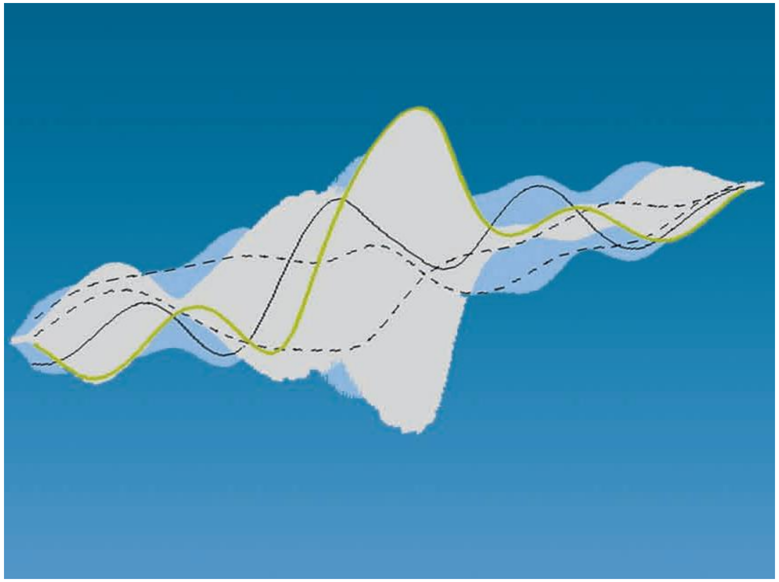
Beam and Spin Dynamics for Hadron Storage Rings



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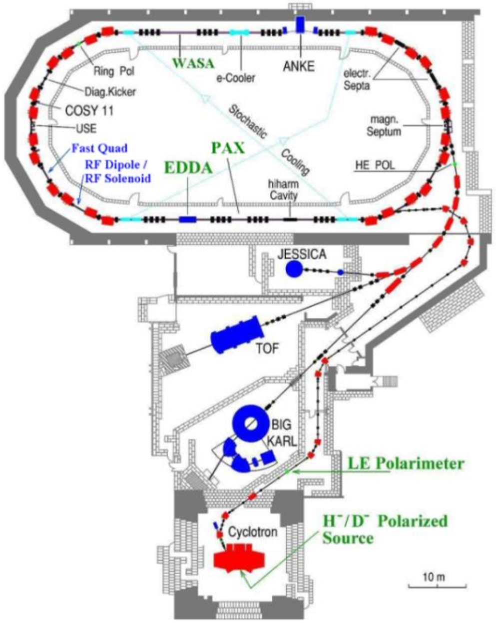
Accelerator Physics at FZ Jülich

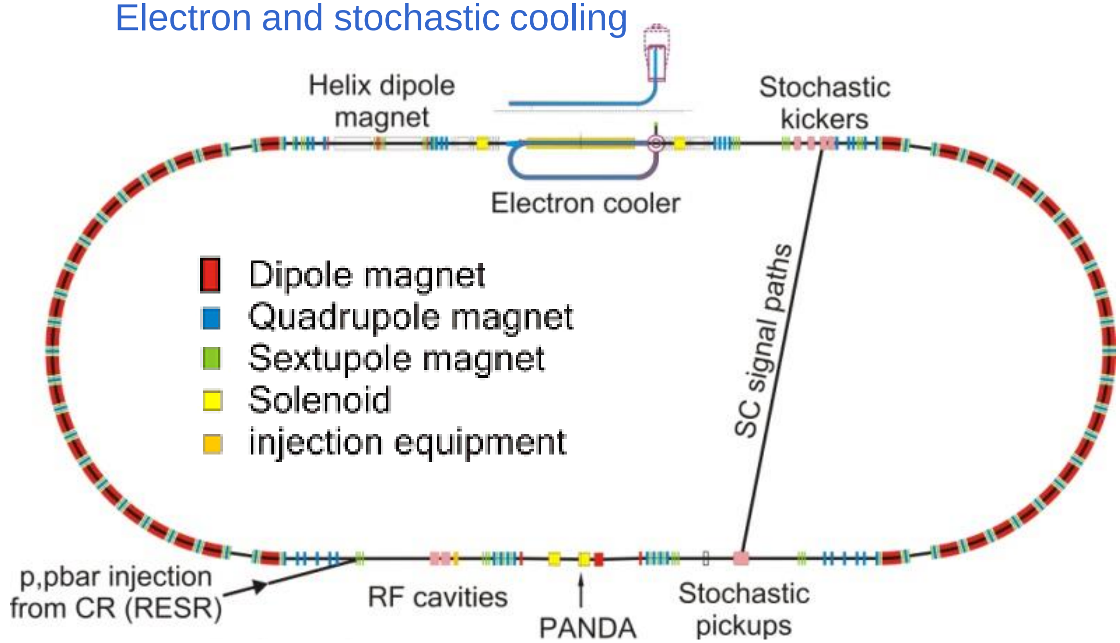
Cooler Synchrotron COSY

Circumference: 184 m
max. magnetic rigidity 12 Tm
Polarized proton and deuteron beams
Electron and stochastic cooling



High-Energy Storage Ring HESR at FAIR

Circumference: 575 m, max. magnetic rigidity 50 Tm
Antiproton and ion beams
Electron and stochastic cooling



Cooperation with Universities:

COSY Association of Networking Universities (CANU)
COSY F&E program
Jülich-Aachen Research Alliance (JARA-FAME)
Student summer school with Univ. of Bonn, Gießen and Bochum
Accelerator physics lectures at Aachen and Bonn
Bachelor/Master and PhD theses in accelerator physics

Beam Dynamics

Single particles (no interaction between particles → lattice design)

- linear beam dynamics: charged particles in bending and focusing fields
- non-linear beam dynamics: higher-order multipoles, dynamic aperture

→ Multipole correction

Ensemble of particles (fields induced by particles → intensity/quality limits)

- space charge: self-fields of particles (non-relativistic effect)
- ring impedances: beam-wall interaction
- trapped electrons/ions

Scattered particles (with each other or material → lifetime, luminosity)

- intra-beam scattering
- rest-gas scattering
- beam-target interaction
- beam-beam interaction

→ Beam cooling

HESR Magnet Specification

Integrated multipole components (units of 10^{-4}) on reference radius r_0 = 33 mm over entire field range

Dipole: 0.17 T to 1.7 T

- $|b_3| < 5$
- $|b_5| < 1$

Quadrupole: 0 to 20 T/m (25 T/m)

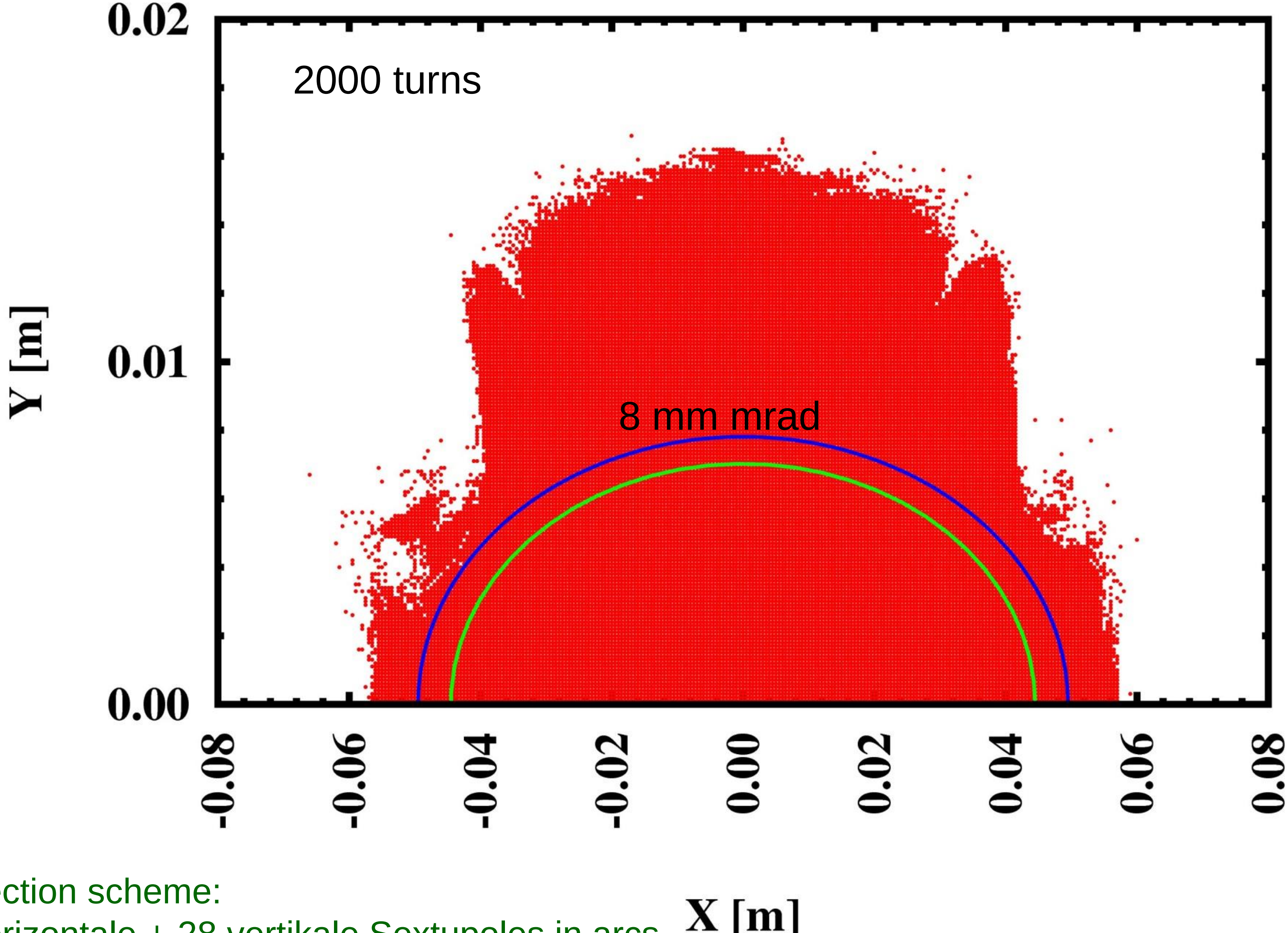
- |b6| < 0.5
- |b10| < 0.05

Sextupole: 0 to 45 T/m^2 (90 T/m^2)

- |b9| < 0.2
- |b15| < 0.01

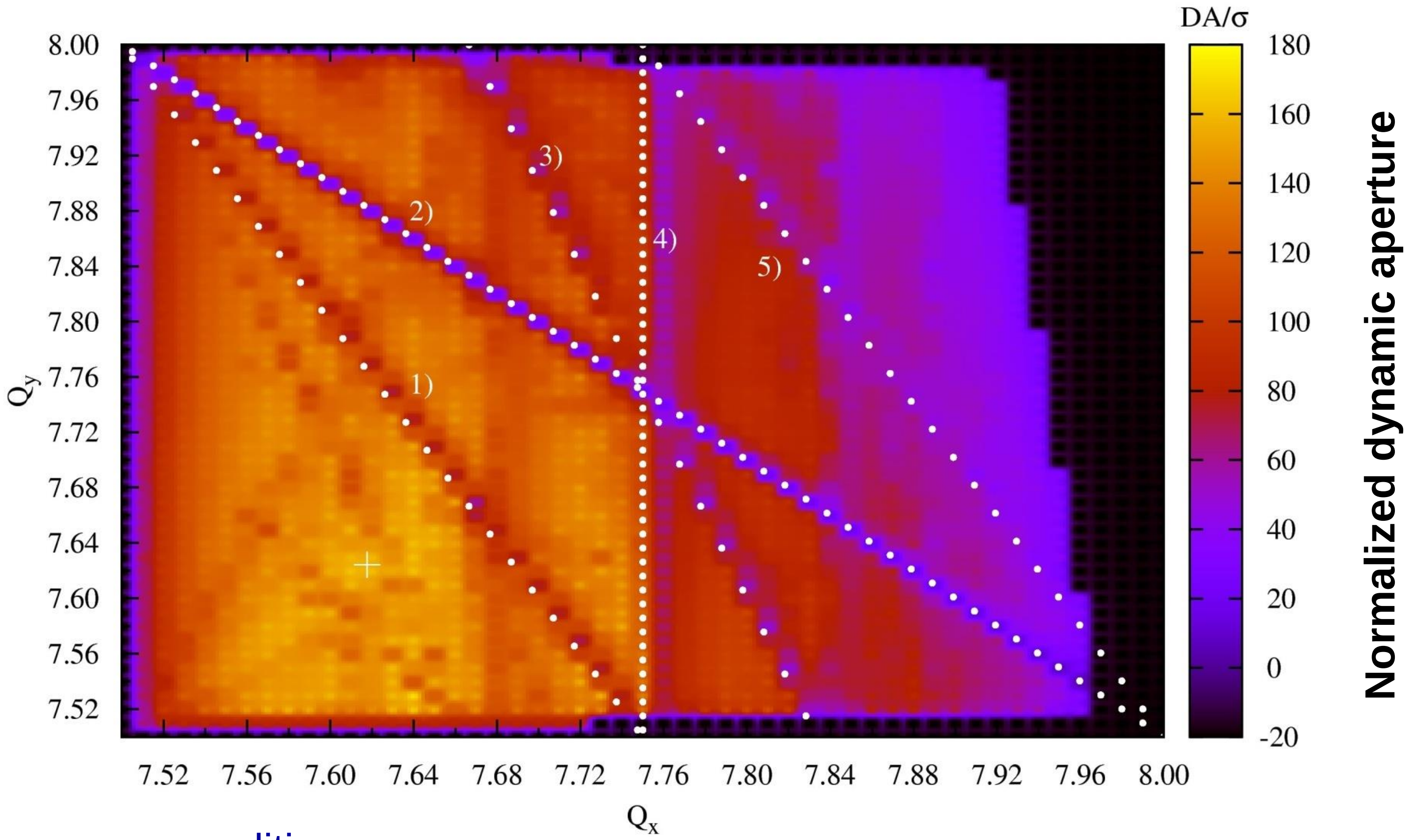
All other multipole components:
$|b_n| < 0.1$

Dynamic Aperture



Correction scheme:
24 horizontale + 28 vertikale Sextupoles in arcs

Tune Diagram (2D)



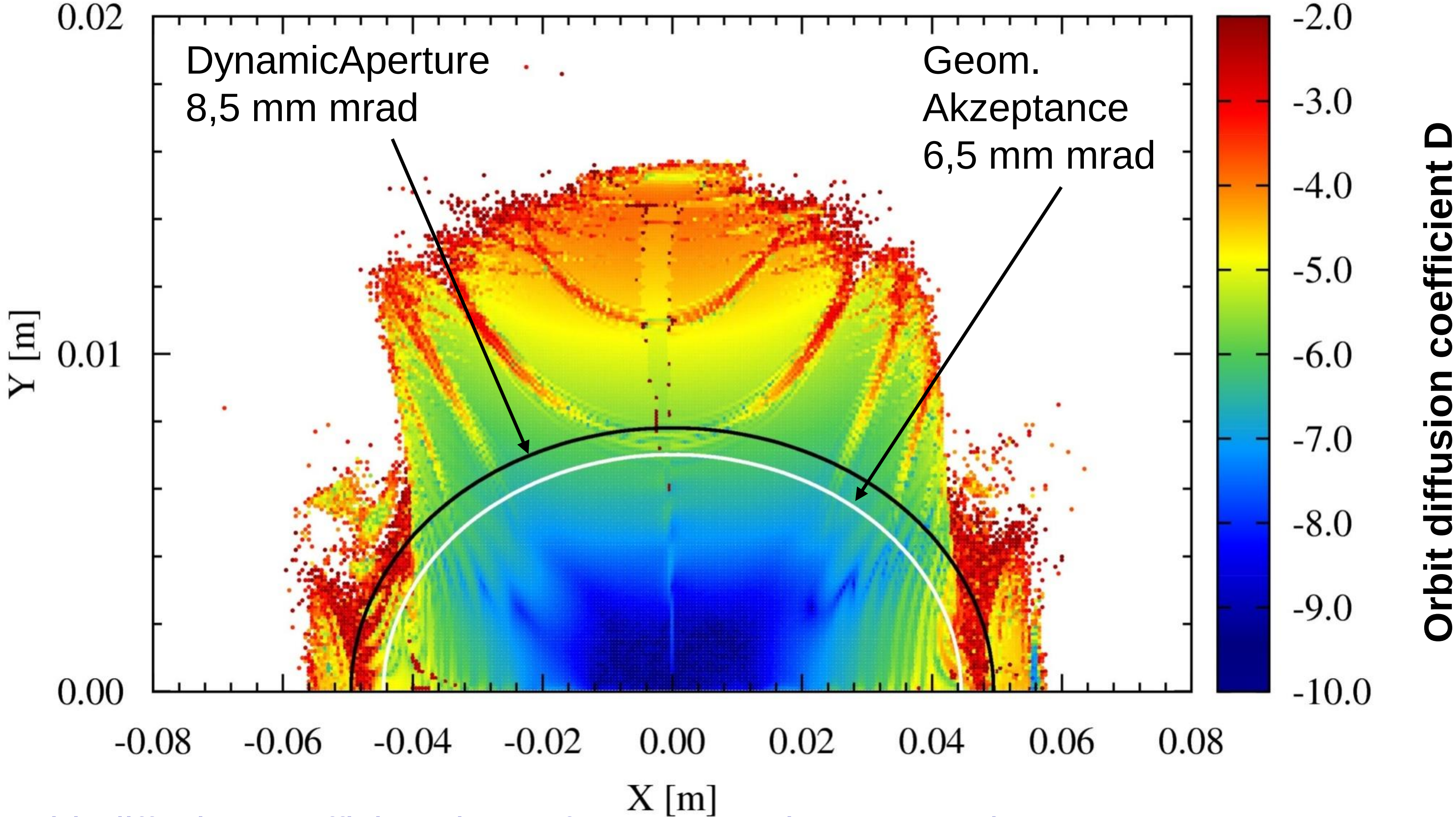
Resonance condition:

$$m, n, p \to m \cdot Q_x + n \cdot Q_y = p$$

1) Skew Sextupole: 2,1,23   2) Octupole: 2,2,31   3) Skew Octupole: 3,1,31

4) Octupole: 4,0,31;   5) 12-pole: 4,2,47

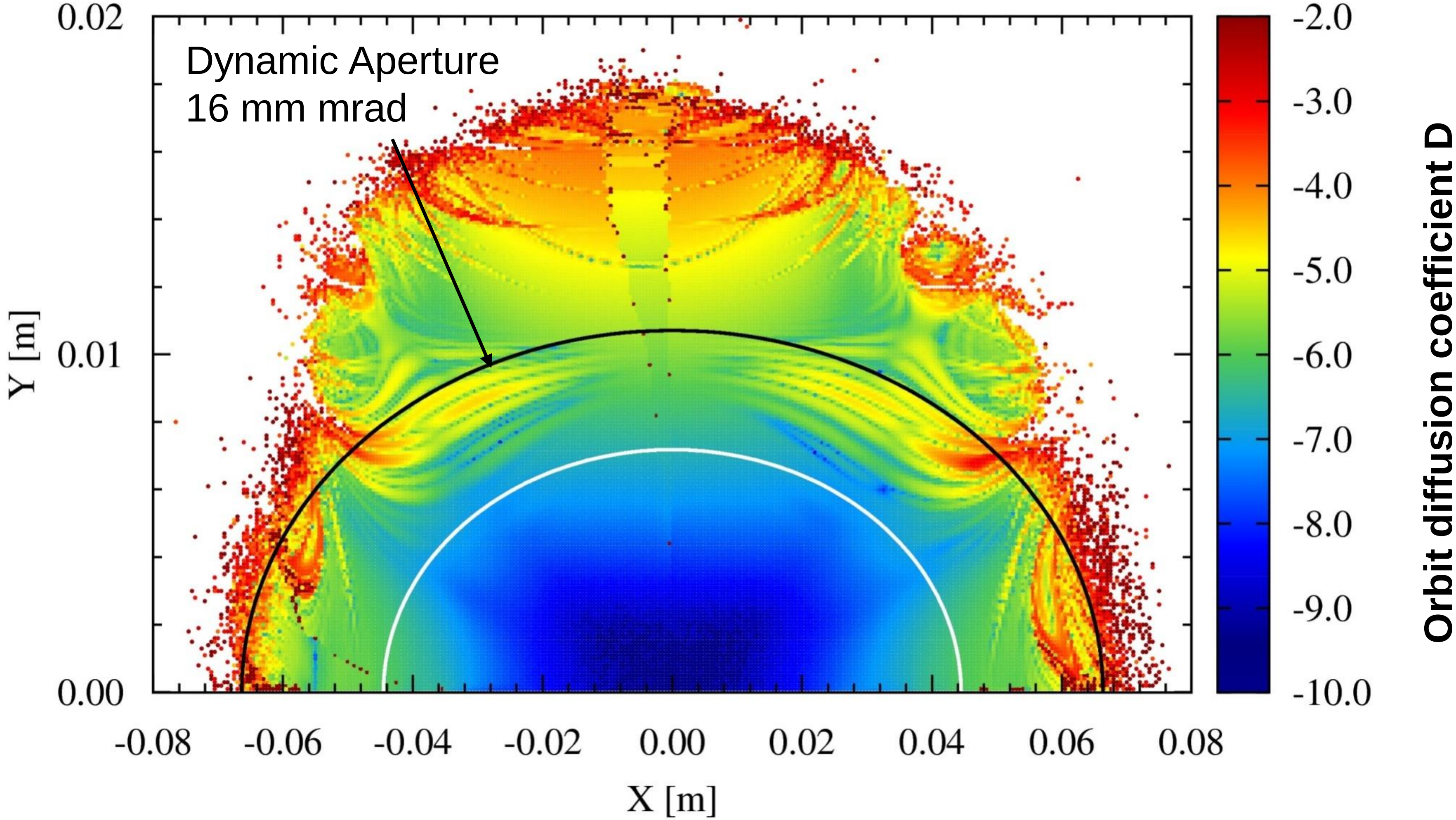
Dynamic Aperture (Design Working Point)



Orbit diffusion coefficient (e.g. after 1000 and 2000 turns):

$$D = \log_{10}\left[\sqrt{\left(Q_x^{(2)} - Q_x^{(1)}\right)^2 + \left(Q_y^{(2)} - Q_y^{(1)}\right)^2}\right]$$

Dynamic Aperture (Optimized)



- Optimized working point and compensation of multipoles
- Dynamic Aperture: 16 mm mrad

D. Welsch, PhD thesis, Univ. Bonn (2010)

Beam Temperature

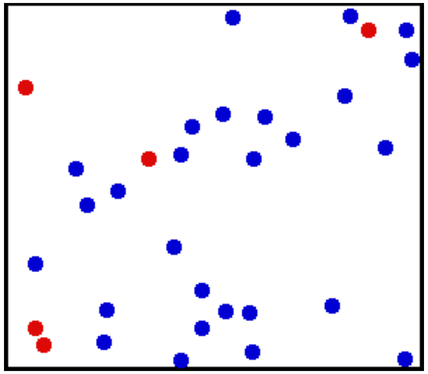
Expression from kinetic theory of gases

Beam temperature

$\leftrightarrow$

Average kinetic energy of particles in the cm system

$$\frac{3}{2}k_B T = \frac{1}{2}m\langle v^2\rangle$$



Relation the beam properties

$$\frac{3}{2}k_B T = k_B T_\perp + \frac{1}{2}k_B T_{||} \approx \frac{1}{2}mc^2(\gamma_r\beta_r)^2\left(\frac{\varepsilon_x}{\beta_x} + \frac{\varepsilon_y}{\beta_y} + \frac{1}{\gamma_r^2}\left(\frac{\sigma_p}{p}\right)^2\right)$$

Cooling Techniques

„Cooling“ a particle beam means reducing is energy spread and emittance → increasing the phase space density is all three dimensions

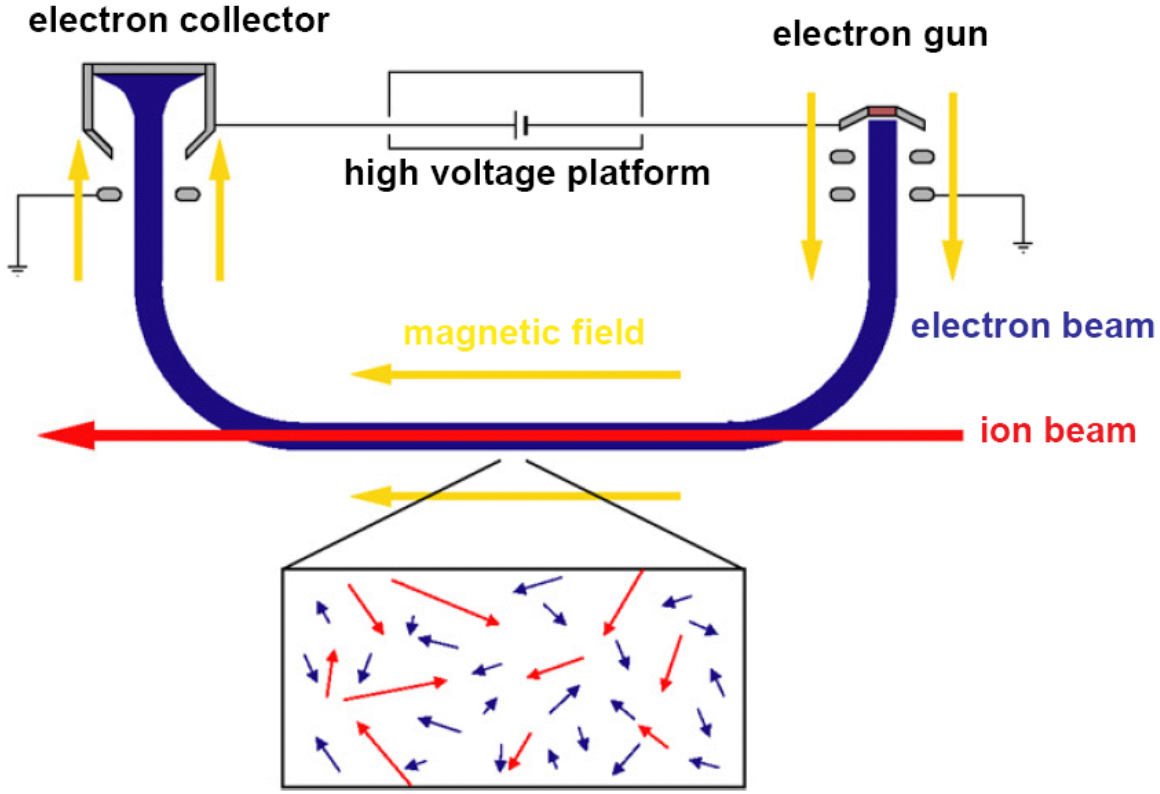
- Liouville-Theorem: Area of phase space ellipse in a system without dissipative forces is invariant

- Dissipative forces for Proton/Ion beams

  Methods: 1.) Electron Cooling
  2.) Stochastic Cooling
  3.) Laser Cooling

- Synchrotron radiation for Electron/Positron beams

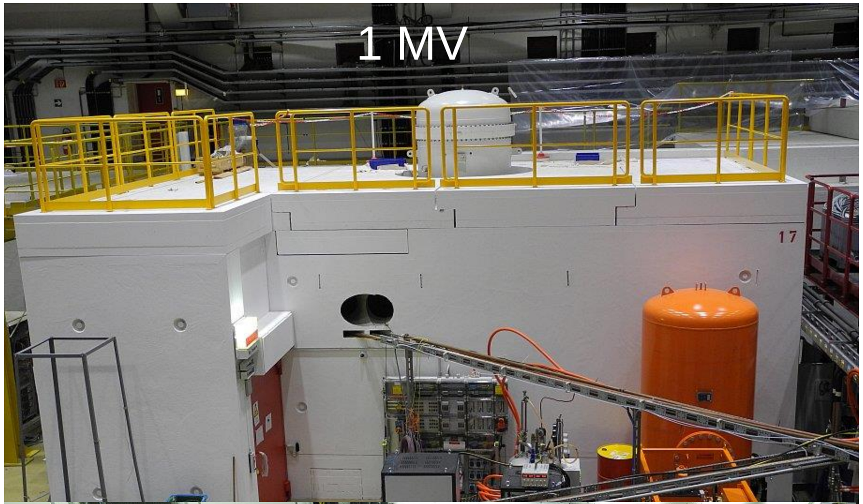
Electron Cooling

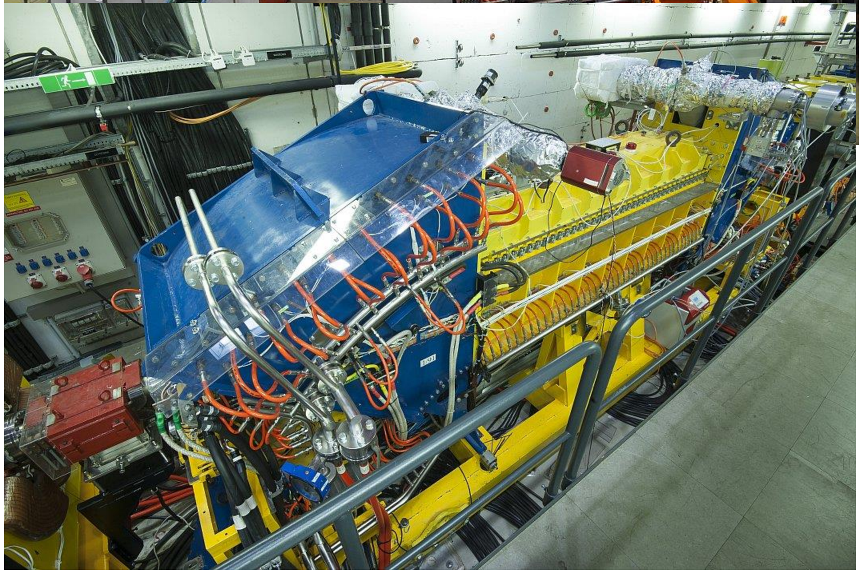


Beam cooling via Coulomb interaction
with cooled electrons (heat exchanger)

G. I. Budker 1966

Electron Cooler (COSY)





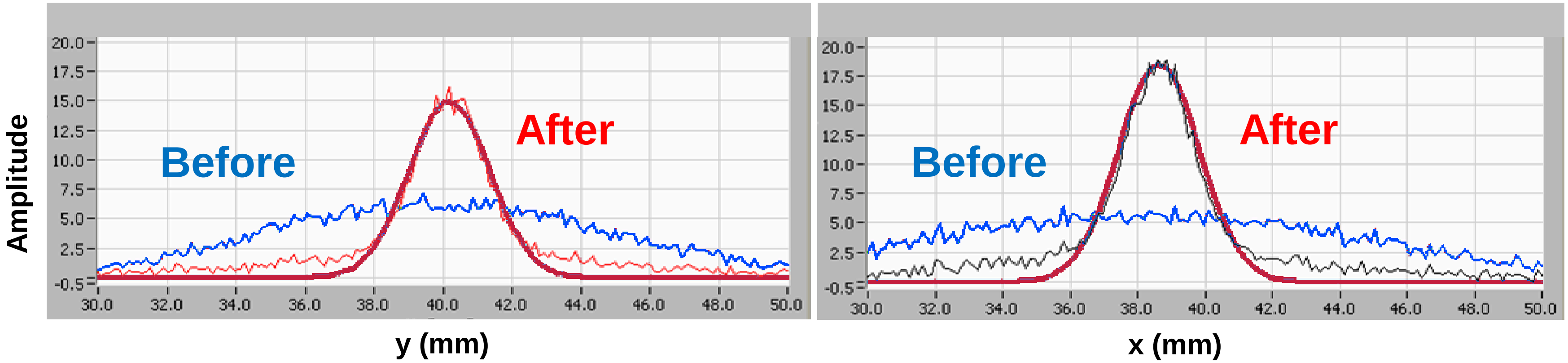
2 MV Electron Cooler

Joint development with Budker Institute (BINP, Russia),
commissioned at COSY, **injection cooler** for HESR

Parameters demonstrated so far:

- Voltage up to 1.4 MV (7 bar SF_6)
- Cooling at 908 kV / 340 mA (1.8 GeV p)

vertical 200 MeV p radial

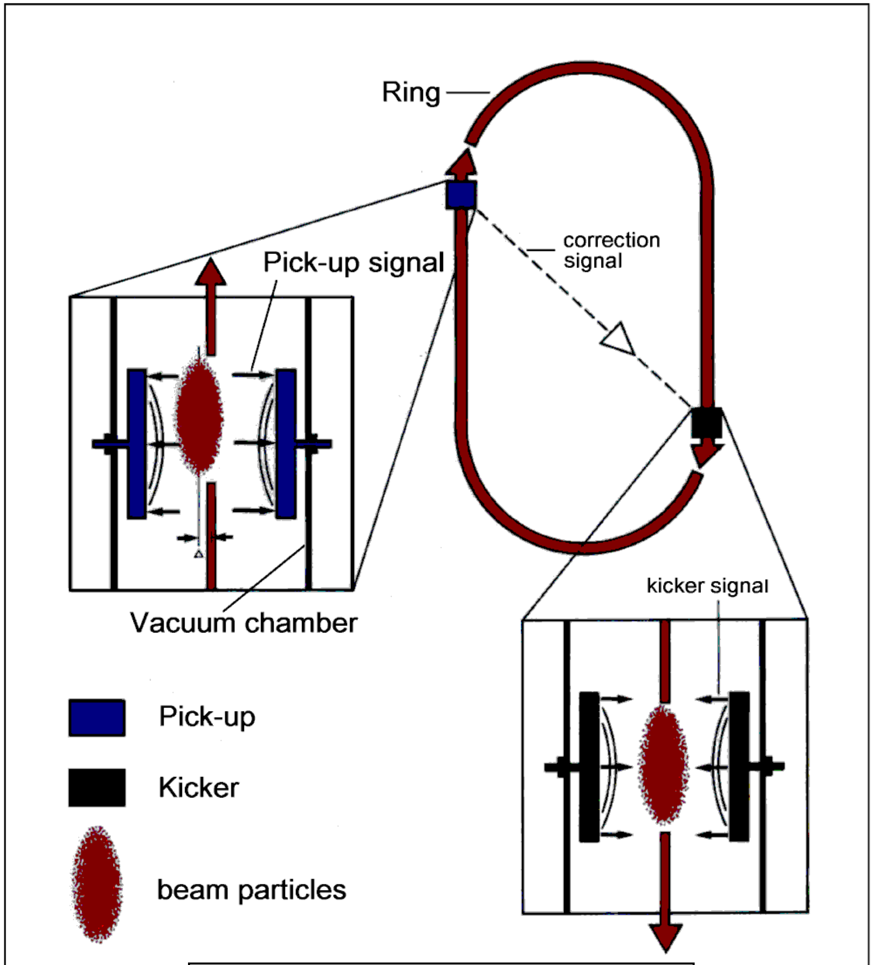


- Electron cooling achieved for 1.8 GeV protons
- Ongoing: commissioning for full COSY energy range (2.8 GeV)

Intermediate step to 4.5 / 8 MeV electron for HESR

Cooperation partners: BINP, JINR, FZJ, HIM

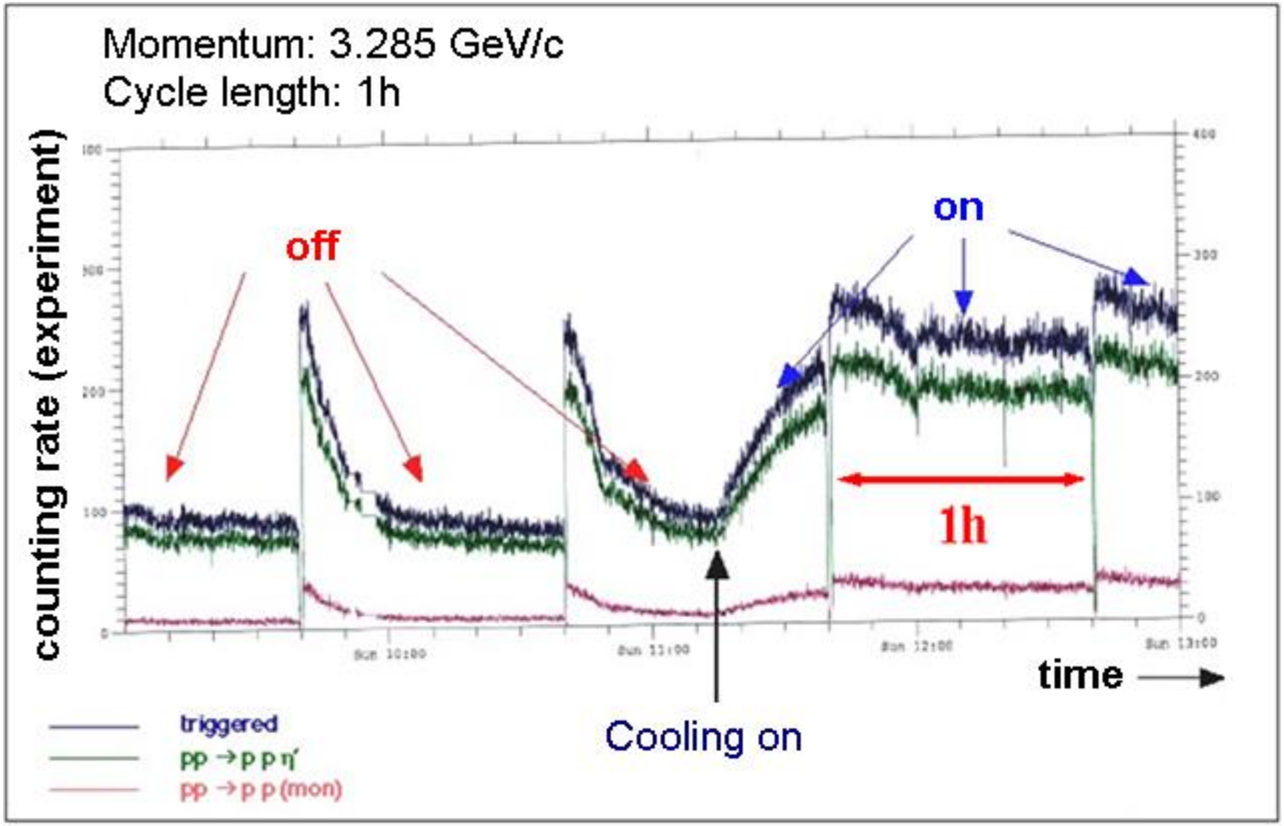
Stochastic Cooling (COSY)



S. van der Meer 1968

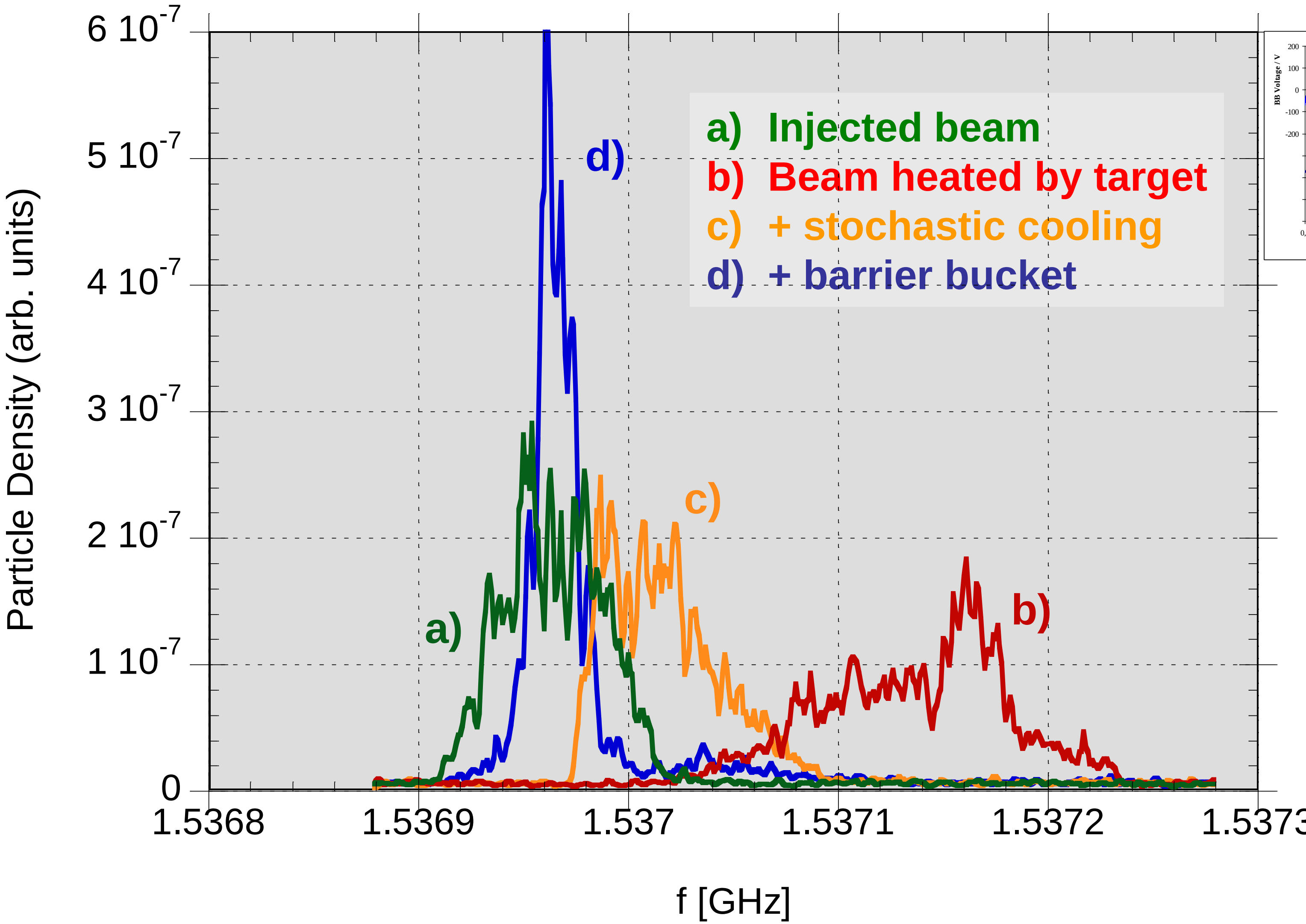
- Transversal and longitudinal
- Frequency range: 1-3 GHz
  2 bands
- RF Power: 500 W
  per plane

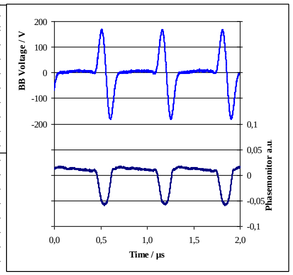
Stochastic Cooling and Luminosity



Compensation of emittance and momentum spread growth by an internal target

Beam Dynamics Studies at COSY





Magnetic Moment

Spin motion in magnetic fields due to magnetic moment:

$$\frac{d\vec{S}}{dt} = \vec{\mu} \times \vec{B}^*$$

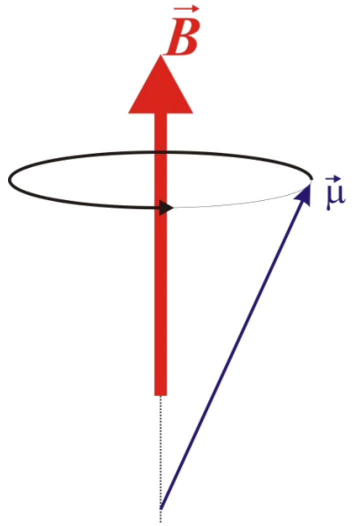
Spin ↔ magnetic moment:

$$\vec{\mu} = g \frac{q}{2m} \vec{S}$$

→ Equation for spin motion in external magnetic fields:

$$\frac{d\vec{S}}{dt} = g \frac{e}{2m} \vec{S} \times \vec{B}^*$$

cm system

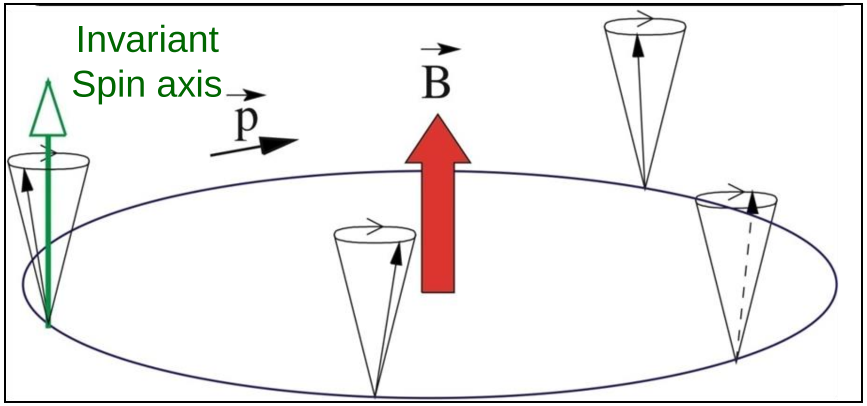


$$\vec{\Omega} = g \frac{e}{2m} \vec{B}^*$$

Particle at rest

Spin Precession in a Circular Ring
(Thomas-BMT Equation)

Spin motion:



$$\frac{d\vec{S}}{dt} = \vec{S} \times \vec{\Omega},$$

$$\vec{\Omega} = \frac{q}{m\gamma} \left[(1+\gamma G)\,\vec{B}_\perp + (1+G)\vec{B}_{||} - \left(\gamma G + \frac{\gamma}{1+\gamma}\right) \frac{\vec{\beta} \times \vec{E}}{c} \right]$$

lab system

Gyromagnetic anomaly: $G = \frac{g-2}{2},\ G_p = 1.7928473,\ G_d = -0.142987,\ a := G_e = 0.001159652193$

Vertical guiding field: spin rotates γG times faster than motion

→ spin tune $\nu_{sp} = \gamma G$

Spinor Formalism

Spinor representation for Spin-1/2 particles:

$$\Psi = \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix}$$

$$S_i = \langle \Psi | \sigma_i | \Psi \rangle = \Psi^+ \sigma_i \Psi$$

Hamilton Formalism to describe spinor motion using a modified Schrödinger equation:

$$\frac{d\Psi}{d\theta} = -\frac{i}{2} H\Psi$$

The T-BMT equation can be expressed in terms of a wave equation:

$$\frac{d\Psi}{d\theta} = -\frac{i}{2}(\vec{\Omega}\cdot\vec{\sigma}) = -\frac{i}{2}\begin{pmatrix} \gamma G & -\xi(\theta) \\ -\xi^*(\theta) & -\gamma G \end{pmatrix}$$

with the perturbing fields ξ in a Fourier series:

$$\xi = \sum_k \varepsilon_r e^{-i\nu_r\theta}$$

Spin Resonances

Horizontal fields lead to beam depolarization for the following spin resonance conditions:

Imperfection resonance:
Field and positioning errors of magnets

Resonance strength $\varepsilon_r \sim y_{rms}$

$$\gamma G = k$$

k: integer

Intrinsic resonance:
Horizontal fields for vertical focusing

Resonance strength $\varepsilon_r \sim \sqrt{\varepsilon_y}$

$$\gamma G = (kP \pm Q_y)$$

P: super-periodicity
Q_y: vertical tune

Higher-order resonance:
Higher-order field errors of magnets and synchrotron motion

$$\gamma G = (k \pm lQ_x \pm mQ_y \pm nQ_s)$$

k, l, m, n: integer
Q_x, Q_y, Q_s: transverse and longitudinal tunes

Spin Resonance Crossing

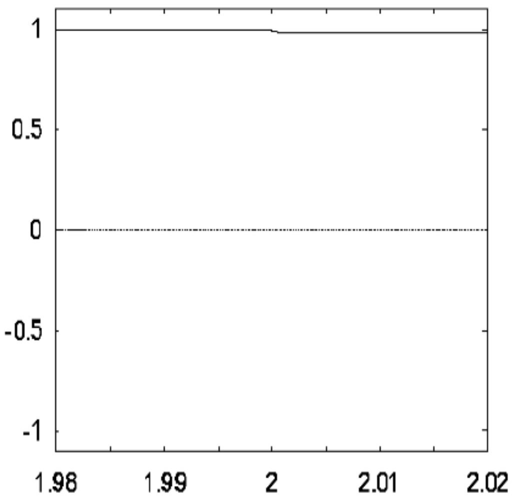
Froissart-Stora formula:

$$\frac{P_f}{P_i} = 2 \cdot e^{\frac{-\pi|\varepsilon_r|^2}{2\alpha}} - 1$$

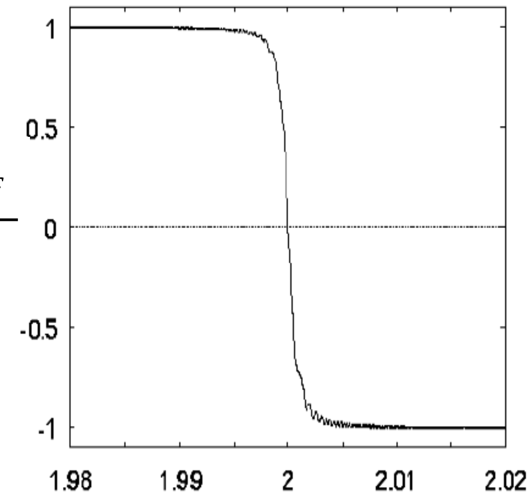
Crossing speed:

$$\alpha = \frac{\Delta\gamma \cdot G \pm \Delta Q_y}{2\pi}$$

fast crossing: $\alpha >> \varepsilon_r^2$

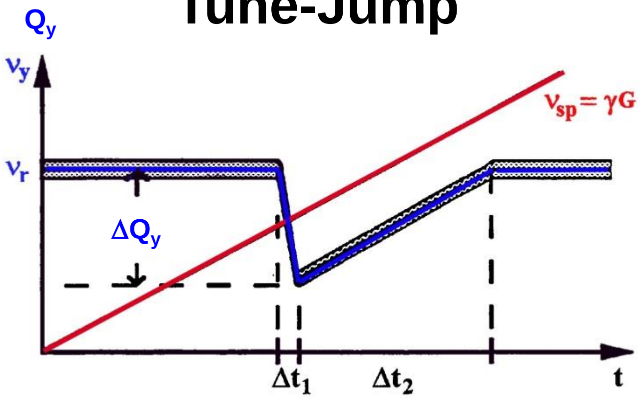


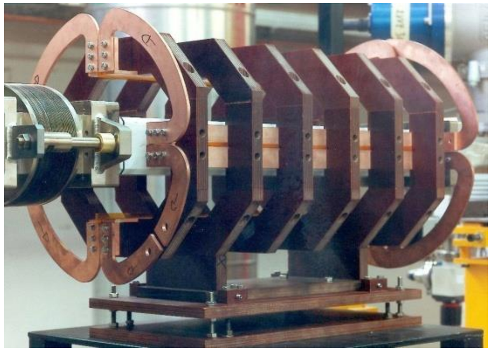
adiabatic spin flip: $\alpha << \varepsilon_r^2$



Polarized Beams at COSY

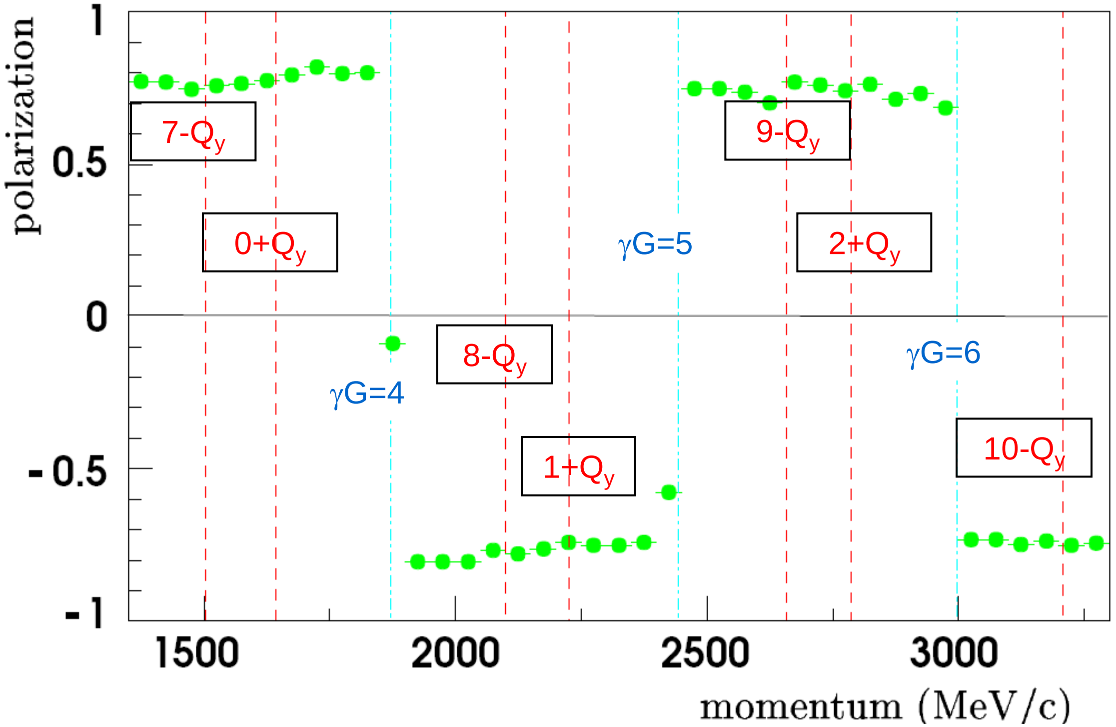
Tune-Jump





- Length 0.6 m
- Max. current ±3100 A
- Max gradient 0.45 T/m
- Rise time 10 μs

Polarization during acceleration



Intrinsic resonances → tune jumps

Imperfection resonances → vertical orbit excitation

P > 75% at 3.3 GeV/c

Spin Precession with Electric Dipole Moment

Spin precession for particles at rest in electric and magnetic fields:

$$\frac{d\vec{S}}{dt} = \vec{\mu} \times \vec{B}^* + \vec{d} \times \vec{E}^*$$

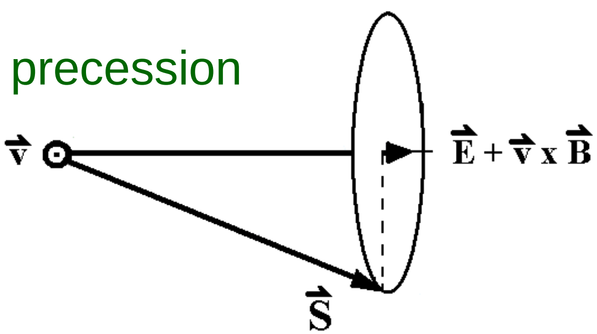
(* rest frame)

In a real neutral particle EDM experiment for non-relativistic particles $\gamma \to 1$ the spin precession is given by:

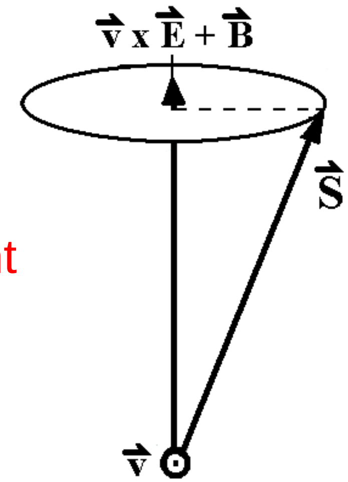
$$\frac{d\vec{S}}{dt} = \vec{\mu} \times (\vec{B} - \vec{v} \times \vec{E}) + \vec{d} \times (\vec{E} + \vec{v} \times \vec{B})$$

Ideal horizontal E-Fields and vertical B-Fields

EDM precession



Precession via magnetic moment



Spin Precession with EDM

Equation for spin motion of relativistic particles in storage rings

for $\vec{\beta}\cdot\vec{B}=\vec{\beta}\cdot\vec{E}=0$.

The spin precession relative to the momentum direction is given by:

$$\frac{d\vec{S}}{dt}=\vec{\Omega}\times\vec{S}$$

$$\vec{\Omega}=\frac{q}{m}\left\{G\vec{B}+\left(G-\frac{1}{\gamma^2-1}\right)\left(\vec{v}\times\vec{E}\right)+\frac{\eta}{2}\left(\vec{E}+\vec{v}\times\vec{B}\right)\right\}.$$

Magnetic Moment — Electric Dipole Moment

$$G=\frac{g-2}{2},\ \vec{\mu}=2(G+1)\frac{q}{2m}\vec{S},\ \text{and}\ \vec{d}=\eta\frac{q}{2m}\vec{S}.$$

Conclusion

- Dynamic aperture calculations to optimized beam performance

- Highest luminosity utilizing phase-space cooling

- Highest beam polarization by optimization of spin resonance crossing

- Electric dipole moment measurement

  → Talk by Marcel Rosenthal