Mitglied der Helmholtz-Gemeinschaft

JÜLICH FORSCHUNGSZENTRUM

Lectures on the Theory of hadronic electric-dipole moments

Lake Bazaleti, Georgia | 26 – 28 September 2013 | Andreas Wirzba

Outline:

Outline:

Outline:

1. Motivation: Matter–Antimatter Asymmetry in the Universe
2. The Permanent Electric Dipole Moment and its Features
3. CP-Violating Sources *in* the Standard Model (SM)
4. CP-Violating Sources *Beyond* the Standard Model (BSM)
5. Electric Dipole Moments (EDMs) of the Nucleon
6. Electric Dipole Moments of the Deuteron and Helium-3
7. Conclusions and Outlook

Outline:

Outline:

Outline:

Outline:

1. Motivation: Matter–Antimatter Asymmetry in the Universe
2. The Permanent Electric Dipole Moment and its Features
3. CP-Violating Sources *in* the Standard Model (SM)
4. CP-Violating Sources *Beyond* the Standard Model (BSM)
5. Electric Dipole Moments (EDMs) of the Nucleon
6. Electric Dipole Moments of the Deuteron and Helium-3
7. Conclusions and Outlook

Some questions that will hopefully be answered:

1. Why is $\cancel{CP}$ beyond the Standard Model expected?
2. How can a point-particle (*e.g.* an electron) support an EDM?
3. Why don't the EDMs of certain molecules predict a strong $\cancel{CP}$?
4. What is the natural scale of a neutron EDM?
5. How large is the EDM window for *New Physics* searches?
6. How can the EDM-producing sources be discriminated?
7. Why is low-energy Effective Field Theory needed here?

Some questions that will hopefully be answered:

1. Why is $\cancel{CP}$ beyond the Standard Model expected?
2. How can a point-particle (*e.g.* an electron) support an EDM?
3. Why don't the EDMs of certain molecules predict a strong $\cancel{CP}$?
4. What is the natural scale of a neutron EDM?
5. How large is the EDM window for *New Physics* searches?
6. How can the EDM-producing sources be discriminated?
7. Why is low-energy Effective Field Theory needed here?

Some questions that will hopefully be answered:

1. Why is $\not{CP}$ beyond the Standard Model expected?
2. How can a point-particle (*e.g.* an electron) support an EDM?
3. Why don't the EDMs of certain molecules predict a strong $\not{CP}$?
4. What is the natural scale of a neutron EDM?
5. How large is the EDM window for *New Physics* searches?
6. How can the EDM-producing sources be discriminated?
7. Why is low-energy Effective Field Theory needed here?

Some questions that will hopefully be answered:

1. Why is $\cancel{CP}$ beyond the Standard Model expected?
2. How can a point-particle (*e.g.* an electron) support an EDM?
3. Why don't the EDMs of certain molecules predict a strong $\cancel{CP}$?
4. What is the natural scale of a neutron EDM?
5. How large is the EDM window for *New Physics* searches?
6. How can the EDM-producing sources be discriminated?
7. Why is low-energy Effective Field Theory needed here?

Some questions that will hopefully be answered:

1. Why is $\cancel{CP}$ beyond the Standard Model expected?
2. How can a point-particle (*e.g.* an electron) support an EDM?
3. Why don't the EDMs of certain molecules predict a strong $\cancel{CP}$?
4. What is the natural scale of a neutron EDM?
5. How large is the EDM window for *New Physics* searches?
6. How can the EDM-producing sources be discriminated?
7. Why is low-energy Effective Field Theory needed here?

Some questions that will hopefully be answered:

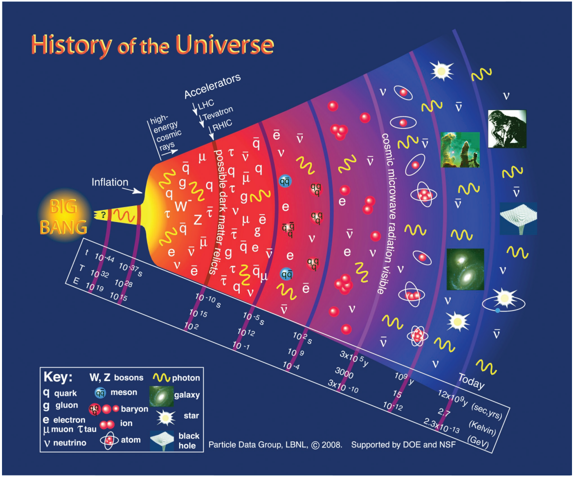
1. Why is $\cancel{CP}$ beyond the Standard Model expected?
2. How can a point-particle (*e.g.* an electron) support an EDM?
3. Why don't the EDMs of certain molecules predict a strong $\cancel{CP}$?
4. What is the natural scale of a neutron EDM?
5. How large is the EDM window for *New Physics* searches?
6. How can the EDM-producing sources be discriminated?
7. Why is low-energy Effective Field Theory needed here?

Some questions that will hopefully be answered:

1. Why is $\cancel{CP}$ beyond the Standard Model expected?
2. How can a point-particle (*e.g.* an electron) support an EDM?
3. Why don't the EDMs of certain molecules predict a strong $\cancel{CP}$?
4. What is the natural scale of a neutron EDM?
5. How large is the EDM window for *New Physics* searches?
6. How can the EDM-producing sources be discriminated?
7. Why is low-energy Effective Field Theory needed here?

Motivation: Matter Excess in the Universe

Baryogenesis, Big Bang Nucleosynthesis & Cosmic Microwave Background



1 End of inflation:

$n_B = n_{\bar{B}}$

2 Big Bang Nucleosynth.

$t_{BBN} \sim 1$ min

3 Cosmic Microwaves

$t_{CMB} \sim 3 \times 10^5$ y:

? SM(s) *prediction:*

$(n_B - n_{\bar{B}})/n_\gamma|_{CMB} \sim 10^{-18}$

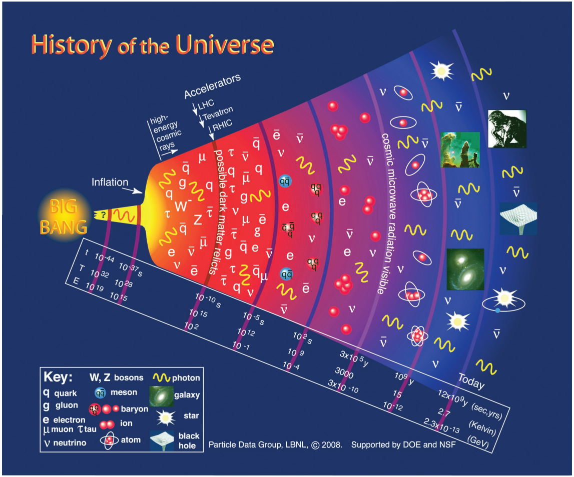
! WMAP+COBE (2003) *observation*:

$n_B/n_\gamma|_{CMB} = (6.1 \pm 0.3) 10^{-10}$

What is missing?

Motivation: Matter Excess in the Universe

Baryogenesis, Big Bang Nucleosynthesis & Cosmic Microwave Background



1 End of inflation:
$n_B = n_{\bar{B}}$

2 Big Bang Nucleosynth.
$t_{BBN} \sim 1$ min

3 Cosmic Microwaves
$t_{CMB} \sim 3 \times 10^5$ y:

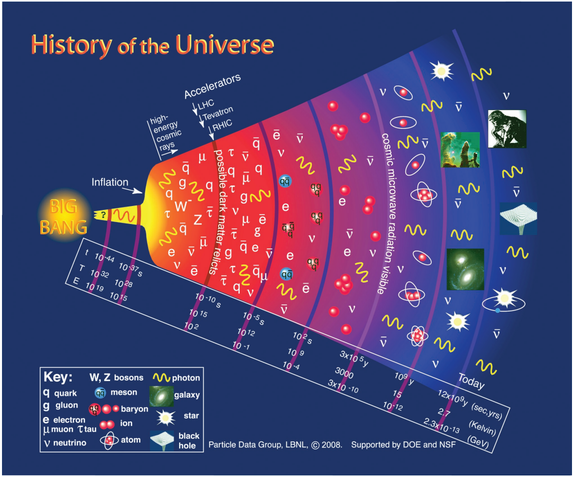
? SM(s) *prediction*:
$(n_B - n_{\bar{B}})/n_\gamma|_{CMB} \sim 10^{-18}$

! WMAP+COBE (2003) *observation*:
$n_B/n_\gamma|_{CMB} = (6.1 \pm 0.3) 10^{-10}$

What is missing?

Motivation: Matter Excess in the Universe

Baryogenesis, Big Bang Nucleosynthesis & Cosmic Microwave Background



1 End of inflation:
$n_B = n_{\bar{B}}$

2 Big Bang Nucleosynth.
$t_{BBN} \sim 1$ min

3 Cosmic Microwaves
$t_{CMB} \sim 3 \times 10^5$ y:

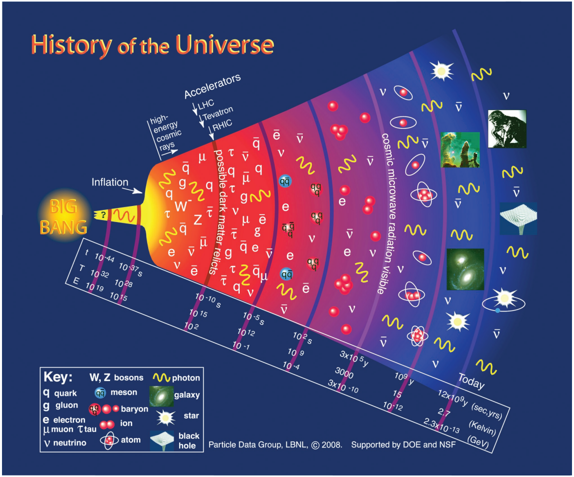
? SM(s) *prediction*:
$(n_B - n_{\bar{B}})/n_\gamma|_{CMB} \sim 10^{-18}$

! WMAP+COBE (2003) *observation*:
$n_B/n_\gamma|_{CMB} = (6.1 \pm 0.3) 10^{-10}$

What is missing?

Motivation: Matter Excess in the Universe

Baryogenesis, Big Bang Nucleosynthesis & Cosmic Microwave Background



1. End of inflation:
   $n_B = n_{\bar{B}}$

2. Big Bang Nucleosynth.
   $t_{BBN} \sim 1$ min

3. Cosmic Microwaves
   $t_{CMB} \sim 3 \times 10^5$ y:

? SM(s) *prediction:*
$(n_B - n_{\bar{B}})/n_\gamma|_{CMB} \sim 10^{-18}$

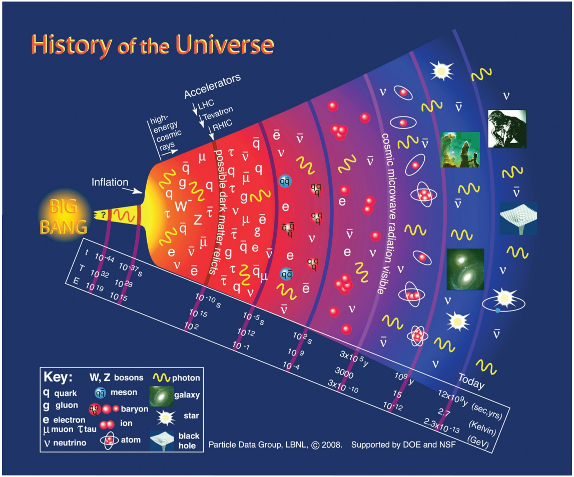
! WMAP+COBE (2003) *observation*:
$n_B/n_\gamma|_{CMB} = (6.1 \pm 0.3) 10^{-10}$

What is missing?

FORSCHUNGSZENTRUM JÜLICH

Motivation: Matter Excess in the Universe

Baryogenesis, Big Bang Nucleosynthesis & Cosmic Microwave Background



1. End of inflation:
   $n_B = n_{\bar{B}}$
2. Big Bang Nucleosynth.
   $t_{BBN} \sim 1$ min
3. Cosmic Microwaves
   $t_{CMB} \sim 3 \times 10^5$ y:

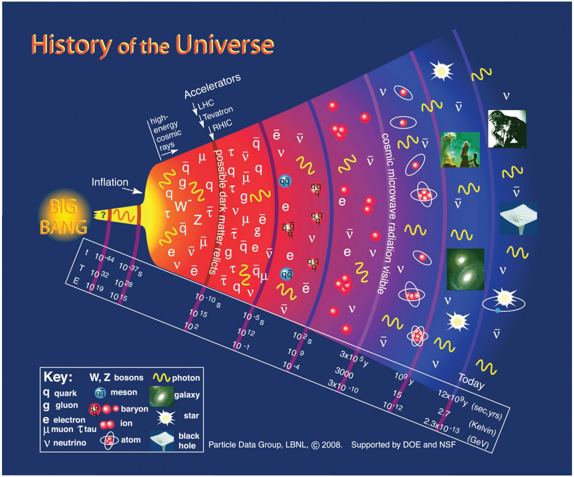
? SM(s) *prediction:*
$(n_B - n_{\bar{B}})/n_\gamma|_{CMB} \sim 10^{-18}$

! WMAP+COBE (2003) *observation*:
$n_B/n_\gamma|_{CMB} = (6.1 \pm 0.3) 10^{-10}$

What is missing?

Motivation: Matter Excess in the Universe

Baryogenesis, Big Bang Nucleosynthesis & Cosmic Microwave Background



1. End of inflation:
   $n_B = n_{\bar{B}}$

2. Big Bang Nucleosynth.
   $t_{BBN} \sim 1$ min

3. Cosmic Microwaves
   $t_{CMB} \sim 3 \times 10^5$ y:

? SM(s) *prediction:*
$(n_B - n_{\bar{B}})/n_\gamma|_{CMB} \sim 10^{-18}$

! WMAP+COBE (2003) *observation*:
$n_B/n_\gamma|_{CMB} = (6.1 \pm 0.3) 10^{-10}$

What is missing?

Motivation: Baryon Asymmetry in the Universe

Nature has probably violated CP when generating the Baryon asymmetry !?

Observed*:
$(n_B - n_{\bar{B}}) / n_\gamma = 6 \times 10^{-10}$

SM expectation:
$(n_B - n_{\bar{B}}) / n_\gamma \sim 10^{-18}$

Sakharov 1967:
B-violation
C & CP-violation
non-equilibrium
[JETP Lett. 5 (1967) 24]

* WMAP + COBE, 2003
$n_B / n_\gamma = (6.1 \pm^{0.3}_{0.2}) \times 10^{-10}$

$(6.19 \pm 0.15) \times 10^{-10}$
[E. Komatsu et al. 2011 ApJS 192]

(adapted from Klaus Kirch (PSI), Fermilab, Feb. 13, 2013)

Dynamical generation of net baryon number

requires the concurrence of three (sufficient) conditions:

Sakharov Conditions JETP Lett. 5 (1967) 24

1. baryon number B violation .... to depart from initial $B = 0$
2. C and CP violation .... to distinguish B and $\bar{B}$ production rates
3. violation of thermal equilibrium ... to escape $\langle B \rangle = 0$ if CPT holds

- Investigation of ~~CP~~: possible window to *physics beyond SM*
- Complementary processes:
  - high-energy collider experiments (new particles, EWSB, ...)

    $\longleftrightarrow$

  - high-precision low-energy experiments
    (flavor-neutral EDMs, $\mu \to e\gamma$ search, $(g-2)_\mu$, ...)

Dynamical generation of net baryon number

requires the concurrence of three (sufficient) conditions:

Sakharov Conditions

JETP Lett. 5 (1967) 24

1. baryon number B violation .... to depart from initial $B = 0$
2. C and CP violation ...... to distinguish B and $\bar{B}$ production rates
3. violation of thermal equilibrium ... to escape $\langle B \rangle$=0 if CPT holds

- Investigation of ~~CP~~: possible window to *physics beyond SM*
- Complementary processes:
  - high-energy collider experiments (new particles, EWSB, ...)

    $\longleftrightarrow$
  - high-precision low-energy experiments
    (flavor-neutral EDMs, $\mu \to e\gamma$ search, $(g-2)_\mu$, ...)

Dynamical generation of net baryon number

requires the concurrence of three (sufficient) conditions:

Sakharov Conditions

JETP Lett. 5 (1967) 24

1. baryon number B violation . . . . to depart from initial $B = 0$
2. C and CP violation . . . . to distinguish B and $\bar{B}$ production rates
3. violation of thermal equilibrium . . . to escape $\langle B \rangle = 0$ if CPT holds

- Investigation of $\cancel{CP}$: possible window to *physics beyond SM*
- Complementary processes:
  - high-energy collider experiments (new particles, EWSB, . . .)

    $\longleftrightarrow$

  - high-precision low-energy experiments

    (flavor-neutral EDMs, $\mu \to e\gamma$ search, $(g-2)_\mu$, . . .)

Dynamical generation of net baryon number

requires the concurrence of three (sufficient) conditions:

Sakharov Conditions

JETP Lett. 5 (1967) 24

1. baryon number B violation . . . . to depart from initial $B = 0$
2. C and CP violation . . . . . . to distinguish B and $\bar{B}$ production rates
3. violation of thermal equilibrium . . . to escape $\langle B \rangle = 0$ if CPT holds

- Investigation of $\cancel{CP}$: possible window to *physics beyond SM*
- Complementary processes:
  - high-energy collider experiments (new particles, EWSB, . . .)

    $\longleftrightarrow$

  - high-precision low-energy experiments
    (flavor-neutral EDMs, $\mu \to e\gamma$ search, $(g-2)_\mu$, . . .)

Dynamical generation of net baryon number

requires the concurrence of three (sufficient) conditions:

Sakharov Conditions

JETP Lett. 5 (1967) 24

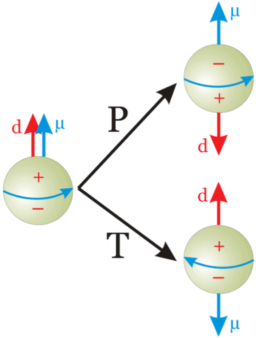
1. baryon number B violation . . . . to depart from initial $B = 0$
2. C and CP violation . . . . . . to distinguish B and $\bar{B}$ production rates
3. violation of thermal equilibrium . . . to escape $\langle B \rangle$=0 if CPT holds

- Investigation of ~~CP~~: possible window to *physics beyond SM*
- Complementary processes:
  - high-energy collider experiments (new particles, EWSB, . . .)

    $\longleftrightarrow$

  - high-precision low-energy experiments
    (flavor-neutral EDMs, $\mu \to e\gamma$ search, $(g-2)_\mu$, . . .)

The Electric Dipole Moment (EDM)



EDM: $\vec{d} = \sum_i \vec{r}_i\, e_i \xrightarrow[\text{particles}]{\text{subatomic}} d \cdot \vec{S}/|\vec{S}|$

(polar) (axial)

$$\mathcal{H} = -\mu \frac{\vec{S}}{S} \cdot \vec{B} - d \frac{\vec{S}}{S} \cdot \vec{E}$$

$$\mathrm{P}: \quad \mathcal{H} = -\mu \frac{\vec{S}}{S} \cdot \vec{B} + d \frac{\vec{S}}{S} \cdot \vec{E}$$

$$\mathrm{T}: \quad \mathcal{H} = -\mu \frac{\vec{S}}{S} \cdot \vec{B} + d \frac{\vec{S}}{S} \cdot \vec{E}$$

Any non-vanishing EDM of some subatomic particle violates P & T

- Assuming CPT to hold, CP is violated as well
  $\hookrightarrow$ subatomic EDMs: "rear window" to CP violation in early universe
- Strongly suppressed in SM (CKM-matrix): $d_n \sim 10^{-31} e\,\mathrm{cm}$, $d_e \sim 10^{-38} e\,\mathrm{cm}$
- Current bounds: $d_n < 3 \cdot 10^{-26} e\,\mathrm{cm}$, $d_p < 8 \cdot 10^{-25} e\,\mathrm{cm}$, $d_e < 1 \cdot 10^{-27} e\,\mathrm{cm}$

n: Baker et al. (2006), *p* prediction: Dimitriev & Sen'kov (2003)*, *e*: Baron et al. (2013)†

* input from ^{199}Hg atom EDM measurement of Griffith et al. (2009)

† input from polar THO molecule measurement of the ACME@Harvard coll. (2013)

A *naive* estimate of the scale of the nucleon EDM

Khriplovich & Lamoreaux (1997); Kolya Nikolaev (2012)

- CP & P conserving magnetic moment ~ nuclear magneton μ_N

$$\mu_N = \frac{e}{2m_p} \sim 10^{-14}\, e\,\mathrm{cm}\,.$$

- A nonzero EDM requires

  parity P violation: the price to pay is $\sim 10^{-7}$

  $(\,G_F \cdot m_\pi^2 \sim 10^{-7}$ with $G_F \approx 1.166 \cdot 10^{-5}\mathrm{GeV}^{-2}\,)$,

  and CP violation: the price to pay is $\sim 10^{-3}$

  $(\,|\eta_{+-}| \equiv |\mathcal{A}(K_L^0 \to \pi^+\pi^-)|\,/\,|\mathcal{A}(K_S^0 \to \pi^+\pi^-)| = (2.232 \pm 0.011) \cdot 10^{-3}\,)$.

- In summary: $\boxed{d_N \sim 10^{-7} \times 10^{-3} \times \mu_N \sim 10^{-24}\, e\,\mathrm{cm}}$

- In SM (without θ term): extra $m_\pi^2 G_F$ factor to undo flavor change

  $\hookrightarrow \quad \boxed{d_N^{\mathrm{SM}} \sim 10^{-7} \times 10^{-24}\, e\,\mathrm{cm} \sim 10^{-31}\, e\,\mathrm{cm}}$

$\hookrightarrow$ *The empirical window* for search of physics BSM(θ=0) is

$$\boxed{10^{-24}\, e\,\mathrm{cm} > d_N > 10^{-30}\, e\,\mathrm{cm}.}$$

A *naive* estimate of the scale of the nucleon EDM

Khriplovich & Lamoreaux (1997); Kolya Nikolaev (2012)

- CP & P conserving magnetic moment ~ nuclear magneton μ_N

$$\mu_N = \frac{e}{2m_p} \sim 10^{-14}\, e\,\mathrm{cm}\,.$$

- A nonzero EDM requires

  parity P violation: the price to pay is $\sim 10^{-7}$

  ($G_F \cdot m_\pi^2 \sim 10^{-7}$ with $G_F \approx 1.166 \cdot 10^{-5}\mathrm{GeV}^{-2}$),

  and CP violation: the price to pay is $\sim 10^{-3}$

  ($|\eta_{+-}| \equiv |\mathcal{A}(K_L^0 \to \pi^+\pi^-)| / |\mathcal{A}(K_S^0 \to \pi^+\pi^-)| = (2.232 \pm 0.011) \cdot 10^{-3}$).

- In summary: $\boxed{d_N \sim 10^{-7} \times 10^{-3} \times \mu_N \sim 10^{-24}\, e\,\mathrm{cm}}$

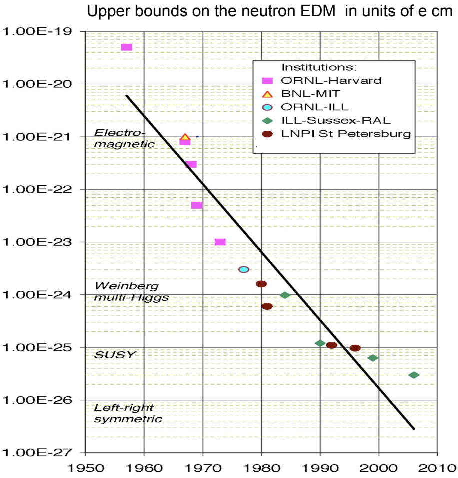
- In SM (without θ term): extra $m_\pi^2 G_F$ factor to undo flavor change

$$\hookrightarrow \quad \boxed{d_N^{\mathrm{SM}} \sim 10^{-7} \times 10^{-24}\, e\,\mathrm{cm} \sim 10^{-31}\, e\,\mathrm{cm}}$$

$\hookrightarrow$ *The empirical window* for search of physics BSM(θ=0) is

$$\boxed{10^{-24}\, e\,\mathrm{cm} > d_N > 10^{-30}\, e\,\mathrm{cm}.}$$

Chronology of upper bounds on the neutron EDM



Smith, Purcell, Ramsey (1957) ………………… Baker et al. (2006)

$\hookrightarrow$ 5 to 6 orders above SM predictions which are out of reach !

Theorem: Permanent EDMs of *non*-selfconjugate* particles *with* spin $j \neq 0$

Let $\langle j^{\mathrm{P}} | \vec{d} \, | j^{\mathrm{P}} \rangle = d \, \langle j^{\mathrm{P}} | \vec{J} \, | j^{\mathrm{P}} \rangle$ with $\vec{d} = \int \vec{r} \rho(\vec{r}) \mathrm{d}^3 r$ an EDM operator in a stationary state $|j^{\mathrm{P}}\rangle$ of definite parity P and nonzero spin j, such that

$$\vec{d} \to \mp \vec{d} \quad \& \quad \vec{J} \to \pm \vec{J} \qquad \text{under} \quad \begin{cases} \text{space reflection,} \\ \text{time reversal.} \end{cases}$$

If $d \neq 0$ and $|j^{\mathrm{P}}\rangle$ has *no* degeneracy (besides rotational), *then* $\cancel{\mathrm{P}}$ & $\cancel{\mathrm{T}}$.

* *non*-selfconjugate particle is *not* its own antiparticle ⇒ at least one “charge" *non*-zero

Theorem: Permanent EDMs of *non*-selfconjugate* particles *with* spin $j \neq 0$

Let $\langle j^{\mathrm{P}} | \vec{d} \, | j^{\mathrm{P}} \rangle = d \, \langle j^{\mathrm{P}} | \vec{J} \, | j^{\mathrm{P}} \rangle$ with $\vec{d} = \int \vec{r} \rho(\vec{r}) \mathrm{d}^3 r$ an EDM operator in a stationary state $|j^{\mathrm{P}}\rangle$ of definite parity P and nonzero spin j, such that

$$\vec{d} \to \mp \vec{d} \quad \& \quad \vec{J} \to \pm \vec{J} \qquad \text{under} \quad \begin{cases} \text{space reflection,} \\ \text{time reversal.} \end{cases}$$

$\boxed{\textit{If } d \neq 0 \text{ and } |j^{\mathrm{P}}\rangle \text{ has } \textit{no} \text{ degeneracy (besides rotational), } \textit{then } \cancel{P} \,\&\, \cancel{T}.}$

* *non*-selfconjugate particle is *not* its own antiparticle ⇒ at least one “charge" *non*-zero

State $|j^{\mathrm{P}}\rangle$ can be *‘elementary’* particle (quark, charged lepton, $W^{\pm}$ boson, Dirac neutrino, ...)

Theorem: Permanent EDMs of *non*-selfconjugate* particles *with* spin $j\neq 0$

Let $\langle j^{\mathrm{P}}|\vec{d}\,|j^{\mathrm{P}}\rangle = d\,\langle j^{\mathrm{P}}|\vec{J}\,|j^{\mathrm{P}}\rangle$ with $\vec{d} = \int \vec{r}\rho(\vec{r})\mathrm{d}^3 r$ an EDM operator in a stationary state $|j^{\mathrm{P}}\rangle$ of definite parity P and nonzero spin j, such that

$$\vec{d} \to \mp\vec{d} \quad \& \quad \vec{J} \to \pm\vec{J} \qquad \text{under} \quad \begin{cases} \text{space reflection,} \\ \text{time reversal.} \end{cases}$$

$\boxed{\textit{If } d\neq 0 \text{ and } |j^{\mathrm{P}}\rangle \text{ has } \textit{no} \text{ degeneracy (besides rotational), } \textit{then } \cancel{P}\ \&\ \cancel{T}.}$

* *non*-selfconjugate particle is *not* its own antiparticle ⇒ at least one “charge" *non*-zero

State $|j^{\mathrm{P}}\rangle$ can be *‘elementary’* particle (quark, charged lepton, $W^{\pm}$ boson, Dirac neutrino, ...)

‘Isn’t an elementary particle a point-particle without structure?
How can such a particle be polarized and support an EDM?’

Theorem: Permanent EDMs of *non*-selfconjugate* particles *with* spin $j\neq 0$

Let $\langle j^{\mathrm{P}}|\vec{d}\,|j^{\mathrm{P}}\rangle = d\,\langle j^{\mathrm{P}}|\vec{J}\,|j^{\mathrm{P}}\rangle$ with $\vec{d} = \int \vec{r}\rho(\vec{r})\mathrm{d}^3r$ an EDM operator in a stationary state $|j^{\mathrm{P}}\rangle$ of definite parity P and nonzero spin j, such that

$$\vec{d} \to \mp\vec{d} \quad \& \quad \vec{J} \to \pm\vec{J} \qquad \text{under} \quad \begin{cases} \text{space reflection,} \\ \text{time reversal.} \end{cases}$$

If $d\neq 0$ and $|j^{\mathrm{P}}\rangle$ has *no* degeneracy (besides rotational), *then* $\cancel{\mathrm{P}}$ & $\cancel{T}$.

* *non*-selfconjugate particle is *not* its own antiparticle $\Rightarrow$ at least one “charge" *non*-zero

State $|j^{\mathrm{P}}\rangle$ can be *‘elementary’* particle (quark, charged lepton, $W^{\pm}$ boson, Dirac neutrino, ...)

‘Isn’t an elementary particle a point-particle without structure?
How can such a particle be polarized and support an EDM?’

There are always vacuum polarizations with rich short-distance structure

($g-2$ of the electron and muon aren’t exactly zero either)

Theorem: Permanent EDMs of *non*-selfconjugate* particles *with* spin $j \neq 0$

Let $\langle j^{\rm P} | \vec{d} \, | j^{\rm P} \rangle = d \, \langle j^{\rm P} | \vec{J} \, | j^{\rm P} \rangle$ with $\vec{d} = \int \vec{r} \rho(\vec{r}) \mathrm{d}^3 r$ an EDM operator in a stationary state $|j^{\rm P}\rangle$ of definite parity P and nonzero spin j, such that

$$\vec{d} \to \mp \vec{d} \quad \& \quad \vec{J} \to \pm \vec{J} \qquad \text{under} \quad \begin{cases} \text{space reflection,} \\ \text{time reversal.} \end{cases}$$

$\boxed{\textit{If } d \neq 0 \text{ and } |j^{\rm P}\rangle \text{ has } \textit{no} \text{ degeneracy (besides rotational), } \textit{then } \cancel{P} \,\&\, \cancel{T}.}$

* *non*-selfconjugate particle is *not* its own antiparticle ⇒ at least one “charge" *non*-zero

State $|j^{\rm P}\rangle$ can be *‘elementary’* particle (quark, charged lepton, $W^{\pm}$ boson, Dirac neutrino, ...) or a *‘composite’* neutron, proton, nucleus, atom, molecule.

Theorem: Permanent EDMs of *non*-selfconjugate* particles *with* spin $j \neq 0$

Let $\langle j^{\mathrm{P}} | \vec{d} | j^{\mathrm{P}} \rangle = d \, \langle j^{\mathrm{P}} | \vec{J} | j^{\mathrm{P}} \rangle$ with $\vec{d} = \int \vec{r} \rho(\vec{r}) \mathrm{d}^3 r$ an EDM operator in a stationary state $|j^{\mathrm{P}}\rangle$ of definite parity P and nonzero spin j, such that

$$\vec{d} \to \mp \vec{d} \quad \& \quad \vec{J} \to \pm \vec{J} \qquad \text{under} \quad \begin{cases} \text{space reflection,} \\ \text{time reversal.} \end{cases}$$

If $d \neq 0$ and $|j^{\mathrm{P}}\rangle$ has *no* degeneracy (besides rotational), *then* $\cancel{P}$ & $\cancel{T}$.

* *non*-selfconjugate particle is *not* its own antiparticle $\Rightarrow$ at least one “charge" *non*-zero

State $|j^{\mathrm{P}}\rangle$ can be *‘elementary’* particle (quark, charged lepton, $W^{\pm}$ boson, Dirac neutrino, ...) or a *‘composite’* neutron, proton, nucleus, atom, molecule.

‘What about the huge EDMs of H_2O or NH_3 molecules?’

Theorem: Permanent EDMs of *non*-selfconjugate* particles *with* spin $j \neq 0$

Let $\langle j^{\mathrm{P}} | \vec{d} \, | j^{\mathrm{P}} \rangle = d \, \langle j^{\mathrm{P}} | \vec{J} \, | j^{\mathrm{P}} \rangle$ with $\vec{d} = \int \vec{r} \rho(\vec{r}) \mathrm{d}^3 r$ an EDM operator in a stationary state $|j^{\mathrm{P}}\rangle$ of definite parity P and nonzero spin j, such that

$$\vec{d} \to \mp \vec{d} \quad \& \quad \vec{J} \to \pm \vec{J} \qquad \text{under} \quad \begin{cases} \text{space reflection,} \\ \text{time reversal.} \end{cases}$$

If $d \neq 0$ and $|j^{\mathrm{P}}\rangle$ has *no* degeneracy (besides rotational), *then* $\cancel{P}$ & $\cancel{T}$.

* *non*-selfconjugate particle is *not* its own antiparticle $\Rightarrow$ at least one "charge" *non*-zero

State $|j^{\mathrm{P}}\rangle$ can be *'elementary'* particle (quark, charged lepton, $W^{\pm}$ boson, Dirac neutrino, ...) or a *'composite'* neutron, proton, nucleus, atom, molecule.

'What about the huge EDMs of H_2O or NH_3 molecules?'

The ground states of these molecules at non-zero temperatures or strong E-fields are mixtures of at least 2 opposite parity states:

The theorem doesn't apply for degenerate states: neither $\cancel{T}$ nor $\cancel{P}$!

Theorem: Permanent EDMs of *non*-selfconjugate* particles *with* spin $j \neq 0$

Let $\langle j^{\mathrm{P}} | \vec{d} \, | j^{\mathrm{P}} \rangle = d \, \langle j^{\mathrm{P}} | \vec{J} \, | j^{\mathrm{P}} \rangle$ with $\vec{d} = \int \vec{r} \rho(\vec{r}) \mathrm{d}^3 r$ an EDM operator in a stationary state $|j^{\mathrm{P}}\rangle$ of definite parity P and nonzero spin j, such that

$$\vec{d} \to \mp \vec{d} \quad \& \quad \vec{J} \to \pm \vec{J} \qquad \text{under} \quad \begin{cases} \text{space reflection,} \\ \text{time reversal.} \end{cases}$$

$\boxed{\textit{If } d \neq 0 \text{ and } |j^{\mathrm{P}}\rangle \text{ has } \textit{no} \text{ degeneracy (besides rotational), } \textit{then } \cancel{\mathrm{P}} \ \& \ \cancel{\mathrm{T}}.}$

* *non*-selfconjugate particle is *not* its own antiparticle ⇒ at least one "charge" *non*-zero

State $|j^{\mathrm{P}}\rangle$ can be *'elementary'* particle (quark, charged lepton, $W^{\pm}$ boson, Dirac neutrino, ...) or a *'composite'* neutron, proton, nucleus, atom, molecule.

'But what about the induced EDM (polarization)?'

Theorem: Permanent EDMs of *non*-selfconjugate* particles *with* spin $j \neq 0$

Let $\langle j^{\rm P} | \vec{d} | j^{\rm P} \rangle = d \, \langle j^{\rm P} | \vec{J} | j^{\rm P} \rangle$ with $\vec{d} = \int \vec{r} \rho(\vec{r}) {\rm d}^3 r$ an EDM operator in a stationary state $|j^{\rm P}\rangle$ of definite parity P and nonzero spin j, such that

$$\vec{d} \to \mp \vec{d} \quad \& \quad \vec{J} \to \pm \vec{J} \qquad \text{under} \quad \begin{cases} \text{space reflection,} \\ \text{time reversal.} \end{cases}$$

$$\boxed{\textit{If } d \neq 0 \text{ and } |j^{\rm P}\rangle \text{ has } \textit{no} \text{ degeneracy (besides rotational), } \textit{then } \cancel{P} \;\&\; \cancel{T}.}$$

* *non*-selfconjugate particle is *not* its own antiparticle ⇒ at least one “charge" *non*-zero

State $|j^{\rm P}\rangle$ can be *‘elementary’* particle (quark, charged lepton, $W^\pm$ boson, Dirac neutrino, ...) or a *‘composite’* neutron, proton, nucleus, atom, molecule.

‘But what about the induced EDM (polarization)?’

The induced EDM is *quadratic* in the electric field and *not* $\cancel{P}$ or $\cancel{T}$

induced EDM	$\longleftrightarrow$	quadratic Stark effect ($\propto E^2$)
permanent EDM	$\longleftrightarrow$	linear Stark effect ($\propto E$)

Theorem: Permanent EDMs of *non*-selfconjugate* particles *with* spin $j \neq 0$

Let $\langle j^{\mathrm{P}} | \vec{d} | j^{\mathrm{P}} \rangle = d \, \langle j^{\mathrm{P}} | \vec{J} | j^{\mathrm{P}} \rangle$ with $\vec{d} = \int \vec{r} \rho(\vec{r}) \mathrm{d}^3 r$ an EDM operator in a stationary state $|j^{\mathrm{P}}\rangle$ of definite parity P and nonzero spin j, such that

$$\vec{d} \to \mp \vec{d} \quad \& \quad \vec{J} \to \pm \vec{J} \qquad \text{under} \quad \begin{cases} \text{space reflection,} \\ \text{time reversal.} \end{cases}$$

If $d \neq 0$ and $|j^{\mathrm{P}}\rangle$ has *no* degeneracy (besides rotational), *then* $\cancel{P}$ & $\cancel{T}$.

* *non*-selfconjugate particle is *not* its own antiparticle ⇒ at least one "charge" *non*-zero

State $|j^{\mathrm{P}}\rangle$ can be *'elementary'* particle (quark, charged lepton, $W^{\pm}$ boson, Dirac neutrino, ...) or a *'composite'* neutron, proton, nucleus, atom, molecule.

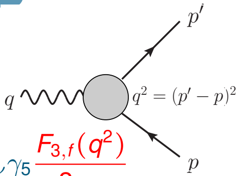
If the interactions are described by an action which is

local, Lorentz-invariant, and hermitian

then CPT invariance holds: thus $\boxed{\cancel{T} \iff \cancel{CP}}$

Permanent EDMs and Form Factors

Here $s = \frac{1}{2}$ fermions (f= quark, lepton, nucleon)



- $\langle f(p')|J^\mu_{\rm em}|f(p)\rangle = \bar{u}_f(p')\,\Gamma^\mu(q^2)\,u_f(p)$

$$\Gamma^\mu_f(q^2) = \gamma^\mu F_{1,f}(q^2) + \mathrm{i}\sigma^{\mu\nu} q_\nu \frac{F_{2,f}(q^2)}{2m_f} + \sigma^{\mu\nu} q_\nu \gamma_5 \frac{F_{3,f}(q^2)}{2m_f} + (\not{q}\, q^\mu - q^2\gamma^\mu)\gamma_5 F_{a,f}(q^2)/m_f^2$$

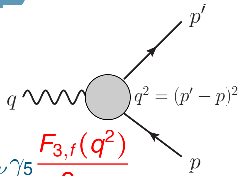
(Dirac $F_1(q^2)$, Pauli $F_2(q^2)$, electric dipole $F_3(q^2)$, and anapole $F_a(q^2)$ FFs)

- Quark, lepton or nucleon EDM $d_f := F_{3,f}(q^2 \to 0)/(2m_f)$

$$\mathcal{H}_{\rm eff} = \mathrm{i}\frac{d_f}{2}\,\bar{f}\sigma^{\mu\nu}\gamma_5 f\, F_{\mu\nu} \xrightarrow{\text{non-rel.}} -d_f\,\langle \boldsymbol{\sigma}\rangle \cdot \mathbf{E} \longrightarrow$$ ▸ *linear* Stark effect

Permanent EDMs and Form Factors

Here $s = \frac{1}{2}$ fermions (f= quark, lepton, nucleon)



- $\langle f(p')|J^\mu_{\rm em}|f(p)\rangle = \bar u_f(p')\,\Gamma^\mu(q^2)\,u_f(p)$

$$\Gamma^\mu_f(q^2) = \gamma^\mu F_{1,f}(q^2) + \mathrm{i}\sigma^{\mu\nu}q_\nu \frac{F_{2,f}(q^2)}{2m_f} + \sigma^{\mu\nu}q_\nu\gamma_5\frac{F_{3,f}(q^2)}{2m_f} + (\not{q}\,q^\mu - q^2\gamma^\mu)\gamma_5 F_{a,f}(q^2)/m_f^2$$

(Dirac $F_1(q^2)$, Pauli $F_2(q^2)$, electric dipole $F_3(q^2)$, and anapole $F_a(q^2)$ FFs)

- Quark, lepton or nucleon EDM $d_f := F_{3,f}(q^2\to 0)/(2m_f)$

$$\mathcal{H}_{\rm eff} = \mathrm{i}\frac{d_f}{2}\,\bar f\sigma^{\mu\nu}\gamma_5 f\,F_{\mu\nu} \xrightarrow{\text{non-rel.}} -d_f\,\langle\boldsymbol{\sigma}\rangle\cdot\mathbf{E} \longrightarrow$$ ▸ *linear* Stark effect

- Likewise chromo quark EDM with ~~CP~~ gluon-quark-quark vertex:

$$\mathrm{i}\frac{d_{cq}}{2}\,\bar q\sigma^{\mu\nu}\gamma_5 T^a G^a_{\mu\nu} q$$

or weak dipole moment (WDM) with Z-boson f-f vertex: $\mathrm{i}\frac{d_f^Z}{2}\,\bar f\sigma^{\mu\nu}\gamma_5 f\,Z_{\mu\nu}$.

Generic features of EDM, chromo EDM or WDM

$$\mathcal{L}_{\rm EDM} = -\mathrm{i}\frac{d_f}{2}\,\bar{f}\sigma_{\mu\nu}\gamma_5 f\,F^{\mu\nu} = -\mathrm{i}\frac{d_f}{2}\,\bar{f}_{\rm L}\,\sigma_{\mu\nu} f_{\rm R}\,F^{\mu\nu} + \mathrm{i}\frac{d_f}{2}\,\bar{f}_{\rm R}\,\sigma_{\mu\nu} f_{\rm L}\,F^{\mu\nu}$$

1. Sum of the mass dimension of these fields: $\frac{3}{2} + \frac{3}{2} + 2 = 5$,
   $\hookrightarrow$ $\dim(d_f) = e \times \text{length} = e \times \text{mass}^{-1}$ (such that $\int d^4x\,\mathcal{L} \sim \text{mass}^0$)
   $\hookrightarrow$ non-renormalizable *effective* interaction

2. For any non-zero EDM (or WDM), $\cancel{CP}$ is flavor *diagonal*!

   Note that $\cancel{CP}$ in SM model (via CKM matrix) is flavor *changing*.
   $\hookrightarrow$ extra $\sim 10^{-7}$ factor multiplies naive estimate $d_n \simeq 10^{-24}\,e\,\mathrm{cm}$.

3. *Chirality* in $\mathcal{L}_{\rm EDM}$ flipped: $\frac{1}{2}(\mathbb{1}-\gamma_5)f = f_{\rm L} \leftrightarrow f_{\rm R} = \frac{1}{2}(\mathbb{1}+\gamma_5)f$

   $\Rightarrow$ fermion mass m_f insertion (e.g. via Higgs mechanism) needed:

   $$d_f \propto m_f^n\,, \qquad n = 1, 2, 3 \qquad \text{(depending on the model of } \cancel{CP}\text{)}$$

   $\hookrightarrow$ $\cancel{CP}$ beyond SM: $\mathcal{L}_{\rm BSM}^{\cancel{CP}} = \cancel{\frac{1}{M_{\rm T\,viol}}\mathcal{L}_{\dim 5}} + \frac{1}{M^2_{\rm T\,viol}}\mathcal{L}_{\dim 6} + \ldots$

CP violation in the Standard Model

The conventional source: Kobayashi-Maskawa mechanism

Empirical facts: 3 generations of u/d quarks (& e/ν leptons)

- $\boxed{u:}$ $m_u < m_c < m_t$, $\boxed{d:}$ $m_d < m_s < m_b$, and $\boxed{l:}$ $m_e < m_\mu < m_\tau$
- quarks & leptons in mass basis $\neq$ quarks & leptons in weak-int. basis
- $\mathcal{L}_{\rm SM} = \mathcal{L}_{\rm gauge} + \mathcal{L}_{\rm gauge\text{-}fermion} + \mathcal{L}_{\rm gauge\text{-}Higgs} + \mathcal{L}_{\rm Higgs\text{-}fermion}$ is CP inv.,
  - with the exception of the θ term of QCD (see later)

    and the charged-weak-current interaction ($\subset \mathcal{L}_{\rm gauge\text{-}fermion}$)

$$\mathcal{L}_{\rm c\text{-}w\text{-}c} = -\frac{g_w}{\sqrt{2}} \sum_{ij=1}^{3} \bar{d}_{Li}\gamma^\mu V_{ij}\, u_{Lj}\, W_\mu^- - \frac{g_w}{\sqrt{2}} \sum_{ij=1}^{3} \bar{\ell}_{Li}\gamma^\mu U_{ij}\, \nu_{Lj}\, W_\mu^- + \text{h.c.}$$

  - V: 3×3 unitary quark-mixing matrix (Cabibbo-Kobayashi-Maskawa m.)

    U: 3×3 unitary lepton-mixing matrix (Maki-Nakagawa-Sakata matrix)

3 angles + 1 $\cancel{CP}$ phase δ_{KM}

3 angles +1(3) $\cancel{CP}$ phase(s) for Dirac (Majorana) ν_i's

$\cancel{CP}$ and EDMs and in the SM with $J_{\mathbf{KM}} = \mathbf{Im}(V_{tb}V_{td}^* V_{cd}V_{cb}^*) \simeq 3 \cdot 10^{-5}$

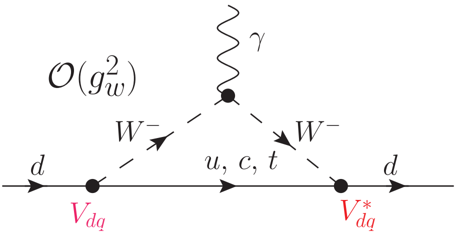
$$\propto \left(\frac{m_t^2-m_c^2}{M_{EW}^2}\right)\left(\frac{m_c^2-m_u^2}{M_{EW}^2}\right)\left(\frac{m_t^2-m_u^2}{M_{EW}^2}\right)\cdot\left(\frac{m_b^2-m_s^2}{M_{EW}^2}\right)\left(\frac{m_s^2-m_d^2}{M_{EW}^2}\right)\left(\frac{m_b^2-m_d^2}{M_{EW}^2}\right)\cdot J_{\mathrm{KM}} \simeq 10^{-15} J_{\mathrm{KM}},$$

Jarlskog (1985)

$\hookrightarrow$ $\boxed{(n_B - n_{\bar{B}})/n_\gamma|^{\mathrm{SM}}_{T\sim 20\mathrm{MeV}} \sim 10^{-20} \text{ and } d_n^{\mathrm{SM}} \sim 10^{-20} \cdot 10^{-14}\, e\,\mathrm{cm} \sim 10^{-34}\, e\,\mathrm{cm}}$

EDM flavor-neutral $\Rightarrow$ KM predictions tiny: $\mathcal{O}(G_F^2) \sim \mathcal{O}(g_W^4)$

1 loop:



$\hookrightarrow$ $\cancel{CP}$ phase δ_{KM} cancels $\rightarrow$ prefactor real $\Rightarrow$ $\boxed{d_q^{\text{1-loop}} = 0}$

$\cancel{CP}$ and EDMs and in the SM with $J_{\mathrm{KM}} = \mathrm{Im}(V_{tb} V_{td}^{*} V_{cd} V_{cb}^{*}) \simeq 3 \cdot 10^{-5}$

$$\propto \left(\frac{m_t^2 - m_c^2}{M_{EW}^2}\right)\left(\frac{m_c^2 - m_u^2}{M_{EW}^2}\right)\left(\frac{m_t^2 - m_u^2}{M_{EW}^2}\right) \cdot \left(\frac{m_b^2 - m_s^2}{M_{EW}^2}\right)\left(\frac{m_s^2 - m_d^2}{M_{EW}^2}\right)\left(\frac{m_b^2 - m_d^2}{M_{EW}^2}\right) \cdot J_{\mathrm{KM}} \simeq 10^{-15} J_{\mathrm{KM}},$$

Jarlskog (1985)

$$\hookrightarrow \boxed{(n_B - n_{\bar{B}})/n_\gamma\Big|^{\mathrm{SM}}_{T\sim 20\,\mathrm{MeV}} \sim 10^{-20} \text{ and } d_n^{\mathrm{SM}} \sim 10^{-20} \cdot 10^{-14}\, e\,\mathrm{cm} \sim 10^{-34}\, e\,\mathrm{cm}}$$

EDM flavor-neutral $\Rightarrow$ KM predictions tiny: $\mathcal{O}(G_F^2) \sim \mathcal{O}(g_W^4)$

2 loops:

$$\boxed{d_{\text{quark}}^{\text{2-loop}} = d_{\text{chromo q}}^{\text{2-loop}} = 0}$$

Shabalin (1978)

$\cancel{CP}$ and EDMs and in the SM with $J_{\mathrm{KM}} = \mathrm{Im}(V_{tb} V_{td}^* V_{cd} V_{cb}^*) \simeq 3 \cdot 10^{-5}$

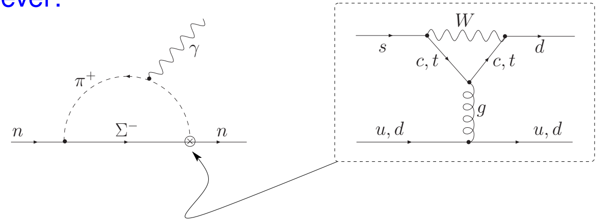
$$\propto \left(\frac{m_t^2 - m_c^2}{M_{EW}^2}\right)\left(\frac{m_c^2 - m_u^2}{M_{EW}^2}\right)\left(\frac{m_t^2 - m_u^2}{M_{EW}^2}\right)\cdot\left(\frac{m_b^2 - m_s^2}{M_{EW}^2}\right)\left(\frac{m_s^2 - m_d^2}{M_{EW}^2}\right)\left(\frac{m_b^2 - m_d^2}{M_{EW}^2}\right)\cdot J_{\mathrm{KM}} \simeq 10^{-15} J_{\mathrm{KM}},$$

Jarlskog (1985)

$$\hookrightarrow \boxed{(n_B - n_{\bar{B}})/n_\gamma\big|^{\mathrm{SM}}_{T\sim 20\mathrm{MeV}} \sim 10^{-20} \text{ and } d_n^{\mathrm{SM}} \sim 10^{-20} \cdot 10^{-14}\, e\,\mathrm{cm} \sim 10^{-34}\, e\,\mathrm{cm}}$$

EDM flavor-neutral $\Rightarrow$ KM predictions tiny: $\mathcal{O}(G_F^2) \sim \mathcal{O}(g_W^4)$

however:



$\mathcal{O}(g_W^4 g_s^2)$

$d_n^{\mathrm{KM}} \simeq 10^{-32}\, e\,\mathrm{cm}$ because of long-range pion & ‘strong penguin’

Gavela; Khriplovich & Zhitnitsky (’82)

$\cancel{CP}$ and EDMs and in the SM with $J_{\rm KM} = \mathbf{Im}(V_{tb}V_{td}^*V_{cd}V_{cb}^*) \simeq 3\cdot 10^{-5}$

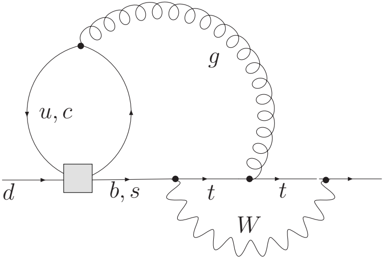
$$\propto \left(\frac{m_t^2-m_c^2}{M_{EW}^2}\right)\left(\frac{m_c^2-m_u^2}{M_{EW}^2}\right)\left(\frac{m_t^2-m_u^2}{M_{EW}^2}\right)\cdot\left(\frac{m_b^2-m_s^2}{M_{EW}^2}\right)\left(\frac{m_s^2-m_d^2}{M_{EW}^2}\right)\left(\frac{m_b^2-m_d^2}{M_{EW}^2}\right)\cdot J_{\rm KM} \simeq 10^{-15} J_{\rm KM},$$

Jarlskog (1985)

$$\hookrightarrow \boxed{(n_B-n_{\bar B})/n_\gamma|^{\rm SM}_{T\sim 20{\rm MeV}} \sim 10^{-20} \text{ and } d_n^{\rm SM} \sim 10^{-20}\cdot 10^{-14}\,e\,{\rm cm} \sim 10^{-34}\,e\,{\rm cm}}$$

EDM flavor-neutral $\Rightarrow$ KM predictions tiny: $\mathcal{O}(G_F^2) \sim \mathcal{O}(g_W^4)$

at $\geq$ 3 loops:



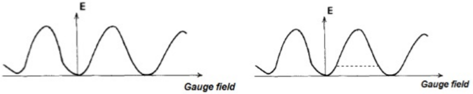
$\mathcal{O}(g_W^4 g_s^2)$

$d_n^{\rm KM} \simeq 10^{-34}\ldots10^{-31}\,e\,{\rm cm}$ $\quad$ $(d_e^{\rm KM} \sim 10^{-38}\ldots10^{-40}\,e\,{\rm cm}$ since 4 loops & $\mathcal{O}(g_W^6))$

Khriplovich (1986); Czarnecki & Krause ('97) (Khriplovich & Pospelov (1992))

EDMs in the SM: unconventional θ-term mechanism

The topologically non-trivial vacuum structure of QCD



induces a direct $\cancel{P}\&\cancel{T} \rightsquigarrow \cancel{CP}$ interaction with a new parameter θ:

$$\mathcal{L}_{QCD} = \mathcal{L}_{QCD}^{CP} + \theta \frac{g_s^2}{64\pi^2} \epsilon^{\mu\nu\rho\sigma} G^a_{\mu\nu} G^a_{\rho\sigma} \qquad (\text{note: } \epsilon^{0123} = -\epsilon_{0123})$$

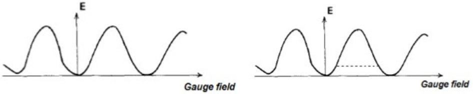
- Anomalous $U_A(1)$ quark-rotations induce mixing with 'mass' term

$$\theta \frac{g_s^2}{64\pi^2} \epsilon^{\mu\nu\rho\sigma} G^a_{\mu\nu} G^a_{\rho\sigma} \xrightarrow{U_A(1)} -\bar{\theta} m_q^* \sum_f \bar{q}_f i\gamma_5 q_f \qquad (m_q^* = \frac{m_u m_d}{m_u+m_d} \text{ reduced mass})$$

$\hookrightarrow$ *unknown* coupling constant is actually $\boxed{\bar{\theta} = \theta + \arg\det \mathcal{M}_{\text{quark}}}$

EDMs in the SM: unconventional θ-term mechanism

The topologically non-trivial vacuum structure of QCD



induces a direct $\cancel{P}$&$\cancel{T}$ $\rightsquigarrow$ $\cancel{CP}$ interaction with a new parameter θ:

$$\mathcal{L}_{QCD} = \mathcal{L}_{QCD}^{CP} + \theta \frac{g_s^2}{64\pi^2} \epsilon^{\mu\nu\rho\sigma} G^a_{\mu\nu} G^a_{\rho\sigma} \qquad (\text{note: } \epsilon^{0123} = -\epsilon_{0123})$$

- Anomalous $U_A(1)$ quark-rotations induce mixing with 'mass' term

$$\theta \frac{g_s^2}{64\pi^2} \epsilon^{\mu\nu\rho\sigma} G^a_{\mu\nu} G^a_{\rho\sigma} \xrightarrow{U_A(1)} -\bar{\theta} m_q^* \sum_f \bar{q}_f i\gamma_5 q_f \qquad (m_q^* = \frac{m_u m_d}{m_u + m_d} \text{ reduced mass})$$

$\hookrightarrow$ *unknown* coupling constant is actually $\boxed{\bar{\theta} = \theta + \arg\det \mathcal{M}_{\text{quark}}}$

- Naive Dimensional Analysis (NDA) estimate of $\bar{\theta}$-induced n EDM:

$$d_n^{\bar{\theta}} \sim \bar{\theta} \cdot \frac{m_q^*}{m_n} \cdot \frac{e}{2m_n} \sim \bar{\theta} \cdot 10^{-2} \cdot 10^{-14}\, e\,\text{cm} \sim \bar{\theta} \cdot 10^{-16}\, e\,\text{cm} \quad \text{with } \bar{\theta} \sim \mathcal{O}(1).$$

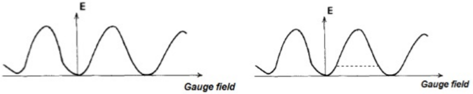
$$d_n^{emp} < 2.9 \cdot 10^{-26}\, e\,\text{cm} \rightsquigarrow \boxed{|\bar{\theta}| < 10^{-10}}$$

strong CP problem

(Baker et al. ('06))

EDMs in the SM: unconventional θ-term mechanism

The topologically non-trivial vacuum structure of QCD



induces a direct $\cancel{P}\&\cancel{T} \rightsquigarrow \cancel{CP}$ interaction with a new parameter θ:

$$\mathcal{L}_{QCD} = \mathcal{L}_{QCD}^{CP} + \theta \frac{g_s^2}{64\pi^2} \epsilon^{\mu\nu\rho\sigma} G^a_{\mu\nu} G^a_{\rho\sigma} \qquad (\text{note: } \epsilon^{0123} = -\epsilon_{0123})$$

- Anomalous $U_A(1)$ quark-rotations induce mixing with 'mass' term

$$\theta \frac{g_s^2}{64\pi^2} \epsilon^{\mu\nu\rho\sigma} G^a_{\mu\nu} G^a_{\rho\sigma} \xrightarrow{U_A(1)} -\bar{\theta} m_q^* \sum_f \bar{q}_f i\gamma_5 q_f \qquad (m_q^* = \frac{m_u m_d}{m_u + m_d} \text{ reduced mass})$$

  $\hookrightarrow$ *unknown* coupling constant is actually $\boxed{\bar{\theta} = \theta + \arg\det \mathcal{M}_{\text{quark}}}$

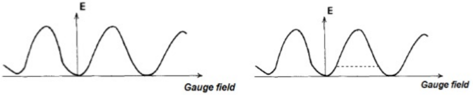
- Naive Dimensional Analysis (NDA) estimate of $\bar{\theta}$-induced n EDM:

$$d_n^{\bar{\theta}} \sim \bar{\theta} \cdot \frac{m_q^*}{m_n} \cdot \frac{e}{2m_n} \sim \bar{\theta} \cdot 10^{-2} \cdot 10^{-14}\, e\,\text{cm} \sim \bar{\theta} \cdot 10^{-16}\, e\,\text{cm} \quad \text{with } \bar{\theta} \sim \mathcal{O}(1).$$

If also $\cancel{P} \sim 10^{-7}$ were included, the bound on n EDM effectively adds *only* 3 orders of magnitude to further constrain $\bar{\theta}$ (?)

EDMs in the SM: unconventional θ-term mechanism

The topologically non-trivial vacuum structure of QCD



induces a direct $\cancel{P}\&\cancel{T} \rightsquigarrow \cancel{CP}$ interaction with a new parameter θ:

$$\mathcal{L}_{QCD} = \mathcal{L}^{CP}_{QCD} + \theta \frac{g_s^2}{64\pi^2}\epsilon^{\mu\nu\rho\sigma} G^a_{\mu\nu} G^a_{\rho\sigma} \qquad (\text{note: } \epsilon^{0123} = -\epsilon_{0123})$$

- Anomalous $U_A(1)$ quark-rotations induce mixing with 'mass' term

$$\theta \frac{g_s^2}{64\pi^2}\epsilon^{\mu\nu\rho\sigma} G^a_{\mu\nu} G^a_{\rho\sigma} \xrightarrow{U_A(1)} -\bar\theta m_q^* \sum_f \bar q_f i\gamma_5 q_f \qquad (m_q^* = \frac{m_u m_d}{m_u+m_d} \text{ reduced mass})$$

$\hookrightarrow$ *unknown* coupling constant is actually $\boxed{\bar\theta = \theta + \arg\det\mathcal{M}_{\text{quark}}}$

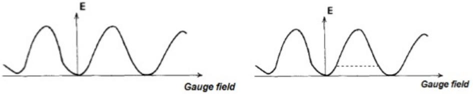
- Naive Dimensional Analysis (NDA) estimate of $\bar\theta$-induced n EDM:

$$d_n^{\bar\theta} \sim \bar\theta \cdot \frac{m_q^*}{m_n} \cdot \frac{e}{2m_n} \sim \bar\theta \cdot 10^{-2} \cdot 10^{-14}\, e\,\text{cm} \sim \bar\theta \cdot 10^{-16}\, e\,\text{cm} \quad \text{with } \bar\theta \sim \mathcal{O}(1).$$

$10^{-10} > |\bar\theta| > 10^{-14}$ eventually measurable via nonzero EDM, but because of $\Lambda_{EWSB} \gg \Lambda_{\chi SB}$ it doesn't explain the cosmic matter surplus.

EDMs in the SM: unconventional θ-term mechanism

The topologically non-trivial vacuum structure of QCD



▸ induces a direct $\not{P}$&$\not{T}$ $\rightsquigarrow$ $\not{CP}$ interaction with a new parameter θ:

$$\mathcal{L}_{QCD} = \mathcal{L}_{QCD}^{CP} + \theta \frac{g_s^2}{64\pi^2} \epsilon^{\mu\nu\rho\sigma} G^a_{\mu\nu} G^a_{\rho\sigma} \qquad (\text{note: } \epsilon^{0123} = -\epsilon_{0123})$$

- ▸ Anomalous $U_A(1)$ quark-rotations induce mixing with 'mass' term

$$\theta \frac{g_s^2}{64\pi^2} \epsilon^{\mu\nu\rho\sigma} G^a_{\mu\nu} G^a_{\rho\sigma} \xrightarrow{U_A(1)} -\bar{\theta} m_q^* \sum_f \bar{q}_f i\gamma_5 q_f \qquad (m_q^* = \frac{m_u m_d}{m_u + m_d} \text{ reduced mass})$$

  $\hookrightarrow$ *unknown* coupling constant is actually $\boxed{\bar{\theta} = \theta + \arg\det \mathcal{M}_{\text{quark}}}$

- Naive Dimensional Analysis (NDA) estimate of $\bar{\theta}$-induced n EDM:

$$d_n^{\bar{\theta}} \sim \bar{\theta} \cdot \frac{m_q^*}{m_n} \cdot \frac{e}{2m_n} \sim \bar{\theta} \cdot 10^{-2} \cdot 10^{-14}\, e\,\text{cm} \sim \bar{\theta} \cdot 10^{-16}\, e\,\text{cm} \quad \text{with } \bar{\theta} \sim \mathcal{O}(1).$$

Thus $\not{CP}$ by new physics (NP) (*i.e.* dimension ≥ 6 sources beyond SM) needed to explain the cosmic matter-antimatter asymmetry.

How to handle CP-violating sources beyond SM?

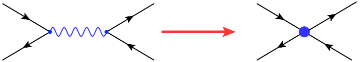
i.e. New Physics (NP) as e.g. SUSY, multi-Higgs, Left-Right-Symmetric Models?

Two-step procedure:

1. Pick NP models (or classes of models) at a scale $\Lambda_{NP} > m_{\text{Higgs}}$
   - integrate out BSM fields in running down to Higgs scale
   - integrate out Higgs boson by running down to EW scale
   - integrate out $Z, W^{\pm}$, heavy quarks by running down to 1 GeV scale

   $\rightsquigarrow$ dimension-6 EDM operators with specified weights formulated in terms of quarks and gluons

2. Transcribe these EDM operators into hadronic language using:
   - (non)-relativistic quark models
     e.g. $d_n = \frac{4}{3}d_d - \frac{1}{3}d_u$ and $d_p = \frac{4}{3}d_u - \frac{1}{3}d_d$ (QM)
   - QCD sum rules
   - Lattice QCD calculations
   - Effective Field Theory (EFT)



How to handle CP-violating sources beyond SM?

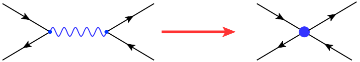
i.e. New Physics (NP) as e.g. SUSY, multi-Higgs, Left-Right-Symmetric Models?

Two-step procedure:

1. Pick NP models (or classes of models) at a scale $\Lambda_{NP} > m_{\text{Higgs}}$
   - integrate out BSM fields in running down to Higgs scale
   - integrate out Higgs boson by running down to EW scale
   - integrate out $Z, W^{\pm}$, heavy quarks by running down to 1 GeV scale

   $\rightsquigarrow$ dimension-6 EDM operators with specified weights formulated in terms of quarks and gluons

2. Transcribe these EDM operators into hadronic language using:
   - (non)-relativistic quark models
     e.g. $d_n = \frac{4}{3}d_d - \frac{1}{3}d_u$ and $d_p = \frac{4}{3}d_u - \frac{1}{3}d_d$ ▸ (QM)
   - QCD sum rules
   - Lattice QCD calculations
   - Effective Field Theory ▸ (EFT)



How to handle CP-violating sources beyond SM?

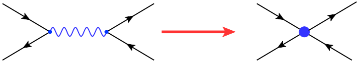
i.e. New Physics (NP) as e.g. SUSY, multi-Higgs, Left-Right-Symmetric Models?

Two-step procedure:

1. Pick NP models (or classes of models) at a scale $\Lambda_{NP} > m_{\text{Higgs}}$
   - integrate out BSM fields in running down to Higgs scale
   - integrate out Higgs boson by running down to EW scale
   - integrate out $Z, W^{\pm}$, heavy quarks by running down to 1 GeV scale

   $\rightsquigarrow$ dimension-6 EDM operators with specified weights formulated in terms of quarks and gluons

2. Transcribe these EDM operators into hadronic language using:
   - (non)-relativistic quark models
     e.g. $d_n = \frac{4}{3}d_d - \frac{1}{3}d_u$ and $d_p = \frac{4}{3}d_u - \frac{1}{3}d_d$ ▸ (QM)
   - QCD sum rules
   - Lattice QCD calculations
   - Effective Field Theory ▸ (EFT)



How to handle CP-violating sources beyond SM?

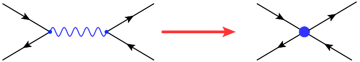
i.e. New Physics (NP) as e.g. SUSY, multi-Higgs, Left-Right-Symmetric Models?

Two-step procedure:

1. Pick NP models (or classes of models) at a scale $\Lambda_{NP} > m_{\text{Higgs}}$
   - integrate out BSM fields in running down to Higgs scale
   - integrate out Higgs boson by running down to EW scale
   - integrate out $Z, W^{\pm}$, heavy quarks by running down to 1 GeV scale

   $\rightsquigarrow$ dimension-6 EDM operators with specified weights formulated in terms of quarks and gluons

2. Transcribe these EDM operators into hadronic language using:
   - (non)-relativistic quark models
     e.g. $d_n = \frac{4}{3}d_d - \frac{1}{3}d_u$ and $d_p = \frac{4}{3}d_u - \frac{1}{3}d_d$ ▸ (QM)
   - QCD sum rules
   - Lattice QCD calculations
   - Effective Field Theory ▸ (EFT)



How to handle CP-violating sources beyond SM?

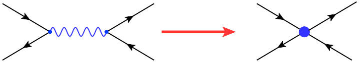
i.e. New Physics (NP) as e.g. SUSY, multi-Higgs, Left-Right-Symmetric Models?

Two-step procedure:

1. Pick NP models (or classes of models) at a scale $\Lambda_{NP} > m_{\text{Higgs}}$
   - integrate out BSM fields in running down to Higgs scale
   - integrate out Higgs boson by running down to EW scale
   - integrate out $Z, W^{\pm}$, heavy quarks by running down to 1 GeV scale

   $\rightsquigarrow$ dimension-6 EDM operators with specified weights formulated in terms of quarks and gluons

2. Transcribe these EDM operators into hadronic language using:
   - (non)-relativistic quark models
     e.g. $d_n = \frac{4}{3} d_d - \frac{1}{3} d_u$ and $d_p = \frac{4}{3} d_u - \frac{1}{3} d_d$ (QM)
   - QCD sum rules
   - Lattice QCD calculations
   - Effective Field Theory (EFT)



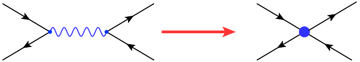
How to handle CP-violating sources beyond SM?

i.e. New Physics (NP) as e.g. SUSY, multi-Higgs, Left-Right-Symmetric Models?

Two-step procedure:

1. Pick NP models (or classes of models) at a scale $\Lambda_{NP} > m_{\text{Higgs}}$
   - integrate out BSM fields in running down to Higgs scale
   - integrate out Higgs boson by running down to EW scale
   - integrate out $Z, W^{\pm}$, heavy quarks by running down to 1 GeV scale

   $\rightsquigarrow$ dimension-6 EDM operators with specified weights formulated in terms of quarks and gluons
2. Transcribe these EDM operators into hadronic language using:
   - (non)-relativistic quark models
     e.g. $d_n = \frac{4}{3} d_d - \frac{1}{3} d_u$ and $d_p = \frac{4}{3} d_u - \frac{1}{3} d_d$ (QM)
   - QCD sum rules
   - Lattice QCD calculations
   - Effective Field Theory (EFT)



How to handle CP-violating sources beyond SM?

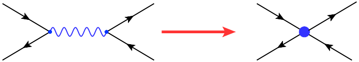
i.e. New Physics (NP) as e.g. SUSY, multi-Higgs, Left-Right-Symmetric Models?

Two-step procedure:

1. Pick NP models (or classes of models) at a scale $\Lambda_{NP} > m_{\text{Higgs}}$
   - integrate out BSM fields in running down to Higgs scale
   - integrate out Higgs boson by running down to EW scale
   - integrate out $Z, W^{\pm}$, heavy quarks by running down to 1 GeV scale

   $\rightsquigarrow$ dimension-6 EDM operators with specified weights formulated in terms of quarks and gluons

2. Transcribe these EDM operators into hadronic language using:
   - (non)-relativistic quark models
     e.g. $d_n = \frac{4}{3}d_d - \frac{1}{3}d_u$ and $d_p = \frac{4}{3}d_u - \frac{1}{3}d_d$ (QM)
   - QCD sum rules
   - Lattice QCD calculations
   - Effective Field Theory (EFT)



How to handle CP-violating sources beyond SM?

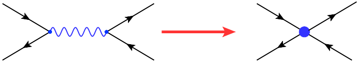
i.e. New Physics (NP) as e.g. SUSY, multi-Higgs, Left-Right-Symmetric Models?

$\boxed{\text{Two-step procedure:}}$

1. Pick NP models (or classes of models) at a scale $\Lambda_{NP} > m_{\rm Higgs}$
   - integrate out BSM fields in running down to Higgs scale
   - integrate out Higgs boson by running down to EW scale
   - integrate out $Z, W^{\pm}$, heavy quarks by running down to 1 GeV scale

   $\rightsquigarrow$ dimension-6 EDM operators with specified weights
   formulated in terms of quarks and gluons

2. Transcribe these EDM operators into hadronic language using:
   - (non)-relativistic quark models
     e.g. $d_n = \frac{4}{3} d_d - \frac{1}{3} d_u$ and $d_p = \frac{4}{3} d_u - \frac{1}{3} d_d$ (QM)
   - QCD sum rules
   - Lattice QCD calculations
   - Effective Field Theory (EFT)



How to handle CP-violating sources beyond SM?

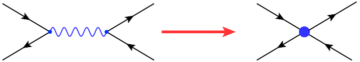
i.e. New Physics (NP) as e.g. SUSY, multi-Higgs, Left-Right-Symmetric Models?

Two-step procedure:

1. Pick NP models (or classes of models) at a scale $\Lambda_{NP} > m_{\text{Higgs}}$
   - integrate out BSM fields in running down to Higgs scale
   - integrate out Higgs boson by running down to EW scale
   - integrate out $Z, W^{\pm}$, heavy quarks by running down to 1 GeV scale

   $\rightsquigarrow$ dimension-6 EDM operators with specified weights formulated in terms of quarks and gluons

2. Transcribe these EDM operators into hadronic language using:
   - (non)-relativistic quark models
     e.g. $d_n = \frac{4}{3}d_d - \frac{1}{3}d_u$ and $d_p = \frac{4}{3}d_u - \frac{1}{3}d_d$ (QM)
   - QCD sum rules
   - Lattice QCD calculations
   - Effective Field Theory (EFT)



Outline:

1. Motivation: Matter–Antimatter Asymmetry in the Universe
2. The Permanent Electric Dipole Moment and its Features
3. CP-Violating Sources *in* the Standard Model (SM)
4. CP-Violating Sources *Beyond* the Standard Model (BSM)
5. Electric Dipole Moments (EDMs) of the Nucleon
6. Electric Dipole Moments of the Deuteron and Helium-3
7. Conclusions and Outlook

Outline:

1. Motivation: Matter–Antimatter Asymmetry in the Universe
2. The Permanent Electric Dipole Moment and its Features
3. CP-Violating Sources *in* the Standard Model (SM)
4. CP-Violating Sources *Beyond* the Standard Model (BSM)
5. Electric Dipole Moments (EDMs) of the Nucleon
6. Electric Dipole Moments of the Deuteron and Helium-3
7. Conclusions and Outlook

Outline:

Some questions that will hopefully be answered:

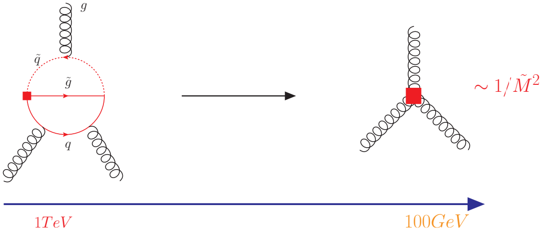
1. Why is $\cancel{CP}$ beyond the Standard Model expected?
2. How can a point-particle (*e.g.* an electron) support an EDM?
3. Why don't the EDMs of certain molecules predict a strong $\cancel{CP}$?
4. What is the natural scale of a neutron EDM?
5. How large is the EDM window for *New Physics* searches?
6. How can the EDM-producing sources be discriminated?
7. Why is low-energy Effective Field Theory needed here?

Some questions that will hopefully be answered:

1. Why is $\cancel{CP}$ beyond the Standard Model expected?
2. How can a point-particle (*e.g.* an electron) support an EDM?
3. Why don't the EDMs of certain molecules predict a strong $\cancel{CP}$?
4. What is the natural scale of a neutron EDM?
5. How large is the EDM window for *New Physics* searches?
6. How can the EDM-producing sources be discriminated?
7. Why is low-energy Effective Field Theory needed here?

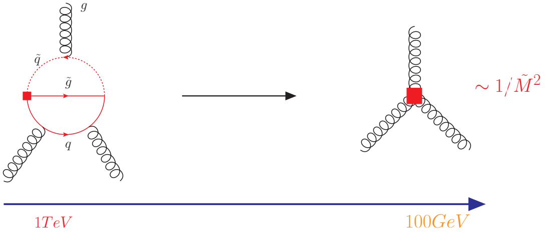
BSM physics and SM treated as effective field theories

- All degrees of freedom beyond a specified scale are integrated out:
  $\hookrightarrow$ remaining theory contains relevant degrees o.f. and *'irrelevant'* contact terms governed by relevant (Lorentz + SM) symmetries
- Write down *all* interactions among the relevant degrees of freedom that respect the relevant symmetries
- Need a power-counting scheme to order these infinite # interactions
- Relics of eliminated BSM physics 'remembered' by the values of the low-energy constants (LECs) of the CP-violating contact terms, e.g.



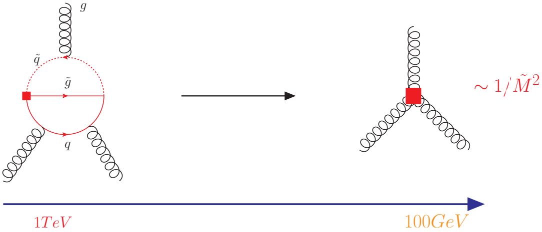
BSM physics and SM treated as effective field theories

- All degrees of freedom beyond a specified scale are integrated out:
  ↪ remaining theory contains relevant degrees o.f. and *'irrelevant'* contact terms governed by relevant (Lorentz + SM) symmetries
- Write down *all* interactions among the relevant degrees of freedom that respect the relevant symmetries
- Need a power-counting scheme to order these infinite # interactions
- Relics of eliminated BSM physics 'remembered' by the values of the low-energy constants (LECs) of the CP-violating contact terms, e.g.



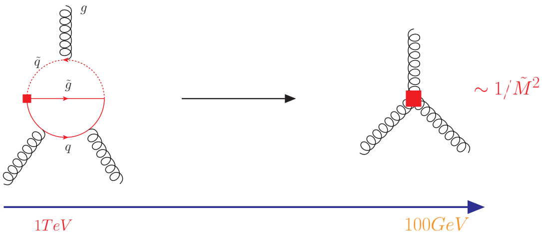
BSM physics and SM treated as effective field theories

- All degrees of freedom beyond a specified scale are integrated out:
  $\hookrightarrow$ remaining theory contains relevant degrees o.f. and *'irrelevant'* contact terms governed by relevant (Lorentz + SM) symmetries
- Write down *all* interactions among the relevant degrees of freedom that respect the relevant symmetries
- Need a power-counting scheme to order these infinite # interactions
- Relics of eliminated BSM physics 'remembered' by the values of the low-energy constants (LECs) of the CP-violating contact terms, e.g.



BSM physics and SM treated as effective field theories

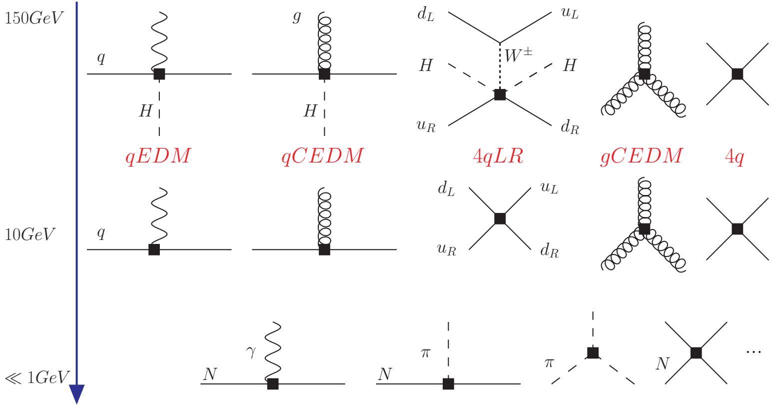
- All degrees of freedom beyond a specified scale are integrated out:
  $\hookrightarrow$ remaining theory contains relevant degrees o.f. and *'irrelevant'* contact terms governed by relevant (Lorentz + SM) symmetries
- Write down *all* interactions among the relevant degrees of freedom that respect the relevant symmetries
- Need a power-counting scheme to order these infinite # interactions
- Relics of eliminated BSM physics 'remembered' by the values of the low-energy constants (LECs) of the CP-violating contact terms, e.g.



CP-violating sources beyond SM

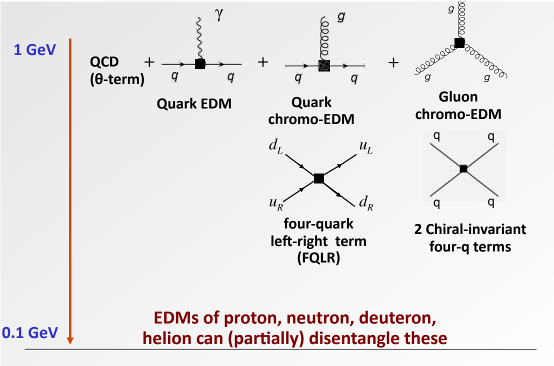
Removal of the Higgs and transition to hadronic fields (plus mixing)

Add to SM all possible T- and P-odd contact interactions

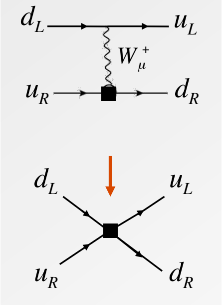


Relevant $\not{T}$ & $\not{P}$ quark sources up to dimension 6

W. Dekens & J. de Vries (2013)



FLQR term based on:



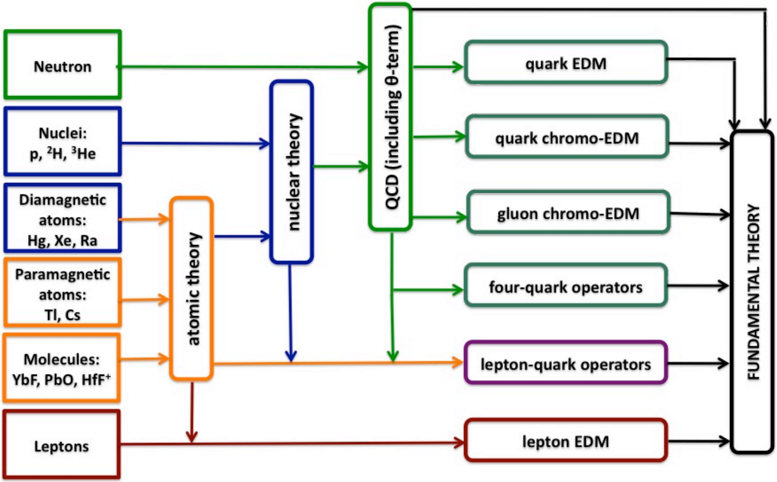
(adapted from Jordy de Vries, Jülich, March 14, 2013)

$$\text{Total \#} = 1(\bar\theta) + 2(\text{qEDM}) + 2(\text{qCEDM}) + 1(\text{gCEDM}) + 1(\text{FQLR}) + 2(\text{4q}) \ \ [+3(\text{semi-lept})]$$
$$= 1(\text{dim-four}) + 8[+3](\text{dim-six})$$

Caveat: implicit assumption that $m_s \gg m_u, m_d$

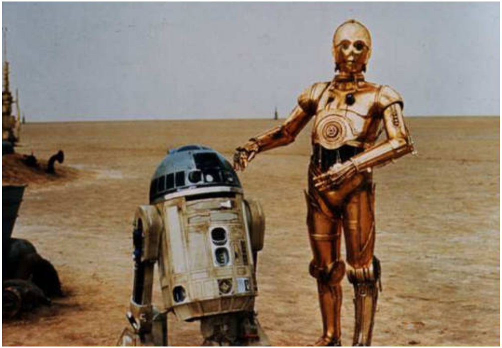
Road map from EDM Measurements to EDM Sources

Experimentalist's point of view →   ← Theorist's point of view

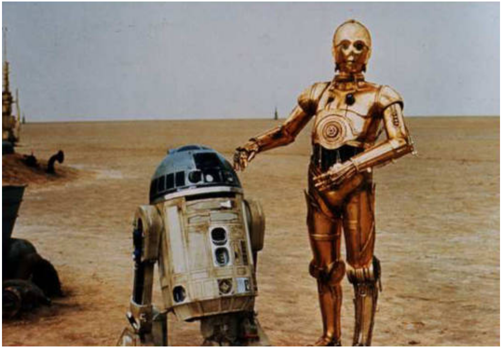


(adapted from Jordy de Vries, Jülich, March 14, 2013)

EDM Translator
from ‘quarkish/machine’ to ‘hadronic/human’ language?



EDM Translator
from ‘quarkish/machine’ to ‘hadronic/human’ language?



$\hookrightarrow$ Symmetries (esp. chiral one) and Goldstone Theorem
Low-Energy Effective Field Theory with External Sources
i.e. Chiral Perturbation Theory (suitably extended)

Hierachy among the sources at the hadronic EFT level

Each source transforms differently under chiral and isospin symmetry

$\cancel{CP}$, I $\qquad$ $\cancel{CP}$, $\cancel{I}$ $\qquad$ $\cancel{CP}$, I + $\cancel{I}$

$$\mathcal{L}_{\mathrm{EFT}}^{\cancel{CP}} = g_0 N^\dagger \vec{\pi}\cdot\vec{\tau} N + g_1 N^\dagger \pi_3 N + N^\dagger(b_0+b_1\tau_3)S^\mu v^\nu F_{\mu\nu} N + \cdots$$

$\Downarrow \qquad \Downarrow \qquad \Downarrow$

…

dominant for $\bar{\theta}$ term $\qquad$ suppressed for $\bar{\theta}$ term $\qquad$ suppressed by $m_q^* \sim m_\pi^2$

- $\mathcal{L}^{\theta}_{QCD} = -\bar{\theta} m^* \sum_f \bar{q}_f i\gamma_5 q_f$: $\quad$ $\cancel{CP}$, I $\quad \Leftrightarrow \quad \mathcal{M} \to \mathcal{M} + \bar{\theta} m^* i\gamma_5\,, \quad m^* = \frac{m_u m_d}{m_u+m_d}$

  $\hookrightarrow$ $\bar{\theta}$ source breaks chiral symmetry ($\propto m^*$) but *conserves* the isospin one:

  $\hookrightarrow$ $\boxed{g_0^\theta \gg g_1^\theta}$: NDA estimate: $g_1^\theta/g_0^\theta \sim \mathcal{O}(m_\pi^2/m_n^2)$ $\qquad$ de Vries et al. (2011)

  resonance saturation: $g_1^\theta/g_0^\theta \sim \mathcal{O}(m_\pi/m_n)$! $\qquad$ Bsaisou et al. (2013)

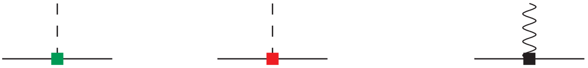
Hierachy among the sources at the hadronic EFT level

Each source transforms differently under chiral and isospin symmetry

$\cancel{CP}$, I | $\cancel{CP}$, $\cancel{\mathrm{I}}$ | $\cancel{CP}$, I + $\cancel{\mathrm{I}}$

$$\mathcal{L}^{\cancel{CP}}_{\mathrm{EFT}} = g_0 N^\dagger \vec{\pi}\cdot\vec{\tau} N \;+\; g_1 N^\dagger \pi_3 N \;+\; N^\dagger(b_0 + b_1\tau_3) S^\mu v^\nu F_{\mu\nu} N \;+\cdots$$

$\Downarrow$ $\Downarrow$ $\Downarrow$



...

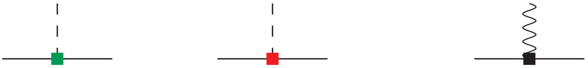
dominant for chromo qEDM source	dominant for chromo qEDM source	$m_q \sim m_\pi^2$ suppressed for chromo qEDM source

- chromo quark EDM: chiral & isospin symmetries are broken
  because of quark masses $\rightsquigarrow$ Goldstone theorem respected
- 4quark Left-Right EDM: explicit breaking of chiral & isospin symmetries
  because of underlying W boson exchange $\rightsquigarrow$ Goldstone theorem does not apply

Hierachy among the sources at the hadronic EFT level

Each source transforms differently under chiral and isospin symmetry

$$\begin{array}{ccccccc} & & \cancel{CP},\ \mathrm{I} & & \cancel{CP},\ \cancel{\mathrm{I}} & & \cancel{CP},\ \mathrm{I}+\cancel{\mathrm{I}} \\ \mathcal{L}_{\mathrm{EFT}}^{\cancel{CP}} & = & g_0 N^\dagger \vec{\pi}\cdot\vec{\tau} N & + & g_1 N^\dagger \pi_3 N & + & N^\dagger (b_0+b_1\tau_3) S^\mu v^\nu F_{\mu\nu} N \quad + \cdots \\ & & \Downarrow & & \Downarrow & & \Downarrow \end{array}$$



…

suppressed for quark EDM source	suppressed for quark EDM source	dominating for quark EDM source

- quark EDM: $N\pi$ (and NN) interactions are suppressed by $\alpha_{\mathrm{em}}/(4\pi)$
- gluon color EDM (and chiral-4quark EDM): relative $\mathcal{O}(m_\pi^2)$ suppression of $N\pi$ interactions because of Goldstone theorem

Summary of scalings of $\cancel{CP}$ hadronic vertices

from θ to BSM sources

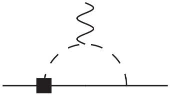
	$g_0: \cancel{CP}, \mathrm{I}$	$g_1: \cancel{CP}, \not{\mathrm{I}}$	$d_0, d_1: \cancel{CP}, \mathrm{I} + \not{\mathrm{I}}$	$C_{3\pi}: \cancel{CP}, \not{\mathrm{I}}$
$\mathcal{L}_{\mathrm{EFT}}^{\cancel{CP}}$:				

	g_0	g_1	d_0, d_1	$C_{3\pi}$
θ-term:	$\mathcal{O}(1)$	$\mathcal{O}(M_\pi/m_N)$	$\mathcal{O}(M_\pi^2/m_N^2)$	$\mathcal{O}(\epsilon M_\pi^2/m_N^2)$
qEDM:	$\mathcal{O}(\alpha_{EM}/(4\pi))$	$\mathcal{O}(\alpha_{EM}/(4\pi))$	$\mathcal{O}(1)$	$\mathcal{O}(\alpha_{EM}/(4\pi))$
qCEDM:	$\mathcal{O}(1)$	$\mathcal{O}(1)$	$\mathcal{O}(M_\pi^2/m_N^2)$	$\mathcal{O}(\epsilon M_\pi^2/m_N^2)$
4qLR:	$\mathcal{O}(M_\pi^2/m_n^2)$	$\mathcal{O}(1)$	$\mathcal{O}(M_\pi^2/m_N^2)$	$\mathcal{O}(1)$
gCEDM:	$\mathcal{O}(M_\pi^2/m_N^2)^*$	$\mathcal{O}(M_\pi^2/m_N^2)^*$	$\mathcal{O}(1)$	$\mathcal{O}(\epsilon M_\pi^2/m_N^2)$
4q:	$\mathcal{O}(M_\pi^2/m_N^2)^*$	$\mathcal{O}(M_\pi^2/m_N^2)^*$	$\mathcal{O}(1)$	$\mathcal{O}(\epsilon M_\pi^2/m_N^2)$

*: Goldstone theorem → relative $\mathcal{O}(M_\pi^2/m_n^2)$ suppression of $N\pi$ interactions

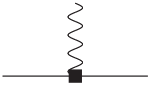
θ-Term Induced Nucleon EDM

single nucleon EDM:



"controlled"

isovector $\approx$ $\ll$ isoscalar



two "unknown" coefficients

Guo & Meißner (2012): also in SU(3) case

$$d_n\Big|_{\text{loop}}^{\text{isovector}} = e\frac{g_{\pi NN}\, g_0^{\theta}}{4\pi^2}\frac{\ln(M_N^2/m_\pi^2)}{2M_N} \sim \bar\theta\, m_\pi^2 \ln m_\pi^2$$

Crewther, di Vecchia, Veneziano & Witten (1979); Pich & de Rafael (1991); Ottnad et al. (2010)

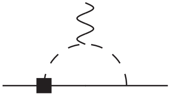
$$g_0^{\theta} = \frac{(m_n - m_p)^{\text{strong}}(1-\epsilon^2)}{4F_\pi\epsilon}\,\bar\theta \approx (-0.018 \pm 0.007)\bar\theta \qquad \left(\text{where } \epsilon \equiv \frac{m_u - m_d}{m_u + m_d}\right)$$

$$\hookrightarrow d_n\Big|_{\text{loop}}^{\text{isovector}} \sim -(2.1 \pm 0.9)\cdot 10^{-16}\,\bar\theta\,\mathrm{e\,cm}$$

Ottnad et al. (2010); Bsaisou et al. (2013)

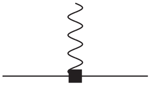
θ-Term Induced Nucleon EDM

single nucleon EDM:



"controlled"

isovector $\approx$ $\ll$ isoscalar



two "unknown" coefficients

Guo & Meißner (2012): also in SU(3) case

$$d_n\Big|^{\text{isovector}}_{\text{loop}} = e\frac{g_{\pi NN}\, g_0^\theta}{4\pi^2}\,\frac{\ln(M_N^2/m_\pi^2)}{2M_N} \sim \bar\theta\, m_\pi^2 \ln m_\pi^2$$

Crewther, di Vecchia, Veneziano & Witten (1979); Pich & de Rafael (1991); Ottnad et al. (2010)

$$g_0^\theta = \frac{(m_n - m_p)^{\text{strong}}(1-\epsilon^2)}{4F_\pi \epsilon}\,\bar\theta \approx (-0.018 \pm 0.007)\bar\theta \qquad \left(\text{where } \epsilon \equiv \frac{m_u - m_d}{m_u + m_d}\right)$$

$$\hookrightarrow d_n\Big|^{\text{isovector}}_{\text{loop}} \sim -(2.1 \pm 0.9)\cdot 10^{-16}\,\bar\theta\,\mathrm{e\,cm}$$

Ottnad et al. (2010); Bsaisou et al. (2013)

But what about the two "unknown" coefficients of the contact terms?

We'll always have ... the lattice

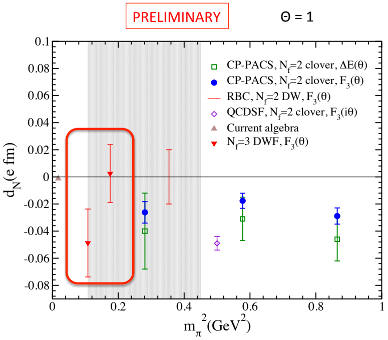
We'll always have ... the lattice

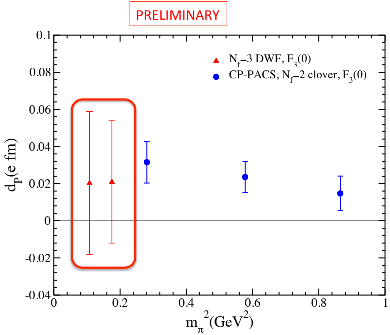
However, *It's a long way to Tipperary ...*

Results from *full* QCD calculations (no systematical errors!) for the

neutron EDM and proton EDM

(note $F_{3,p}$ mixes with $F_{1,p}$)





(adapted from Taku Izubuchi (BNL), *Lattice-QCD calculations for EDMs*, Fermilab, Feb. 14, 2013)

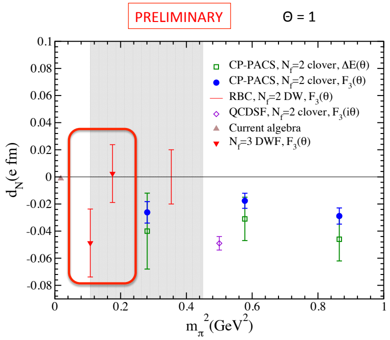
We'll always have ... the lattice

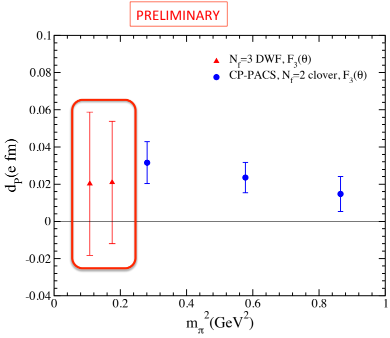
However, *It's a long way to Tipperary ...*

Results from *full* QCD calculations (no systematical errors!) for the

neutron EDM and proton EDM

(note $F_{3,p}$ mixes with $F_{1,p}$)





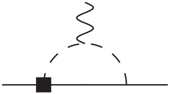
(adapted from Taku Izubuchi (BNL), *Lattice-QCD calculations for EDMs*, Fermilab, Feb. 14, 2013)

Don't mention the ... light nuclei

θ-Term Induced Nucleon EDM:

Crewther, di Vecchia, Veneziano & Witten (1979); Pich & de Rafael (1991); Ottnad et al. (2010)

single nucleon EDM:

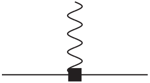


isovector

$\approx$

$\ll$

isoscalar



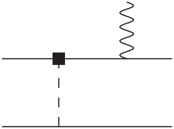
“controlled” “unknown" coefficients

$\longrightarrow$ lattice QCD required

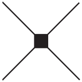
Guo, Meißner (2012)

two nucleon EDM:

Sushkov, Flambaum, Khriplovich (1984)



$\gg$

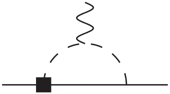


controlled unknown coefficient

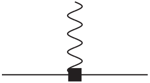
θ-Term Induced Nucleon EDM:

Crewther, di Vecchia, Veneziano & Witten (1979); Pich & de Rafael (1991); Ottnad et al. (2010)

single nucleon EDM:



isovector $\approx$ $\ll$ isoscalar



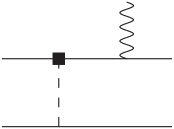
“controlled” “unknown" coefficients

$\longrightarrow$ lattice QCD required

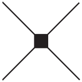
Guo, Meißner (2012)

two nucleon EDM:

Sushkov, Flambaum, Khriplovich (1984)



$\gg$



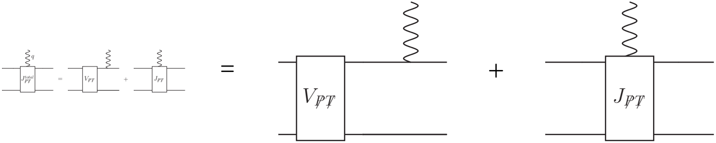
controlled unknown coefficient

EDM of the Deuteron:

Deuteron (D) as Isospin Filter

note:  $= \frac{ie}{2}(1+\tau_3)$

2N-system: $I + S + L$=odd



total current | irred. potential | irred. current

I = 0 I = 0 | I = 0 → I = 1 → I = 0 | I = 0 I = 0

isospin selection rules!

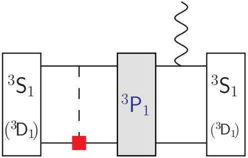
$\Downarrow$

$\cancel{g_0^\theta N^\dagger \vec{\pi}\cdot\vec{\tau} N}$ at leading order (LO)

$\Downarrow$

subleading (NLO) $g_1^\theta N^\dagger \pi_3 N$ acts as *'new'* leading order (LO) for D

EDM of the Deuteron at LO: quantitative θ-term results



LO: $\cancel{g_0^\theta N^\dagger \vec{\pi}\cdot\vec{\tau} N}$ $(\cancel{CP}, I) \rightarrow$ Isospin excl.

NLO: $g_1^\theta N^\dagger \pi_3 N$ $(\cancel{CP}, \cancel{I})$ $\rightarrow$ "LO"

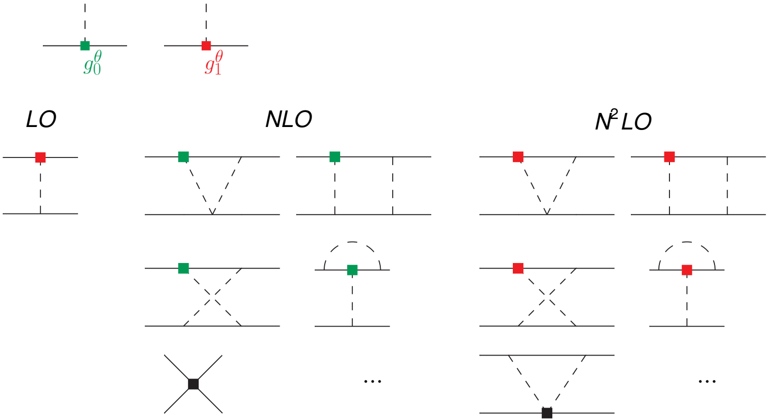
in units of $g_1^\theta\, e \cdot \mathrm{fm} \cdot (g_A\, m_N/F_\pi)$

Ref.	potential	no $^3P_1$-int	with $^3P_1$-int	total
JBC (2013)*	Av_{18}	-1.93×10^{-2}	$+0.48 \times 10^{-2}$	-1.45×10^{-2}
JBC (2013)	CD Bonn	-1.95×10^{-2}	$+0.51 \times 10^{-2}$	-1.45×10^{-2}
JBC (2013)*	ChPT $(N^2LO)^\dagger$	-1.94×10^{-2}	$+0.65 \times 10^{-2}$	-1.29×10^{-2}
Song (2013)	Av_{18}	-	-	-1.45×10^{-2}
Liu (2004)	Av_{18}	-	-	-1.43×10^{-2}
Afnan (2010)	Reid 93	-1.93×10^{-2}	$+0.40 \times 10^{-2}$	-1.43×10^{-2}

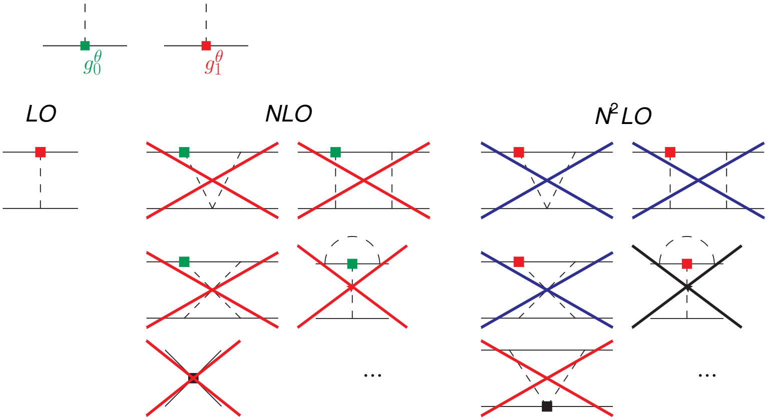
*: in preparation †: cutoffs at 600 MeV (LS) and 700 MeV (SFR)

$\boxed{\text{BSM } \cancel{CP} \text{ sources:}}$ LO g_1 πNN-vertex also exists in qCEDM and 4qLR cases

EDM of the Deuteron: *NLO*- and *N²LO*-Potentials



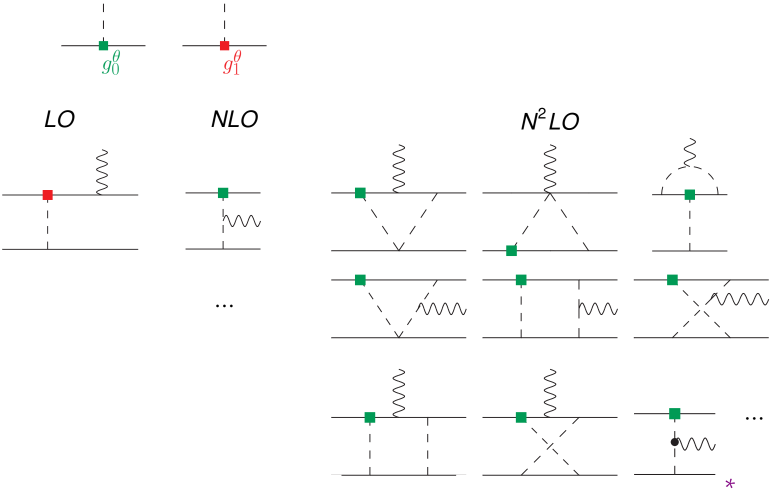
EDM of the Deuteron: *NLO*- and N^2LO-Potentials



- ×: vanishing by selection rules, ×: sum of diagrams vanishes
  ×: vertex correction

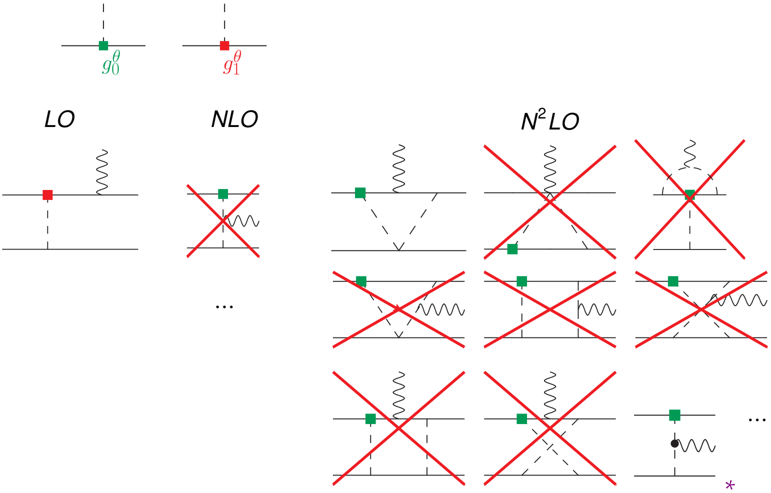
LO NLO N^2LO

EDM of the Deuteron: *NLO*- and N^2LO-Currents



$\ast$: de Vries et al. (2011), Bsaisou et al. (2013)

EDM of the Deuteron: *NLO*- and N^2LO-Currents



∗: de Vries et al. (2011), Bsaisou et al. (2013)

- ×: vanishing by selection rules, ×: sum of diagrams vanishes

Deuteron EDM from the $\bar{\theta}$-term

Bsaisou et al. (2013)

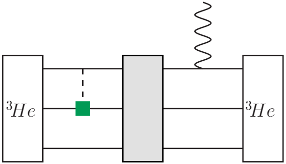
total deuteron EDM: $d_D = d_n + d_p + d_D(2N)$

- single-nucleon contribution: EFT *alone* has no predictive power
  $\longrightarrow$ *Experiment* or *Lattice QCD* needed in addition

- two-nucleon contribution $d_D(2N)$: EFT *has* predictive power

$$d_D(2N) = \underbrace{-(0.59 \pm 0.39) \cdot 10^{-16}\,\bar{\theta}\,e\,\mathrm{cm}}_{\mathrm{LO}} \quad + \quad \underbrace{(0.05 \pm 0.02) \cdot 10^{-16}\,\bar{\theta}\,e\,\mathrm{cm}}_{\mathrm{N^2LO}}$$

^3He EDM: quantitative results for g_0 exchange



$g_0 N^\dagger \vec{\pi}\cdot\vec{\tau} N \quad (\cancel{CP}, \mathrm{I})$

θ-term, qCEDM $\rightarrow$ LO

4qLR $\rightarrow$ N^{2}LO

units: $g_0 (g_A m_N / F_\pi)\, e\,\mathrm{fm}$

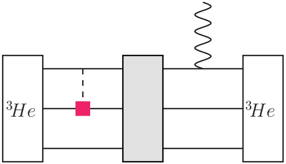
author	potential	no int.	with int.	total
JBC (2013)*	Av_{18}UIX	-0.45×10^{-2}	-0.13×10^{-2}	-0.57×10^{-2}
JBC (2013)*	CD BONN TM	-0.56×10^{-2}	-0.12×10^{-2}	-0.67×10^{-2}
JBC (2013)*	ChPT (N^2LO)†	-0.56×10^{-2}	-0.19×10^{-2}	-0.76×10^{-2}
Song (2013)	Av_{18}UIX	-	-	-0.59×10^{-2}
Stetcu (2008)	Av_{18} UIX	-	-	-1.21×10^{-2}

*: in preparation †: cutoffs at 600 MeV (LS) and 700 MeV (SFR)

Results for ^3H also available (not shown)

Note: calculation finally under control !

^3He EDM: quantitative results for g_1 exchange



$g_1 N^\dagger \pi_3 N$ ($\not{CP}$, $\not{I}$)
θ-term $\rightarrow$ NLO
qCEDM, 4qLR $\rightarrow$ LO !

units: $g_1 (g_A m_N / F_\pi)\, e\,\mathrm{fm}$

Ref.	potential	no int.	with int.	total
JBC (2013)*	Av_{18}UIX	-1.09×10^{-2}	-0.02×10^{-2}	-1.11×10^{-2}
JBC(2013)*	CD BONN TM	-1.11×10^{-2}	-0.03×10^{-2}	-1.14×10^{-2}
JBC (2013)*	ChPT (N^2LO)†	-1.09×10^{-2}	-0.14×10^{-2}	-0.96×10^{-2}
Song (2013)	Av_{18}UIX	-	-	-1.08×10^{-2}
Stetcu (2008)	Av_{18} UIX	-	-	-2.20×10^{-2}

*: in preparation †: cutoffs at 600 MeV (LS) and 700 MeV (SFR)

Results for ^3H also available (not shown)

In the pipeline: $\not{CP}$ 3π-vertex contribution (4qLR: LO)

Quantitative EDM results in the θ-term scenario

Single Nucleon (with adjusted signs for consistency; note here $e < 0$):

$$-d_1^{\text{loop}} \equiv \frac{1}{2}(d_n - d_p)^{\text{loop}}$$
$$= (2.1 \pm 0.9) \cdot 10^{-16}\,\bar{\theta}\,e\,\text{cm} \qquad \text{(Bsaisou et al. (2013))}$$

$$d_n = +(2.9 \pm 0.9) \cdot 10^{-16}\,\bar{\theta}\,e\,\text{cm} \qquad \text{(Guo \& Meißner (2012))}$$
$$d_p = -(1.1 \pm 1.1) \cdot 0^{-16}\,\bar{\theta}\,e\,\text{cm} \qquad \text{(Guo \& Meißner (2012))}$$

Deuteron:

$$d_D = d_n + d_p - \left[(0.59 \pm 0.39) - (0.05 \pm 0.02)\right] \cdot 10^{-16}\,\bar{\theta}\,e\,\text{cm}$$
$$= d_n + d_p - (0.54 \pm 0.39) \cdot 10^{-16}\,\bar{\theta}\,e\,\text{cm} \qquad \text{(Bsaisou et al. (2013))}$$

Helium-3:

$$d_{^3He} = \tilde{d}_n + \left[(1.52 \pm 0.60) - (0.46 \pm 0.30)\right] \cdot 10^{-16}\,\bar{\theta}\,e\,\text{cm}$$
$$= \tilde{d}_n + (1.06 \pm 0.67) \cdot 10^{-16}\,\bar{\theta}\,e\,\text{cm} \qquad \text{(JBC (2013))}$$

with $\tilde{d}_n = 0.88 d_n - 0.047 d_p$ (de Vries et al. (2011))

Testing Strategies in the θ EDM scenario

Remember:

$$
\begin{aligned}
d_D &= d_n + d_p &- (0.54 \pm 0.39) \cdot 10^{-16}\, \bar\theta\, e\,\mathrm{cm} \quad &\text{(Bsaisou et al. (2013))}\\
d_{^3He} &= \tilde d_n &+ (1.06 \pm 0.67) \cdot 10^{-16}\, \bar\theta\, e\,\mathrm{cm} \quad &\text{(JBC (2013))}
\end{aligned}
$$

Testing strategies:

- plan A: measure d_n, d_p, and d_D $\xrightarrow{d_D(2N)} \bar\theta \xrightarrow{\text{test}} d_{^3He}$
- plan A': measure d_n, (d_p), and $d_{^3He}$ $\xrightarrow{d_{^3He}(2N)} \bar\theta \xrightarrow{\text{test}} d_D$
- plan B: measure d_n (or d_p) + Lattice QCD $\rightsquigarrow \bar\theta \xrightarrow{\text{test}} d_D$
- plan B': measure d_n (or d_p) + Lattice QCD $\rightsquigarrow \bar\theta \xrightarrow{\text{test}} d_p$ (or d_n)

If $\bar{\theta}$-term tests fail: effective BSM dim. 6 sources: de Vries et al. (2011)

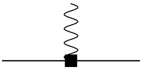
qEDM	qCEDM	4qLR	gCEDM + 4qEDM
$d_D \approx d_p + d_n$	$d_D > d_p + d_n$	$d_D > d_p + d_n$	$d_D \sim d_p + d_n$
$d_{^3He} \approx d_n$	$d_{^3He} > d_n$	$d_{^3He} > d_n$	$d_{^3He} \sim d_n$

$\longrightarrow \quad g_0,\, g_1 \propto \alpha/(4\pi)$

$2N$ contribution suppressed by photon loop!

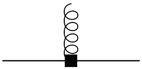
here: only absolute values considered

If $\bar{\theta}$-term tests fail: effective BSM dim. 6 sources: de Vries et al. (2011)



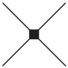
$qEDM$

$$d_D \approx d_p + d_n \qquad d_{^3He} \approx d_n$$



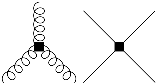
$qCEDM$

$$d_D > d_p + d_n \qquad d_{^3He} > d_n$$



$4qLR$

$$d_D > d_p + d_n \qquad d_{^3He} > d_n$$



$gCEDM + 4qEDM$

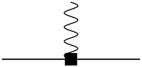
$$d_D \sim d_p + d_n \qquad d_{^3He} \sim d_n$$

$\longrightarrow \quad g_0\,,\ g_1$

$2N$ contribution enhanced!

here: only absolute values considered

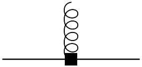
If $\bar{\theta}$-term tests fail: effective BSM dim. 6 sources: de Vries et al. (2011)



qEDM

$d_D \approx d_p + d_n$

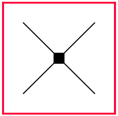
$d_{^3He} \approx d_n$



qCEDM

$d_D > d_p + d_n$

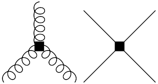
$d_{^3He} > d_n$



4qLR

$d_D > d_p + d_n$

$d_{^3He} > d_n$



gCEDM + 4qEDM

$d_D \sim d_p + d_n$

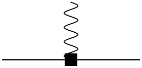
$d_{^3He} \sim d_n$

$\longrightarrow$ $g_1 \gg g_0$; 3π-coupling (unsuppressed)

$2N$ contribution enhanced!

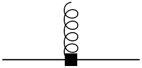
here: only absolute values considered

If $\bar{\theta}$-term tests fail: effective BSM dim. 6 sources: de Vries et al. (2011)



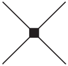
qEDM

$d_D \approx d_p + d_n$
$d_{^3He} \approx d_n$



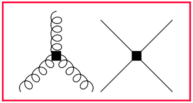
qCEDM

$d_D > d_p + d_n$
$d_{^3He} > d_n$



4qLR

$d_D > d_p + d_n$
$d_{^3He} > d_n$



gCEDM + 4qEDM

$d_D \sim d_p + d_n$
$d_{^3He} \sim d_n$

$\longrightarrow g_1, g_0, 4N$ – coupling

$2N$ contribution difficult to asses!

here: only absolute values considered

Summary and Outlook

- θEDM: relevant low-energy couplings quantifiable

  strategy A: measure d_n, d_p, d_D $\xrightarrow{d_D(2N)}$ $\bar{\theta}$ $\xrightarrow{\text{test}}$ $d_{^3He}$

  strategy B: measure d_n (or d_p) + Lattice QCD $\rightsquigarrow$ $\bar{\theta}$ $\xrightarrow{\text{test}}$ d_D

  strategy B': measure d_n (or d_p) + Lattice QCD $\rightsquigarrow$ $\bar{\theta}$ $\xrightarrow{\text{test}}$ d_p (or d_n)

- qEDM, qCEDM, 4QLR:
  - NDA required to asses sizes of low-energy couplings
  - disentanglement possible by measurements of d_n, d_p, d_D & $d_{^3He}$

- gCEDM, 4quark chiral singlet:

  controlled calculation/disentanglement difficult (lattice ?)

- Ultimate progress may eventually come from Lattice QCD
  $\hookrightarrow$ the $\cancel{CP}$ $N\pi$ couplings g_0 & g_1 may be accessible even for dim-6 $\hookrightarrow$ *quantifiable* d_D ($d_{^3He}$) EFT predictions may be feasible in BSM case

Conclusions

- (Hadronic) EDMs play a key role in probing new sources of $\not{C}\not{P}$
- May be relevant for the Baryon Asymmetry of the Universe (BAU)

  However, no theorem which directly links BAU with the EDMs.
  (Remember *e.g.* the $\bar{\theta}$ scenario with EDMs without BAU)

  Moreover, there are no *smoking guns* so far
- Measurements of hadronic EDMs are low-energy measurements
  ↪ Predictions have to be given in the *empirical language of hadrons*
  ↪ only reliable methods: *ChPT* or *Lattice QCD*
- EDMs of light nuclei provide independent information to nucleon EDMs and may be even larger and even simpler
- Deuteron and helium-3 nuclei serve as isospin filters for EDMs

At least the EDMs of p, n, d, and ^{3}He
have to be measured
to disentangle the underlying physics

Many thanks to my colleagues

in Jülich: Jan Bsaisou, Christoph Hanhart, Susanna Liebig,
Ulf-G. Meißner, Andreas Nogga, and Jordy de Vries

in Bonn: Feng-Kun Guo, Bastian Kubis, Ulf-G. Meißner

and: Werner Bernreuther, Bira van Kolck, Kolya Nikolaev

- J. Bsaisou, C. Hanhart, S. Liebig, U.-G. Meißner, A. Nogga and A. Wirzba, *The electric dipole moment of the deuteron from the QCD θ-term,* Eur. Phys. J. A **49** (2013) 31 [arXiv:1209.6306 [hep-ph]].
- K. Ottnad, B. Kubis, U.-G. Meißner and F.-K. Guo, *New insights into the neutron electric dipole moment,* Phys. Lett. B **687** (2010) 42 [arXiv:0911.3981 [hep-ph]].
- F.-K. Guo and U.-G. Meißner, *Baryon electric dipole moments from strong CP violation,* JHEP **1212** (2012) 097 [arXiv:1210.5887 [hep-ph]].
- W. Dekens and J. de Vries, *Renormalization Group Running of Dimension-Six Sources ...,* JHEP **1305** (2013) 149 [arXiv:1303.3156 [hep-ph]].
- J. de Vries, R. Higa, C.-P. Liu, E. Mereghetti, I. Stetcu, R. Timmermans, U. van Kolck, *Electric Dipole Moments of Light Nuclei From Chiral Effective Field Theory,* Phys. Rev. C **84** (2011) 065501 [arXiv:1109.3604 [hep-ph]].

Naive Quark Model results for a nucleon with N_c quarks

A proton (neutron) with isospin $I = \frac{1}{2}$ and spin $J = \frac{1}{2}$ contains $\frac{N_c+1}{2}$ quarks of u (d) flavor and $\frac{N_c-1}{2}$ quarks of d (u) flavor.

Because of spin-flavor *symmetry* the total spin $\vec{J}_u$ ($\vec{J}_d$) of all u (d) quarks satisfies $J_u = \frac{N_c+1}{4}$ ($J_d = \frac{N_c-1}{4}$) s.t. $J_z = \pm(J_u - J_d) = \pm\frac{1}{2}$ and

$$
\begin{aligned}
\langle n|\vec{J}_d|n\rangle &\equiv \langle p|\vec{J}_u|p\rangle \equiv \lambda_u^p\langle p|\vec{J}|p\rangle = \frac{N_c+5}{6}\langle p|\vec{J}|p\rangle \xrightarrow{N_c=3} \frac{4}{3}\langle p|\vec{J}|p\rangle \\
\langle n|\vec{J}_u|n\rangle &\equiv \langle p|\vec{J}_d|p\rangle \equiv \lambda_d^p\langle p|\vec{J}|p\rangle = -\frac{N_c-1}{6}\langle p|\vec{J}|p\rangle \xrightarrow{N_c=3} -\frac{1}{3}\langle p|\vec{J}|p\rangle
\end{aligned}
$$

$$
\hookrightarrow \frac{\mu_n}{\mu_p} = \frac{\left[\frac{2e}{3}\lambda_u^n - \frac{e}{3}\lambda_d^n\right]}{\left[\frac{2e}{3}\lambda_u^p - \frac{e}{3}\lambda_d^p\right]} = \frac{\left[\frac{2e}{3}\frac{-1}{3} - \frac{e}{3}\frac{4}{3}\right]}{\left[\frac{2e}{3}\frac{4}{3} - \frac{e}{3}\frac{-1}{3}\right]} = -\frac{2}{3}\ (!) \quad \left(-\frac{(N_c+1)^2-4}{(N_c+1)^2+2} \text{ in general}\right),
$$

$$
g_A^p = \lambda_u^p - \lambda_d^p = \frac{4}{3} - \frac{-1}{3} = \lambda_u^n - \lambda_d^n = g_A^n = \frac{5}{3} \quad \left(\frac{N_c+2}{3} \text{ in general}\right),
$$

$$
d_p = \lambda_u^p d_u + \lambda_d^p d_d = \frac{4}{3}d_u - \frac{1}{3}d_d \quad \text{such that } \boxed{d_p - d_n = \frac{N_c+2}{3}(d_u - d_d)}
$$

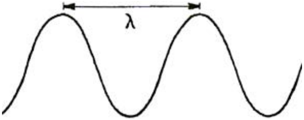
$$
d_n = \lambda_u^n d_u + \lambda_d^n d_d = -\frac{1}{3}d_u + \frac{4}{3}d_d \quad \text{and only } \boxed{d_p + d_n = d_u + d_d}.
$$

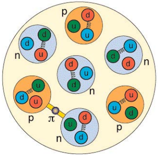
back

What are Effective Field Theories (EFT)?

- Different areas in physics describe phenomena at very disparate scales (of length, time, energy, mass)
- Very intuitive idea: scales much smaller / much bigger than the ones of interest shouldn't matter much
  - *e.g.* masses in particle physics: $m_e \approx 0.511\,\mathrm{MeV} \ldots m_t \sim 180\,\mathrm{GeV}$ range nearly six orders of magnitude (even without neutrinos)
  - still hydrogen atom spectrum can be calculated very precisely without knowing m_t at all

$\hookrightarrow$ Separation of scales: $1/k = \lambda \gg R_{\text{substructure}}$



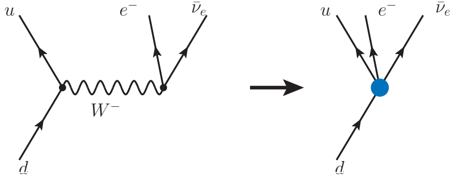


back

EFT example: weak interactions for $E \ll M_W$

Weak decays:

- mediated by the $W^\pm$ boson, $M_W \approx 80\,\text{GeV}$
- energy release in neutron decay: $\approx 1\,\text{MeV}$
- energy release in kaon decays: $\approx$ few $100\,\text{MeV}$



$$\frac{e^2}{8\sin\theta_W} \times \frac{1}{M_W^2 - q^2} \xrightarrow{q^2 \ll M_W^2} \underbrace{\frac{e^2}{8M_W^2\sin\theta_w}\left(1 + \frac{q^2}{M_W^2} + \dots\right)}_{G_F/\sqrt{2} \quad + \quad \mathcal{O}(q^2/M_W^2)}$$

$\hookrightarrow$ Fermi's current-current interaction

back

JÜLICH FORSCHUNGSZENTRUM

Effective Field Theory: Weinberg's conjecture

Quantum Field Theory has no content besides
unitarity, analyticity, cluster decomposition and symmetries

Weinberg 1979

To calculated the S-matrix for any theory below some scale, simply use the most general effective Lagrangian consistent with these principles in terms of the appropriate asymptotic states

(*i.e.* the general S-matrix can be obtained by perturbation theory using some effective lagrangian from the free theory — Witten (2001))

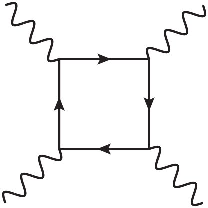
Power-law expand the amplitudes in *energy(momentum) / scale.*

- Physics at specific energy scale described by relevant d.o.f.
- Unresolved substructure incorporated via low-energy const(s)
- Systematic approach $\rightsquigarrow$ estimate of uncertainty possible

back

EFT example: light-by-light scattering

Euler, Heisenberg (1936)



- only one scale: m_e
- consider energies $\omega \ll m_e$
- $\mathcal{L}_{QED}[\underbrace{\psi, \bar{\psi}}_{\text{matter}}, \underbrace{A_\mu}_{\text{light}}] \to \mathcal{L}_{\text{eff}}[A_\mu]$
- invariants: $F_{\mu\nu}F^{\mu\nu} \propto \vec{E}^2 - \vec{B}^2$ & $F_{\mu\nu}\tilde{F}^{\mu\nu} \propto \vec{E} \cdot \vec{B}$

$$\mathcal{L}_{\text{eff}} = \frac{1}{2}\left(\vec{E}^2 - \vec{B}^2\right) + \frac{e^4}{16\pi^2 m_e^4}\left[a\left(\vec{E}^2 - \vec{B}^2\right)^2 + b\left(\vec{E} \cdot \vec{B}\right)^2\right] + \ldots$$

- calculation form the underlying theory, QED, yields $7a = b = 14/45$
- energy power law expansion: $(\omega/m_e)^{2n}$
- $\mathcal{L}_{\text{eff}}$ more efficient than full *QED* for calculating cross sections etc.

back

The symmetries of QCD

$$\mathcal{L}_{\rm QCD} = -\tfrac{1}{2}\mathrm{Tr}\,(G_{\mu\nu}G^{\mu\nu}) + \sum_f \bar{q}_f(\mathrm{i}\not{D} - m_f)q_f + \ldots$$

$$D_\mu = \partial_\mu - igA_\mu \equiv \partial_\mu - igA^a_\mu \frac{\lambda^a}{2}\,, \qquad G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]$$

- Lorentz-invariance, P, C, T invariance, $SU(3)_c$ gauge invariance
- The masses of the u, d, s quarks are small: $m_{u,d,s} \ll 1\,\mathrm{GeV} \approx \Lambda_{\rm hadron}$.
- Chiral decomposition of quark fields:

$$q = \tfrac{1}{2}(1-\gamma_5)q + \tfrac{1}{2}(1+\gamma_5)q = q_L + q_R\,.$$

- For massless fermions: left-/right-handed fields do not interact

$$\mathcal{L}[q_L, q_R] = i\bar{q}_L\not{D}q_L + i\bar{q}_R\not{D}q_R - m\,(\bar{q}_Lq_R + \bar{q}_Rq_L)$$

and $\mathcal{L}^0_{QCD}$ invariant under (global) chiral $U(3)_L \times U(3)_R$ transformations:

$\hookrightarrow$ rewrite $U(3)_L \times U(3)_R = SU(3)_V \times SU(3)_A \times U(1)_V \times U(1)_A$.

- $SU(3)_V = SU(3)_{R+L}$: still conserved for $m_u = m_d = m_s > 0$
- $U(1)_V = U(1)_{R+L}$: quark or baryon number is conserved
- $U(1)_A = U(1)_{R-L}$: broken by quantum effects ($U(1)_A$ anomaly + instantons)

Hidden Symmetry and Goldstone Bosons

$$[Q_V^a, H] = 0, \quad \text{and} \quad e^{-iQ_V^a}|0\rangle = |0\rangle \Leftrightarrow Q_V^a|0\rangle = 0 \quad \text{(Wigner-Weyl realization)}$$
$$[Q_A^a, H] = 0, \quad \text{but} \quad e^{-iQ_A^a}|0\rangle \neq |0\rangle \Leftrightarrow Q_A^a|0\rangle \neq 0 \quad \text{(Nambu-Goldstone realiz.)}$$

- Consequence: $e^{-iQ_A^a}|0\rangle \neq |0\rangle$ is not the vacuum, but

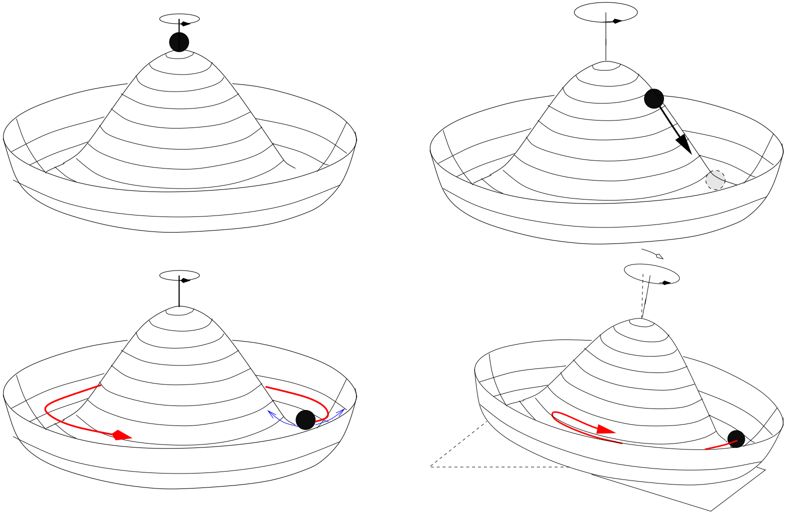
$$H\, e^{-iQ_A^a}|0\rangle = e^{-iQ_A^a} H|0\rangle = 0 \quad \text{is a massless state!}$$

Goldstone theorem: *continuous* global symmetry that does *not* leave the ground state invariant ('hidden' or 'spontaneously broken' symm.)

- mass- and spinless particles, "Goldstone bosons" (GBs)
- number of GBs = number of broken symmetry generators
- axial generators broken $\Rightarrow$ GBs should be pseudoscalars
- finite masses via (small) quark masses
  $\hookrightarrow$ 8 lightest hadrons: $\pi^\pm$, π^0, $K^\pm$, K^0, $\bar{K}^0$, η (not η')
- Goldstone bosons decouple (non-interacting) at vanishing energy

back

Illustration: spontaneous symmetry breaking (SSB)



back

Decoupling theorem of Goldstone bosons

Goldstone bosons do not interact at zero energy/momentum

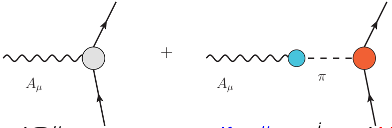
1. $Q_A^a|0\rangle \neq 0 \quad \Rightarrow \quad Q_A^a$ creates GB $\quad \Rightarrow \quad \langle \pi^a | Q_A^a | 0 \rangle \neq 0.$
2. Lorentz invariance $\rightsquigarrow \langle \pi^a(q) | A_b^\mu(x) | 0 \rangle = -i f_\pi \, q^\mu \, \delta_b^a \, e^{iq\cdot x} \neq 0\,!$

   A_b^μ axial current

   $\hookrightarrow f_\pi \neq 0$ necessary for SSB (order parameter)

   (pion decay constant $f_\pi = 92\,\mathrm{MeV}$ from weak decay $\pi^+ \to \mu^+ \nu_\mu$)
3. Coupling of axial current A_μ to matter fields (and/or pions)

$i\mathcal{A}^\mu =$



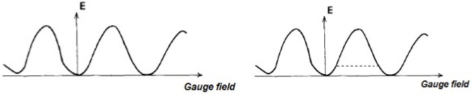
$$= \quad i\mathcal{R}^\mu \ \text{(non-sing.)} \quad + \quad -i f_\pi \, q^\mu \, \frac{i}{q^2 - m_\pi^2 + i\epsilon} \, i\,V \quad (V\text{: coupling of GB to matter fields})$$

4. Conservation of axial current $\partial_\mu A_b^\mu(x) = 0$: $\Rightarrow m_\pi^2 = 0$ and $q_\mu \mathcal{A}^\mu = 0$:

$$0 = q_\mu \mathcal{R}^\mu - f_\pi \frac{q^2}{q^2} V \ \xrightarrow{q\to 0}\ 0 = -f_\pi \lim_{q\to 0} V \ \xrightarrow{f_\pi \neq 0}\ \lim_{q\to 0} V = 0 \ \Rightarrow \ \text{decoupling!}$$

θ vacua in strong interaction physics

The topologically non-trivial vacuum structure of QCD



induces winding number n and strong gauge transformation (instanton)

$$\Omega_1 : |n\rangle \to |n+1\rangle$$

Naive vacuum therefore unstable (and violates cluster decomposition).

Thus true vacuum must be a superposition of the various $|n\rangle$ vacua
$\leadsto$ Theta vacuum:

$$|vac\rangle_\theta = \sum_{n=-\infty}^{+\infty} e^{in\theta}|n\rangle \qquad \text{with} \quad \Omega_1 : |vac\rangle_\theta \to e^{-i\theta}|vac\rangle_\theta \quad \text{(phase shift)}$$

Note

$${}_{\theta'}\langle vac|e^{-iHt}|vac\rangle_\theta = \delta_{\theta-\theta'} \times {}_\theta\langle vac|e^{-iHt}|vac\rangle_\theta$$

such that θ unique parameter of strong interaction physics which can be absorbed into the effective Lagrangian

$$\mathcal{L}_{\text{eff}} = \mathcal{L} + \frac{\theta}{16\pi^2 g^2}\text{Tr}\left(G_{\mu\nu}\tilde{G}^{\mu\nu}\right)$$

back

θ term

$$\mathcal{L}_{QCD} = \mathcal{L}_{QCD}^{\mathrm{CP}} + \theta \frac{g_s^2}{32\pi^2} \tilde{G}^a_{\mu\nu} G^{a,\mu\nu} = \mathcal{L}_{QCD}^{\mathrm{CP}} + \theta \frac{g_s^2}{32\pi^2} \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} G^a_{\mu\nu} G^a_{\alpha\beta}$$

- Under $U_A(1)$ rotation of the quark fields $q_f \to \mathrm{e}^{\mathrm{i}\alpha\gamma_5/2} q_f \approx (1 + \mathrm{i}\frac{1}{2}\alpha\gamma_5) q_f$:

$$\mathcal{L}_{QCD} \quad \to \quad \mathcal{L}_{QCD}^{\mathrm{CP}} - \alpha \sum_f m_f \bar{q} i\gamma_5 q + (\theta - N_f \alpha) \frac{g_s^2}{32\pi^2} \tilde{G}^a_{\mu\nu} G^{a,\mu\nu}$$

$\hookrightarrow \mathcal{L}_{SM}^{\mathrm{str}\,\cancel{CP}} = \mathcal{L}_{SM}^{\mathrm{CP}} - \bar{\theta} m^* \sum_f \bar{q}_f i\gamma_5 q_f$ with $\bar{\theta} = \theta + \arg\det\mathcal{M}$ and $m^* = \frac{m_u m_d}{m_u + m_d}$

back

Strong CP problem
Peccei-Quinn symmetry and axions

R. Peccei & H. Quinn (1977)

Consider adding a new field ϕ_A, the axion field, to the QCD action

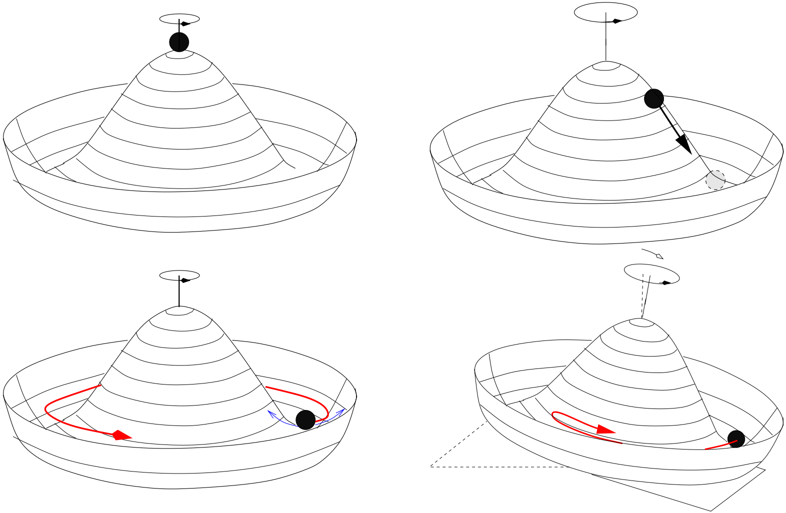
$$\mathcal{L}_{\rm axion} = \bar\psi(\mathcal{M}e^{-i\phi_A/f_A})\psi + \tfrac{1}{2}\partial_\mu\phi_A\partial^\mu\phi_A$$

- The axion arises as Goldstone boson of the new broken U(1) symmetry of the quark and the Higgs sector.
- Perform further axial U(1) transformation on quark fields to eliminate the $G\tilde{G}$ term entirely

  $\hookrightarrow$ new phase of quark mass term: $e^{i(\theta+\arg\det\mathcal{M}-\phi_A/f_A)}$
- Make the trivial U(1) shift $\phi_A \to \phi_A + \theta + (\arg\det\mathcal{M})f_A$.
  The kinetic term is invariant under this shift (ϕ_A massless to LO)
- The axion acquires its mass as $m_A f_A \approx m_\pi f_\pi$ with $f_A \gg v_{\rm weak}$
- New Problem: frustrating search for a (light) axion !

back

Illustration: spontaneous symmetry breaking (SSB)



back

EDM Measurements of Neutral Particles

and indirect EDM measurements of charged particles

- Neutron EDM experiments at ILL, SNS, PSI, TRIUMF
  - current $d_n = \big(0.2 \pm 1.5(\text{stat.}) \pm 0.7(\text{sys.})\big) \cdot 10^{-26}\,e\,\text{cm}$

    Baker et al. [ILL] (2006)

  - proposed $\searrow \sim 10^{-28}\,e\,\text{cm}$

- Diagmagnetic atoms
  - current $d(^{199}\text{Hg} \leq 3.1 \cdot 10^{-29}\,e\,\text{cm}$ (95%C.L.)

    Griffith et al. [UW] (2009)

    inferred $d_p \leq 7.9 \cdot 10^{-25}\,e\,\text{cm}$

    Dmitriev + Sen'kov (2003)

  - Ongoing experiments on Ra, Rn, Xe, . . .

- Dipolar YbF molecule measurement

  Hudson et al. (2011)

  - inferred $d_e \leq 1 \cdot 10^{-27}\,e\,\text{cm}$

Storage-Ring EDM Measurement of Charged Particles

Thomas–Bargmann-Michel-Telegdi (BMT) equation

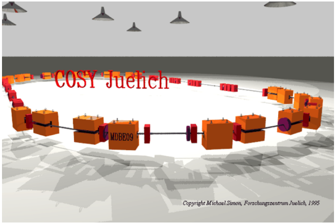
Farley et al. (2004)

$$\frac{d\vec{S}}{dt} = \vec{S}\times\vec{\Omega}\,, \qquad \vec{\Omega} = \frac{q}{m}\left[\underbrace{a}_{\substack{\text{anom.}\\ \text{magn.}\\ \text{mom.}}}\vec{B} + \left(\frac{1}{v^2\gamma^2} - a\right)\vec{v}\times\vec{E}\right] + \frac{1}{S}\underbrace{d_q}_{\text{EDM}}\left(\vec{E} + \vec{v}\times\vec{B}\right)$$

- Limit on muon EDM: $d_\mu \leq 1.8\cdot 10^{-19}\,e\,\mathrm{cm}$ (95% C.L.)
- Proposed storage ring exp.s of proton/deuteron EDMs $\sim 10^{-29}\,e\,\mathrm{cm}$

Counter-circling proton ring at Brookhaven or Fermilab ?

All-purpose ring (p, D, ^{3}He) at COSY ?



Non-relativistic reduction of

$$\mathcal{H}_{\rm eff} = -\tfrac{a_f}{2}\bar{f}\sigma^{\mu\nu} f\, F_{\mu\nu}, \quad a_f = \frac{F_2(0)}{2m_f}$$

$$
\begin{aligned}
& -\int d^3x\, \frac{a_f}{2}\, \bar\psi_f\, \sigma^{ij} \psi_f\, F_{ij} + \ldots \\
\to \quad & -\frac{a_f}{2}\int d^3x\, \bar\psi_f\, \epsilon^{ijk} \begin{pmatrix} \sigma^k & 0 \\ 0 & \sigma^k \end{pmatrix} \psi_f\, F_{ij} \\
= \quad & -\frac{a_f}{2}\int d^3x\, \bar\psi_f \begin{pmatrix} \sigma^k & 0 \\ 0 & \sigma^k \end{pmatrix} \psi_f\, \underbrace{\epsilon^{ijk} F_{ij}}_{-2B^k} \\
\to \quad & a_f \int d^3x\, \bar\psi_f\, \vec\sigma\, \psi_f \cdot \vec B \\
= \quad & a_f\, \langle\vec\sigma\rangle \cdot \vec B \\
= \quad & a_f\, g\, \langle\vec S\rangle \cdot \vec B\,, \qquad g = 2
\end{aligned}
$$

$$\mathcal{H}_{\rm eff} = \mathrm{i}\, \tfrac{d_f}{2}\bar{f}\sigma^{\mu\nu}\gamma_5 f\, F_{\mu\nu}, \quad d_f \equiv \frac{F_3(0)}{2m_f}$$

$$
\begin{aligned}
& \mathrm{i}\int d^3x\, \frac{d_f}{2}\, \bar\psi_f\, \sigma^{0i}\gamma_5\, \psi_f\, F_{0i} \times 2 + \ldots \\
\to \quad & \mathrm{i} d_f \int d^3x\, \bar\psi_f\, \mathrm{i} \begin{pmatrix} 0 & \sigma_i \\ \sigma^i & 0 \end{pmatrix} \gamma_5\, \psi_f\, F^{i0} \\
= \quad & \mathrm{i} d_f \int d^3x\, \bar\psi_f\, \mathrm{i} \begin{pmatrix} \sigma^i & 0 \\ 0 & \sigma^i \end{pmatrix} \psi_f\, \underbrace{F^{i0}}_{E^i} \\
\to \quad & \mathrm{i}^2 d_f \int d^3x\, \bar\psi_f\, \vec\sigma\, \psi_f \cdot \vec E \\
= \quad & -d_f\, \langle\vec\sigma\rangle \cdot \vec E \\
= \quad & -d_f\, \langle\vec S/S\rangle \cdot \vec E
\end{aligned}
$$

back

Construction of the CKM matrix

Since weak interactions do not respect the global flavor symmetry, there is mixing within the groups of quarks with the same charge:

$$U \equiv \begin{pmatrix} u \\ c \\ t \end{pmatrix} \to \tilde{U} = M_U U\,, \qquad D \equiv \begin{pmatrix} d \\ s \\ b \end{pmatrix} \to \tilde{D} = M_D D\,,$$

where M_U & M_D are 3×3 unitary matrices

$\hookrightarrow$ charged weak current: $J_\mu = \bar{\tilde{U}}^\mu \gamma_\mu (1-\gamma_5) \tilde{D}^\mu = \bar{U} \gamma_\mu (1-\gamma_5) \underbrace{M_U^\dagger M_D}_{\text{CKM matrix } M} D\,.$

- M unitary $N_f \times N_f$ matrix for N_f quark families $\rightsquigarrow$ $\boxed{N_f^2 \text{ real parameters}}$.
- $2N_f - 1$ of these can be absorbed by the relative phases of the quark wave functions $\rightsquigarrow$ $\boxed{(N_f - 1)^2}$ remaining parameters:

  $N_f = 2$: one remaining real parameter: *Cabibbo angle*
  $N_f = 3$: $O(3)$ matrix with $\frac{1}{2} 3 \cdot (3-1) = $ 3 angles plus 1 $\cancel{CP}$ phase
- Lepton case: neutrinos may be Majoranas: $\rightsquigarrow$ 3 angles plus 3 $\cancel{CP}$ phases
- If phase(s) present, M complex matrix, whereas CP invariance $\rightsquigarrow$ $M^* = M$!

Transformation Properties of the Form Factor Γ^μ

$A_\mu \langle f(p')|J^\mu_{\rm em}|f(p)\rangle = A_\mu\, \bar u_f(p')\,\Gamma^\mu(q^2)\, u_f(p)$ with

$$\Gamma^\mu(q^2) = \gamma^\mu F_1(q^2) + i\sigma^{\mu\nu} q_\nu \frac{F_2(q^2)}{2m_f} + \sigma^{\mu\nu} q_\nu \gamma_5 \frac{F_3(q^2)}{2m_f} + (\not{q}\, q^\mu - q^2\gamma^\mu)\gamma_5 F_a(q^2)/m_f^2$$

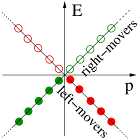
Op.	P	C	CP	T	CPT
A_μ	A^μ	$-A_\mu$	$-A^\mu$	A^μ	$-A_\mu$
γ^μ	γ_μ	$-\gamma^\mu$	$-\gamma_\mu$	γ_μ	$-\gamma^\mu$
$\gamma^\mu\gamma_5$	$-\gamma_\mu\gamma_5$	$\gamma^\mu\gamma_5$	$-\gamma_\mu\gamma_5$	$\gamma_\mu\gamma_5$	$-\gamma^\mu\gamma_5$
$\sigma^{\mu\nu}$	$\sigma_{\mu\nu}$	$-\sigma^{\mu\nu}$	$-\sigma_{\mu\nu}$	$-\sigma_{\mu\nu}$	$\sigma^{\mu\nu}$
$\sigma^{\mu\nu} q_\nu$	$\sigma_{\mu\nu} q^\nu$	$-\sigma^{\mu\nu} q_\nu$	$-\sigma_{\mu\nu} q^\nu$	$-\sigma_{\mu\nu} q^\nu$	$\sigma^{\mu\nu} q_\nu$
$i\sigma^{\mu\nu} q_\nu$	$i\sigma_{\mu\nu} q^\nu$	$-i\sigma^{\mu\nu} q_\nu$	$-i\sigma_{\mu\nu} q^\nu$	$i\sigma_{\mu\nu} q^\nu$	$-i\sigma^{\mu\nu} q_\nu$
$\sigma^{\mu\nu} q_\nu\gamma_5$	$-\sigma_{\mu\nu} q^\nu\gamma_5$	$-\sigma^{\mu\nu} q_\nu\gamma_5$	$\sigma_{\mu\nu} q^\nu\gamma_5$	$-\sigma_{\mu\nu} q^\nu\gamma_5$	$-\sigma^{\mu\nu} q_\nu\gamma_5$

For EDMs of charged particles both $F_1(q^2)$ and $F_3(q^2)$ are present at the same time $\rightsquigarrow$ mixing

back

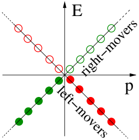
$U_A(1)$ anomaly in 1+1 D

1. Dispersion for massless fermions in 1+1 D:



$U_A(1)$ anomaly in 1+1 D

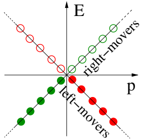
1. Dispersion for massless fermions in 1+1 D:



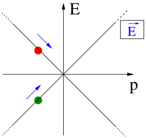
2. Add electric field
   to a single right/left-movers:

$U_A(1)$ anomaly in 1+1 D

1. Dispersion for massless fermions in 1+1 D:



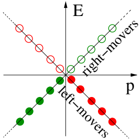
2. Add electric field
to a single right/left-movers:



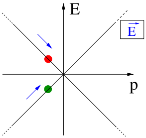
Nothing happens to single states, except energy change.

$U_A(1)$ anomaly in 1+1 D

1. Dispersion for massless fermions in 1+1 D:



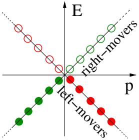
2. Add electric field
   to a single right/left-movers:



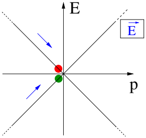
Nothing happens to single states, except energy change.

$U_A(1)$ anomaly in 1+1 D

1. Dispersion for massless fermions in 1+1 D:



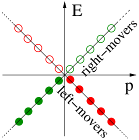
2. Add electric field
to a single right/left-movers:



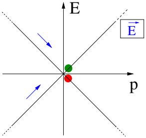
Nothing happens to single states,
except energy change.

$U_A(1)$ anomaly in 1+1 D

1. Dispersion for massless fermions in 1+1 D:



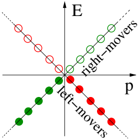
2. Add electric field
   to a single right/left-movers:



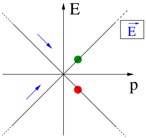
Nothing happens to single states, except energy change.

$U_A(1)$ anomaly in 1+1 D

1. Dispersion for massless fermions in 1+1 D:



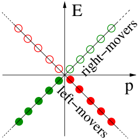
2. Add electric field
   to a single right/left-movers:



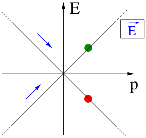
Nothing happens to single states, except energy change.

$U_A(1)$ anomaly in 1+1 D

1. Dispersion for massless fermions in 1+1 D:



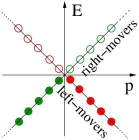
2. Add electric field
   to a single right/left-movers:



Nothing happens to single states, except energy change.

$U_A(1)$ anomaly in 1+1 D

1. Dispersion for massless fermions in 1+1 D:

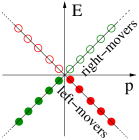


2. Add electric field
to a single right/left-movers:

Nothing happens to single states,
except energy change.

$U_A(1)$ anomaly in 1+1 D

1. Dispersion for massless fermions in 1+1 D:



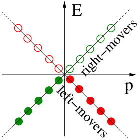
2. Add electric field
   to a single right/left-movers:

Nothing happens to single states,
except energy change.

3. However, for *infinite mode sums* the level shifts induce a surplace of right-movers:

$U_A(1)$ anomaly in 1+1 D

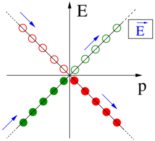
1 Dispersion for massless fermions in 1+1 D:



2 Add electric field
to a single right/left-movers:

Nothing happens to single states,
except energy change.

3 However, for *infinite mode sums* the level shifts induce a surplace of right-movers:



$$\dot{Q}_V = \dot{Q}_R + \dot{Q}_L = 0$$

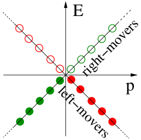
(vector charge still conserved)

$$\dot{Q}_A = \dot{Q}_R - \dot{Q}_L \neq 0$$

(axial charge not conserved)

$U_A(1)$ anomaly in 1+1 D

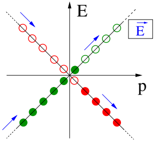
1. Dispersion for massless fermions in 1+1 D:



2. Add electric field
to a single right/left-movers:

Nothing happens to single states,
except energy change.

3. However, for *infinite mode sums* the level shifts induce a surplace of right-movers:



$$\dot{Q}_V = \dot{Q}_R + \dot{Q}_L = 0$$

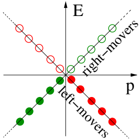
(vector charge still conserved)

$$\dot{Q}_A = \dot{Q}_R - \dot{Q}_L \neq 0$$

(axial charge not conserved)

$U_A(1)$ anomaly in 1+1 D

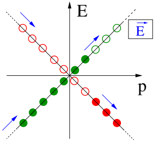
1. Dispersion for massless fermions in 1+1 D:



2. Add electric field
to a single right/left-movers:

Nothing happens to single states,
except energy change.

3. However, for *infinite mode sums* the level shifts induce a surplace of right-movers:



$$\dot{Q}_V = \dot{Q}_R + \dot{Q}_L = 0$$

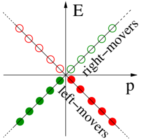
(vector charge still conserved)

$$\dot{Q}_A = \dot{Q}_R - \dot{Q}_L \neq 0$$

(axial charge not conserved)

$U_A(1)$ anomaly in 1+1 D

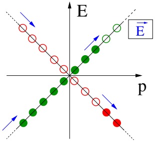
1. Dispersion for massless fermions in 1+1 D:



2. Add electric field
to a single right/left-movers:

Nothing happens to single states,
except energy change.

3. However, for *infinite mode sums* the level shifts induce a surplace of right-movers:



$$\dot{Q}_V = \dot{Q}_R + \dot{Q}_L = 0$$

(vector charge still conserved)

$$\dot{Q}_A = \dot{Q}_R - \dot{Q}_L \neq 0$$

(axial charge not conserved)