(n) = \frac{1}{2\pi} \delta \omega_{\nu_s}(n) = \frac{1}{2\pi} \frac{d \delta \phi_{\nu_s}(n)}{dn}
\]  

(6)

The spin tune is defined as \( \nu_s = \frac{f_{\text{spin}}}{f_{\text{beam}}} \), therefore

\[
\frac{\Delta \nu_s}{\nu_s} = \frac{\Delta f_{\text{spin}}}{f_{\text{spin}}} - \frac{\Delta f_{\text{beam}}}{f_{\text{beam}}}
\]

\[
= \frac{1}{2\pi \nu_s} \frac{\partial \phi_{\nu_s}(n)}{\partial n}
\]

(7)
Spin Tune Determination

- The turn dependent spin tune is therefore given by

\[ \nu_s(n) = \nu_s^0 + \delta \nu_s = \nu_s^0 + \frac{1}{2\pi} \frac{\partial \phi_{\nu_s}(n)}{\partial n} \]  

(8)

where \( \nu_s^0 \) is the initial spin tune

- Therefore, the phase \( \phi_{\nu_s}(n) \), which is calculated using the Fourier spectrum, is plotted against the turn number and fitted with a polynomial (order 2)

- From this fit, we get the slope of the phase with respect to turn number, i.e \( \frac{\partial \phi_{\nu_s}(n)}{\partial n} \)

- Using this, we calculate the spin tune \( \nu_s(n) \)
The turn dependent phase is calculated by varying the frequency, hence Fourier coefficients, and plotting them at different turn numbers \( \phi(n) = \text{atan2}(b_{\nu_s}, a_{\nu_s}) \)

The peak frequency from the Fourier spectrum is taken as the centre and a frequency range is created for frequencies lesser than and greater than the peak.

Varying the frequency changes the overall slope of the phase evolution.

If the frequency at which the phase is calculated and the spin tune frequency match then the phase remains constant.
Phase of the Fourier Spectrum

- The figure shows the phase distribution for a frequency of \( \nu = 0.161000509 \) for one bunch in one cycle, which is then fitted with a second order polynomial.

\[
\chi^2 / \text{ndf} \quad 12.46 / 5 \\
p0 \quad 0.9218 \pm 0.0888 \\
p1 \quad 4.344e-09 \pm 8.340e-10 \\
p2 \quad 1.736e-17 \pm 1.655e-18
\]
Phase in Different Bunches

Figure: Phase plots and fits in 4 bunches
Figure: Spin tune drift during the cycle in one bunch
Figure: Spin tune evolution in different bunches in one polarized cycle
Fitting a polynomial of order 2 to the phase \( \phi(n) = c_0 + c_1 n + c_2 n^2 \)
\[ \Rightarrow \frac{\partial \phi(n)}{\partial n} = c_1 + 2c_2 n \]

Therefore, the spin tune is given by \( \nu_s(n) = \nu_s^0 + \frac{1}{2\pi} (c_1 + 2c_2 n) \)

The error on the spin tune is given by

\[
\sigma_{\nu_s}^2 = \frac{1}{4\pi^2} \left( \sigma_{c_1}^2 + 4\sigma_{c_2}^2 n^2 + 4ncov(c_1, c_2) \right)
\]  \( \text{(9)} \)

Earlier spin tune measurements with cycles \( \sim 100 \) s had achieved an uncertainty in spin tune of the order of \( 10^{-10} \)

The minimum uncertainty observed for this cycle is in the order of \( \sim 10^{-11} \)
Figure: Uncertainty in the spin tune during the cycle in one bunch
Uncertainty in Spin Tune in Bunches

Figure: Uncertainty in spin tune in different bunches
Comparing to Expected Uncertainty

- The uncertainty in the phase is given by $\sigma^2_\phi = \frac{2}{NP^2}$ where $N$ is the number of events and $P$ is the polarization.

- If the variance in the time values when the phase measurements took place is $V(t)$, the uncertainty in the spin tune is given by

  \[
  \sigma^2_{\nu_s} = \frac{\sigma^2_\phi}{N_{bin} V(t)} = \frac{2}{NP^2 N_{bin} V(t)}
  \]  

  (10)

- $V(t) \propto T_{cycle}^2$, $N_{bin} \cdot N = N_{tot}$, implies

  \[
  \sigma^2_{\nu_s} \propto \frac{2}{P^2 T_{cycle}^2 N_{tot}} \implies \sigma_{\nu_s} \propto \frac{1}{T_{cycle}}
  \]

  (11)

- Therefore, if $T_{cycle}$ is ten times greater (1000 s vs. 100 s), $\sigma_{\nu_s}$ is expected to drop by an order of 10, which is what is observed.
The spin tune is an important quantity in the study of EDMs. Longer cycles are expected to help make more precise calculations of the spin tune. 1000 s cycles were analyzed using the Fourier method. The uncertainty in the spin tune was observed to have gone down to $10^{-11}$, compared to $10^{-10}$ for 100 s cycles.