Spin Dynamics in Storage Rings in Application to Searches for EDM

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Outlook: selected issues from JEDI driven activity

- EDM: why? Fundamental symmetries and baryogenesis
- EDM: how? Spin rotation by electric fields
- JEDI@COSY: selected record setting results
- ---- all have been reviewed in the plenary talk by Frank Rathmann
- Systematic background from MDM in imperfection magnetic rings is an evil
- JEDI@COSY: spintune mapping as a tool to quantify imperfection fields
- Impact of synchrotron oscillations on spin coherence time: nonexponential decay of polarization and spin echo
- Gravity induced spin rotation as a false EDM signal: de Siitter spin-orbit interaction and imperfection fields from focusing to compensate for the free fall

http://collaborations.fz-juelich.de/ikp/jedi/documents/colpapers.shtml

Why: EDM and baryogenesis

- Sakharov (1967): CP violation is imperative for baryogenesis in the Big Bang Cosmology

	observed	*SM prediction*
$\frac{n_B - n_{\overline{B}}}{n_\gamma}$	$(6.1 \pm 0.3) \times 10^{-10}$	10^{-18}
neutron EDM limit $(e \cdot cm)$	3×10^{-26}	10^{-31}

- EDM as a high-precision window at physics Beyond Standard Model
- nEDM: plans to increase sensitivity by 1 order in magnitude
- pEDM: statistical accuracy of 10^{-29} is aimed at dedicated all-electric storage rings
- dEDM and pEDM in precursor experiment at COSY: dEDM $\sim 10^{-20}$ is within reach?
- Sequel to JEDI: CPEDM & prototype 30 VeV pure electric ring (at CERN? at COSY?...) --- big international effort, CDR under preparation for the fall 2018

EDM vs. MDM (learnt from Lev Okun in 60's)

- MDM: allowed by all symmetries, a scale is set by a nuclear magneton μ_N
- Buy CPT: EDM is P and CP/T forbidden
- Price for the PV: 10^{-7} , for CPV extra 10^{-3} from K-decays
- Natural scale $d_N = \mu_N \times 10^{-7} \times 10^{-3} \sim 10^{-24} e \cdot cm$
- The SM: CPV linked to the flavor change. Pay 10^{-7} more to neutralize the flavor change

$$d_{N,SM} \sim \mu_N \times 10^{-7} \times 10^{-3} \times 10^{-7} \sim 10^{-31} e \cdot cm$$

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Why charged particles besides neutrons?

- Neutrons are record holders: next generation expts in the pipeline wherever ultracold neutrons are available (PNPI, Grenoble, Oak Ridge, PSI, Triumph…), excellent overview by Klaus Kirch.

- Isotopic properties of CP violation Beyond the Standard Model are entirely unknown: $d_p \gg d_n$ is not excluded

- Even with CP violation from **isoscalar** QCD θ-term the theory predicts $d_p \neq d_n$
- (e.g. Bonn-Juelich Collab. in the EFT approach, ask Andreas Wirzba)

- Deuteron: besides d_p and d_n the deuteron d_d may receive new contributions from T- and CP –violating np-interaction --- basically an open issue

- The same is true for helium-3 and other nuclei

A principle of EDM measurement: spin rotation by EDM-interaction with E-fields

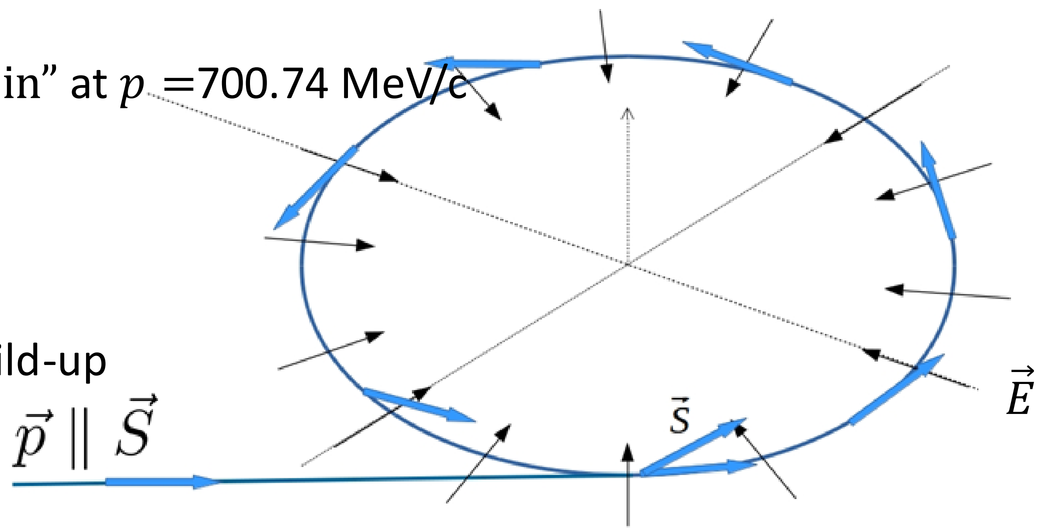
- FT-BMT eqn :
- $$\frac{d\vec{S}}{dt} = \vec{\Omega} \times \vec{S}(t) = -\frac{q}{m}\Big(\underbrace{G\vec{B} + \Big(\frac{1}{\gamma^2-1} - G\Big)\vec{\beta} \times \vec{E}}_{\text{MDM}} + \underbrace{\frac{1}{2}\eta(\vec{E} + \vec{\beta} \times \vec{B})}_{\text{EDM}}\Big) \times \vec{S}(t)$$

MDM

EDM $d = \frac{\eta \hbar q}{2mc}$

All-electric ring is ideal for protons (Yu Orlov et al, srEDM at BNL)

- MDM-term → 0 - "frozen spin" at p =700.74 MeV/c
- Longitudinal initial spin
- EDM signal: vertical spin build-up per turn → πη



Ideal experimental setup

- Ideal storage ring (alignment, stability, field homogeneity, no systematics)
- high intensity beams ($N = 4 \times 10^{10}$ per fill)
- polarized hadron beams ($P = 0.8$)
- large electric fields ($E = 10$ MV/m)
- long spin coherence time ($\tau = 1000$ s)
- polarimetry (analyzing power A = 0.6, f = 0.005)

$$\sigma_{\text{stat}} \approx \frac{1}{\sqrt{Nf\tau}PAE} \Rightarrow \sigma_{\text{stat}}(1\text{year}) = 10^{-29}\ e\cdot\text{cm}$$

challenge: get σ_{sys} to the same level

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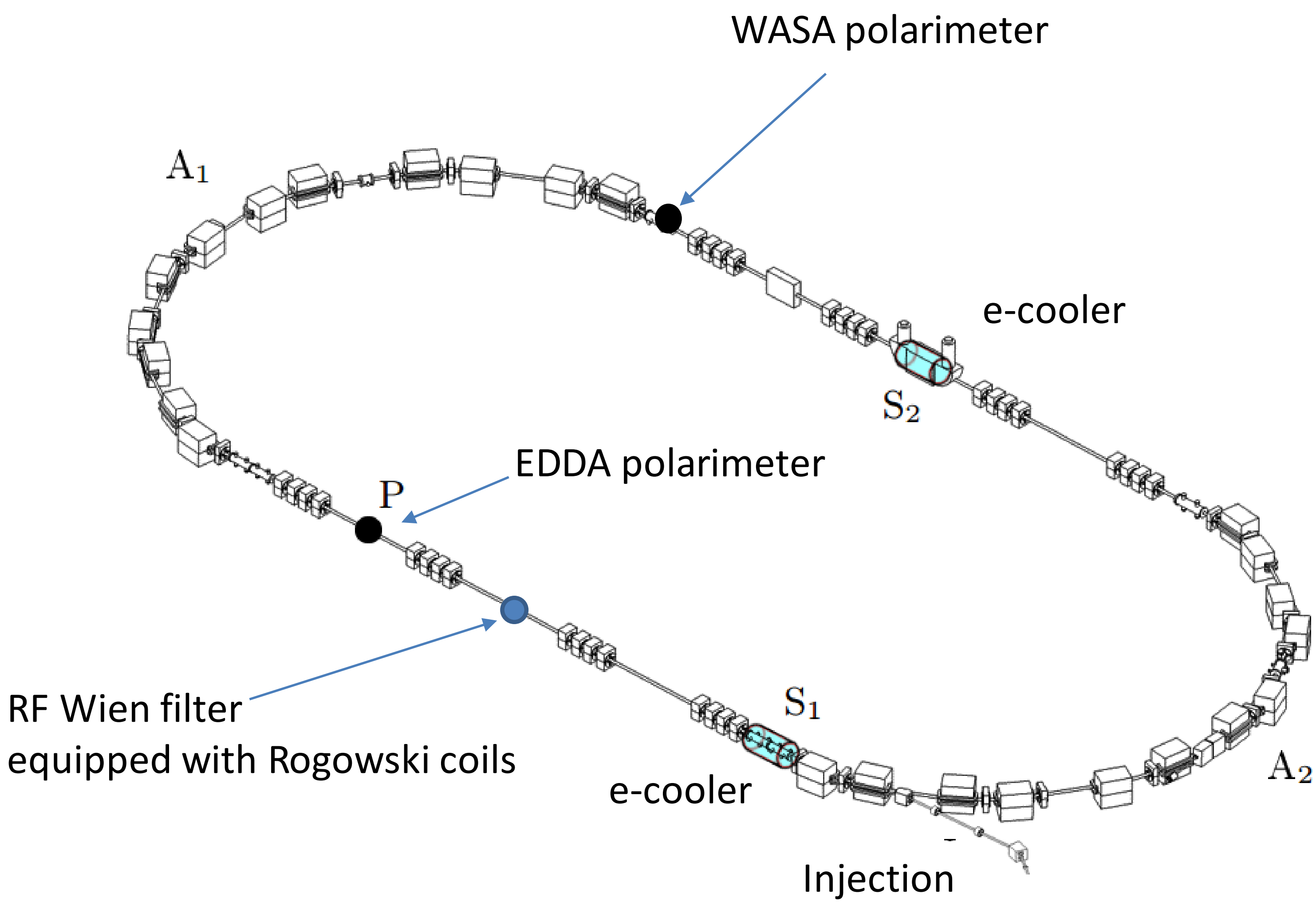
JEDI: EDM searches at COSY
Precursor experiment in the pipeline

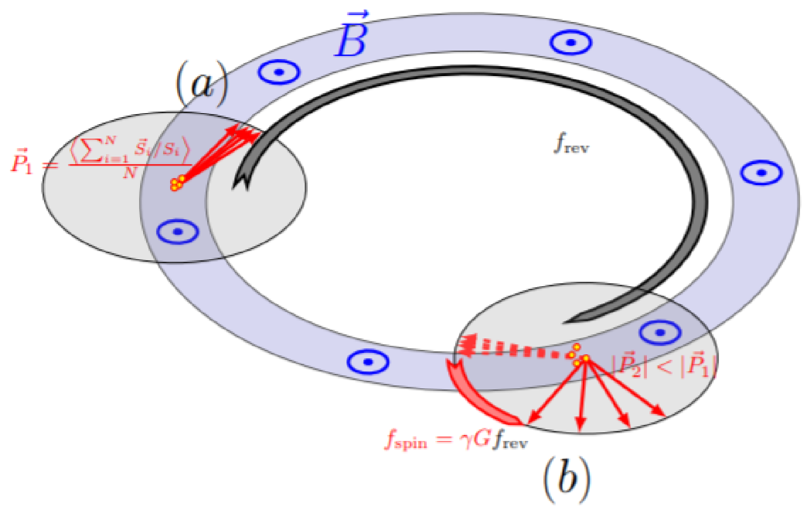
- COSY is all-magnetic storage ring, unique for studying spin dynamics but still needs upgrades for EDM searches
- Statistical accuracy for $d_d = 10^{-24} e \cdot cm$ is reachable at COSY
- Systematic effects: horizontal imperfection magnetic fields are evil because MDM >> EDM and MDM rotations give false EDM signal
- JEDI experimental studies of imperfections: MDM background can be suppressed to 10^{-6} level. Further suppression of systematics is possible
- COSY as is: EDM $\lesssim 10^{-6}$MDM $\cong 10^{-20}\ e \cdot cm$
- More in the plenary talk by Frank Rathmann

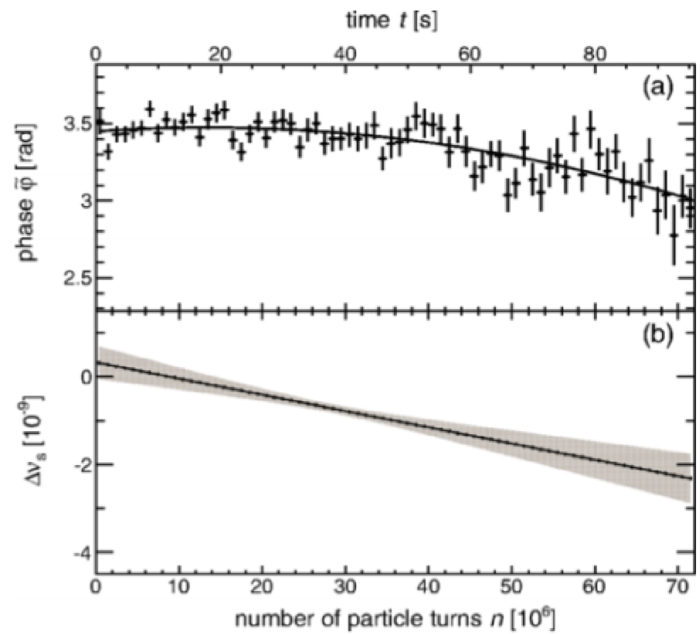
Meanwhile COSY as a Testing Ground



JEDI at COSY: record spin tune precision

Achieved by continuous polarimetry of spin precession with time stamp





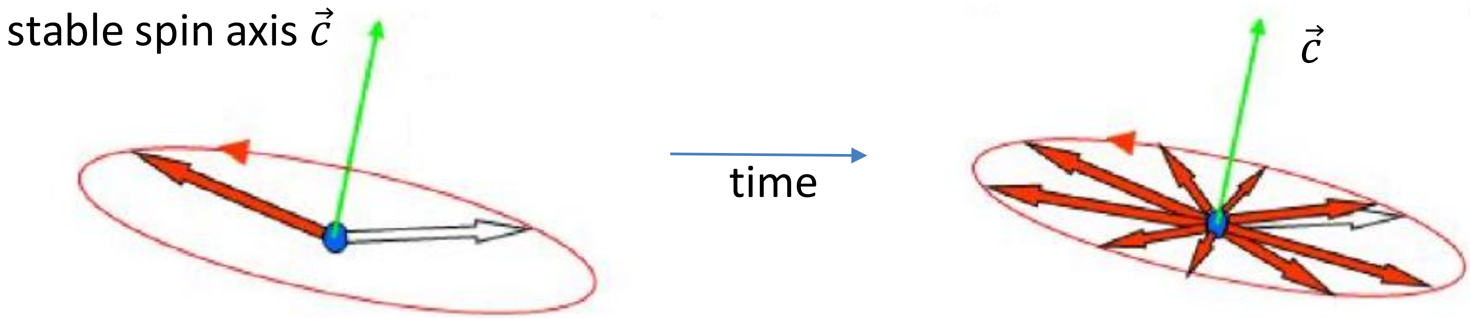
$$\sigma(\nu_s = \gamma G) \approx 10^{-10} \text{ in } 100\,\text{s}$$

$$\text{Note: } \gamma G = \frac{f^{MDM}_{spin}}{f_{rev}}, \qquad \frac{f^{EDM}_{spin}}{f_{rev}} \approx 10^{-10} \text{ for EDM } d = 10^{-24}\, e\,\text{cm}$$

JEDI: PRL 115, 094801 (2015); PRL, 119, 014801 (2017); PRST AB **21**, 042002 (2018)

Spin coherence time

- Long spin coherence is crucial for high sensitivity to EDM signal



Inititally all spins aligned

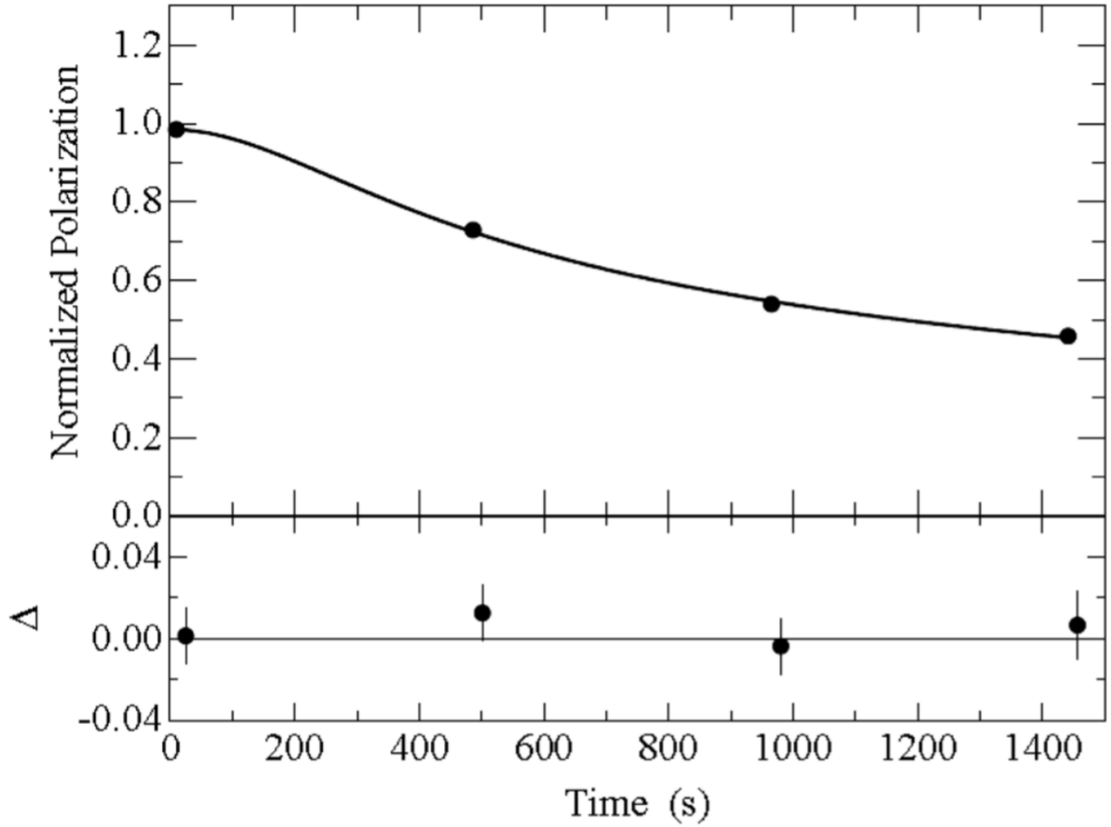
Spins decohered - polarization vanishes

Prerequisites for long SCT:

- use bunched beam
- decrease beam emittance via electron-cooling
- Betatron oscillations: fine-tune sextupole families to suppress chromaticity (old idea by Ivan Koop and Yuri Shatunov (1988))
- *N.B.* **Yuri just turned 75 --- our congratulations to a guru in spin physics at storage rings!**

JEDI: record spin coherence time

- From 2017 on JEDI routinely runs at COSY with SCT of more than 1000 s
- JEDI: PRL 117, 054801 (2016) PR AB 21, 024201 (2018);



EDM effect

- RF Wien-Filter does not disturb a beam orbit but entails a vanishing EDM term in the FT-BMT eqn.

- Still EDM enters via tilt of the stable spin axis $\vec{c}$

$$\vec{c} = \vec{e}_x \sin \xi_{\mathrm{EDM}} + \vec{e}_y \cos \xi_{\mathrm{EDM}}$$

$$\tan \xi_{\mathrm{EDM}} = \frac{\eta\beta}{2G}$$

- RF WF with upright B-field and spin kick χ_{WF} still rotates spin with resonance tune (Morse et al. PRSTAB 16 (2013)114001, NNN (2013) unpublished)

$$\epsilon_{\mathrm{EDM}} = \frac{1}{4\pi} \chi_{\mathrm{WF}} \sin \xi_{\mathrm{EDM}}$$

- Extract EDM from either stable spin axis or resonance tune?

EDM effect

- **A pitfall**: false EDM signal from MDM rotation in imperfection magnetic fields

$$\sin\xi_{\mathrm{EDM}}\vec{e}_x \ \rightarrow\ [c_{\mathrm{x}}(\mathrm{MDM}) + \sin\xi_{\mathrm{EDM}}]\vec{e}_x + c_{\mathrm{z}}(\mathrm{MDM})\vec{e}_z$$

$$\epsilon_0 = \frac{1}{4\pi}\chi_{\mathrm{WF}}|\vec{c}\times\vec{w}|$$

$\vec{w}$ is a WF magnetic field axis

- Spin tune depends on the imperfection fields

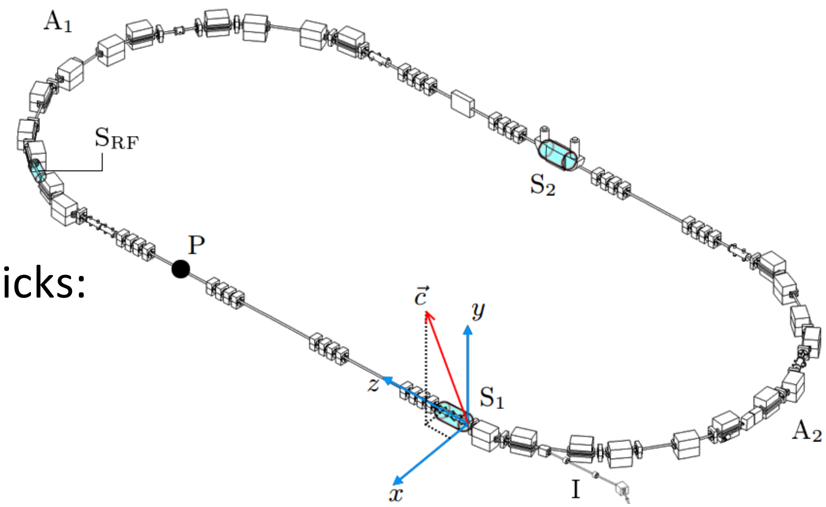
- Spin tune mapping: convert a record precision of spin tune to a tool to determine imperfections $c_{\mathrm{x},z}$.

- Probe imperfection complementing a ring with artificial in-plane magnetic fields

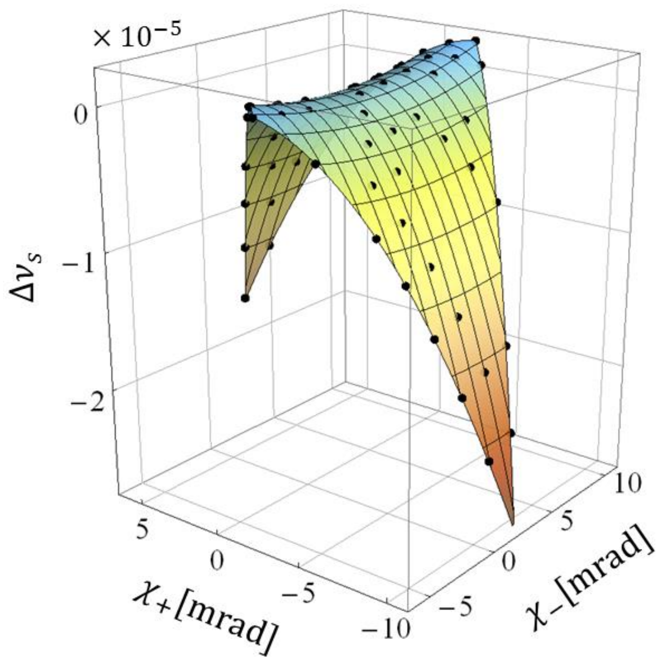
- Realized experimentally: JEDI: Phys.Rev. AB 20, 072801 (2017)

JEDI: spin tune mapping evaluation of imperfection magnetic fields at COSY

- Two cooler solenoids as spin rotators to generate artificial imperfection fields



- Measure spin tune shift vs solenoid spin kicks:



- Position of the saddle point determines a tilt of stable spin axis by magnetic imperfections

- Control of MDM background at level $\Delta c = 2.8 \times 10^{-6}$ rad

- Systematics-limited sensitivity $\sigma_{d_d} \approx 10^{-20} e \cdot cm$

RF WF in the EDM mode (vertical magnetic field axis)

- Spin transfer matrix with running RF WF

$$T(n) = \exp[-i\pi\nu_s n(\vec{\sigma}\cdot\vec{c})]\cdot\exp[-i\pi\epsilon_0 n(\vec{\sigma}\cdot\vec{u})]$$

- Axis of driven spin rotation

$$\vec{u} = \cos\Delta_{WF}\,\vec{m} + \sin\Delta_{WF}\,\vec{k}$$

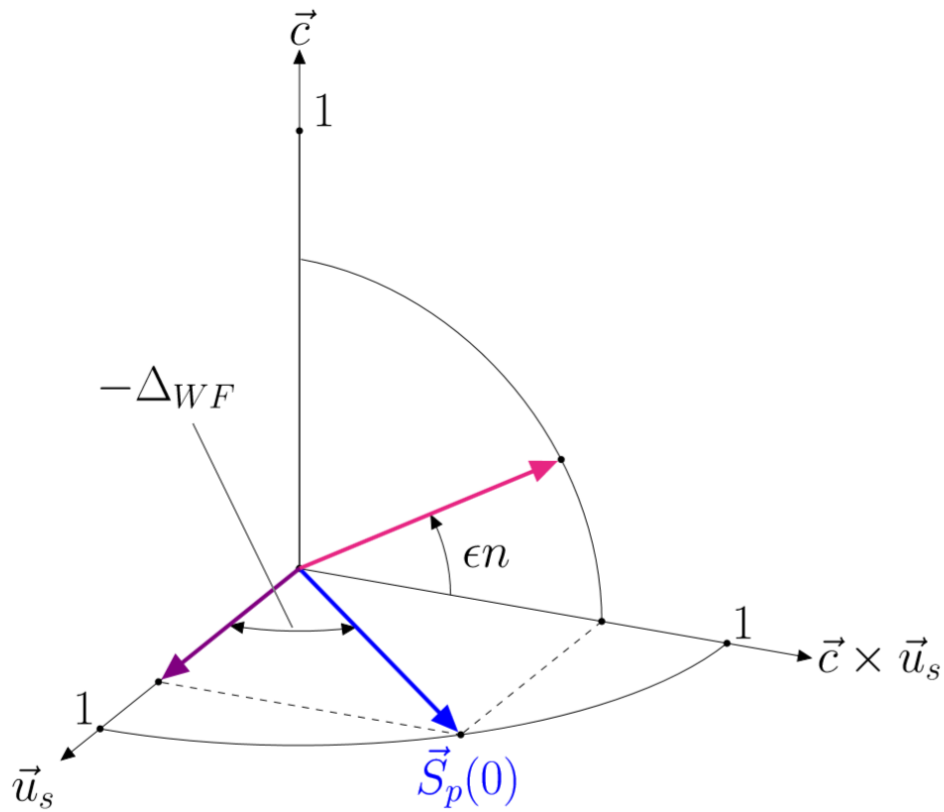
- Δ_{WF} - a phase shift between the spin precession and RF phases

$$\vec{k} = \frac{[\vec{c}\times\vec{w}]}{|\vec{c}\times\vec{w}|}, \qquad m = \vec{k}\times\vec{c}$$

- Idle precession

$$\vec{u}_s(n) = \vec{u}\cos 2\pi\nu_s n + [\vec{c}\times\vec{u}]\sin 2\pi\nu_s n$$

Rotating frame



- Rotating frame: one component of the initial in-plane polarization participates the RF WF driven spin resonance
- The second component keeps idle precession
- The initial vertical polarization does not generate the idle precessing component

Synchrotron oscillations

Synchrotron oscillations: Nikolaev, Saleev, Rathmann, JETP Lett. 106(4), 213-216 (2017)

- Individual spin doesn't decohere, polarization decoherence comes from averaging over an ensemble.

- Bunch density and synchrotron amplitude, a_z, distributions are related by the Abel transform.

- A Gaussian bunch as an working approximation:

$$N_z \propto \exp(-z^2/B^2)$$

$$F\left(\xi = \frac{a_z}{B}\right) = 2\xi \exp(-\xi^2)$$

- Bunch length is related to $\Delta p/p$

Synchrotron oscillations

- Spin phase modulation with two random parameters

$$\Delta\theta_s(n) = \psi_s \xi [\cos(2\pi\nu_z n + \lambda) - \cos\lambda]$$

$$\psi_s = \frac{G\gamma\beta^2\sqrt{2}}{\nu_z} \left\langle \left(\frac{\Delta p}{p}\right)^2 \right\rangle^{1/2}$$

- Related modulation of the RF phase (η_{SF} is a slip factor)

$$\Delta\theta_{WF}(n) - \Delta\theta_S(n) = C_{WF}\Delta\theta_S(n)$$

$$C_{WF} = \frac{\nu_{WF} + K}{\nu_s} \cdot \frac{\eta_{SF}}{\beta^2} - 1$$

- Set of decoherence-free magic energies at $C_{WF} = 0$ (Lehrach et al (2012))

Synchrotron oscillations

- Jittering of the driven spin rotation axis vs. λ

$$\vec{u}(\lambda) = \vec{u}\cos(y\cos\lambda) - [\vec{c} \times \vec{u}]\sin(y\cos\lambda)$$

$$y = C_{WF}\psi_S\xi = y_0\xi$$

- Driven rotation of each individual spin rotation in its own λ -dependent plane.

- Driven resonance tune does not depend on the synchrotron phase λ

$$\epsilon(\xi) = \epsilon J_0(y)$$

- All individual driven rotation planes share the same stable spin axis $\vec{c}$

Synchrotron oscillations and spin echo

- Averaging over synchrotron phase for initial vertical $\vec{S}(0) = \vec{c}$

$$\vec{S}(\vec{c};n) = \cos(2J_0(y)\pi\epsilon_0 n)\,\vec{c} - J_0(y)\sin(2J_0(y)\pi\epsilon_0 n)\,[\vec{c} \times \vec{u}_s(n)]$$

- The $\cos(2J_0(y)\pi\epsilon_0 n)$ and $\sin(2J_0(y)\pi\epsilon_0 n)$ are spin envelopes from RF driven spin resonance

- Extra suppression by $J_0(y) < 1$ of the in-plane polarization from averaging over ensemble of particle-to-particle jittering rotation planes.

- **Spin echo**: while the in-plane polarization decoheres, the amplitude of the vertical polarization stays put at unity

- No idle precessing in-plane component is generated from vertical polarization

Synchrotron oscillations

- The initial in-plane polarization $\vec{S}_p(0)$:

$$\vec{S}(\vec{S}_p; n) = J_0(y) \cos\Delta_{WF} \sin(J_0(y)\Phi)\, \vec{c}$$

$$+\frac{1}{2}\cos(J_0(y)\Phi)\, \{\cos\Delta_{WF} \left(1 - J_0(2y)\right)\vec{u}_s(n) - \sin\Delta_{WF} \left(1 + J_0(2y)\right)[\vec{c} \times \vec{u}_s(n)]\}_{driven}$$

$$+\frac{1}{2}\{\sin\Delta_{WF} \left(1 + J_0(2y)\right)\vec{u}_s(n) - \sin\Delta_{WF} \left(1 - J_0(2y)\right)[\vec{c} \times \vec{u}_s(n)]\}_{idle}$$

- Reminder of the spin echo: in-plane polarization decoheres stronger than the vertical one

- Driven rotation plane and the idle precession are axis rotated by an angle $\sim y^2 \tan\Delta_{WF}$

Damping of driven oscillations

- One-particle resonance strength $\epsilon(\xi) = \epsilon_0 J_0(C_{WF}\psi_s\xi)$

- Spread of driven resonance tunes → decoherence of polarization of an ensemble of particles

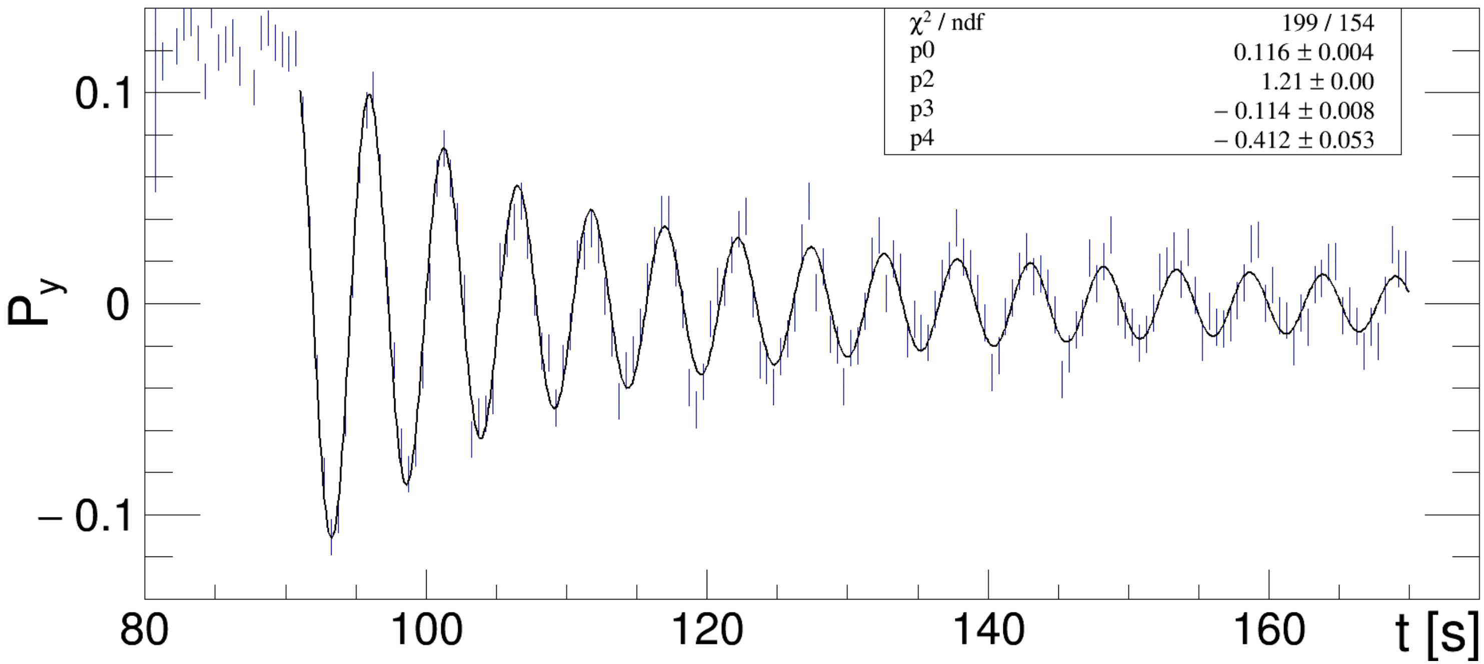
$$
\begin{aligned}
S_y &= \Re\langle \exp[-in\epsilon(\xi)]\rangle_\xi \\
&= \Re\left\langle \exp\left\{-in\epsilon_0\left[1-\frac{1}{4}C_{rf}^2\psi_s^2\xi^2\right]\right\}\right\rangle_\xi \\
&= \frac{1}{\sqrt{1+\rho^2 n^2}}\cos[\epsilon_0 n - \kappa(n)]\,,
\end{aligned}
$$

- Damping parameter $\rho = \frac{1}{4}\epsilon_0 C_{WF}^2 \psi_s^2$

- Phase walk $\kappa(n) = \arctan(\rho n)$

Damping of driven oscillations

- An example of damping of oscillations driven by RF Wien Filter (JEDI, November 2017, very preliminary):



- Exptl confirmation of non-exponential attenuation

- A word of caution: there is certain sensitivity to the bunch shape

Detuned driven spin rotations

- The phase of driven spin rotation: no linear growth of the driven spin phase

$$\epsilon n \Rightarrow \phi = \epsilon_0 \frac{\sin \delta_{WF} n}{\delta_{WF}} \qquad \epsilon(\xi) = \epsilon_0 J_0(C_{WF}\psi_s \xi)$$

- Decoherence

$$S_y = \frac{1}{\sqrt{1+\Phi^2}} \cos[\phi - \kappa(n)]$$

$$\kappa(n) = \arctan(\Phi)\,,$$

$$\rho n \Rightarrow \Phi = \frac{1}{4} C_{WF}^2 \psi_s^2 \phi$$

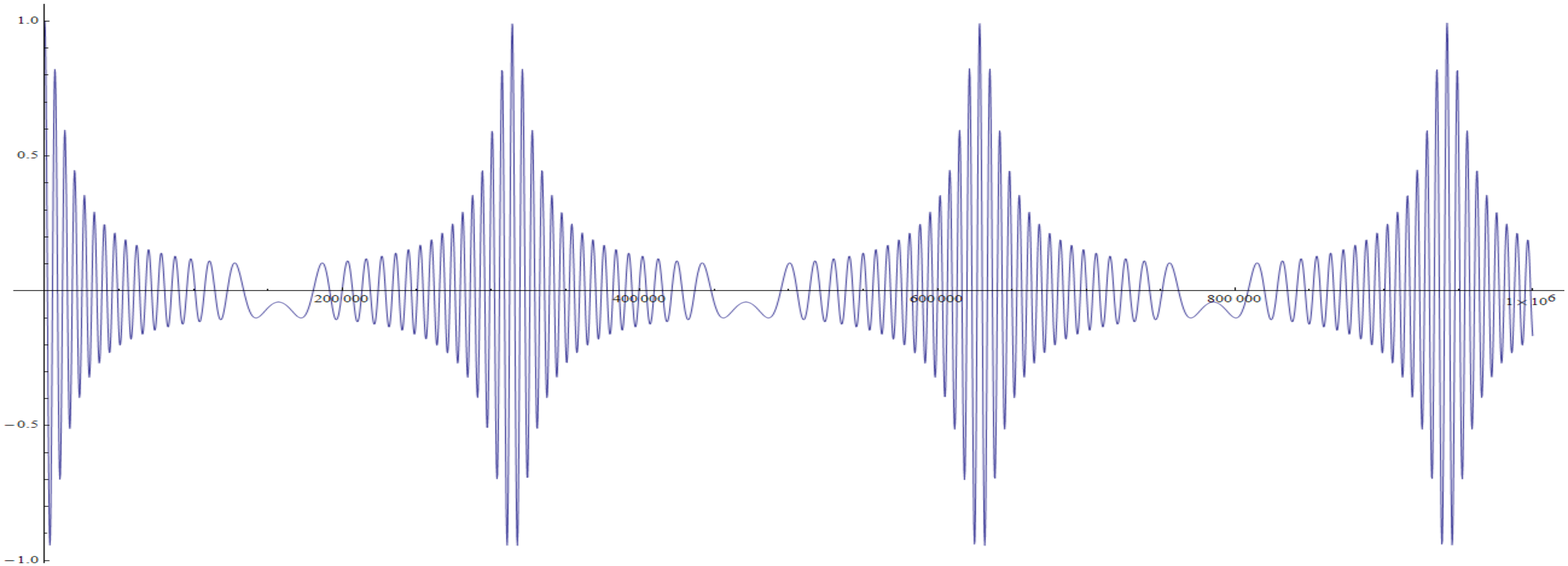
- A **spin echo:** at $\phi = \Phi = 0$, i.e., with the period ,

$$n = \frac{\pi}{\delta_{WF}}$$

spin decoherence and phase walk vanish!

- Similar spin echo in the in-plane polarization (formulas are too lengthy)

Spin echo in vertical polarization under detuning



- Strong detuning for the sake of illustration of the phenomenon
- Variable driven oscillation frequency ~ cos ϕ
- Higher harmonics in the difference of spin and RF frequencies at work

Spin in curved space-time and gravity induced false EDM effects

- New interest inspired by misleading e-prints by T. Morishima et al. PTEP (2018) no.6, 063B07 and references therein
- Promptly refuted by several authors. Good summary in arXiv:1805.01944 [hep-ph] by J. P. Miller and B. Lee Roberts
- My principal task: historical overview and vindication of early results by A. Silenko and O. Teryaev, Phys. Rev. D71 (2005) 064016; Phys.Rev. D76 (2007) 061101;

  Y. Orlov Y, E. Flanagan E and Y. Semertzidis. Phys.Lett. A376 (2012) 2822

Spin in curved space-time and gravity induced false EDM effects

The Earth as a laboratory: storage rings rests on the terrestial surface.

No real need in full machinery of General Relativity: weak field approximation is OK: it suffices to know the free fall acceleration $\vec{g}$

Two principal effects:

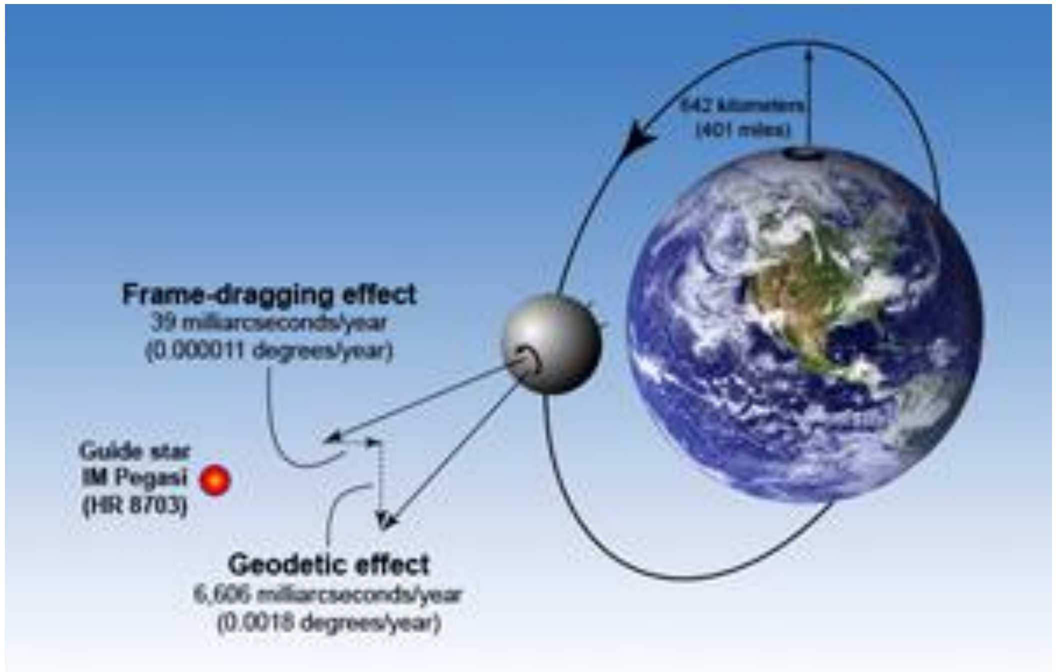
- The spin-orbit interaction in the Earth gravitational field (the de Sitter-Fokker effect, aka the geodetic effect (1916, 1921))
- Focusing EM fields are imperative to impose the closed paricle orbit in a storage ring compensating for the particle weight: first derivation by Silenko & Teryaev (2005) for magnetic case
- The both effects have similar structure and both produce false EDM signal in frozen spin pure electric ring

- No explicit separation of the two in Orlov et al. (2012)

The spin-orbit interaction

Has been tested experimentally by Gravity Probe B

C.W.F Everitt et al. Phys.Rev.Lett. 106 (2011) 221101



	Measured	Predicted
Geodetic precession (mas)	6602 ± 18	6606
Frame-dragging (mas)	37.2 ± 7.2	39.2

De Siiter in relativistic case

The relativistic extension of the spin-orbit interaction result: .

- I.B. Khriplovich, A.A. Pomeransky, J.Exp.Theor.Phys. 86 (1998) 839-849
- A.A. Pomeransky, R.A. Senkov, I.B. Khriplovich, Phys.Usp. 43 (2000) 1055-1066

The precession frequency equals

$$\vec{\Omega}_{LS} = -\frac{2\gamma+1}{\gamma+1}[\vec{v}\times\vec{g}]$$

As $\vec{g}$ is normal to the storage ring plane, $\vec{\Omega}_{LS}$ describes spin precession around the radial axis.

The spin is not quite a classical object. It is usuful to study the Dirac eqn. in a static gravitational field invoking the Foldy-Wouthhuysen representation

Khriplovich-Pomeransky result is fully confirmed (Obukhov, Silenko, Teryyaev (2005,2016))

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The precession frequency equals

$$\vec{\Omega}_{LS} = -\frac{2\gamma + 1}{\gamma + 1}[\vec{v} \times \vec{g}]$$

As $\vec{g}$ is normal to the storage ring plane, $\vec{\Omega}_{LS}$ describes gravity induced spin precession around **the radial axis --- already a false EDM effect!**

The spin is not quite a classical object. Study the Dirac eqn. in a static gravitational field invoking the Foldy-Wouthhuysen representation.

Khriplovich-Pomeransky result is fully confirmed (Obukhov, Silenko, Teryyaev (2005,2016))

Closed orbit in a storage ring

Gravity force

$$\vec{F}_g = \frac{2\gamma^2 - 1}{\gamma} m\vec{g}$$

displaces the orbit w.r.t. the electromagnetic equilibrium one.

- Never has been of any concern to accelerator builders
- Compensation by radial focusing magnetic field (Silenko, Teryaev (2005))

$$\vec{B}_r = \frac{2\gamma^2 - 1}{\gamma v^2} \cdot \frac{m}{e} [\vec{v} \times \vec{g}]$$

- Compensation by vertical focusing electric field (Obukhov et al. (2016), can be digged out also from GR juggling by Orlov et al (2012))

$$\vec{E}_y = -\frac{2\gamma^2 - 1}{\gamma} \cdot \frac{m}{e} \vec{g}$$

In the commoving frame amounts to he motional radial magnetic field $\propto [\vec{v} \times \vec{g}]$ --- false EDM effect

False EDM from gravity

Gravity acts as imperfection radial magnetic field.

- Absolute evil in an all electric EDM ring - false EDM signal

- Obukhov et al. (2016))

$$\vec{\Omega}_{gE} = \frac{1 - G(2\gamma^2 - 1)}{\gamma c^2}[\vec{v} \times \vec{g}]$$

- Upon the frozen spin constraint $v^2 = \frac{1}{1+G}$

$$\vec{\Omega}_{gE} = \frac{g\sqrt{G}}{c}\vec{e}_r$$

- First derived by Orlov et al. (2012) by brute force solution of GR equations without explicit separation of the spin-orbit and focusing effects.

- Similar derivation by Laszlo et al. arXiv: 1803.01395 [gr-qc], Wedn., A11, 17:55

- Orlov et al (2012): gravity effects can be cancelled out with counterrotating beams

Magic ring for deuterons

New result for $G < 0$: frozen spin with crossed E- and B-fields

- Pure magnetic field (Silenko, Teryaev (2005)

$$\vec{\Omega}_{gM} = -\frac{1}{\gamma v^2}\{1 + G(2\gamma^2 - 1)\}[\vec{v} \times \vec{g}]$$

- Frozen spin condition in the E × B ring

$$[\vec{v} \times \vec{B}_y] = \frac{1}{G}\{1 - v^2(1 + G)\}\vec{E}_r$$

- Focusing forces are propto a dispacement from the EM equilibrium orbit

$$\kappa = \frac{vB_r}{E_y} \approx const$$

Depends on the ring design

- Frequency of gravity induced false EDM signal

$$\vec{\Omega}_g = -\frac{1}{1 + \kappa}(\vec{\Omega}_{gE} + \kappa\vec{\Omega}_{gM})$$

Summary:

The srEDM and JEDI = z> new exptl results and theoretical work ideas on spin dynamics in storage rings (record precision in spin tune, record spin coherence time, spin tune mapping, nonexponential spin decoherence, spin echo…)

COSY@Juelich was, is and will remain a unique facility exploring the frontiers of spin dynamics

Systematic backgrounds from ring imperfection effects are and will remain the major concern: only the first scratch of all-magnetic case

Still Terra Incognita for all-electric rings despite first forays

Griavity as an example of unexpected imperfection in all-electric rings: false EDM effects from gravity (first discovered in **1916** !), Orlov-Flanagan-Semertzidis 2012 result is fully vindicated, not an issue for pure electric magic rings

Future: CPEDM (JEDI+CERN) Collaboration in the formative stage

Damping of driven oscillations

- One-particle resonance strength $\epsilon(\xi) = \epsilon_0 J_0(C_{WF}\psi_s\xi)$

- Spread of resonance strengths --→ decoherence of polarization of an ensemble of particles

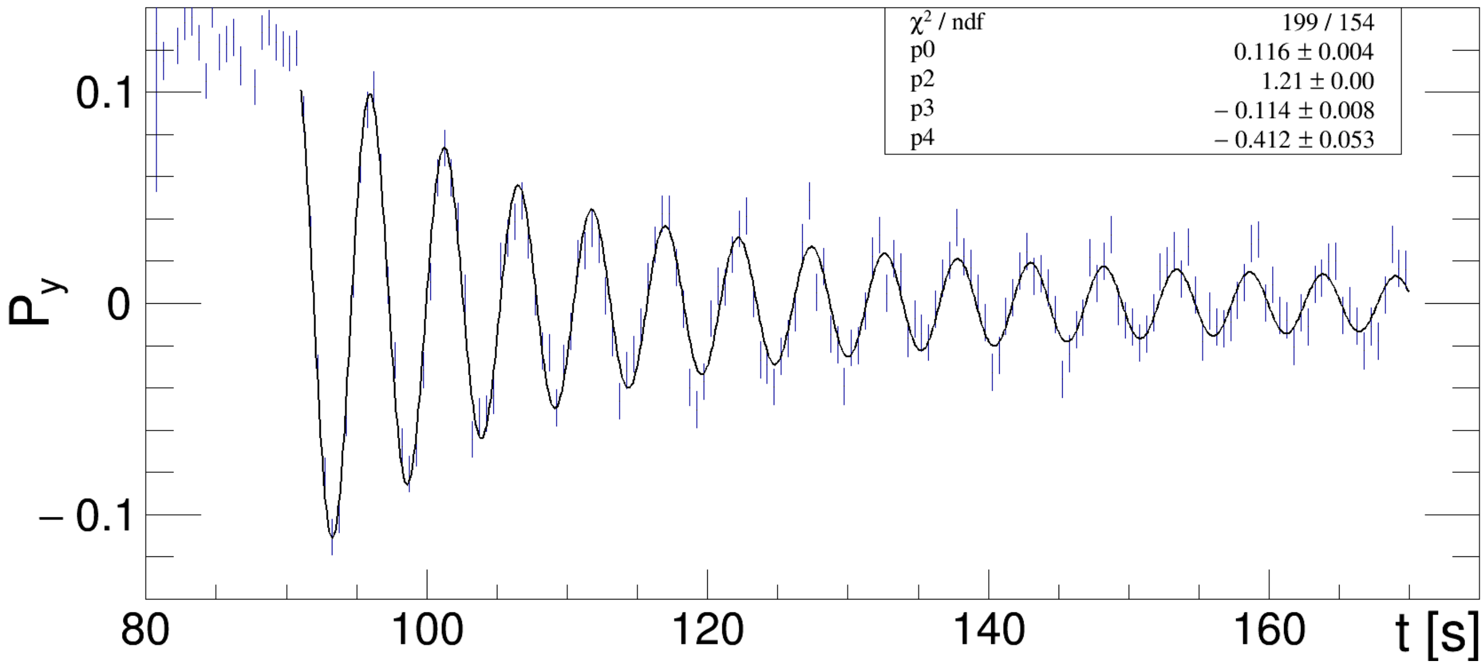
$$
\begin{aligned}
S_y &= \Re\langle \exp[-in\epsilon(\xi)]\rangle_\xi \\
&= \Re\left\langle \exp\left\{-in\epsilon_0\left[1 - \frac{1}{4}C_{rf}^2\psi_s^2\xi^2\right]\right\}\right\rangle_\xi \\
&= \frac{1}{\sqrt{1+\rho^2 n^2}}\cos[\epsilon_0 n - \kappa(n)]\,,
\end{aligned}
$$

- Damping parameter $\rho = \frac{1}{4}\epsilon_0 C_{WF}^2\psi_s^2$

- Phase walk $\kappa(n) = \arctan(\rho n)$

Damping of driven oscillations

- An example of damping of oscillations driven by , RF Wien (JEDI, November 2017, very preliminary):



- p0-initial amplitude of oscillation
- p2-oscillation frequency(*2π)
- p3-parameter of damping
- p4 -normalization for running phase function

JEDI: RF Wien-Filter-based first direct measurement of EDM at COSY

- In pure magnetic storage ring, T-BMT eq.:
- $\frac{d\vec{S}}{dt} = \vec{\Omega} \times \vec{S}(t) = -\frac{q}{m}\left(G\vec{B} + \frac{1}{2}\eta(\vec{E} + \vec{\beta} \times \vec{B})\right) \times \vec{S}(t)$
- EDM effect in the stable spin axis: $\vec{c} = \vec{e}_x \sin \xi_{\mathrm{EDM}} + \vec{e}_y \cos \xi_{\mathrm{EDM}}$

$$\tan \xi_{\mathrm{EDM}} = \frac{\eta\beta}{2G}$$

- EDM signal is a tilt of stable spin axis in- or outwards the ring
- Measure EDM-induced tilt by spin resonance with radiofrequency Wien filter