Model of statistical errors in the search for the deuteron EDM in the storage ring
**Methodology**

When put into an electromagnetic field, the particle spin begins to precess according to the T-BMT equation:

\[
\frac{d \vec{S}}{dt} = \vec{\Omega} \times \vec{S}
\]

\[
\vec{\Omega} = -\frac{e}{m} \left( G \vec{B} + \left( \frac{1}{\gamma^2 - 1} - G \right) \vec{\beta} \times \vec{E} + \frac{\eta}{2} \vec{E} + \vec{\beta} \times \vec{B} \right)
\]

By measuring the beam’s polarization, we can determine the frequency

\[
\vec{\Omega}^\pm = \vec{\Omega}_{MDM} \pm \vec{\Omega}_{EDM}
\]

Comparing the CW vs CCW frequencies, determine \( \Omega_{EDM} \)
Detector counting rate

\[ \tilde{N}(t) = N_0(t) \left[ 1 + P \cdot e^{-t/\tau_d} \cdot \sin(\omega t + \phi) \right] + \epsilon_t \]

Number of counts is Poisson distributed, hence

\[ \sigma_{\tilde{N}_0}^2 = N_0(t) \]

\[ \sigma_{N_0}(t) = \sigma_{\tilde{N}_0}(t) / \sqrt{n_{c/\epsilon}} \]

\[ \frac{\sigma_{N_0}(t)}{N_0(t)} \propto \frac{1}{\sqrt{\Delta t/\epsilon}} \cdot \exp \left( \frac{t}{2 \tau_b} \right) \]
Cross section asymmetry

A measure of polarization

Definition:

\[ A = \frac{N_L - N_R}{N_L + N_R} \]

Model:

\[ A(t) = A(0) \cdot e^{\lambda t} \cdot \sin(\omega t + \phi) \]

\[ \sigma_A^2(t) \approx \frac{1}{2N_0(t)} \]

Error:

\[ \sigma^2[\hat{\omega}] = \frac{\sigma^2[\varepsilon]}{\sum_i f(t_i) \cdot \sigma_w^2[t]} \]
Limiting factors

- Sample Fisher information can be increased by sampling during rapid change
- Limited by polarimetry sampling rate
- Point Fisher information falls exponentially due to decoherence
- Can't economize the beam too much
Time-spread

\[
\sum f(t_i) = n_{\epsilon/\zeta} \cdot x_{01} \cdot \frac{\exp\left(-\frac{\pi}{\omega \tau_d} n_{\zeta}\right) - 1}{\exp\left(-\frac{\pi}{\omega \tau_d}\right) - 1}
\]

\[
t(z) = \tau_d \cdot \ln\left(\frac{1}{1 - z}\right)
\]
Simulation

• Uniform sampling
• 75% of the beam (7.5 $\cdot$ 10^8 useful scatterings)
• 3% initial counting rate error

• Standard error 7.55 $\cdot$ 10^{-7} rad/sec
• If $\omega$ is known down to 10^{-6}, can improve the result by 30%
Thank You