Beam and Spin Dynamics for Hadron Storage Rings

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Cooler Synchrotron COSY
Circumference: 184 m
max. magnetic rigidity 12 Tm
Polarized proton and deuteron beams
Electron and stochastic cooling

High-Energy Storage Ring HESR at FAIR
Circumference: 575 m, max. magnetic rigidity 50 Tm
Antiproton and ion beams
Electron and stochastic cooling

Cooperation with Universities:
COSY Association of Networking Universities (CANU)
COSY F&E program
Jülich-Aachen Research Alliance (JARA-FAME)
Student summer school with Univ. of Bonn, Gießen and Bochum
Accelerator physics lectures at Aachen and Bonn
Bachelor/Master and PhD theses in accelerator physics
Beam Dynamics

Single particles (no interaction between particles → lattice design)
- linear beam dynamics: charged particles in bending and focusing fields
- non-linear beam dynamics: higher-order multipoles, dynamic aperture
  → Multipole correction

Ensemble of particles (fields induced by particles → intensity/quality limits)
- space charge: self-fields of particles (non-relativistic effect)
- ring impedances: beam-wall interaction
- trapped electrons/ions

Scattered particles (with each other or material → lifetime, luminosity)
- intra-beam scattering
- rest-gas scattering
- beam-target interaction
- beam-beam interaction → Beam cooling
HESR Magnet Specification

Integrated multipole components (units of $10^{-4}$) on reference radius $r_0 = 33$ mm over entire field range

Dipole: 0.17 T to 1.7 T
- $|b_3| < 5$
- $|b_5| < 1$

Quadrupole: 0 to 20 T/m (25 T/m)
- $|b_6| < 0.5$
- $|b_{10}| < 0.05$

Sextupole: 0 to 45 T/m$^2$ (90 T/m$^2$)
- $|b_9| < 0.2$
- $|b_{15}| < 0.01$

All other multipole components:
- $|b_n| < 0.1$
Dynamic Aperture

Correction scheme:
24 horizontale + 28 vertikale Sextupoles in arcs
Tune Diagram (2D)

Resonance condition: \[ m, n, p \rightarrow m \cdot Q_x + n \cdot Q_y = p \]

1) Skew Sextupole: 2,1,23  2) Octupole: 2,2,31  3) Skew Octupole: 3,1,31
4) Octupole: 4,0,31;  5) 12-pole: 4,2,47
Dynamic Aperture (Design Working Point)

Orbit diffusion coefficient (e.g. after 1000 and 2000 turns):

\[ D = \log_{10} \left[ \sqrt{(Q_x^{(2)} - Q_x^{(1)})^2 + (Q_y^{(2)} - Q_y^{(1)})^2} \right] \]
Dynamic Aperture (Optimized)

- Optimized working point and compensation of multipoles
- Dynamic Aperture: 16 mm mrad

Beam Temperature

Expression from kinetic theory of gases

Beam temperature

Average kinetic energy of particles in the cm system

\[ \frac{3}{2}k_BT = \frac{1}{2}m\langle v^2 \rangle \]

Relation the beam properties

\[ \frac{3}{2}k_BT = k_BT_\perp + \frac{1}{2}k_BT_\parallel \approx \frac{1}{2}mc^2(\gamma_r\beta_r)^2 \left( \frac{\varepsilon_x}{\beta_x} + \frac{\varepsilon_y}{\beta_y} + \frac{1}{\gamma_r^2} \left( \frac{\sigma_p}{p} \right)^2 \right) \]
Cooling Techniques

„Cooling“ a particle beam means reducing its energy spread and emittance → increasing the phase space density is all three dimensions.

- Liouville-Theorem: Area of phase space ellipse in a system without dissipative forces is invariant.

- Dissipative forces for Proton/Ion beams

  Methods:
  1.) Electron Cooling
  2.) Stochastic Cooling
  3.) Laser Cooling

- Synchrotron radiation for Electron/Positron beams
Electron Cooling

Beam cooling via Coulomb interaction with cooled electrons (heat exchanger)

G. I. Budker 1966
Electron Cooler (COSY)
2 MV Electron Cooler

Joint development with Budker Institute (BINP, Russia), commissioned at COSY, **injection cooler** for HESR

Parameters demonstrated so far:

- Voltage up to 1.4 MV (7 bar SF6)
- Cooling at 908 kV / 340 mA (1.8 GeV p)

**Before**  
**After**

- Electron cooling achieved for 1.8 GeV protons
- Ongoing: commissioning for full COSY energy range (2.8 GeV)

Intermediate step to 4.5 / 8 MeV electron for HESR

Cooperation partners: BINP, JINR, FZJ, HIM

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Stochastic Cooling (COSY)

- Transversal and longitudinal
- Frequency range: 1-3 GHz
- 2 bands
- RF Power: 500 W per plane

S. van der Meer 1968
Stochastic Cooling and Luminosity

Compensation of emittance and momentum spread growth by an internal target
Beam Dynamics Studies at COSY

- a) Injected beam
- b) Beam heated by target
- c) + stochastic cooling
- d) + barrier bucket

Particle Density (arb. units) vs. $f$ [GHz]

Time / µs vs. BB Voltage / V

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Magnetic Moment

Spin motion in magnetic fields due to magnetic moment:

\[
\frac{d\vec{S}}{dt} = \vec{\mu} \times \vec{B}^*
\]

Spin ↔ magnetic moment:

\[
\vec{\mu} = g \frac{q}{2m} \vec{S}
\]

→ Equation for spin motion in external magnetic fields:

\[
\frac{d\vec{S}}{dt} = g \frac{e}{2m} \vec{S} \times \vec{B}^*
\]

Particle at rest

cm system
Spin Precession in a Circular Ring
(Thomas-BMT Equation)

Spin motion:

\[
\frac{d\vec{S}}{dt} = \vec{S} \times \vec{\Omega},
\]

\[
\vec{\Omega} = \frac{q}{m\gamma} \left[ (1 + \gamma G) \vec{B}_\perp + (1 + G) \vec{B}_\parallel - \left( \gamma G + \frac{\gamma}{1 + \gamma} \right) \frac{\vec{\beta} \times \vec{E}}{c} \right]
\]

Gyromagnetic anomaly: \( G = \frac{g - 2}{2}, \ G_p = 1.7928473, \ G_d = -0.142987, \ a := G_e = 0.001159652193 \)

Vertical guiding field: spin rotates \( \gamma G \) times faster than motion
\( \Rightarrow \) spin tune \( \nu_{sp} = \gamma G \)
Spinor Formalism

Spinor representation for Spin-1/2 particles:

\[ \Psi = \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix} \]

\[ S_i = \langle \Psi \mid \sigma_i \mid \Psi \rangle = \Psi^+ \sigma_i \Psi \]

Hamilton Formalism to describe spinor motion using a modified Schrödinger equation:

\[ \frac{d\Psi}{d\theta} = -\frac{i}{2} H\Psi \]

The T-BMT equation can be expressed in terms of a wave equation:

\[ \frac{d\Psi}{d\theta} = -\frac{i}{2} (\vec{\Omega} \cdot \vec{\sigma}) = -\frac{i}{2} \begin{pmatrix} \gamma G & -\xi(\theta) \\ -\xi^*(\theta) & -\gamma G \end{pmatrix} \]

with the perturbing fields \( \xi \) in a Fourier series:

\[ \xi = \sum_k \xi_k e^{-i\nu_k \theta} \]
Spin Resonances

Horizontal fields lead to beam depolarization for the following spin resonance conditions:

**Imperfection resonance:**
- Field and positioning errors of magnets
- Resonance strength $\varepsilon_r \sim \gamma_{rms}$

$$\gamma G = k$$

$k$: integer

**Intrinsic resonance:**
- Horizontal fields for vertical focusing
- Resonance strength $\varepsilon_r \sim \sqrt{\varepsilon_y}$

$$\gamma G = (kP \pm Q_y)$$

$P$: super-periodicity
$Q_y$: vertical tune

**Higher-order resonance:**
- Higher-order field errors of magnets and synchrotron motion

$$\gamma G = (k \pm lQ_x \pm mQ_y \pm nQ_s)$$

$k, l, m, n$: integer
$Q_x, Q_y, Q_s$: transverse and longitudinal tunes
Spin Resonance Crossing

Froissart-Stora formula:

\[ \frac{P_f}{P_i} = 2 \cdot e^{-\frac{-\pi |\varepsilon_r|^2}{2\alpha}} - 1 \]

Crossing speed:

fast crossing: \( \alpha \gg \varepsilon_r^2 \)

adiabatic spin flip: \( \alpha \ll \varepsilon_r^2 \)
Polarized Beams at COSY

**Tune-Jump**

- Length 0.6 m
- Max. current ±3100 A
- Max gradient 0.45 T/m
- Rise time 10 µs

**Intrinsic resonances → tune jumps**
**Imperfection resonances → vertical orbit excitation**

P > 75% at 3.3 GeV/c

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Spin Precession with Electric Dipole Moment

Spin precession for particles at rest in electric and magnetic fields:

$$\frac{d\vec{S}}{dt} = \vec{\mu} \times \vec{B} + \vec{d} \times \vec{E}^*$$

(* rest frame)

In a real neutral particle EDM experiment for non-relativistic particles $\gamma \to 1$ the spin precession is given by:

$$\frac{d\vec{S}}{dt} = \vec{\mu} \times (\vec{B} - \vec{v} \times \vec{E}) + \vec{d} \times (\vec{E} + \vec{v} \times \vec{B})$$

Ideal horizontal E-Fields and vertical B-Fields

EDM precession

Precession via magnetic moment
Spin Precession with EDM

Equation for spin motion of relativistic particles in storage rings for \( \vec{\beta} \cdot \vec{B} = \vec{\beta} \cdot \vec{E} = 0 \).

The spin precession relative to the momentum direction is given by:

\[
\frac{d \vec{S}}{dt} = \vec{\Omega} \times \vec{S}
\]

\[
\vec{\Omega} = \frac{q}{m} \left\{ GB + \left( G - \frac{1}{\gamma^2 - 1} \right)(\vec{v} \times \vec{E}) + \frac{\eta}{2} \left( \vec{E} + \vec{v} \times \vec{B} \right) \right\}.
\]

- Magnetic Moment
- Electric Dipole Moment

\[
G = \frac{g-2}{2}, \quad \vec{\mu} = 2(G+1) \frac{q}{2m} \vec{S}, \text{ and } \vec{d} = \eta \frac{q}{2m} \vec{S}.
\]

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Conclusion

• Dynamic aperture calculations to optimized beam performance

• Highest luminosity utilizing phase-space cooling

• Highest beam polarization by optimization of spin resonance crossing

• Electric dipole moment measurement
  → Talk by Marcel Rosenthal