Optimization of Spin Coherence Time for Electric Dipole Moment measurements at Storage Rings

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In this presentation…

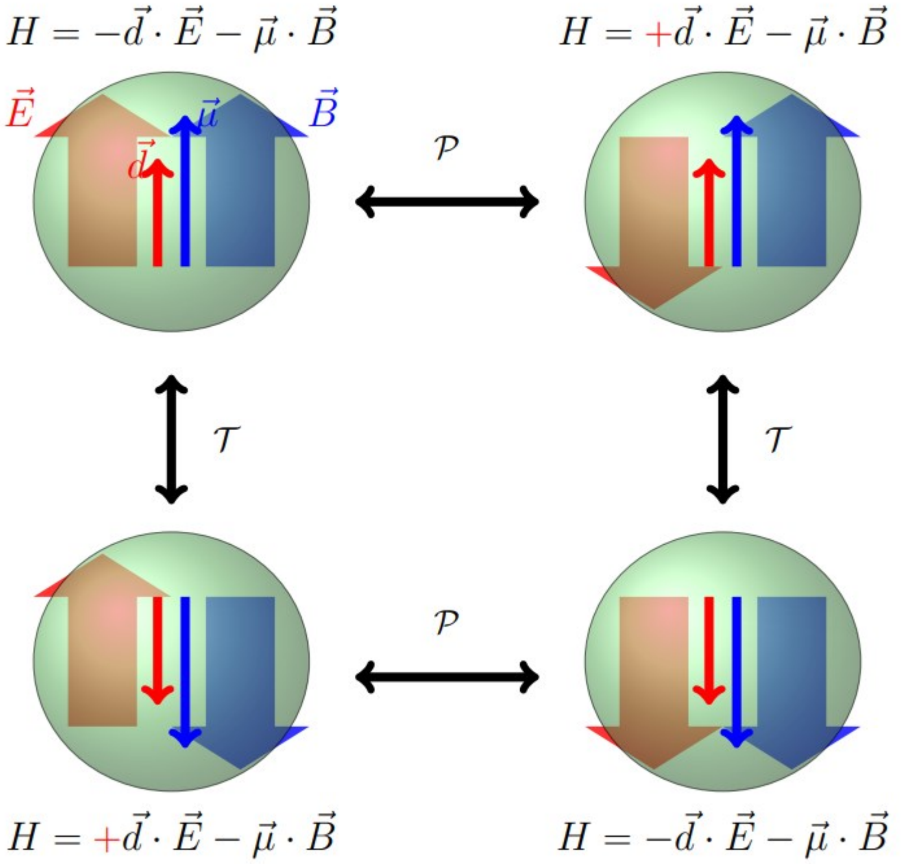
Introduction
- CP violation
- Frozen spin method
- Prototype Ring

Spin tune model
- Empirical spin tune model
- Longitudinal focusing constraint

Spin coherence time optimization
- Brute force
- Optimization using the spin tune model

Summary
- Conclusions
- Ongoing work

What is EDM?



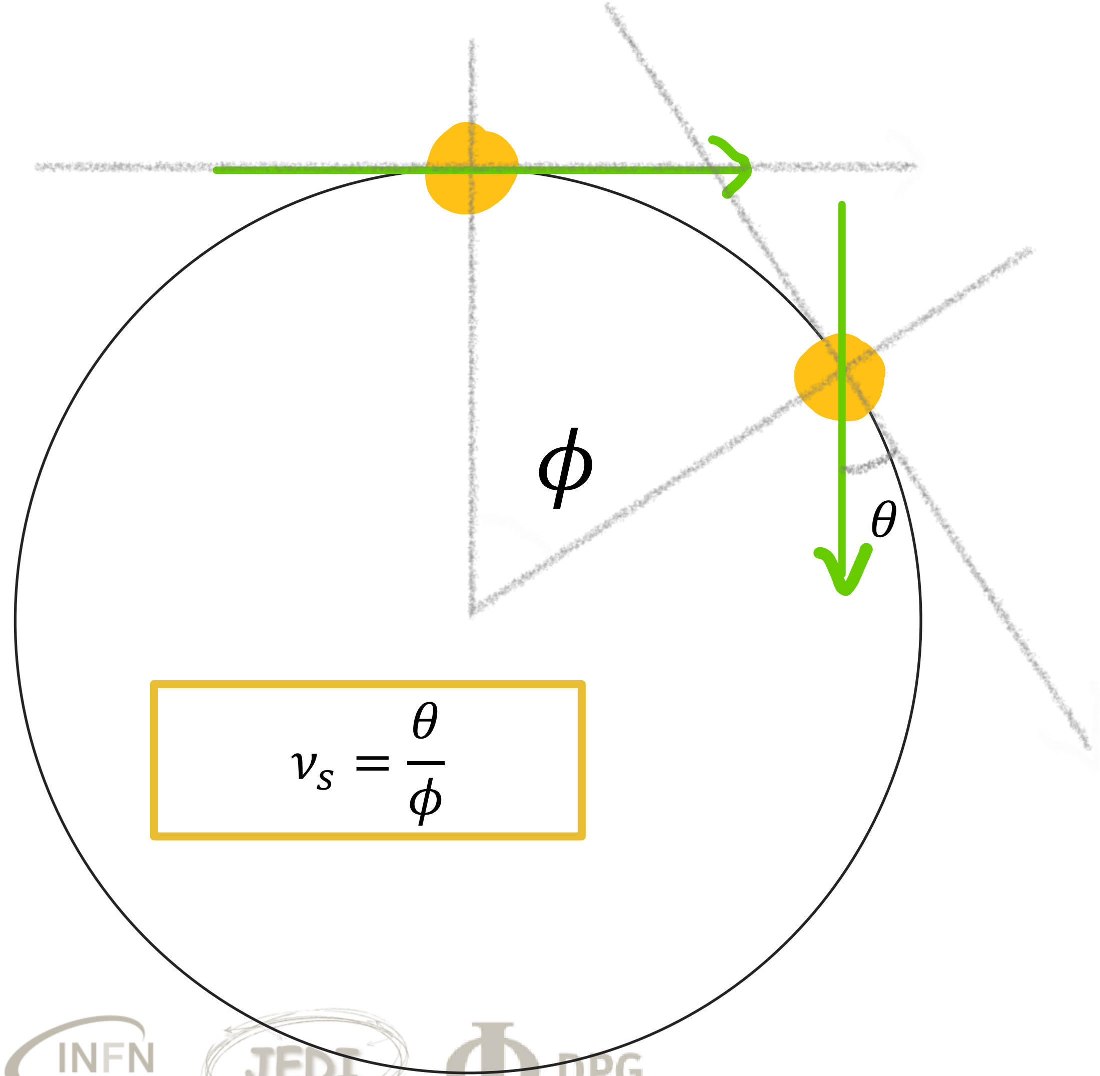
[1]

- Permanent separation of + and - charge
- Fundamental property of particles (like magnetic moment, mass, charge)
- Possible via violation of time-reversal (T) and parity (P)

EDM meas. test violation of P and T symmetries ($\overset{CPT}{=}$ CP)

EDM aligned with spin $\Rightarrow$ Spin Tracking

But what is a "spin tune"?



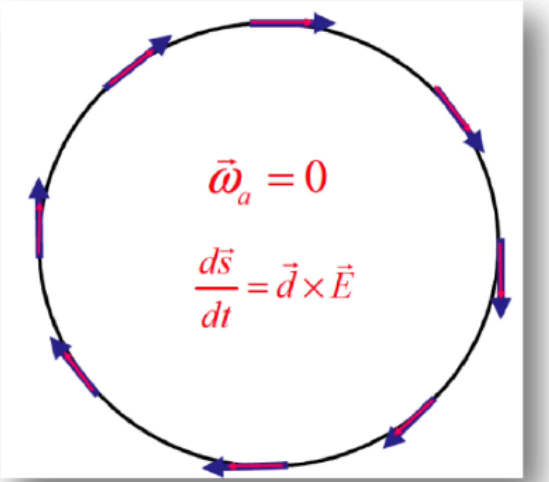
$$\nu_s = \frac{\theta}{\phi}$$

For a single particle, the angular change in the horizontal spin orientation in its rest frame for every radian covered by the particle in the ring is called the “spin tune”.

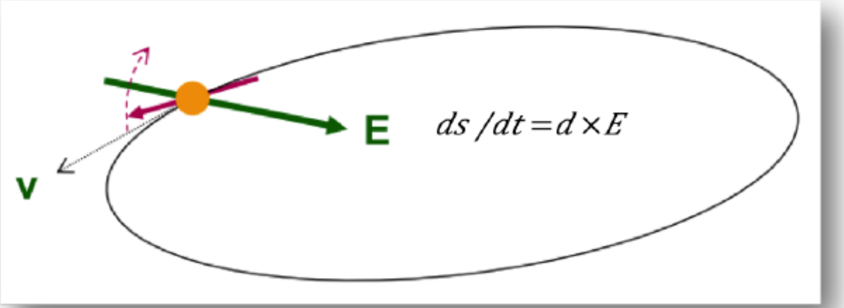
EDM Measurement with the Frozen Spin method

Measurement concept

1. Inject particles in storage ring
2. Align spin along momentum ($\rightarrow$ freeze horiz. spin-precession)
3. Search for time development of vertical polarization



@ Frozen Spin: $\boldsymbol{\nu_s = 0}$



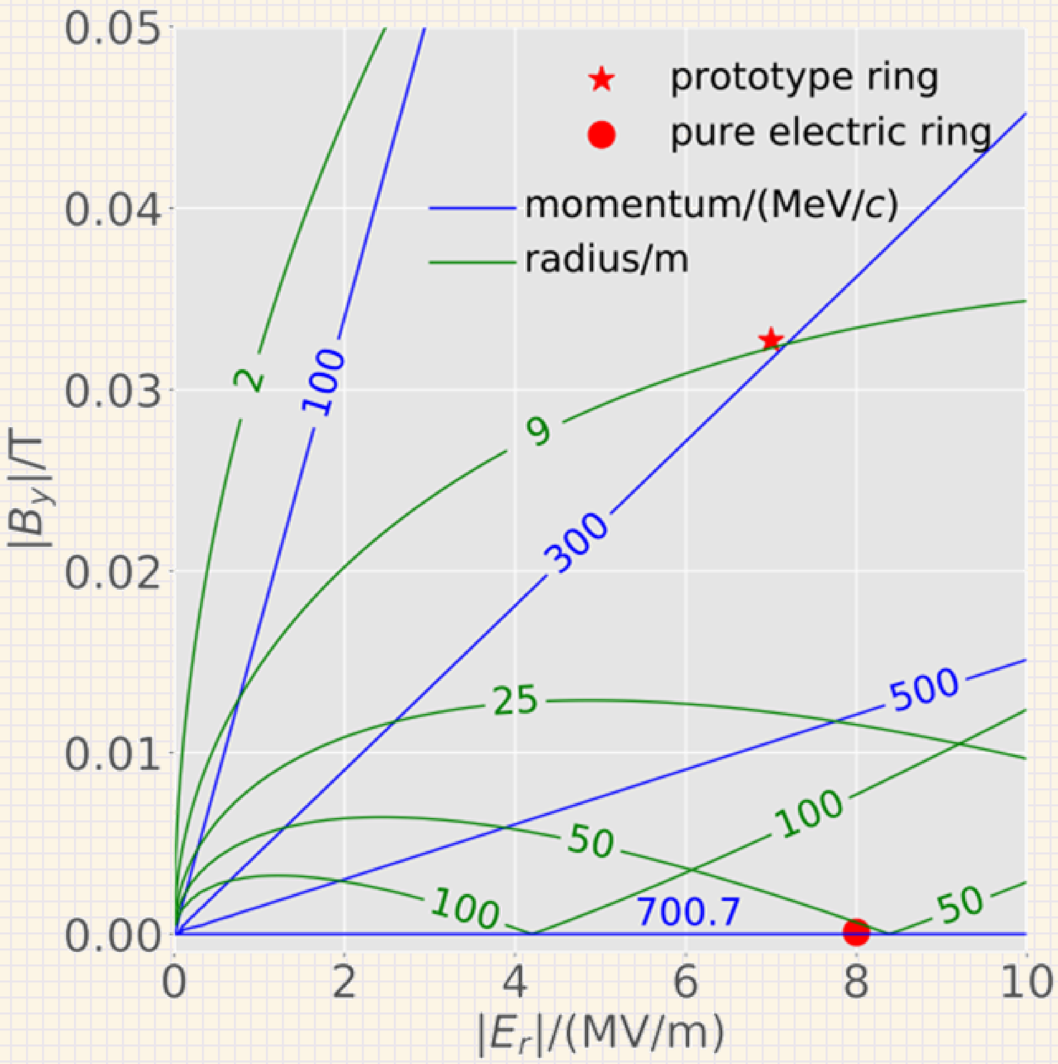
“Frozen spin” dynamics

From the BMT equation, for a pure magnetic ring: $\nu_s = \gamma G$

For combined EM ring:

$$\nu_s = \gamma G - \frac{r(G+1)}{\gamma(\beta + r)}; \quad r = \frac{E}{cB}$$

At frozen spin, $\nu_s = 0$.



The Prototype EDM Ring

Diameter $\approx 29\ m$

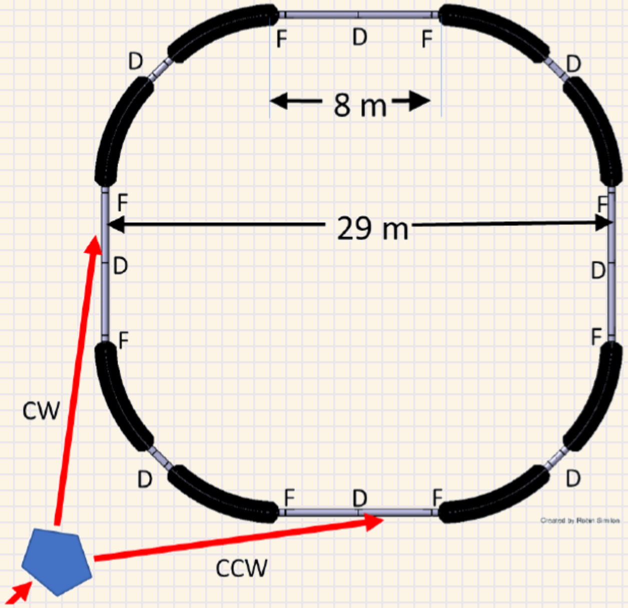
Bending Radius: $8.86\ m$

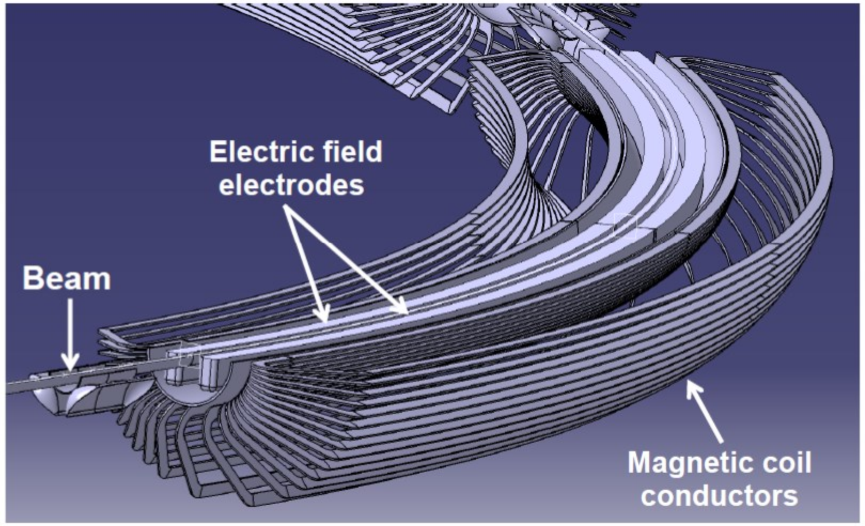
Design momentum for frozen-spin: $294.057\ MeV/c$

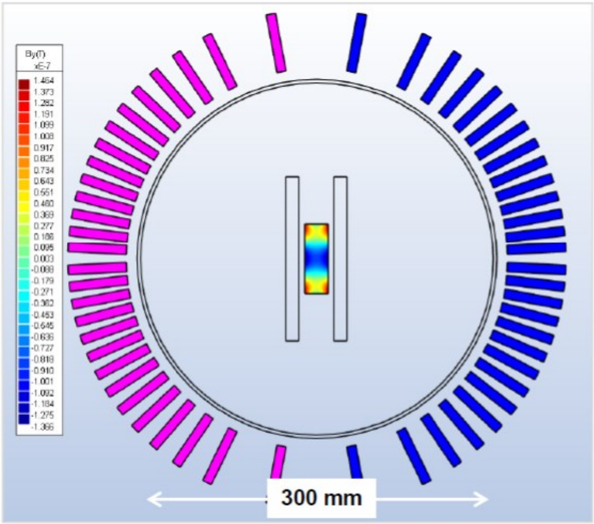
Electric Field: $7\ MV/m$

Magnetic Field: $32.7\ mT$

Non-Simultaneous ↻ and ↺ beams







The Hybrid EDM Ring

Diameter $\approx 255\ m$

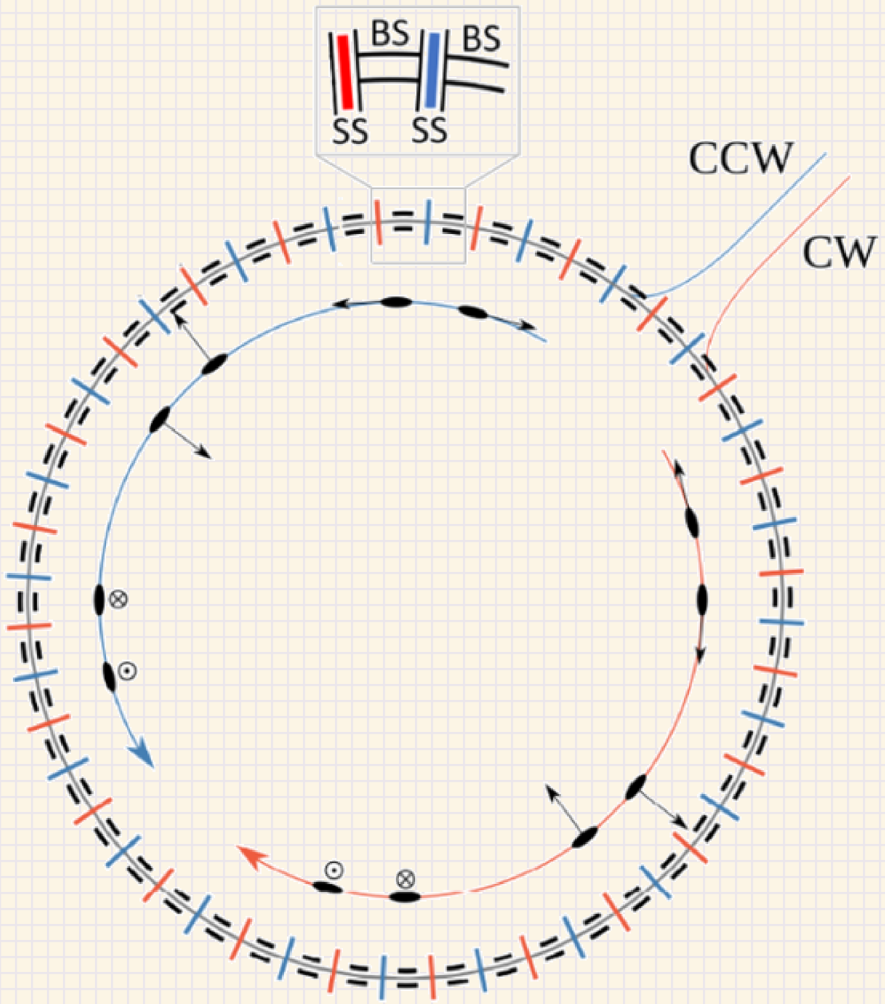
Bending Radius: $95.49\ m$

Design momentum for frozen-spin: $\approx 700\ MeV/c$

Electric Field: $4.4\ \ MV/m$

Magnetic Field: $0\ mT$

Simultaneous ↻ and ↺ beams



Spin Tune Model

A DATA-DRIVEN APPROACH

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Parameter Space

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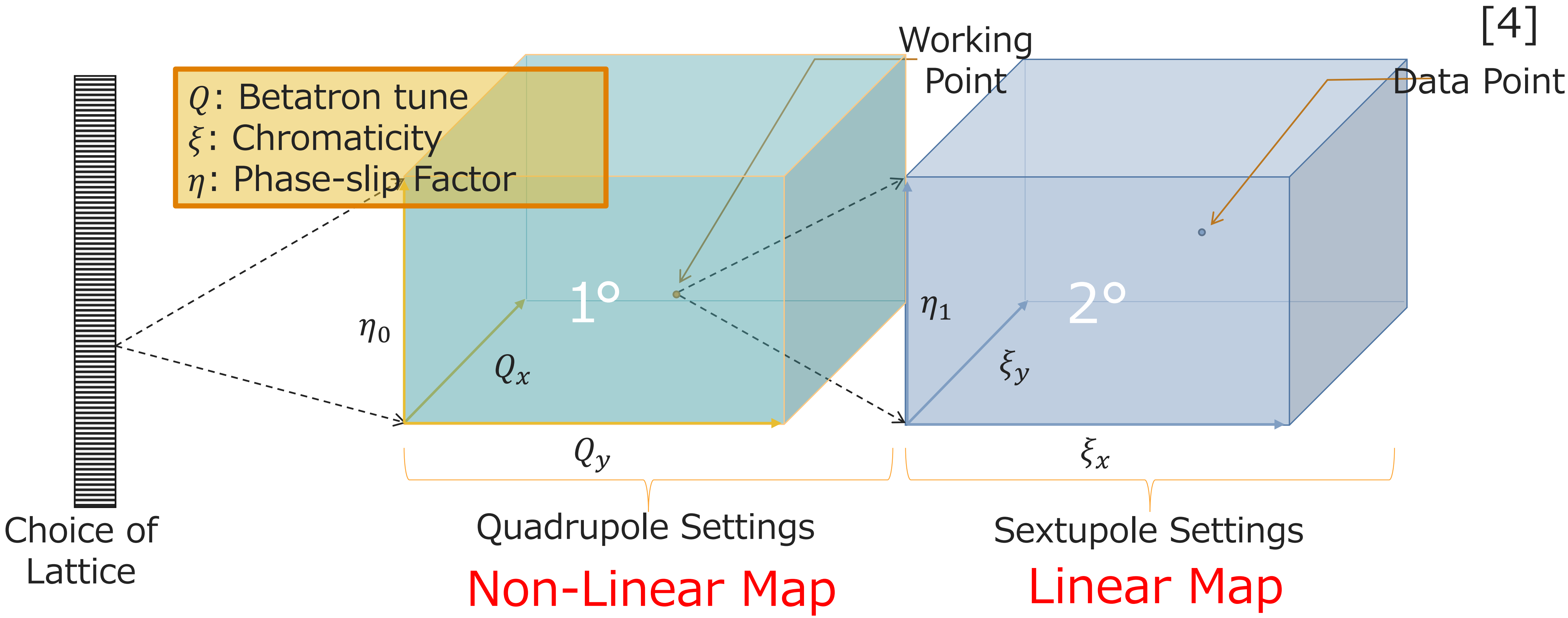


The empirical spin tune model

3×3 Matrix of Lattice properties (coefficients)

1° spin-tune factor

Natural offset vector (coefficients)

Vector of phase-space properties (emittances)

Spin tune of single particle

Momentum offset

Vector of 2° optical properties (chromaticity and phase slip)

$$\nu_s = \sigma_0 \delta + a_i \epsilon^i + V_{ij} \epsilon^i \xi^j$$

$$[\epsilon^i] = \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \delta^2 \end{bmatrix}$$

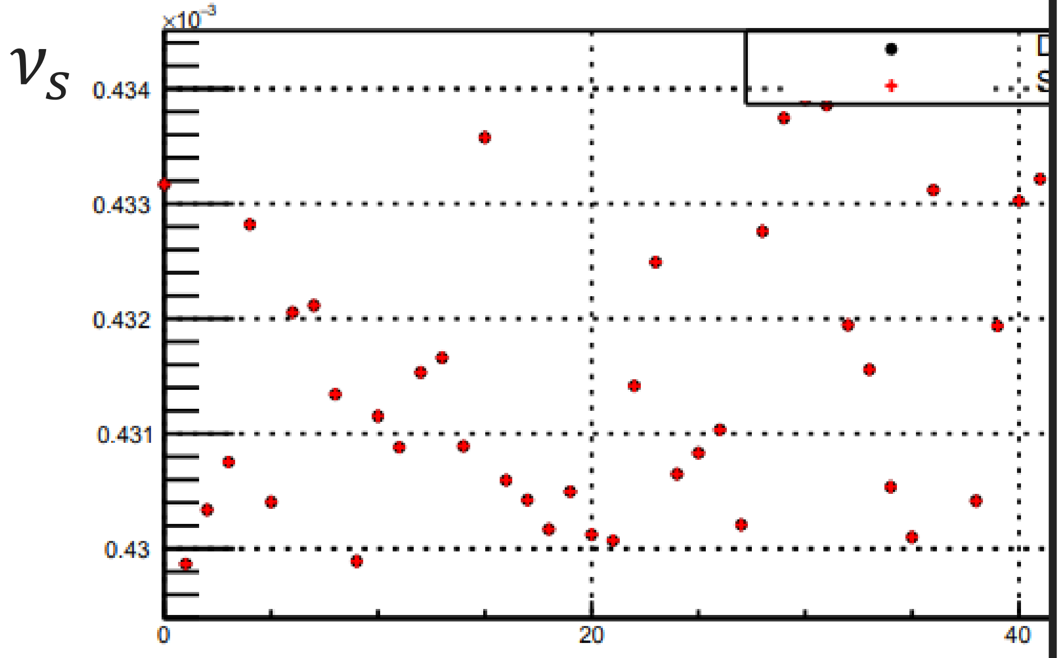
$$[\xi^j] = \begin{bmatrix} \xi_x \\ \xi_y \\ \eta_1 \end{bmatrix}$$

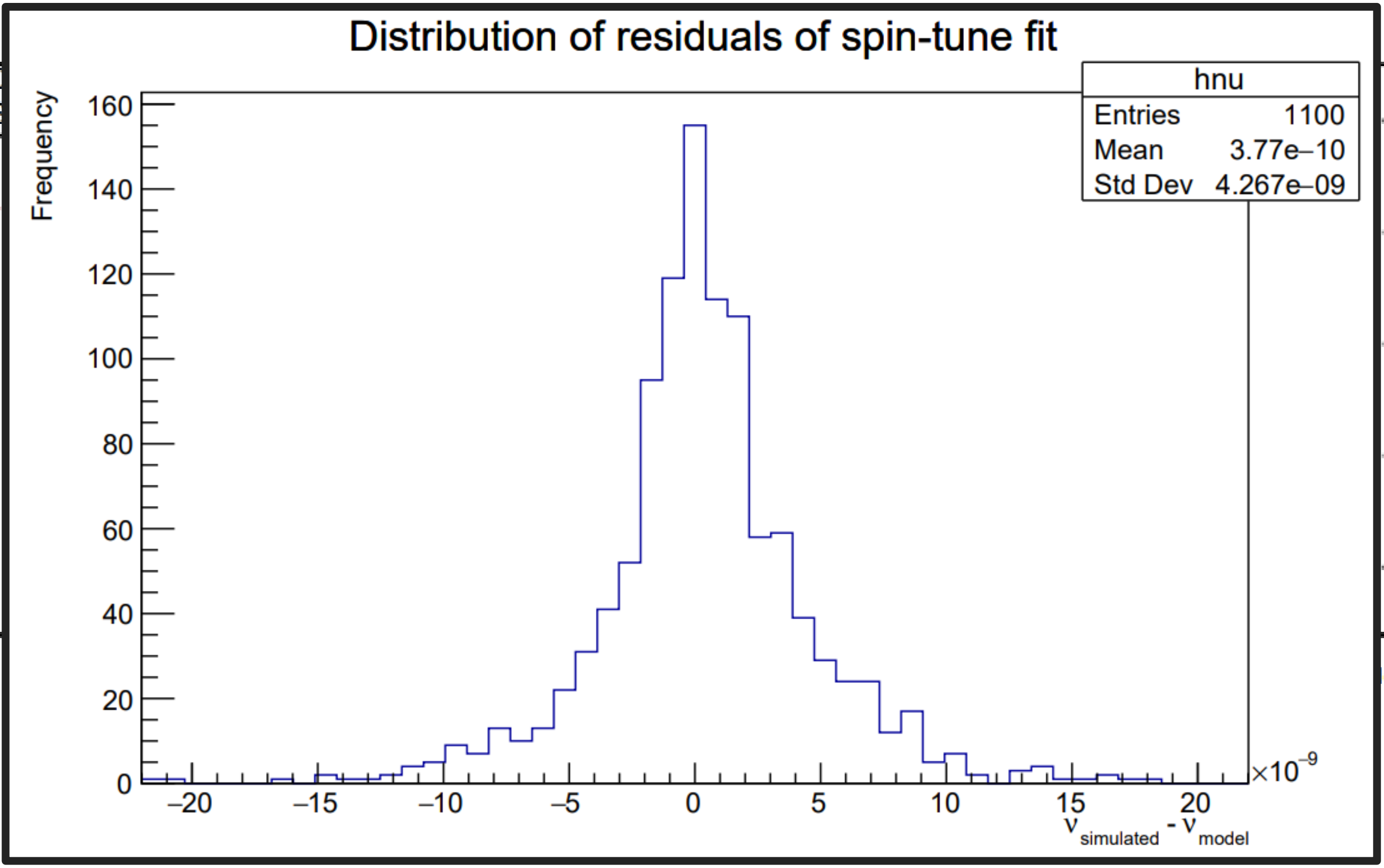
$$[V^i{}_j] = \begin{bmatrix} V^1{}_1 & V^1{}_2 & V^1{}_3 \\ V^2{}_1 & V^2{}_2 & V^2{}_3 \\ V^3{}_1 & V^3{}_2 & V^3{}_3 \end{bmatrix}$$

$$[a_i] = [a_1 \quad a_2 \quad a_3]$$

Testing the accuracy of spin tune model (Prototype)

$$\nu_s = \sigma_0\delta + a_i\epsilon^i + V_{ij}\epsilon^i\xi^j$$



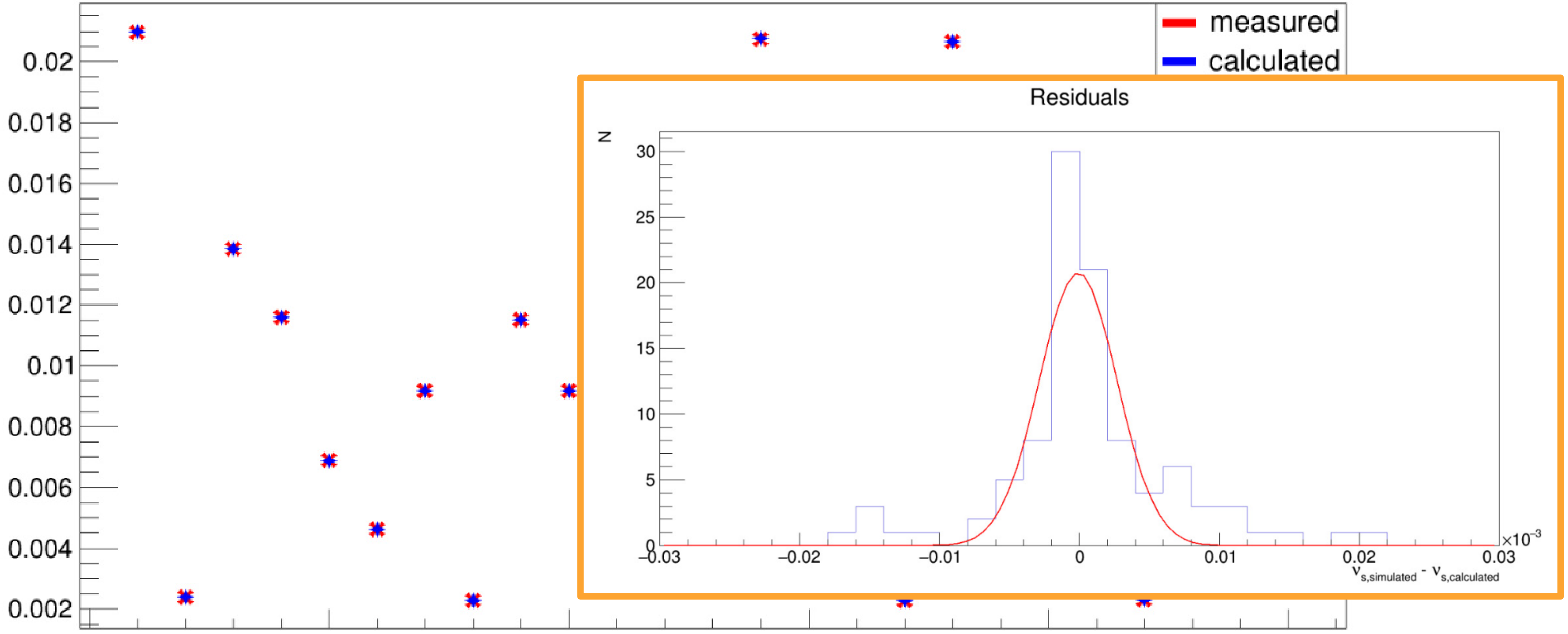


$\xi_x, \xi_y, \eta_1 \in (1.0, -1.0)$

Testing the accuracy of spin tune model (Hybrid)

$$\nu_s = \sigma_0 \delta + a_i \epsilon^i + V_{ij} \epsilon^i \xi^j$$

Std dev: $\approx 10^{-6}$



$\xi_x, \xi_y, \eta_1 \in (-4.0, 4.0)$

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Empirical Spin Model (Time-averaged for longitudinal bunching)

$$\left\langle \frac{\Delta T}{T} \right\rangle = 0$$

Natural offset vector (coefficients)

Vector of time-averaged phase-space properties (emittances)

3×3 Matrix of Lattice properties (coefficients)

Time-averaged spin tune of single particle

Vector of 2° optical properties (chromaticity and phase slip)

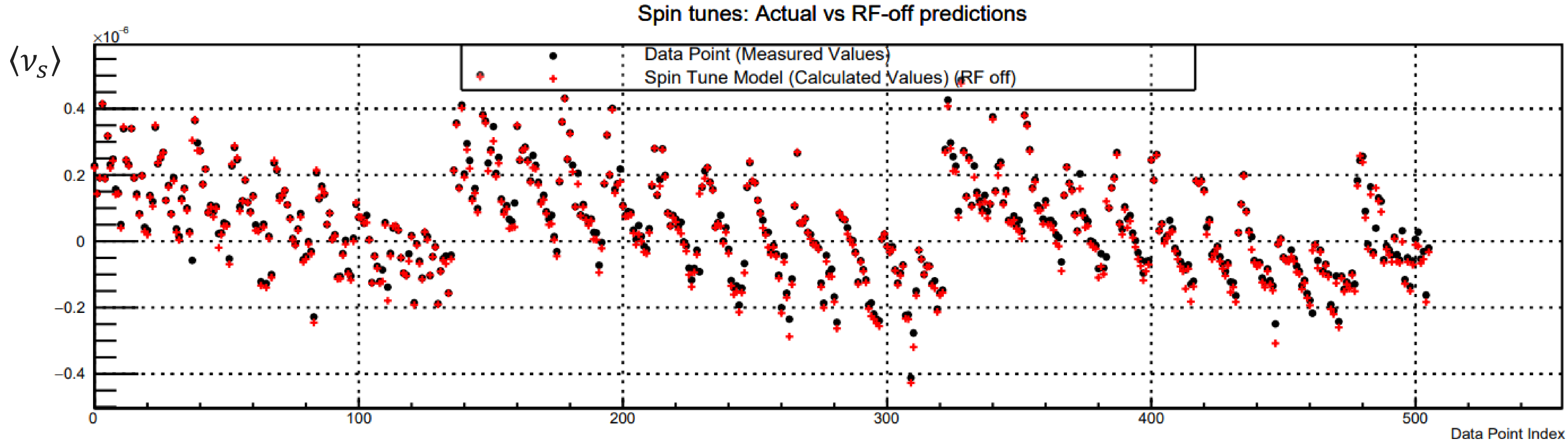
$$\langle \nu_s \rangle = a_i \langle \epsilon \rangle^i + M_{ij} \langle \epsilon \rangle^i \xi^j$$

$$[\langle \epsilon \rangle^i] = \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \delta_a^2/_2 \end{bmatrix} \qquad [\xi^j] = \begin{bmatrix} \xi_x \\ \xi_y \\ \eta_1 \end{bmatrix}$$

Testing the accuracy of phase slip model (RF on)

$$\langle \nu_s \rangle = a_i \langle \epsilon \rangle^i + M_{ij} \langle \epsilon \rangle^i \xi^j$$

Std dev: $\approx 10^{-9}$



$\xi_x, \xi_y, \eta_1 \in (-4.0, 4.0)$

Testing the accuracy of phase slip model (RF on)

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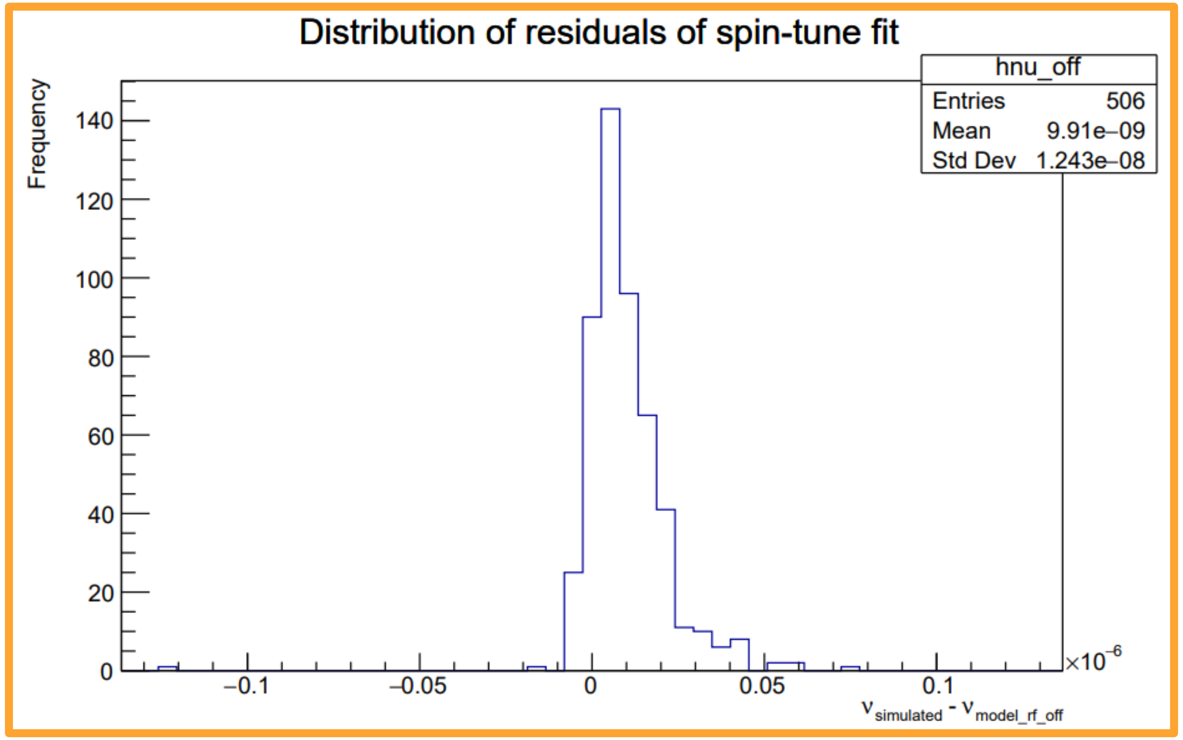
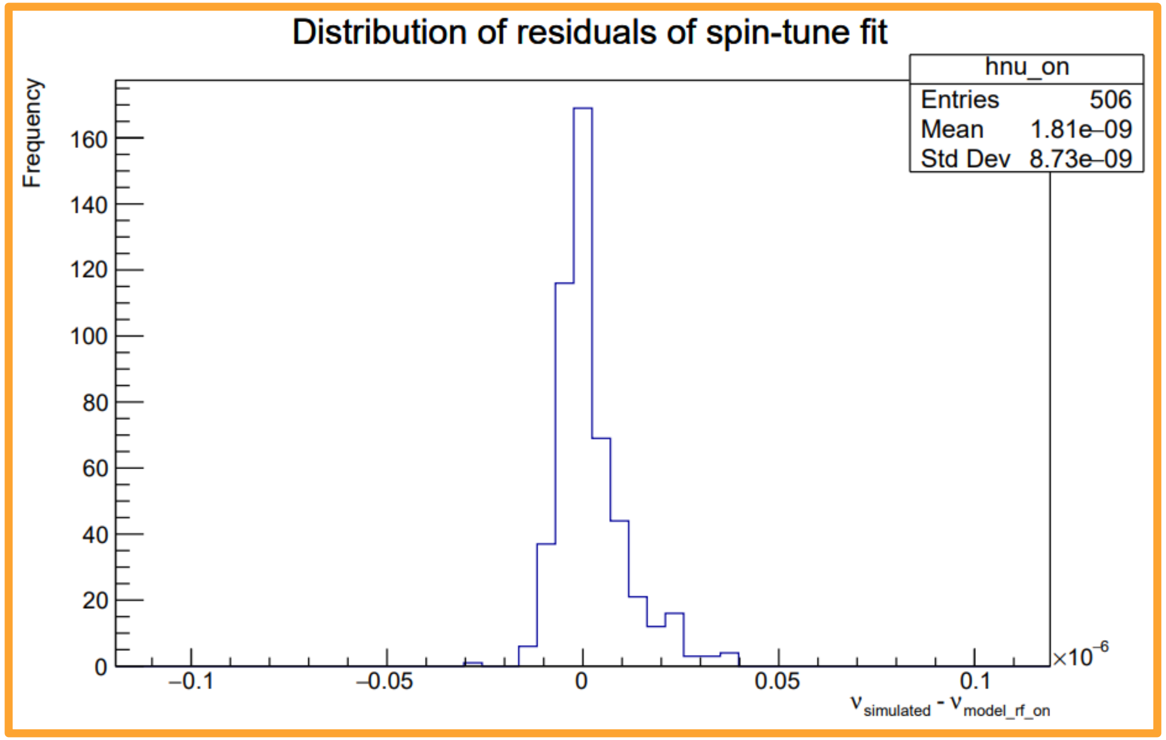
Std dev: $\approx 10^{-9}$

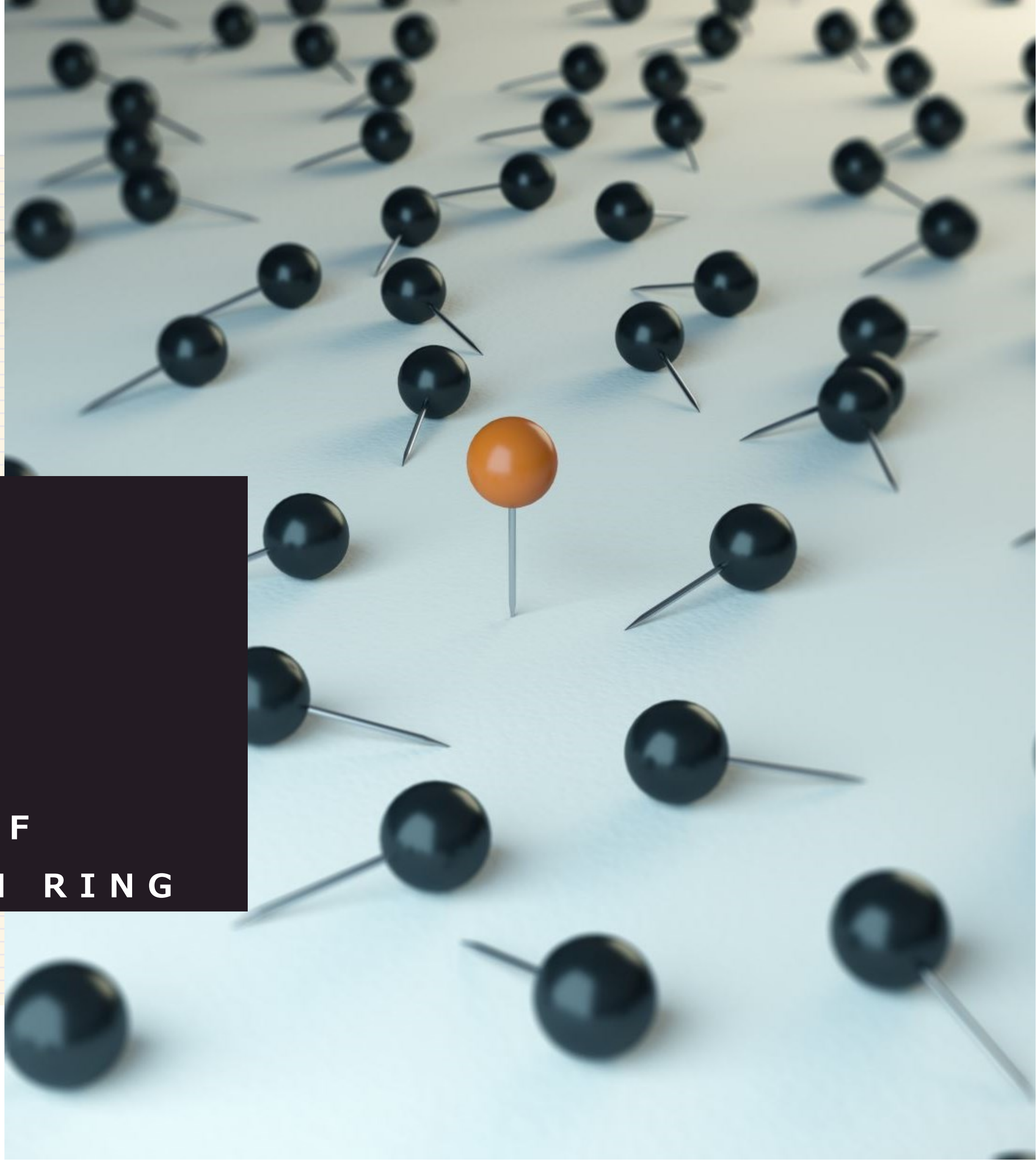


$\xi_x, \xi_y, \eta_1 \in (-4.0, 4.0)$



Spin Coherence Time

BEHAVIOR OF AN ENSEMBLE OF PARTICLES IN A FROZEN-SPIN RING



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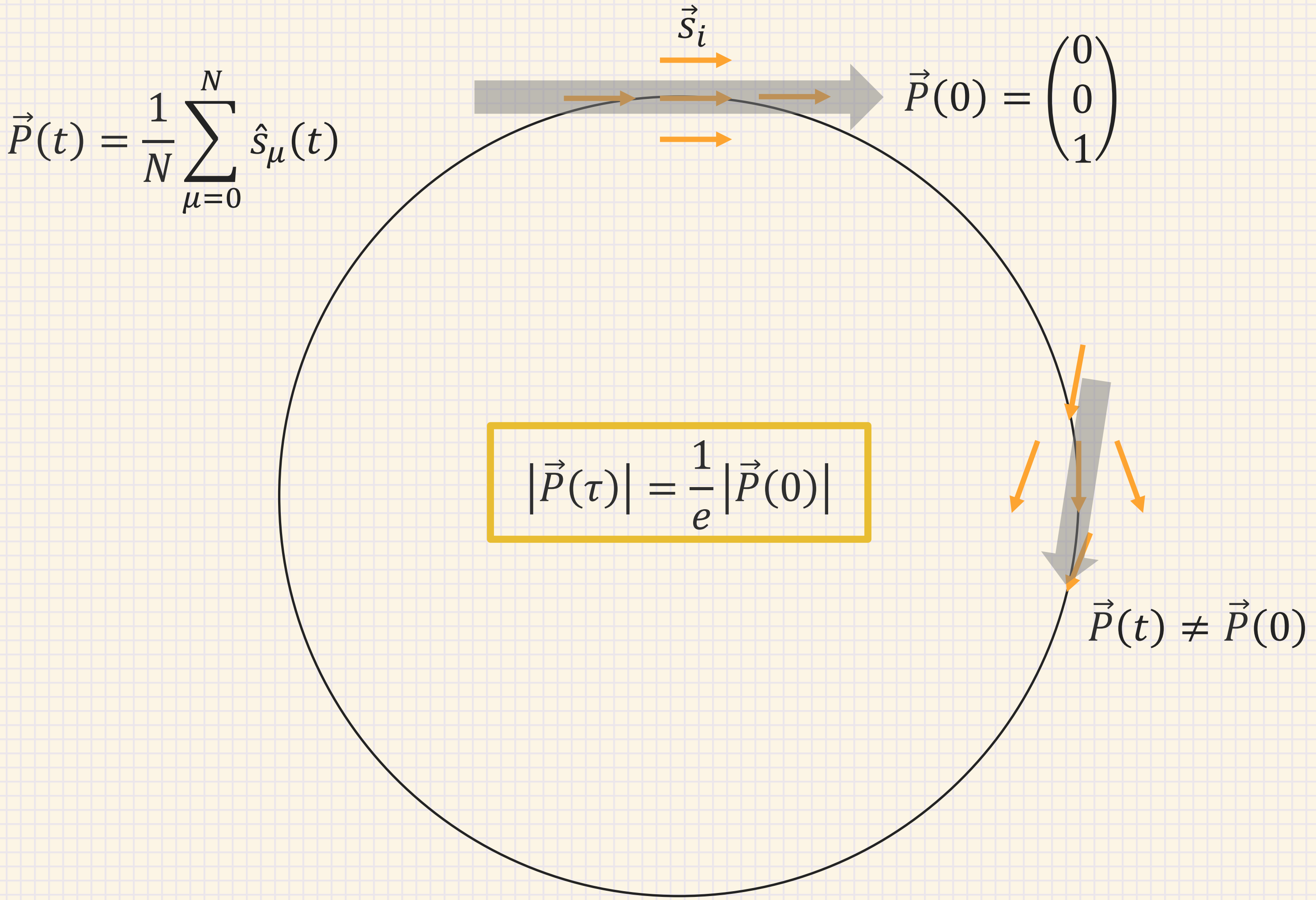
What is Spin Coherence Time

Time taken for the polarisation vector $\vec{P}(t)$ of an ensemble of particles to reduce to $1/e$ of its initial value due to decoherence.

For the required precision in EDM measurement, we require $\tau \geq 1000s$ at a frozen-spin storage ring.



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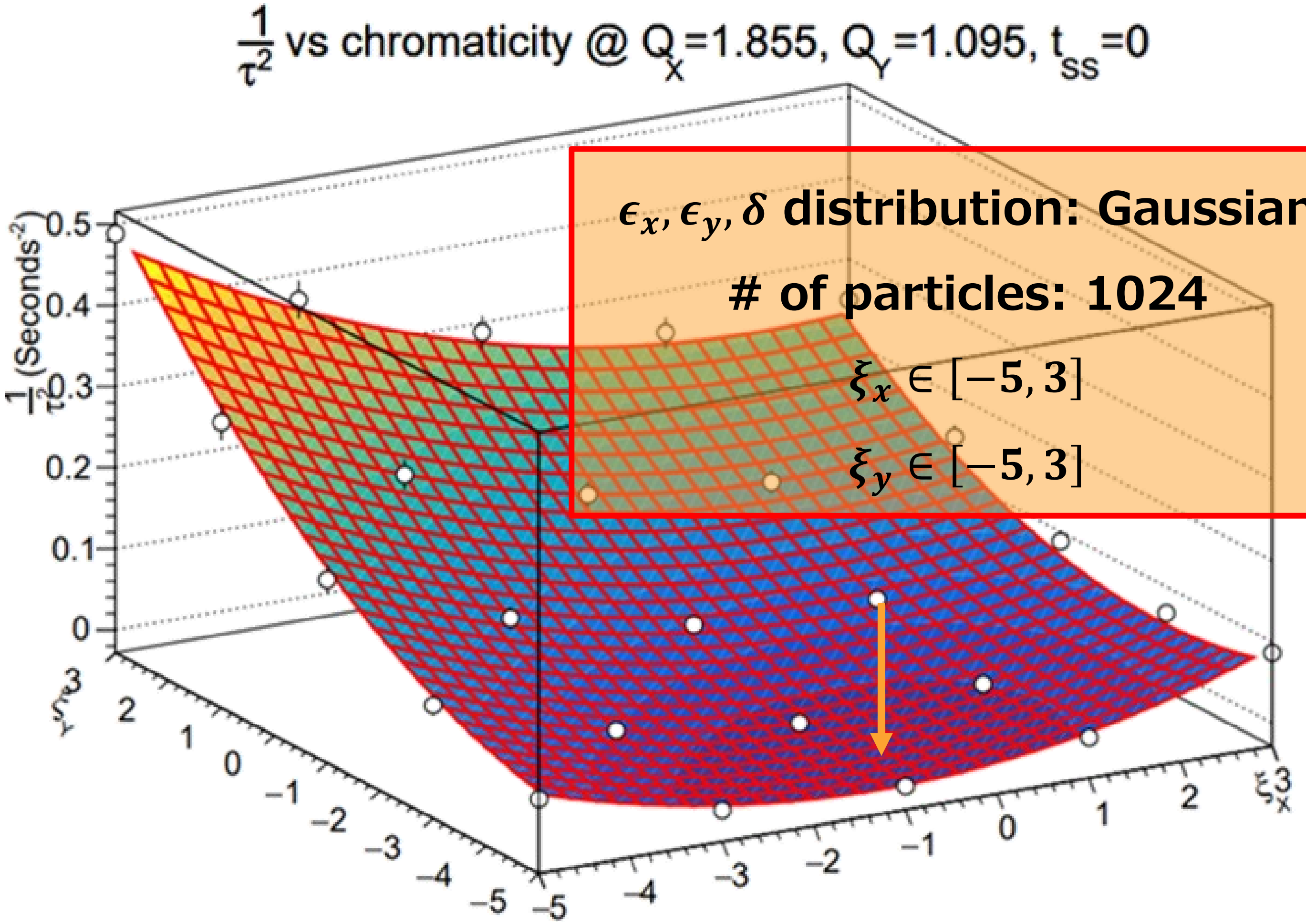
Optimization of Spin Coherence Time



$\epsilon_x, \epsilon_y, \delta$ **distribution: Gaussian**

# of particles: 1024

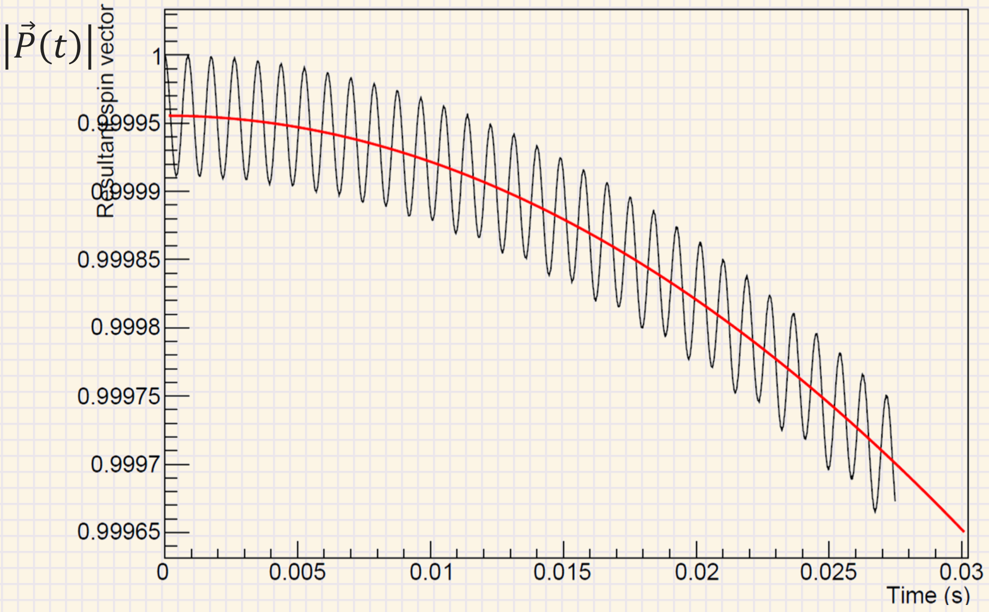
$\xi_x \in [-5, 3]$

$\xi_y \in [-5, 3]$

$$|\vec{P}(t)| = \left| \frac{1}{N} \sum_{\mu=0}^{N} \hat{s}_\mu(t) \right|$$

Spin tune of single particle:

$$\langle \nu_s \rangle = a_i \langle \epsilon \rangle^i + M_{ij} \langle \epsilon \rangle^i \xi^j$$



Optimization of Spin Coherence Time: Brute-force Search

Q_x	Q_y	κ_F	κ_D	ξ_x^o	ξ_y^o	α_1^o	τ_{model}	τ_{low}	Δ_{1000}^{ξ}
1.855	0.723	0.062	-0.165	-1.845	-2.831	0.2997	1801	1173	0.010
1.855	0.823	0.068	-0.188	-1.635	-3.469	0.1365	2834	2769	0.022
1.855	0.923	0.072	-0.209	-1.480	-4.009	-0.0282	667	666	
1.855	1.023	0.075	-0.228	-1.351	-4.527	-0.0451	3470	2413	0.025
1.855	1.095	0.077	-0.242	-1.271	-4.904	-0.0756	1473	1472	0.020
1.855	1.123	0.077	-0.247	-1.253	-5.036	-0.156	877	1614	0.016
1.855	1.223	0.079	-0.264	-1.167	-5.626	-0.211	3721	3237	0.013
1.823	0.723	0.036	-0.140	-1.483	-1.353	0.0543	677	676	
1.823	0.823	0.055	-0.177	-1.465	-2.389	0.0132	6105	4154	0.016
1.823	0.923	0.062	-0.201	-1.337	-2.983	-0.0573	5647	3870	0.019
1.823	1.023	0.066	-0.223	-1.221	-3.489	-0.1584	1439	1347	0.015
1.823	1.123	0.070	-0.243	-1.124	-4.006	-0.2142	4449	4420	0.022
1.823	1.223	0.072	-0.261	-1.038	-4.508	-0.2572	2575	2571	0.019

Optimization using the spin tune model

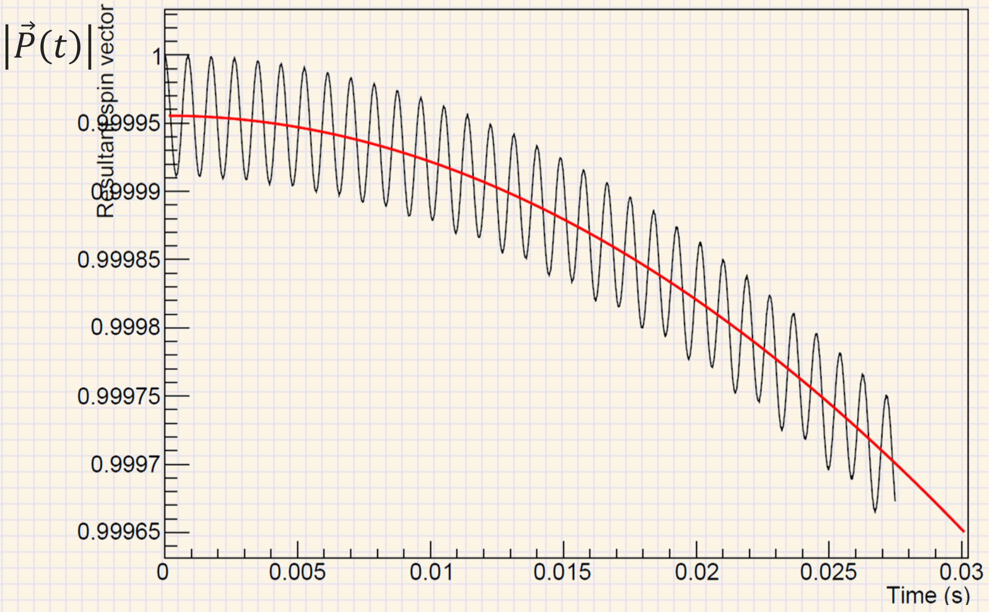
$$\left|\vec{P}(t)\right| = \left|\frac{1}{N}\sum_{\mu=0}^{N}\hat{s}_\mu(t)\right|$$

Spin tune of single particle:

$$\boldsymbol{\langle \nu_s \rangle = a_i \langle \epsilon \rangle^i + M_{ij} \langle \epsilon \rangle^i \xi^j}$$

Factoring out the phase-space term:

$$\langle \nu_s \rangle = \langle \epsilon \rangle^i \left(a_i + M_{ij}\xi^j\right)$$



Therefore, the condition for high spin coherence time:

$$a_i + M_{ij}\xi^j = 0$$

...where the spin tune of a particle **regardless of its phase-space properties** vanishes.

This would occur at:

$$\xi^j = -(M^{-1})^{ji} a_i$$

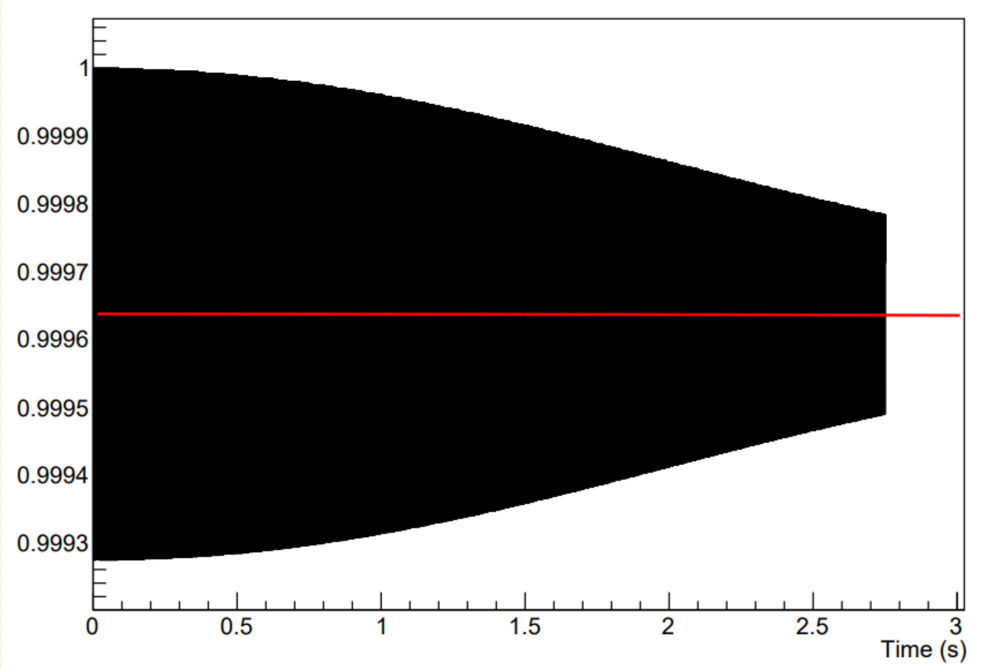
Results of an attempt at model-based optimization

Brute-force

$$[\xi^j] = \begin{bmatrix} -1.351 \\ -4.527 \\ 1.136 \end{bmatrix}$$

$\tau > 2000\ s$

$|\vec{P}(t)|$

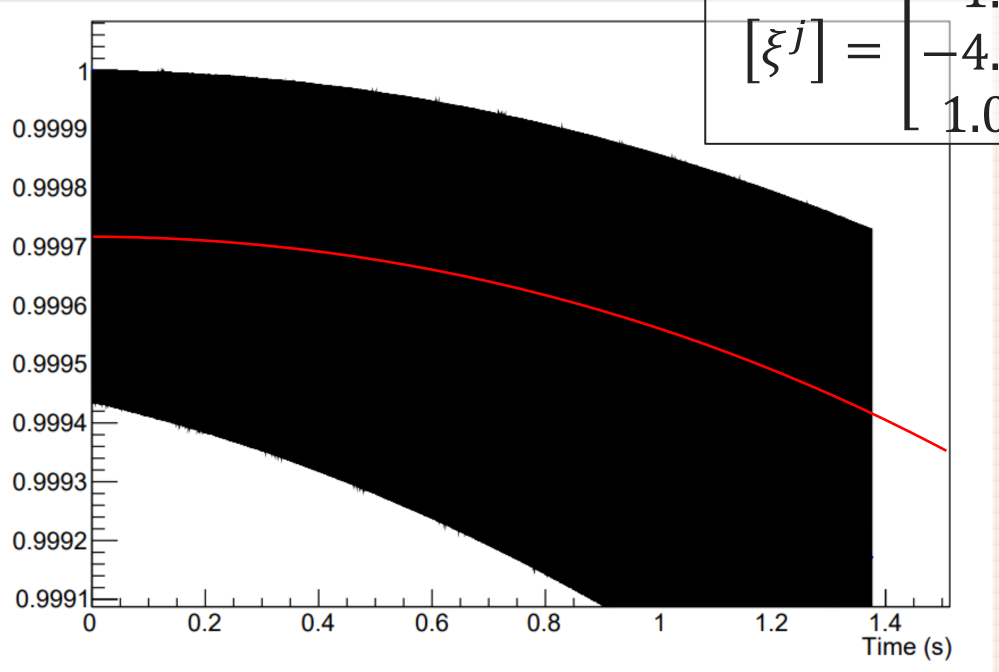


Prediction of spin tune model

$\tau = 91\ s$

$$[\xi^j] = \begin{bmatrix} -1.597 \\ -4.306 \\ 1.012 \end{bmatrix}$$

$|\vec{P}(t)|$



$\epsilon_x, \epsilon_y, \delta$ distribution: Gaussian

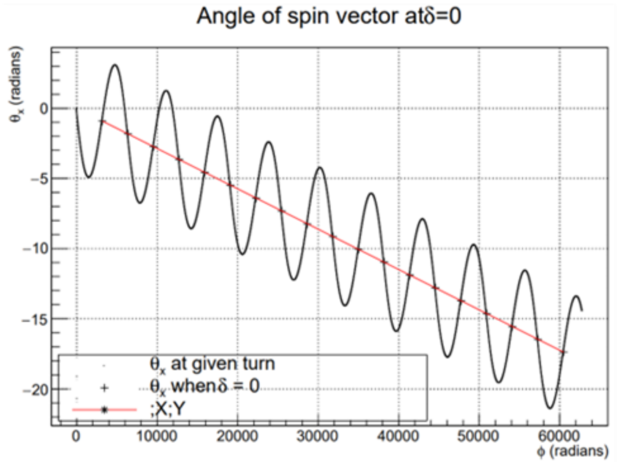
# of particles: 1024

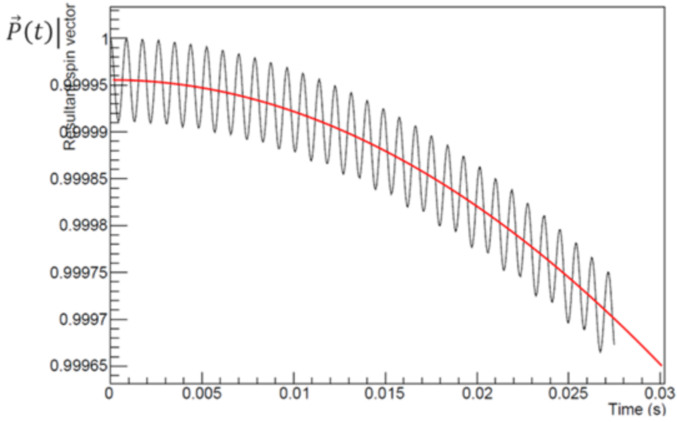
Summary

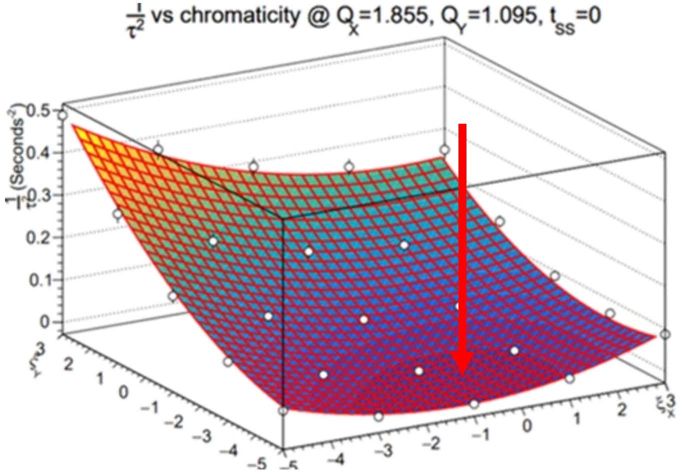
AND FUTURE WORK

Conclusions

- **Spin tune model accurate for spin tracking of single particles.**
  - Also works for the Hybrid ring (Lattice independence?)
- **Can be used to optimize the spin coherence time, if at least three sextupole families are present.**
  - ...however, not very accurate (yet).
- **Work on this is currently ongoing.**







Thank you for your attention…

QUESTIONS AND SUGGESTIONS ARE WELCOME...

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