Accelerator Physics Limitations on an EDM ring Design

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  Optics code: http://www-ap.fnal.gov/~ostiguy/OptiM/
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- Suppression of vertical magnetic field
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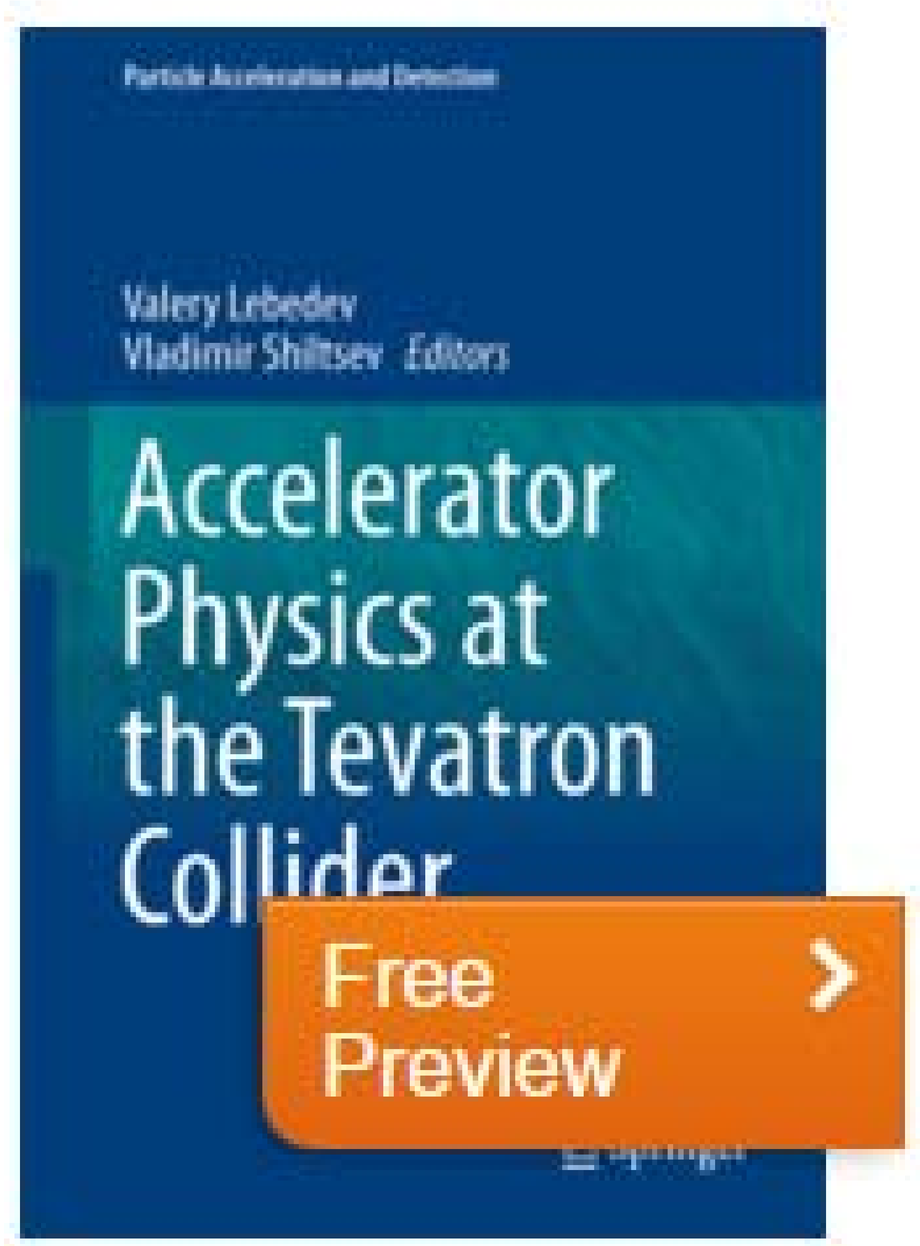
This presentation is aimed to discuss issues related accelerator physics

It is not aimed to make a credible suggestion for a ring

Coupling formalizm and IBS are described in:
http://www.springer.com/gp/book/9781493908844
Electronic copy is also available there

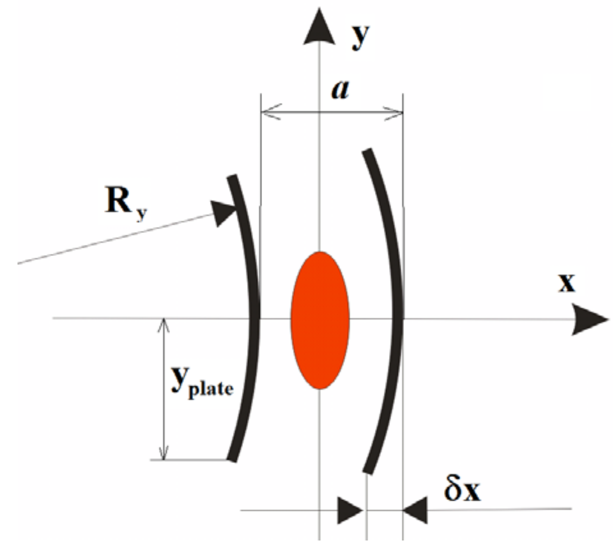


Electrostatic bends do not change IBS formulas derived for circular machines based on magnetic field!!!

Bending and Focusing with Pure Electric Field

- Electric field in electrostatic bend
  - Non linearities are present at fundamental level

$$\varphi(r,z)=A_1\ln\left(\frac{r}{R}\right)+\sum_{n=2}^{\infty}A_n\left(y^n-\frac{n(n-1)}{4}y^{n-2}r^2\right)$$



  - If required E_y can be made linear:

$$\Phi(x,y)=E_0R_0\left(\left(1+\frac{m}{2}\right)\ln\left(1+\frac{x}{R_0}\right)-\frac{m}{4}\left(\left(1+\frac{x}{R_0}\right)^2-1\right)+\frac{my^2}{2R_0^{\,2}}\right)$$

$$\xrightarrow{x\ll R_0}E_0R_0\left(\frac{x}{R_0}-(1+m)\frac{x^2}{2R_0^{\,2}}+m\frac{y^2}{2R_0^{\,2}}+(2+m)\frac{x^3}{6R_0^{\,3}}+O\left(x^4\right)\right)$$

  - Non-linear contribution to E_x is $\Delta E/E_0\approx(x/R_0)^2\approx 10^{-7}$ ($\Delta G/G\approx(x/R_0)\approx 3\cdot10^{-4}$)
    It is well below expected manufacturing accuracy
- Electric field in electrostatic quadruple

$$\varphi(\rho,\theta)=C_2r^2\cos(2\theta)+C_6r^6\cos(6\theta)+\ldots\quad\Rightarrow\quad\varphi(x,y)=G_0\frac{x^2-y^2}{2}+\ldots$$

  - Motion non-linearity comes from kin. en. change ($\Delta F/F\approx 2\cdot 100$ keV/230 MeV$\approx 10^{-3}$)

$$\frac{d\mathbf{p}}{ds}=-\frac{e}{\mathrm{v}}\nabla\varphi\approx-\frac{e}{\mathrm{v}_0}\left(1-\frac{\delta\mathrm{v}}{\mathrm{v}_0}\right)\nabla\varphi\approx-\frac{e}{\mathrm{v}_0}\left(1+\frac{e\varphi}{mc^2\beta^2\gamma^3}\right)\nabla\varphi=-\frac{e}{\mathrm{v}_0}\left(\nabla\varphi+\frac{e}{2mc^2\beta^2\gamma^3}\nabla\left(\varphi^2\right)\right)$$

    - It cannot be compensated by electrode geometry adjustments: $\Delta(\phi^2)\neq 0$

Linear Optics

- Methods of optics analyses developed for magnet-based beam optics are directly applicable to the optics based on electrostatics
  - Difference comes from kinetic energy change in electric field

$$K = E_0 - e\phi$$

  - Analysis is based on transfer matrix
    - kinetic energy is the same in drifts => K changes are irrelevant
- Transfer matrix of electric bend (see H. Wollnik, "Optics of charged particles")

$$M = \begin{bmatrix} c_x & s_x & 0 & 0 & 0 & d_x N_t \\ -k_x^2 s_x & c_x & 0 & 0 & 0 & s_x N_t / R_0 \\ 0 & 0 & c_y & s_y & 0 & 0 \\ 0 & 0 & -k_y^2 s_y & c_y & 0 & 0 \\ -s_x N_t / R_0 & -d_x N_t & 0 & 0 & 1 & -N_t^2 t_d \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad V = \begin{bmatrix} x \\ \theta_x \\ y \\ \theta_y \\ s \\ \Delta p / p \end{bmatrix}$$

$$k_x = \frac{1}{R_0}\sqrt{1 - m + \frac{1}{\gamma^2}} \qquad k_y = \frac{1}{R_0}\sqrt{m} \qquad N_t = 1 + \frac{1}{\gamma^2}$$

$$c_x = \cos(k_x L) \qquad s_x = \sin(k_x L) \qquad d_x = \frac{1 - c_x}{R_0 k_x^2}$$

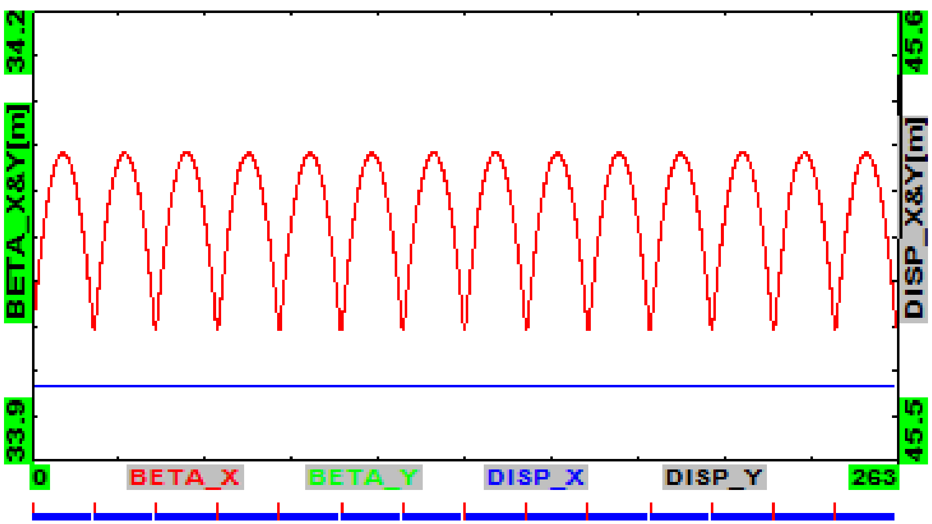
$$c_y = \cos(k_y L) \qquad s_y = \sin(k_y L) \qquad t_d = \frac{k_x L - s_x}{R_0^2 k_x^3}$$

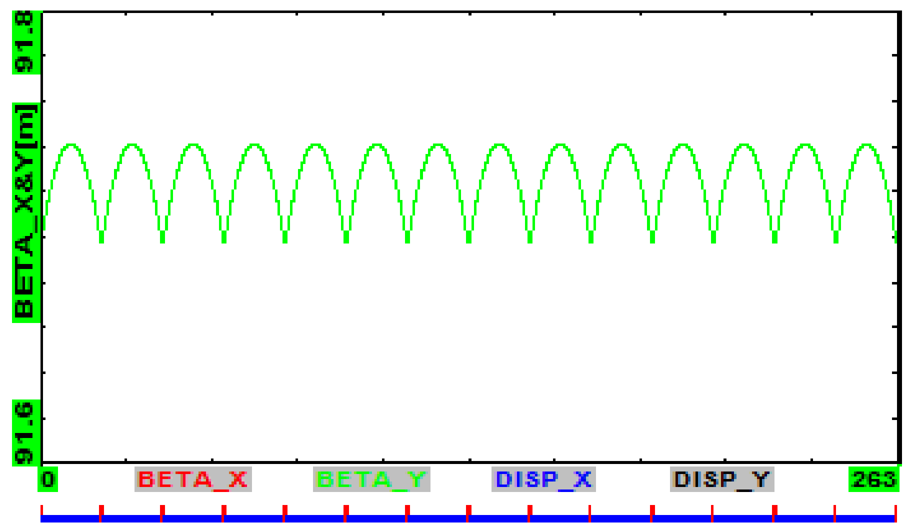
!!! In this definition M_{56} accounts for the effective orbit lengthening

- includes both orbit lengthening and velocity change due to radial displacement in the bend
- Does not include the effect of velocity change due to momentum change outside bend - standard acc. phys. definition.

Weak Focusing Ring

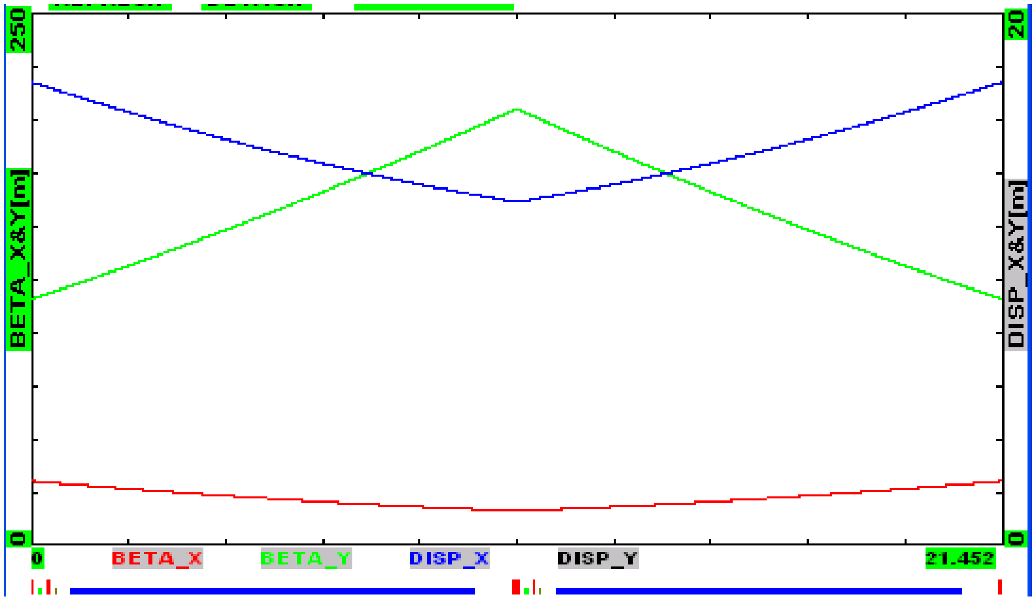
- Major part of focusing comes from bends, m≈0.2
- Ring structure
  - 14 periods:
    - each include electric bending with R_0=40 m and 0.834 m drift
      - o Gap between plates 3 cm, Voltage ±157 kV
- Small variations of β-functions excited by drifts, constant dispersion
  - Large beta-functions => Ring is extremely sensitive to focusing errors
  - Trim quads are required for final tuning and optics correction
    - GdL≈2 kV/cm^2 for tune correction





”Strong” Focusing Ring

- Ring structure
  - Same 14 periods. Each includes:
    - 2 electric bends with R_0=40 m and L=8.97 m
      - Gap between plates 3 cm, V= ±157 kV,
      - m = 0 (no vert. focusing)
    - 2 electric quads, one F and one D
      - L=15 cm, G_F = 17.2 kV/cm^2, G_D = -13.8 kV/cm^2
      - Each quad can be independently adjusted for optics correct.
  - One of two 80 cm gaps between quads and bends are filled with
    - H or V corrector, skew-quad corrector, F or D sextupole, and BPM
    - Other can be used by experiment, + RF cavity



- Circumference – 300 m
- Kinetic energy - 232.79 MeV

Weak Focusing versus "Strong" Focusing

	Weak foc.	Strong foc.
Kinetic energy [MeV]	232.79	
Number of periods	14	14
Circumference [m]	263	300
Focusing parameter in bends, m	0.199	0
Tunes, Q_x / Q_y	1.229 / 0.456	2.32 / 0.31
Maximum beta-function, β_x/β_y [m]	34 / 91.7	29.1 / 204
Dispersion	45.5	17.35
Maximum momentum deviation: $\Delta p/p\|_{max}$	$\pm 3.3\cdot 10^{-4}$ ♦	$\pm 8.6\cdot 10^{-4}$ ♦
Rms momentum spread	$1.1\cdot 10^{-4}$ ♥	$2.9\cdot 10^{-4}$ ♥
Hor. norm. acceptance [mm mrad]	5 ♦	5.8 ♦
Hor. /vert. norm. emittance [mm mrad]	0.56♥/1.52	0.31♥/2.2♥
Revolution frequency [kHz]	682.1	597.3
Momentum compaction, α	1.785	0.51
Slip-factor: $\eta = \alpha - 1/\gamma^2$	1.144	-0.132
Transition energy ($\gamma_{tr} = 1/\sqrt{\alpha}$), [MeV]	N/A *	376

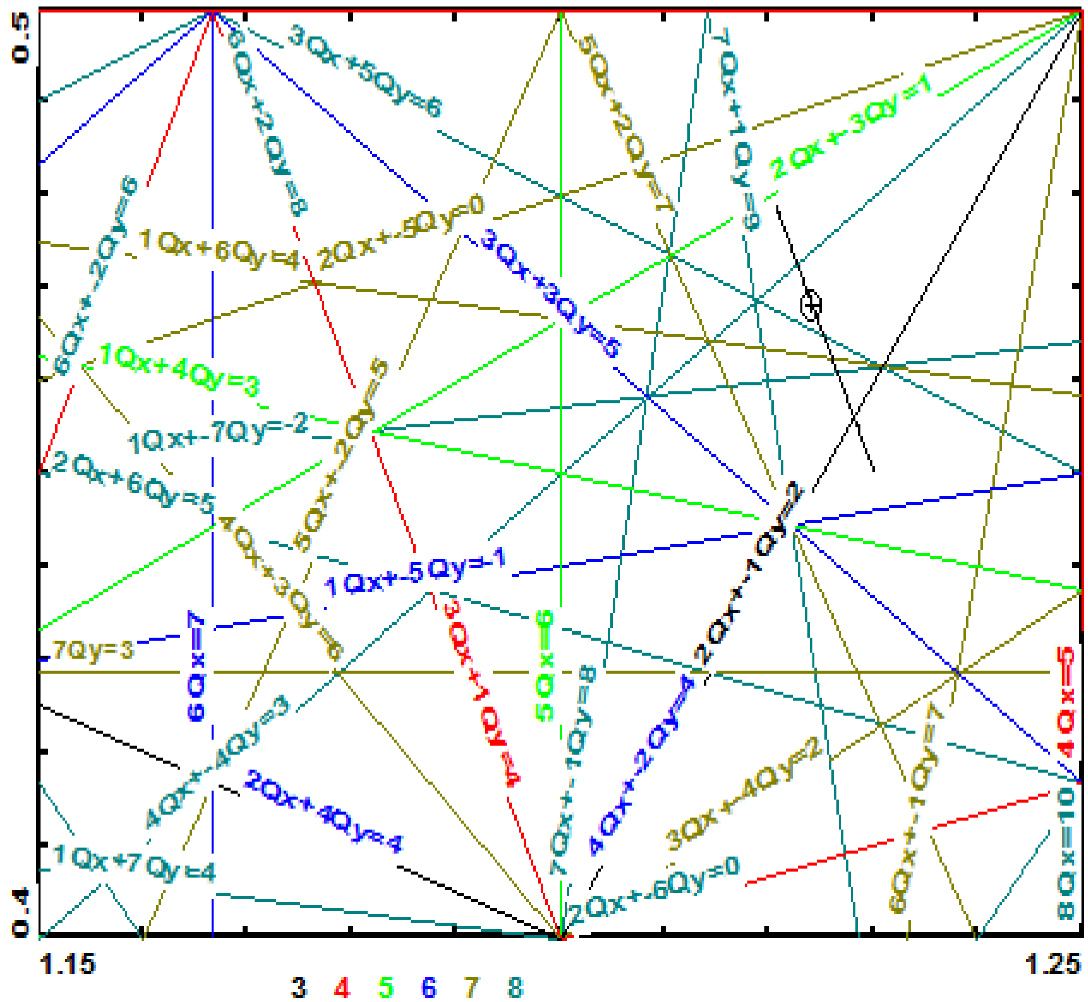
♦ Limited by distance between bending plates ($2a=3$ cm)

* Operation above transition because $\alpha>1$

♥ Set by IBS

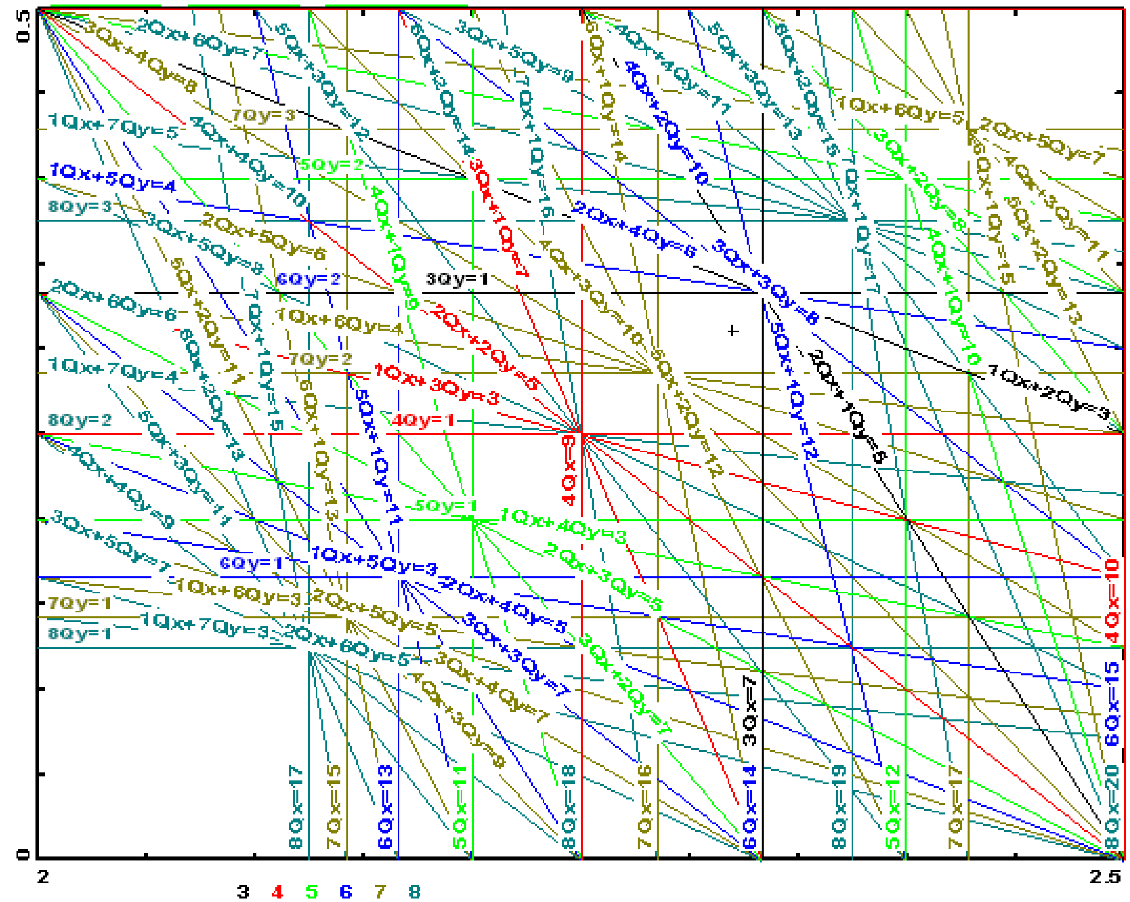
Tune Diagram for Weak Focusing Ring

- Working tunes are very close to 6-th and 8-th order resonances
- Their effect will be suppressed by quite small variations of β-functions. However simulations with realistic errors are required to understand effect of beam space charge on particle stability
  - But actual problem is IBS
- Distance to 4-th order resonance, $3Q_x+Q_y=4$, is ~0.04. It is sufficient to accommodate the space charge tune shifts



Tune Diagram for "Strong" Focusing Ring

- Additional focusing adjustments (F_x & F_y) instead of parameter *m*
  - Larger flexibility in choice of beam optics
- Considerable space in the tune diagram
  - Distance to 4-th order resonances is ~0.06. It is sufficient to accommodate the space charge tune shifts
- Weak vertical focusing was chosen for control of radial magnetic field
  - It results in high sensitivity to focusing errors
- "Strong" focusing ring has
  - larger area in the tune diagram
  - Larger momentum acceptance
  - operates above transition => better for IBS (see below)



RF and Related Parameters

- Synchrotron frequency has to be large enough to minimize spin decoherence within one synchrotron period but small relative to the distance to strong resonances

	Weak foc.	Strong foc.
Bucket height, $\Delta p/p\|_{bucket}$	$4.97\cdot10^{-4}$	$1.3\cdot10^{-3}$
Harmonic number: h	70	
Synchrotron tune, Q_s	0.002	0.006
RF voltage: V_0 [kV]	13	10
RF frequency: f_{RF} [MHz]	47.75	41.81
Bucket length [cm]	375	430
Bunch length: σ_s [cm]	27	31

- Sum of bunch lengths, $n_b\sigma_s$, has to be as large as possible to reduce space charge tune shifts and IBS
  - Bucket height, $\Delta p/p|_{bucket}$, has to be only slightly larger than the longitudinal acceptance, $\Delta p/p|_{max}$, but linearity is still desirable

    => $\Delta p/p|_{bucket}$ / $\Delta p/p|_{max}$=1.5

$$Q_s = \sqrt{\frac{heV_0\eta}{2\pi mc^2\gamma\beta^2}}$$

$$\left.\frac{\Delta p}{p}\right|_{bucket} = \frac{2Q_s}{h\eta}$$

$$\sigma_s = \frac{C\eta\sigma_p}{2\pi Q_s}$$

Coupling between Transverse and Longitudinal Motions

- Large dispersion results in coupling between x and s motions
- However for chosen RF voltage this coupling is sufficiently small even for the soft focusing ring considered above
  - It has a weak dependence on number of cavities => 1 cavity looks OK
  - It results in minor changes in tunes

$N_{cav} = 1$

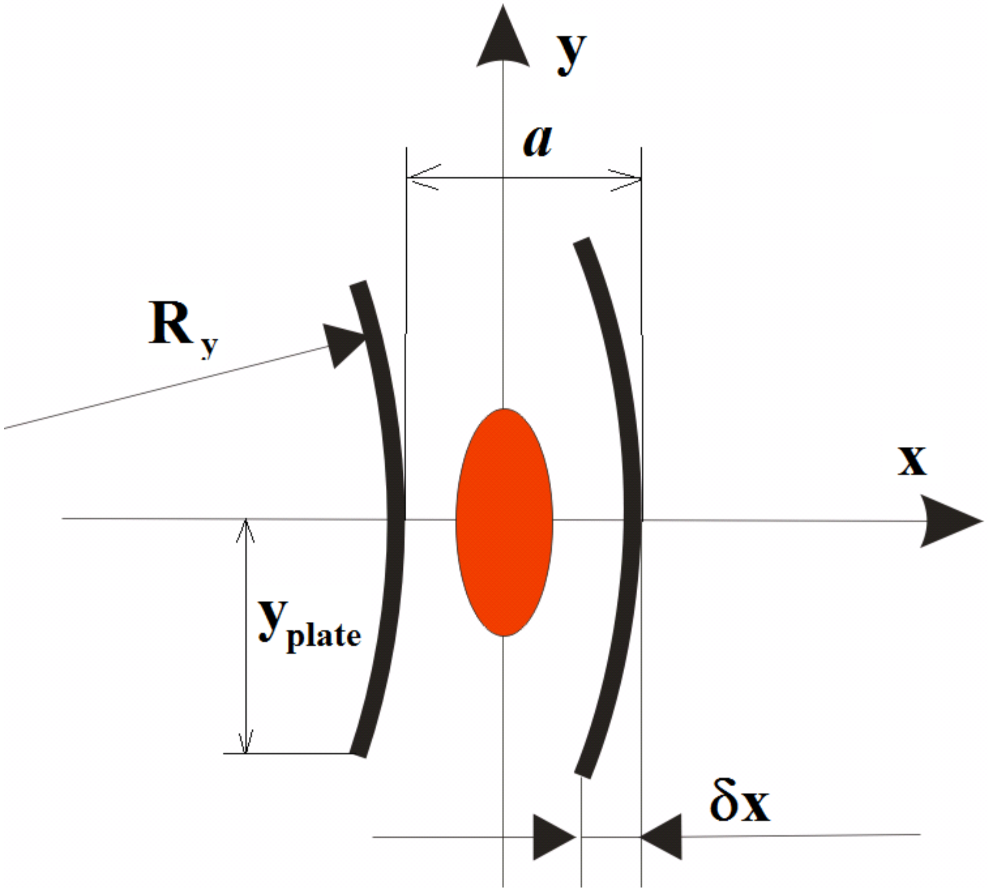
$Q_1 = 0.22858$	$Q_x = 1.22883$
$Q_2 = 0.45634$	$Q_y = 0.45634$
$Q_3 = 0.01991$	$Q_s = 0.01987$

$$\frac{v_1}{v_{1_0}} = \begin{pmatrix} 1 \\ 2.938i\times 10^{-4} \\ 0 \\ 0 \\ 1.349i \\ 4.003\times 10^{-7} \end{pmatrix} \qquad \frac{v_3}{v_{3_4}} = \begin{pmatrix} -0.019i \\ 4.003\times 10^{-7} \\ 0 \\ 0 \\ 1 \\ -4.14i\times 10^{-6} \end{pmatrix}$$

  - and a change in the horizontal beta-function from β_x= 33.99 m to β_{x1}=34.10 m (in Mais-Ripken representation)
- As one can see from the above eigen-vector corresponding to horizontal betatron oscillations, v_1, the betatron motion is accompanied by the longitudinal motion
  - at the cavity location the motions are shifted in phase by 90 deg.
  - 1σ horizontal motion makes 0.025σ synchrotron motion

Requirements to Manufacturing and Installation of Bending Plates

- The plate bending radius in the vertical plane is $R_y = R_0 / m$ = 201 m
  - For vertical displacement y_{plate}=10 cm it yields δx of 25 μm
    - Looks close to impossible to manufacture and install with required accuracy



- Non-parallel plates (non-concentric plate surfaces) create skew-quad field with gradient $G_s = \theta E_0 / a$, where θ is the angle between plates
  - Requirement to have this skew gradient much smaller than the gradient of vertical focusing field, $\theta E_0 / a \ll m E_0 / R_0$, yields: $\theta \ll 1.5 \cdot 10^{-4}$
    - with margin of 100 (skew quads are still required) one obtains very tight requirement: $\theta < 1.5 \cdot 10^{-6}$
- Looks like that the required mechanical and installation accuracies are too tight => "Soft-focusing -> Normal quad focusing machine!!!"

Space Charge Tune Shifts

- Space charge tune shift is weakly affected by ring optics

$$\Delta Q \approx \frac{r_p}{(2\pi)^{3/2}\beta^2\gamma^3}\left(\frac{C}{\sigma_s}\right)\frac{N_p}{2\varepsilon}$$

	Weak foc.	Strong foc.
Protons per bunch: N_p	$1.5\cdot10^8$	$7\cdot10^8$
Beam current, [mA]	1.1	4.7
$\Delta Q_x / \Delta Q_y$, $[10^{-3}]$	4.7/6.6	15/27

  - Exact formulas were used

$$\Delta Q_x = \frac{r_p N_p C}{(2\pi)^{3/2}\beta^2\gamma^3\sigma_s}\left\langle \frac{\beta_x}{(\sigma_x+\sigma_y)\sigma_x}\right\rangle_s$$

$$\Delta Q_y = \frac{r_p N_p C}{(2\pi)^{3/2}\beta^2\gamma^3\sigma_s}\left\langle \frac{\beta_y}{(\sigma_x+\sigma_y)\sigma_y}\right\rangle_s$$

- Beam emittances and momentum spreads are set by aperture (gap)
- Requirement to have sufficiently small IBS sets the beam current for soft-focusing ring
  - Tune shifts due to space charge are the main beam current limitation for strong focusing ring

- The tune shift due to counter rotating beam, $\sqrt{2\pi}C / N_b\sigma_s \approx 5.5$, is smaller and does not represent a problem

IBS

- IBS is the major source of emittance growth and, consequently, the major source of particle loss
- Dependence of potential energy on radial position in the electrostatic bend yields an additional term to the dependence of average particle momentum on radius
  - However it does not change local velocity spreads => "standard" IBS theory is applicable
- For the case when derivatives of dispersions and beta-functions can be neglected the growth rates for Gaussian beam are expressed by comparatively simple formula

$$\frac{d}{dt}\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \sigma_p^{\,2}\end{bmatrix}=\frac{r_p c N_p L_c}{4\sqrt{2}\beta^3\gamma^5\sigma_x\sigma_y\sigma_s\sqrt{\theta_x^{\,2}+\theta_y^{\,2}+\theta_p^{\,2}}}\begin{bmatrix}\left\langle \beta_x\psi\left(\theta_x,\theta_y,\theta_p\right)+\gamma^2\dfrac{D_x^{\,2}}{\beta_x}\psi\left(\theta_p,\theta_x,\theta_y\right)\right\rangle_s\\ \left\langle \beta_y\psi\left(\theta_y,\theta_p,\theta_x\right)\right\rangle_s\\ \left\langle \gamma^2\psi\left(\theta_p,\theta_x,\theta_y\right)\right\rangle_s\end{bmatrix},\quad \begin{matrix}\sigma_x=\sqrt{\varepsilon_x\beta_x+\left(\sigma_p D_x\right)^2},\quad \sigma_x=\sqrt{\varepsilon_y\beta_y},\\ \theta_x=\sqrt{\varepsilon_x/\beta_x},\quad \theta_x=\sqrt{\varepsilon_y/\beta_y},\quad \theta_p=\sigma_p\sqrt{\varepsilon_x\beta_x}\Big/\left(\gamma\sigma_x\right),\\ L_c=\ln\left(\sqrt{\sigma_x\sigma_y}\Big/r_{\min}\right),\quad r_{\min}=2r_p\Big/\left(\beta^2\gamma^2\left(\theta_x^{\,2}+\theta_y^{\,2}+\theta_p^{\,2}\right)\right),\end{matrix}$$

$$\Psi\left(x,y,z\right)=\frac{\sqrt{2}\sqrt{x^2+y^2+z^2}}{3\pi}\left(y^2\,\mathrm{R_D}\left(z^2,x^2,y^2\right)+z^2\,\mathrm{R_D}\left(x^2,y^2,z^2\right)-2x^2\,\mathrm{R_D}\left(y^2,z^2,x^2\right)\right),\mathrm{R_D}\left(x,y,z\right)=\frac{3}{2}\int_0^\infty\frac{dt}{\sqrt{\left(t+x\right)\left(t+y\right)\left(t+z\right)^3}}$$

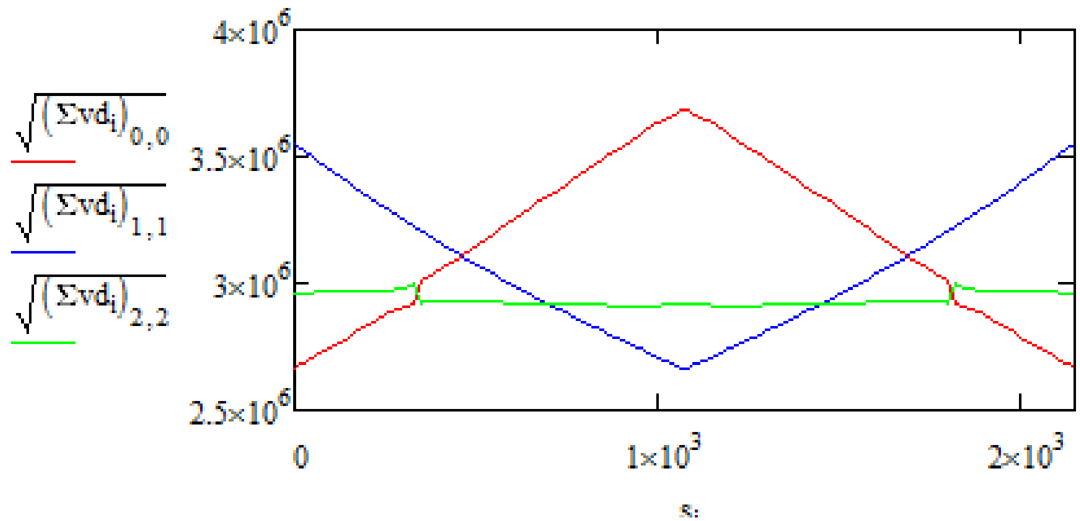
- Growth rates look significantly more complicated in the general case
  - Exact formulas are used in below estimates

IBS for the Weak and Strong Focusing Rings

- Weak focusing ring
  - Operates above transition
    - It is impossible to get to a quasi-equilibrium between temperatures of planes
      $\Rightarrow$ Fast emit. growth
  - Emittance growth leads to particle loss which determines the experiment time scale

	Weak foc.	Strong foc.
Protons per bunch	$1.5 \cdot 10^8$	$7 \cdot 10^8$
$\Delta Q_x / \Delta Q_y$, $[10^{-3}]$	4.7/6.6	15/27
$\tau_x = \varepsilon_x / (d\varepsilon_x/dt)$ [s]	305	7500
$\tau_y = \varepsilon_y / (d\varepsilon_y/dt)$ [s]	-1400	7500
$\tau_s = \varepsilon_s / (d\varepsilon_s/dt)$ [s]	250	7500

  - 1000 s measurement
    => IBS growth rates to be greater or about 300 s (see below)
    => Protons per bunch



- Strong focusing ring
  - IBS is suppressed in a quasi-equilibrium state
  - Beam space charge is the major limitation

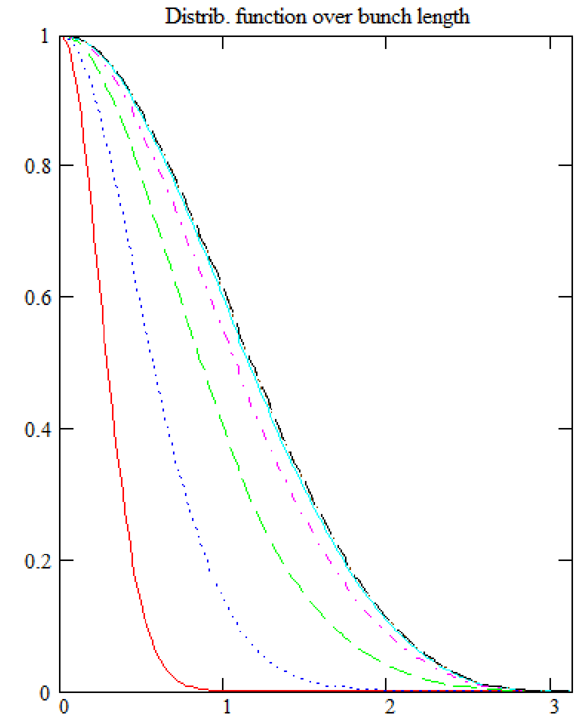
Growth Rates and Beam Lifetime

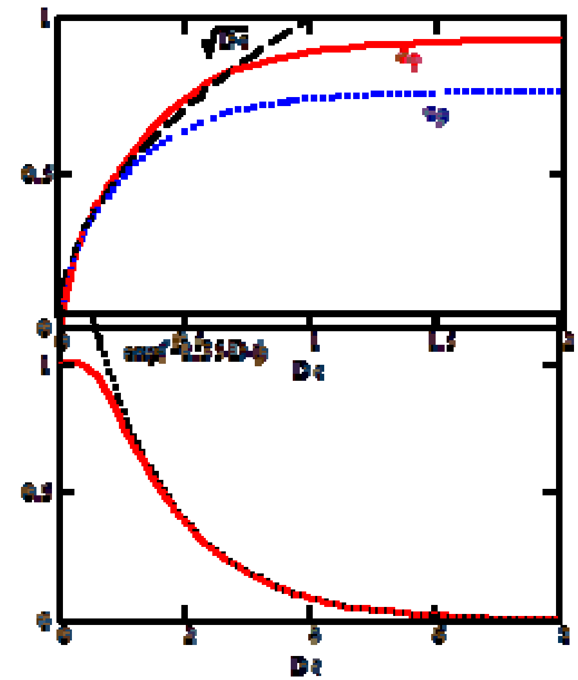
- In the above emit. growth rate calculations we assume (1) the beam distribution stays Gaussian and (2) there is no boundaries, *i.e.* particle loss
- Boundaries => the beam loss
  - When the beam size approaches aperture the particle distribution comes to the equilibrium shape which form does not change
    ⇨ exponential intensity decay
  - Simple model to estimate the beam lifetime
    - Hor. and long. growth rates are equal
    - Vertical growth rate is zero
    - Diffusion does not depend on coordinates

=> Diffusion equation in 4D phase space $(x,\theta_x,s,\Delta p/p)$

$$\frac{\partial f}{\partial t} = D\Delta_4 f$$

  - Here all variables are normalized to the gap between plates and we assume the growth rates inside each plane (x,θ_x) & $(s,\Delta p/p)$ are equal due to mixing by betatron or synchrotron motions





IBS Growth Rates and Beam Lifetime (continue)

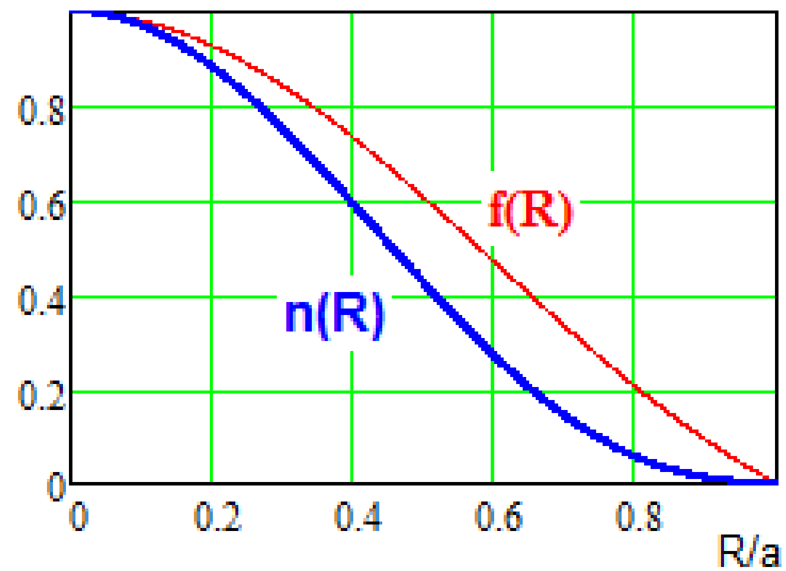
- Taking into account that particle scraping happens due to both betatron and synchrotron motion we obtain the boundary condition

$$f\left(t,R\right)_{R=a}=0,\quad R=\sqrt{x^2+\Theta_x{}^2+s^2+\Theta_s{}^2}$$

- Looking for spherically symmetric solution with exponential decay in time we obtain ordinary differential equation for the distribution function

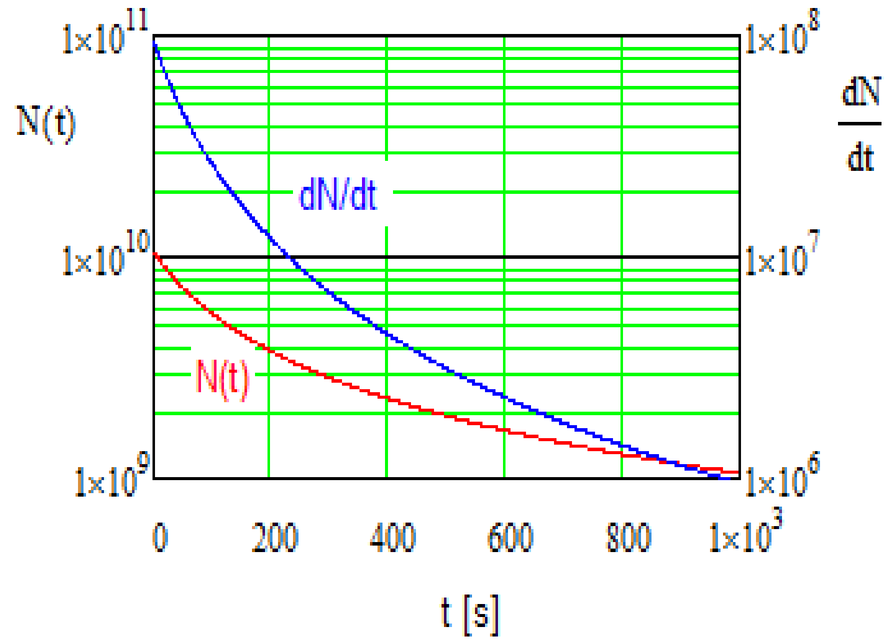
$$-\lambda f = D\frac{1}{R^3}\frac{d}{dR}\left(R^3\frac{df}{dR}\right),\quad f\left(\mathrm{a}\right)=0$$

Solution is $f(R)=2J_1\left(\mu_{11}R/a\right)/\left(R/a\right)$



- Integrating it over s, Θ_x, Θ_s we obtain particle distribution, n(x)
- Taking into account that diffusion depends on N_p we obtain a dependence of particle population on time

$$N(t)=\frac{N_p}{1+\lambda_D t},\quad \lambda_D\approx\frac{\mu_0{}^2}{n_\sigma\left(\tau_x+\tau_s\right)},\quad \mu_{11}\approx 3.832,\quad n_\sigma\approx 2.96,\quad \tau_x\approx\tau_s$$



An independent re-analysis of the storage ring proton EDM concept, Valeri Lebedev,

Effect of Voltage Ripple on Coherent Spin Precession

- Spin precession

$$\frac{d\mathbf{s}}{dt}=\frac{e}{m_p c}\mathbf{s}\times\left(\left(\frac{g_p}{2}-\frac{\gamma-1}{\gamma}\right)\mathbf{B}-\left(\frac{g_p}{2}-1\right)\frac{\gamma(\boldsymbol{\beta}\cdot\mathbf{B})\boldsymbol{\beta}}{\gamma+1}-\left(\frac{g_p}{2}-\frac{\gamma}{\gamma+1}\right)\boldsymbol{\beta}\times\mathbf{E}\right),\quad a_p=\frac{g_p-2}{2}$$

- Neglecting drifts and assuming uniform field variation along the ring

$$\Omega_s-\Omega_0=\frac{e}{m_p c}\left(a_p B_y-\left(\frac{1}{\gamma^2\beta^2}-a_p\right)\beta E_r\right),\quad a_p\approx 1.793$$

  - Magic moment: $\gamma^2\beta^2=1/a_p$ => K = 232.792 MeV, pc = 700.74 MeV/c
- RF frequency in storage ring can be stabilized to very high accuracy
  - That stabilizes the revolution frequency
  - Variation of voltage between plates results in
    - Electric field change

      $\Rightarrow$ Orbit change
    - but the revolution frequency stays the same

$$\Rightarrow\quad \frac{\Omega_s-\Omega_0}{\Omega_0}=\gamma\left(a_p\frac{\beta B_y}{E_0}+\frac{\Delta E+\beta B_y}{E_0}\frac{2}{\gamma^2\beta^2-m}\right)\xrightarrow[m=0.2]{\gamma^2\beta^2=1/a_p}\approx 5.5\frac{B_y}{E_0}+7\frac{\Delta E}{E_0},\quad B_y\ll E_0$$

  - RF frequency stabilization increases magnetic field effect by ~3 times
  - For $\Delta E/E_0=10^{-5}$ we obtain spin precession of 49.7 Hz
    - Accounting for the drifts (soft. foc.) changes this number to 50.3 Hz

Feedback to Suppress Spin-walk due to Voltage Variations

- Definition of spectral density of voltage ripple

$$\frac{\overline{U(t)^2}}{U^2} = \int_{-\infty}^{\infty} P_U(\omega) d\omega \,, \quad K_U(\tau) \equiv \frac{\overline{U(t)U(t+\tau)}}{U^2} = \int_{-\infty}^{\infty} P_U(\omega) \mathrm{e}^{i\omega t} \, d\omega$$

- Feedback effect on the spin-motion (analog system is assumed)

$$\begin{cases} \dfrac{ds_x}{dt} = \kappa_E \left(\dfrac{\Delta U(t)}{U} + V \right) \\ V = -K s_x \end{cases} \Rightarrow \frac{ds_x}{dt} = \kappa_E \left(\frac{\Delta U(t)}{U} - K s_x \right) \Rightarrow s_x(t) = \kappa_E \int_{-\infty}^{t} \frac{\Delta U(t)}{U} e^{-\frac{t-t'}{\tau_0}} dt'$$

where $\tau_0 = 1/(\kappa_E K)$, $|\mathbf{s}| = 1$, $\kappa_E = \dfrac{4\pi\gamma f_0}{\gamma^2 \beta^2 - m}$

- Combining we obtain rms fluctuations of spin deviation from velocity direction due to voltage fluctuations of the bending voltage

$$\left. \overline{\Delta s_x{}^2} \right|_U = \kappa_E{}^2 \int_{-\infty}^{\infty} \frac{P_U(\omega) d\omega}{\omega^2 + 1/\tau_0{}^2}$$

- If the spectral density width is narrower than the system bandwidth, $1/\tau_0$, then the spin motion is determined by rms voltage fluctuations

$$\overline{\Delta s_x{}^2} \approx \kappa_E{}^2 \int_{-\infty}^{\infty} \frac{P_U(\omega) d\omega}{1/\tau_0{}^2} = \kappa_E{}^2 \tau_0{}^2 \frac{\overline{\Delta U^2}}{U^2}$$

Shot Noise in the Spin Measurement System

- Each proton makes pulse $u(t)$ => the spectral density of spin signal is

$$P_u(\omega) = 2\pi\dot{n}\left|S_\omega{}^2\right| , \quad S_\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} u(t)dt$$

  where $\dot{n} \equiv dn/dt$ is the rate of particles in the measurement system

  - It results in the spectral density of V value

$$P_V(\omega) = \left|\frac{S_\omega(\omega)}{S_\omega(0)}\right|^2 \frac{K^2}{2\pi\dot{n}A^2},$$

    where is polarimeter resolution efficiency (A~0.6 used, need more realistic value)

- That yields the rms fluctuations of spin deviation from velocity direction due to shot noise

$$\left.\overline{\Delta s_x{}^2}\right|_S = \frac{1}{2\pi\dot{n}A^2}\int_{-\infty}^{\infty}\left|\frac{S_\omega(\omega)}{S_\omega(0)}\right|^2 \frac{d\omega}{1+\omega^2\tau_0{}^2}$$

  Here we assume that duration of single particle signal is much smaller than the damping time

  - Otherwise more complicated integro-differential equation needs to be used (next slide)

System stability with a finite decay time of single particle signal

- Let's consider system where signal of each particle exponentially decays with time: $u(t) = u_0 \, \mathrm{e}^{-t/t_a}$,
  - ⇨ corresponding spectral density ratio is: $\left|\dfrac{S_\omega(\omega)}{S_\omega(0)}\right|^2 = \dfrac{1}{1+\omega^2\tau_a{}^2}$
- The equation describing the system response is now changing to:

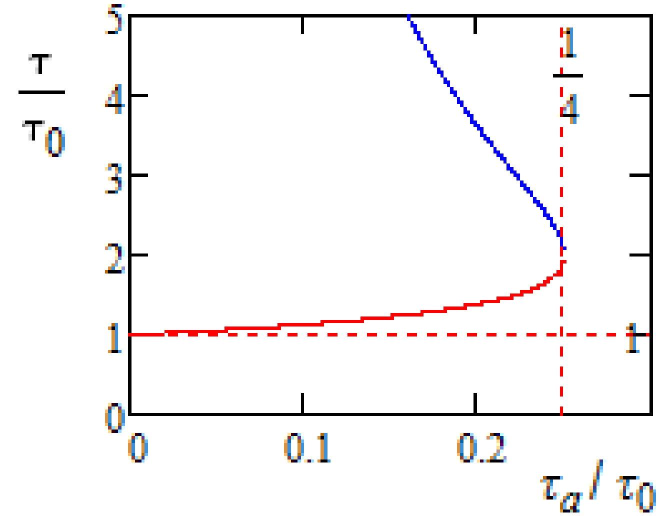
$$\frac{ds_x}{dt} = -\frac{s_x}{\tau_0} \quad \Rightarrow \quad \frac{ds_x}{dt} = -\frac{1}{\tau_0}\int_{-\infty}^{t} s_x(t')e^{-\frac{t-t'}{\tau_a}}\frac{dt'}{\tau_a}$$

- Differentiating and using the original equation one obtains an ordinary differential equation

$$\frac{d^2 s_x}{dt^2} = -\frac{1}{\tau_a}\frac{ds_x}{dt} - \frac{s_x}{\tau_0\tau_a}$$

- The solution is:

$$s_x = s_x \, \mathrm{e}^{-t/\tau} , \quad \frac{1}{\tau} = \frac{1}{2\tau_a} \pm \sqrt{\left(\frac{1}{2\tau_a}\right)^2 - \frac{1}{\tau_0\tau_a}}$$

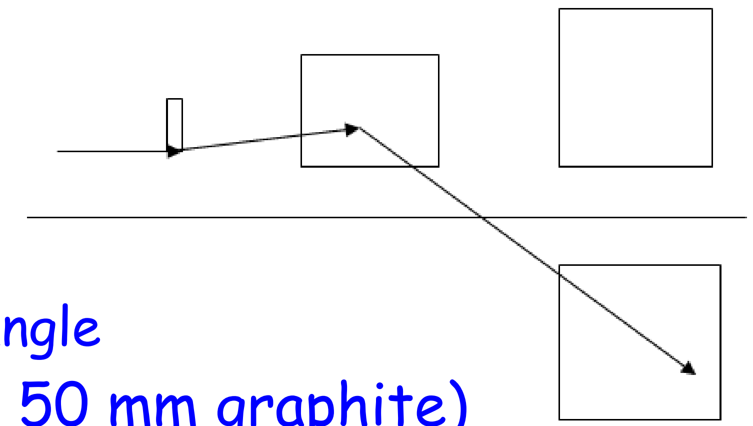


- The system loses stability with $\tau_a > \tau_0 / 4$, otherwise it's response is close to the response for the original first order equation

Optimization of Polarimeter

- Shot noise in polarimeter limits performance of the spin-feedback
  - Maximum possible percentage of lost protons needs to be used in polarimeter
- Two step scattering (similar to Tevatron collimation system)
  - Thin primary target
    - Particle with typical impact parameter of ~ 1 μm does not leave this target through its side

      ⇒ 2 mm graphite yields 1.6 mrad rms angle



  - Thick major target (50 cm downstream, 50 mm graphite)
    - Half of the particles scattered in primary collimator gets to it
    - probability of nuclear scattering in major target ~10%
    - 50 cm between targets is chosen to reduce probability of particle to get out on side
      - It would reduce probability of nuclear interaction
    - Energy lost is ~100 MeV
  - Particle detector intercepts major fraction of particles lost in the target => ~3% of lost particles will make useful signal

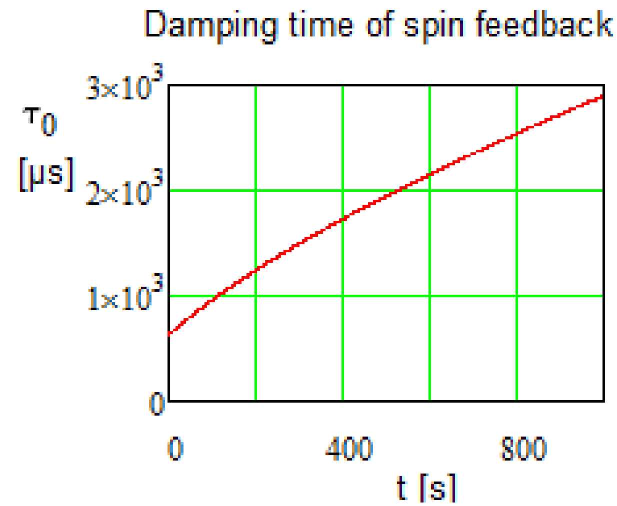
Estimate of Spin Fluctuations due to Voltage Ripple and Shot Noise in the Spin Measurement System

- For exponential decay of single particle system we have

$$\left.\overline{\Delta s_x^{\,2}}\right|_S = \frac{1}{2\pi\dot{n}A^2}\int_{-\infty}^{\infty}\left|\frac{S_\omega(\omega)}{S_\omega(0)}\right|^2\frac{d\omega}{1+\omega^2\tau_0^{\,2}} = \frac{1}{2\pi\dot{n}A^2}\int_{-\infty}^{\infty}\frac{d\omega}{\left(1+\omega^2\tau_0^{\,2}\right)\left(1+\omega^2\tau_a^{\,2}\right)} = \frac{1}{2\dot{n}A^2\left(\tau_0+\tau_a\right)}$$

- For $\tau_a<\tau_0/4$ we can neglect τ_a in denominator. That yields the total spin fluctuations:

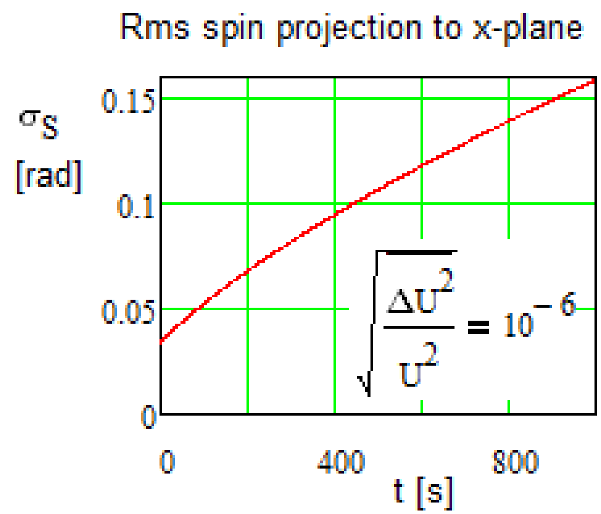
$$\left.\overline{\Delta s_x^{\,2}}\right|_{tot} = \left.\overline{\Delta s_x^{\,2}}\right|_S + \left.\overline{\Delta s_U^{\,2}}\right| \approx \frac{1}{2\dot{n}A^2\tau_0} + \kappa_E^{\,2}\tau_0^{\,2}\frac{\overline{\Delta U^2}}{U^2}$$



- Optimum damping rate and resulting spin motion are:

$$\left.\frac{1}{\tau_0}\right|_{opt} \approx \sqrt[3]{4\dot{n}A^2\kappa_E^{\,2}\frac{\overline{\Delta U^2}}{U^2}}\ ,\qquad \sigma_S \equiv \left.\overline{\Delta s_x^{\,2}}\right|_{@opt} \approx \frac{3}{2}\sqrt[3]{\frac{\kappa_E^{\,2}}{2\dot{n}^2A^4}\frac{\overline{\Delta U^2}}{U^2}}$$

  - ♦ We imply here that 3% of lost particles is intercepted by spin measurement system



- Number of particles left after ~1000 s is insufficient to support further operation of spin feedback

Vertical Magnetic Field Compensation

- Magnetic field does not change as fast as electric field and a suppression its fluctuations with feedback system is easier
- Vertical component of magnetic field
  - To nullify $\langle B_y \rangle$ we can use the second spin feedback which measures the horizontal spin precession of the counter rotating beam and corrects it with vertical magnetic field excited by coils
    - only average magnetic field needs to be compensated
      $\Rightarrow$ 0.1 rad after 1000 s corresponds ΔB_y=0.058 μG
  - Keeping the spin horizontal components equal to zero for both beams keeps us at the magic energy if $dB_y/dx = 0$.
    - Otherwise the beam separation introduced by magnetic field makes average magnetic field different for both beams
      => slightly different energies

Effect of Radial Magnetic Field

- First assume that B_x does not change along circumference. Then, β has only longitudinal component, and the spin precession into vertical plane is:

$$\frac{ds_y}{dt} = \frac{e}{m_p c}\left(\left(\frac{g_p}{2} - \frac{\gamma - 1}{\gamma}\right) B_x s_z + \left(\frac{g_p}{2} - \frac{\gamma}{\gamma + 1}\right)\beta E_y s_z\right)$$

- Equation of motion bounds electrical and magnetic fields
  - That yields

$$\frac{dp_y}{dt} = e\left(E_y + \beta B_x\right) = 0 \quad \Rightarrow \quad E_y = -\beta B_x$$

$$\frac{ds_y}{dt} = \frac{e}{m_p c}\left(\left(\frac{g_p}{2} - \frac{\gamma - 1}{\gamma}\right) - \left(\frac{g_p}{2} - \frac{\gamma}{\gamma + 1}\right)\beta^2\right) B_x s_z \xrightarrow{\text{At magic energy}} \approx \frac{e}{m_p c}\left(2.59 - 0.8\right) B_x s_z$$

    - Thus, electric field which comes from the vertical focusing reduces the spin precession by about 30%
    - Effect has the same sign for both counter-rotating beams. It mimics EDM

- The half-separation of beams comes from: $mE_0 \Delta y / R_0 = \beta B_x$
  - It is directly related to the spin precession to the vertical plan

$$\frac{\Omega_{sy}}{\Omega_0} \equiv \frac{1}{\Omega_0}\frac{ds_y}{s_z dt} = m\gamma\left(\left(\frac{g_p}{2} - \frac{\gamma - 1}{\gamma}\right) - \left(\frac{g_p}{2} - \frac{\gamma}{\gamma + 1}\right)\beta^2\right)\frac{\Delta y}{R_0} \xrightarrow{\text{At magic energy}} \approx 2.24 m \frac{\Delta y}{R_0}$$

⇨ $5 \cdot 10^{-6}$ rad after 1000 s => $B_x \approx 0.29$ pG and $\Delta y \approx 0.1$ pm

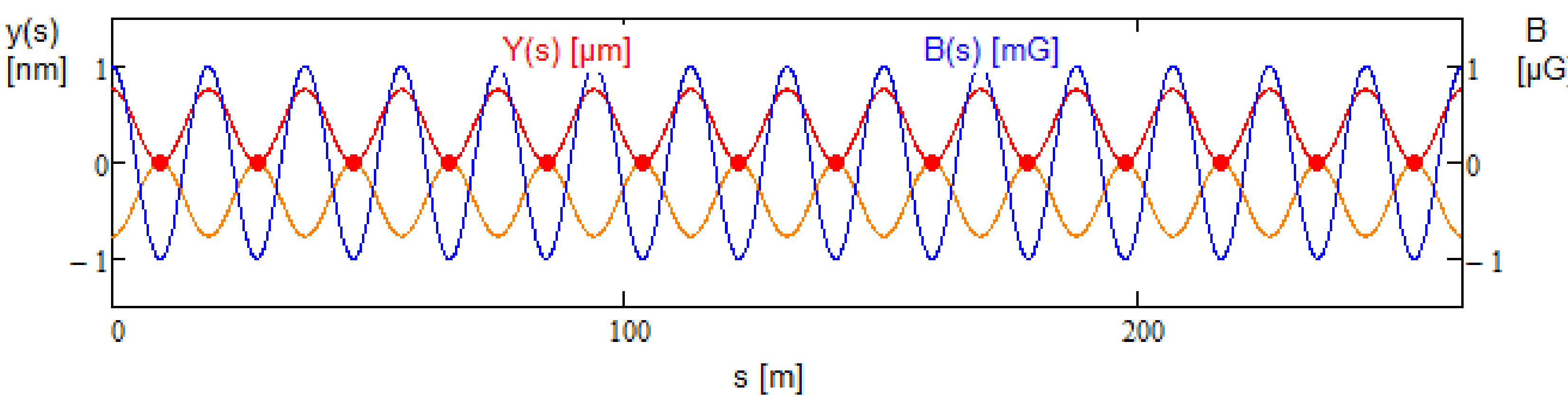
Limitations on Radial Magnetic Field Cancellation

- Typical values of residual magnetic fields
  - in SC cavities ~10 mG
  - in high precision experiments with active suppression in small volume ~ 1 μG
- In our estimate we assume that
  - the magnetic field is suppressed to 1 μG
  - we have 14 infinite accuracy BPMs
  - after correction of differential orbit for counter rotating beams we assume the magnetic field being distributed in worst possible scenario

$$B_x(s) = \frac{m_p c^2 \gamma}{e}\left(\left(\kappa^2 - k^2\right)\cos(\mathrm{ks}) + \kappa^2\right)A_y\ ,\quad k = \frac{2\pi}{C}N_{BPM}\ ,\quad \kappa = \frac{2\pi}{C}Q_y\ ,\quad N_{BPM} = 14$$

which corresponds to the beam displacement of $y(s) = \left(1 + \cos(\mathrm{ks})\right)A_y$

- B_{max}=1 μG yields A_y=380 pm and the average $\overline{B_x} = \Delta B_{\max}\left(Q_y / N_{BPM}\right)^2$ =1.06 nG



Limitations on Radial Magnetic Field Cancellation

- The above estimate results in that it does not make much sense to have
  - BPM accuracy better than ~50 pm and that
  - the best expectations for average magnetic field cancellation is about 1 nG if 1 μG is achieved in non-beam measurements
- This is 4 orders of magnitude worse than the desired values
  - Note that 1 μG field used in the estimate looks as extremely optimistic requirement
  - Note that this residual field is not determined by random fluctuations and therefore cannot be averaged out with more measurements

Sources of Magnetic Field Gradient

- Eddy magnetic field in the cavity is the major source of magnetic field gradient and, consequently, magnetic field
  - Beam crosses the cavity at zero voltage

⇨ $\int G dL \Big|_{cavity} \approx \omega_{RF} V_{RF} / 2c \approx 0.22\ G$ for V_{RF}=13 kV

  - For the differential precession rate <5 nrad/s we obtain the beam offset from the cavity center < **0.35 nm**
    - Typical microseism > 1 μm @ ~1 s

      ⇒ we need 3000 times suppression
- Main limitations on beam position measurements
  - Shot noise after 1 sec averaging ~ 60 pm @ full intensity
  - Thermal noise with 5 kΩ coupling impedance and room temperature amplifier ~ 50 pm in 10 Hz band @ full intensity
- These accuracy limitations correspond to expectations from the previous slide
- Can we achieve such accuracy?
  - Systematic errors are expected to be the main problem

Conclusions

- Overall concept of proton EDM electrostatic machine is not limited by the considered beam-physics issues
- Judged on pure acceleration physics grounds the strong focusing ring looks better than the soft focusing ring
  - Larger momentum acceptance and particle number
  - Suppressed IBS rates

	Soft focusing	Strong focusing
Circumference, m	263	300
Qx/Qy	1.229/0.456	2.32/0.31
Particle per bunch	$1.5\cdot10^{8}$	$7\cdot10^{8}$
Coulomb tune shifts, $\Delta Q_x/\Delta Q_y$	0.0046/0.0066	0.0146/0.0265
Rms emittances, x/y, norm, μm	0.56/1.52	0.31/2.16
Rms momentum spread	$1.1\cdot10^{-4}$	$2.9\cdot10^{-4}$
IBS growth times, x/y/s, s	300/(-1400)/250	7500
RF voltage	13	10.3
Synchrotron tune	0.02	0.006

- Analysis of spin decoherence for both rings is required to see their potential for EDM
  - In particular, the sensitivity of spin decoherence to sextupoles
- Small vertical tune requires exceptionally high mechanical accuracies of bending plates manufacturing
  - Corrections are required for any ring
  - At minimum a standard set: dipole correctors, trim quads, skew-quads and sextupoles.
  - Note that all soft-focusing machines which were built had much larger ratio of the gap to radius
    - It greatly alleviates problems
- 2 feedback systems are required to cancel the vertical magnetic field and keep the spin aligned along velocity
- It is not feasible how the average radial magnetic field can be suppressed below 1 nG
  - It is already unprecedented level of magnetic field suppression for such large vacuum chamber
  - Looks like we are above the desired value by about 4 order of magnitude

500 m Electric Ring: IBS and Beam Parameters

Valeri Lebedev
Fermilab

December 24, 2014

Optics

Main parameters

Beam energy	232.792 MeV
Circumference	500 m
Qx/Qy	2.42/0.44
Number of super-periods	4
FODO sells per super period	6
FODO sell length	20.83333 m
Number of arcs	4
Sells per arc	5
Number of straights	4
Sells per straight	1
Bends per half cell	3
Bending radius	52.3089
Gap	3 cm
Bending voltage	±120 kV
Slip-factor, $\eta=\alpha-1/\gamma^2$	-0.192

Structure of FODO half-cell in arc (other half is mirror symmetric)

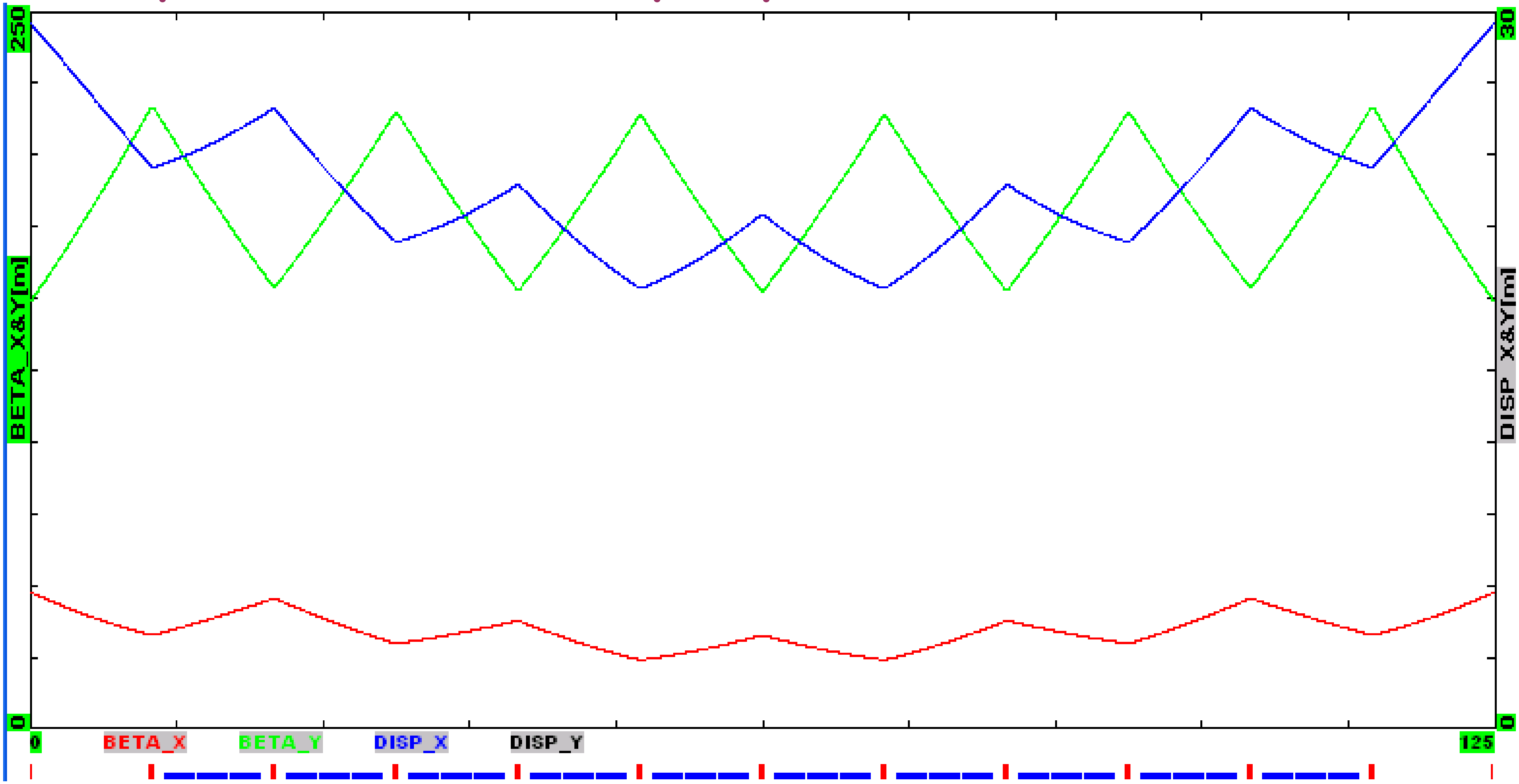
N	Name	S[cm]	L[cm]		
1	LqDh1	1061.67	20		Ge[kV/cm**2]=-3.3918
2	oQ	1141.67	80		
3	Rbend	1415.56	273.889	E[kV/cm]=80.16	Ge[kV/cm**2]=0
4	ob	1425.56	10		
5	Rbend	1699.44	273.889	E[kV/cm]=80.16	Ge[kV/cm**2]=0
6	ob	1709.44	10		
7	Rbend	1983.33	273.889	E[kV/cm]=80.16	Ge[kV/cm**2]=0
8	oQ	2063.33	80		
9	LqF	2083.33	40		Ge[kV/cm**2]=3.7306
10	oQ	2183.33	80		
11	Rbend	2457.22	273.889	E[kV/cm]=80.16	Ge[kV/cm**2]=0
12	ob	2467.22	10		
13	Rbend	2741.11	273.889	E[kV/cm]=80.16	Ge[kV/cm**2]=0
14	ob	2751.11	10		
15	Rbend	3025	273.889	E[kV/cm]=80.16	Ge[kV/cm**2]=0
16	oQ	3105	80		
17	LqDh	3125	20		Ge[kV/cm**2]=-3.2068

Optical structure of in straight lines

1	Rbend	273.889	273.889	E[kV/cm]=80.16	Ge[kV/cm**2]=0
2	oQ	353.889	80		
3	LqD1	393.889	40		Ge[kV/cm**2]=-3.3918
4	oLong	1395.56	1001.67		
5	LqF1	1435.56	40		Ge[kV/cm**2]=4.1756
8	oLong	2437.22	1001.67		
6	LqD1	2477.22	40		Ge[kV/cm**2]=-3.3918
7	oQ	2557.22	80		
8	Rbend	2831.11	273.889	E[kV/cm]=80.16	Ge[kV/cm**2]=0

Optics (3)

Twiss parameters for 1 super-period (out of four)

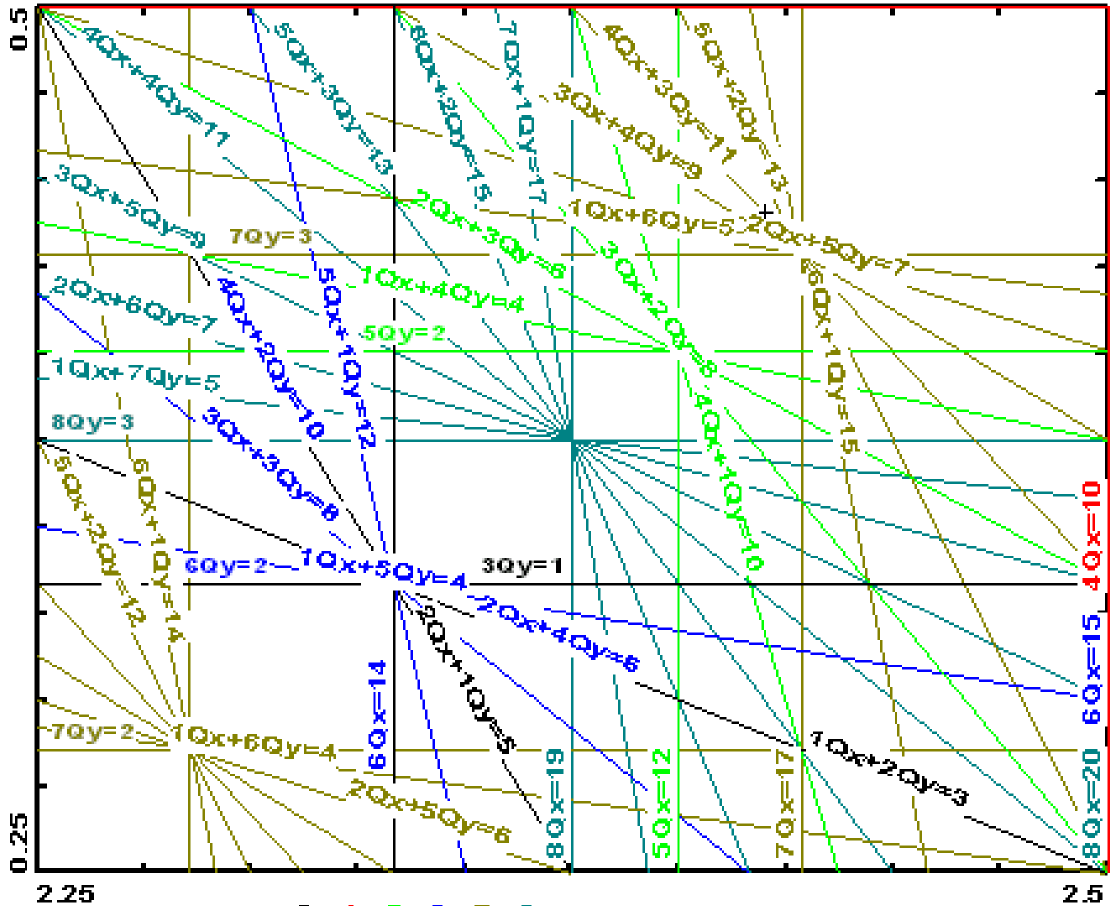


β_{xmax}=47 m, β_{ymax}=216 m, D_{xmax}=29.5 m

Optics (4)

- Ring structure
  - 4 super periods. Each includes:
    - 5 FODO cells with 3 bends per half cell
      - electric bends with R_0=52.3 m and L=2.7389 m
        - Gap between plates 3 cm, V= ±120 kV,
        - m = 0 (no vert. focusing)
    - 1 FODO cell of the same length but without bends (~22 m straight line)
  - One of two 80 cm gaps between quads and bends are filled with
    - H or V corrector, skew-quad corrector, F or D sextupole, and BPM
    - Other can be used by experiment
  - RF cavity, injection kicker and septum are located in the straights
- Large flexibility in adjustments of beam optics
- For chosen optics its major parameters are:
  - β_{xmax}=47 m, β_{ymax}=216 m, D_{xmax}=29.5 m
- Weak vertical focusing was chosen for control of radial magnetic field
  - It results in high sensitivity to focusing errors

Optics (5)

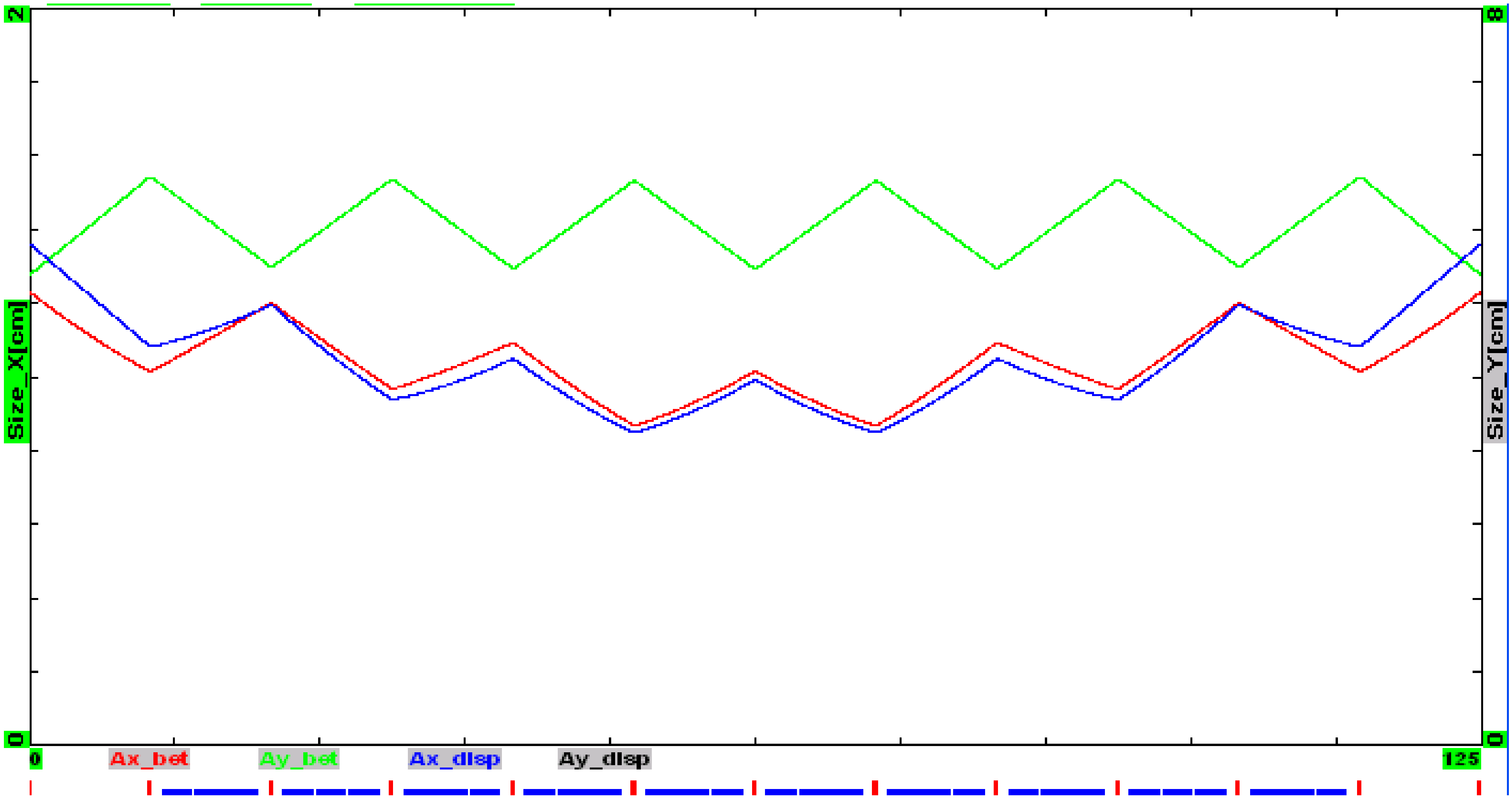


- Tunes were chosen to exclude strong resonances in the working point vicinity
- Resonances of 5-th and 7-the order are located in close vicinity. However they are not excited by the beam space charge and therefore are not expected to limit the dynamic aperture.
- Distance to 4-th order resonances is ~0.07.
  - It is sufficient to accommodate the space charge tune shifts

Acceptances

- Horizontal acceptance is set by the gap between plates of 3 cm and maximum of beta-function in the bends of 44 m
  - Assuming 3 mm orbit error one obtains ε_{Hmax}=3.2 mm mrad
- The maximum dispersion in the bends is 27 m
  - Similarly, one obtains the maximum momentum spread: $(\Delta p/p)_{max}=4.6\cdot10^{-4}$
- We determine the vertical acceptance to be large enough so that the vertical and horizontal degrees of freedom would be in thermal equilibrium, i.e. vertical and transverse velocity spreads would be equal
  - That minimizes the IBS
  - It results in: ε_{Vmax}=17 mm mrad and the maximum beam size (at acceptance boundary) of 6.2 cm which is about 5 times larger than the horizontal beam size

Acceptances (2)



Beam boundary at acceptances:

ε_{Hmax}=3.2 mm mrad, ε_{Vmax}=17.5 mm mrad, $(\Delta p/p)_{max}=4.6\cdot 10^{-4}$

RF and Related Parameters

- Synchrotron frequency has to be large enough to minimize spin decoherence within one synchrotron period but small relative to the distance to strong resonances, Q_s=0.0066 was chosen (ΔQ_{SC}~0.04)
- Sum of bunch lengths, $n_b \sigma_s$, has to be as large as possible to reduce space charge tune shifts and IBS
  - Bucket height, $\Delta p/p|_{bucket}$, has to be only slightly larger than the longitudinal acceptance, $\Delta p/p|_{max}$, but linearity is still desirable
    => $\Delta p/p|_{bucket}$ / $\Delta p/p|_{max}$=1.5
- Main parameters
  - RF voltage: V_0=6 kV
  - Harmonic number: h=100
  - RF frequency: f_{RF}=35.878 MHz
  - Synchrotron tune: Q_s=0.0066
  - Bucket height: $\Delta p/p|_{bucket}$=$6.9\cdot 10^{-4}$
  - Bucket length: 5.0 m
  - Bunch length: σ_s= 32 cm

$$Q_s = \sqrt{\frac{heV_0\eta}{2\pi mc^2\gamma\beta^2}}$$

$$\left.\frac{\Delta p}{p}\right|_{bucket} = \frac{2Q_s}{h\eta}$$

$$\sigma_s = \frac{C\eta\sigma_p}{2\pi Q_s}$$

Space Charge Tune Shifts

- Beam emittances and momentum spreads are set by aperture (gap)
- Tune shifts due to space charge are the main beam current limitation for strong focusing ring
- The tune shifts due to counter rotating beam, $\sqrt{2\pi}C / N_b\sigma_s \approx 40$, are smaller and do not represent a problem

$$\Delta Q_x = \frac{r_p N_p C}{(2\pi)^{3/2} \beta^2 \gamma^3 \sigma_s} \left\langle \frac{\beta_x}{(\sigma_x + \sigma_y)\sigma_x} \right\rangle_s$$

$$\Delta Q_y = \frac{r_p N_p C}{(2\pi)^{3/2} \beta^2 \gamma^3 \sigma_s} \left\langle \frac{\beta_y}{(\sigma_x + \sigma_y)\sigma_y} \right\rangle_s$$

Protons per bunch: N_p	$2.5 \cdot 10^8$
Beam current, [mA]	1.4
Rms bunch length [cm]	32
Rms norm. emittances, $\varepsilon_x/\varepsilon_y$ [μm]	0.12/0.61
$\Delta Q_x / \Delta Q_y$, [10^{-2}]	2.9/5.0

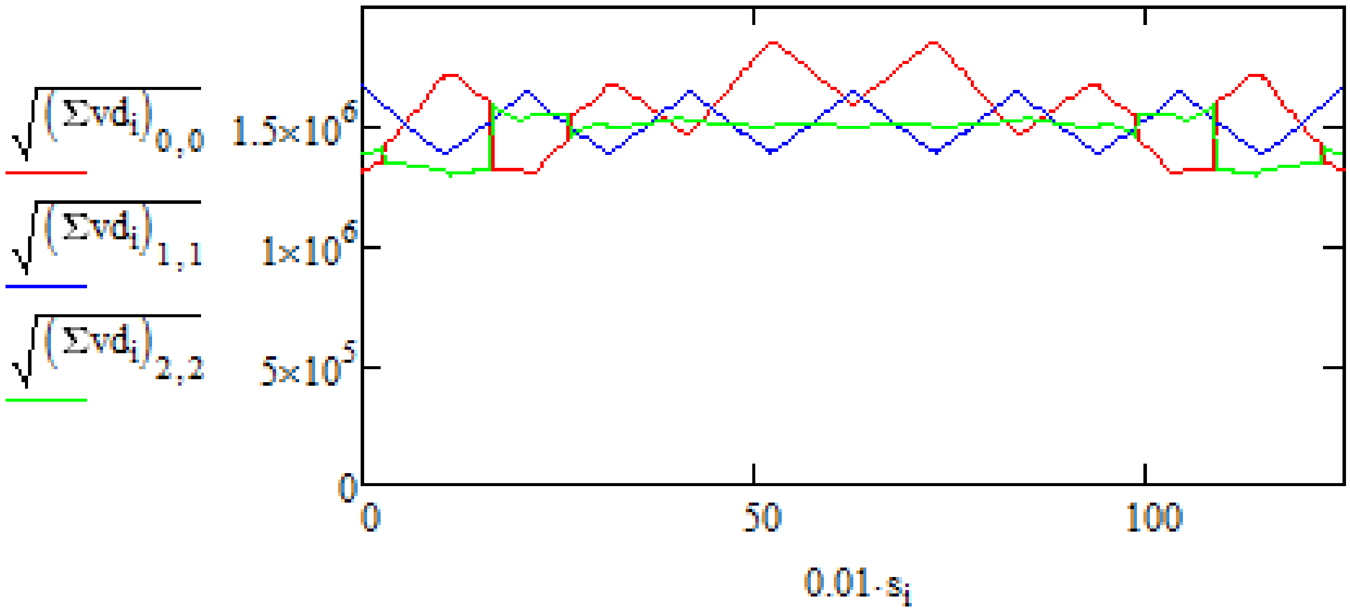
IBS Growth Rates

- Operation below transition greatly reduces IBS growth rates for operation at the thermal equilibrium: $\tau_{x,y,s} \approx 1600$ s (no scraping)
- Temperature exchange between different degrees of freedom proceeds faster by more than an order of magnitude for a states close to the equilibrium
- Collimation stops emittance growth when the rms beam size achieves approximately 1/3 of the aperture
- For chosen optics the major aperture limitation is in the longitudinal plane. That results in a quasi-equilibrium state when the growth rates of horizontal and vertical emittances due to IBS are equal to zero. The IBS driven emittance growth rate for longitudinal plane of 1/830 s^{-1} is stopped by collimation which results in the intensity loss:

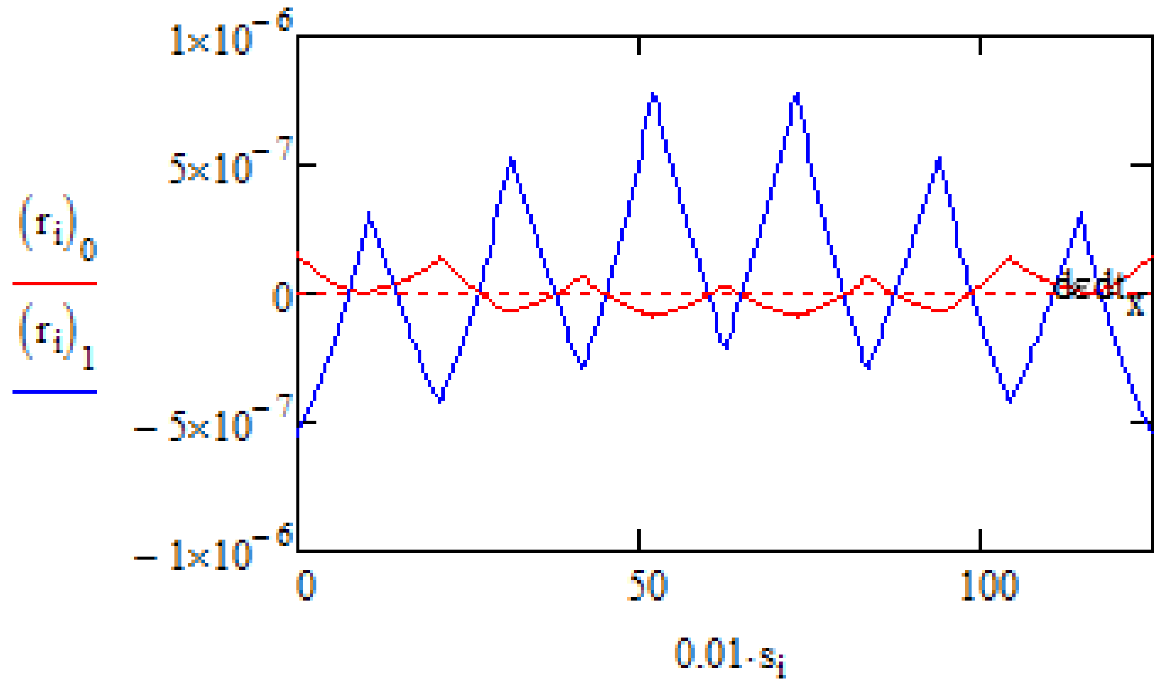
$$N(t) = \frac{N_p}{1+\lambda_D t}, \quad \lambda_D \approx \frac{{\mu_{01}}^2}{n_\sigma \tau_s}, \quad \mu_{01} \approx 2.405, \quad n_\sigma \approx 2.96, \quad \tau_x = \tau_y = 0$$

- The parameters of the quasi-equilibrium state are: ε_x=0.21 μm, ε_y=1.0 μm, $\sigma_p = 1.4\cdot 10^{-4}$ => τ_s=830 s, $1/\lambda_D$=350 s

- An increase of horizontal and vertical emittances at the quasi-equilibrium reduces the space charge tune shifts to: $\Delta Q_{SCx}=0.021$, $\Delta Q_{SCy}=0.032$



Variation of rms velocity spreads in the beam frame along the quarter of the ring. Major axes of 3D velocity ellipsoid are presented



Heating contributions to the horizontal and vertical planes along the quarter of the ring at the equilibrium. One can see that averaging yield much smaller resulting values.