A Small Proton EDM Prototype Ring with 10^{-26}e-cm Precision

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Outline

Reduced energy EDM prototype ring

Spin tunes in superimposed electric and magnetic fields

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3 Field Transformations

The dominant fields in an electric storage ring are radial lab frame electric field $\mathbf{E} = -E\hat{\mathbf{x}}$ and/or vertical lab magnetic field $\mathbf{B} = B\hat{\mathbf{y}}$. Transverse proton rest frame field vectors $\mathbf{E}'$ and $\mathbf{B}'$, and longitudinal components E'_z and B'_z, are related by

$$\mathbf{E}' = \gamma(\mathbf{E} + \boldsymbol{\beta} \times c\mathbf{B}) = -\gamma(E + \beta cB)\,\hat{\mathbf{x}} \tag{1}$$

$$\mathbf{B}' = \gamma(\mathbf{B} - \boldsymbol{\beta} \times \mathbf{E}/c) = \gamma(B + \beta E/c)\,\hat{\mathbf{y}} \tag{2}$$

$$E'_z = E_z, \tag{3}$$

$$B'_z = B_z. \tag{4}$$

Even if lab magnetic field $B = 0$, in the proton rest frame $\mathbf{B}' \neq 0$. Except

in the nonrelativistic regime, the magnetic field in the particle rest frame (and hence the induced spin precessions) are comparable in laboratory electric and magnetic fields.

4 All-electric proton frozen spin parameters

$$
\begin{aligned}
c &= 2.99792458e8\,\mathrm{m/s} \\
m_p c^2 &= 0.93827231\,\mathrm{GeV} \\
\gamma_0 &= 1.248107349 \\
\mathcal{E}_0 = \gamma_0 m_p c^2 &= 1.171064565\,\mathrm{GeV} \\
K_0 = \mathcal{E}_0 - m_p c^2 &= 0.232792255\,\mathrm{GeV} \\
p_0 c &= 0.7007405278\,\mathrm{GeV} \\
\beta_0 &= 0.5983790721 \\
G &= 1.7928474
\end{aligned}
\tag{5}
$$

the last of which is the proton anomalous magnetic moment G. For mnemonic purposes it is enough to remember $\beta_0 \approx 0.6$, $p_0 c \approx 0.7$ GeV and $\gamma_0 \approx 1.25$.

5 Reduced energy EDM prototype ring

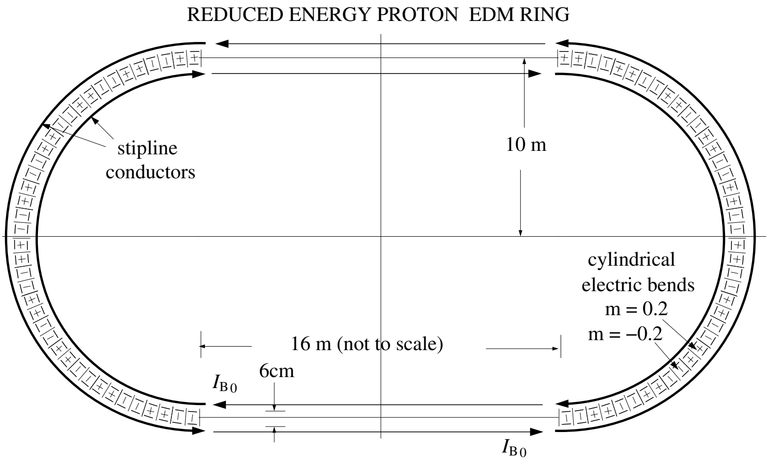


Figure 1: (Reduced energy and circumference) proton EDM prototype ring. Superimposed magnetic field (0.00865 T) is required because the proton 45 MeV kinetic energy is less than the 233 MeV magic energy required to freeze the spins in an all-electric ring.

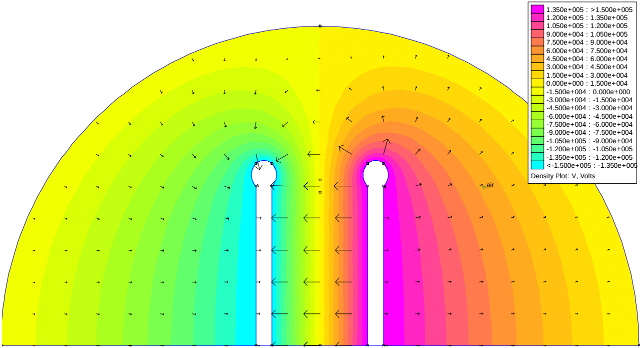


Figure 2: The top 5 cm of cylindrical electrodes is shown. The electrode height can be increased without altering the electric field. A tentative electrode height is $H_{\mathrm{electrode}} = 0.19\,\mathrm{m}$. Bulb-shaped edges maximize the good electric field volume. Longitudinal currents in conductors shaped much like the electrodes, provide the (iron-free) magnetic bending needed to "freeze" the proton spins. These currents are also taulored to provide tunable focusing, avoiding the technically difficult task of deforming the electrodes. This magnetic bending will not be needed in an an eventual larger, higher-energy, more precise ring. Though still needed, the magnetic focusing will be "extrapolated to zero"

7 Parameter table for small proton EDM ring

Table 1: Parameters for 10 m radius proton EDM prototype storage ring. The values in this, and subsequent tables are only crude, because the short drift lengths are being neglected. Since transverse dynamics is purely geometrical, kinematic quantities such as speed and energy, and even particle type, do not enter.

parameter	symbol	unit	value
arcs			2
cells/arc	$N_{\rm cell}$		20
bend radius	r_0	m	10
short drift length	L_D	m	0.30
accumulated drift length		m	32
circumference	$\mathcal{C}$	m	94.8
field index	m		± 0.2
horizontal beta (min/max)	β_x	m	4.0/17.0
vertical beta	β_y	m	600
(outside) dispersion	D_x^O	m	5.2
horizontal tune	Q_x		1.73
vertical tune	Q_y		0.0254
protons per bunch	N_p		1.0×10^8
horz. emittance	ϵ_x	μm	?
vert. emittance	ϵ_y	μm	?
(outside) mom. spread	$\Delta p^O/p_0$		± 0.000082108
(inside) mom. spread	$\Delta p^I/p_0$		± 0.000009853

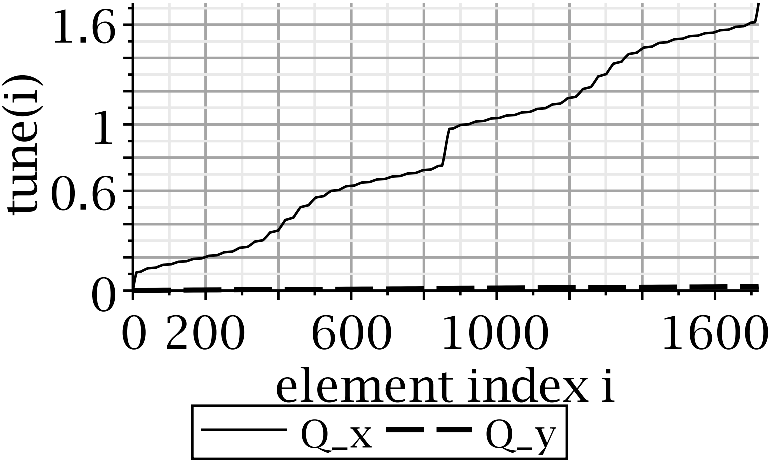


Figure 3: The curves exhibit the circumferential integrations giving the accumulation of incremental horizontal and vertical tune advances to produce tunes of $Q_x = 1.731$ and $Q_y = 0.0253$.

9 Horizontal beta function

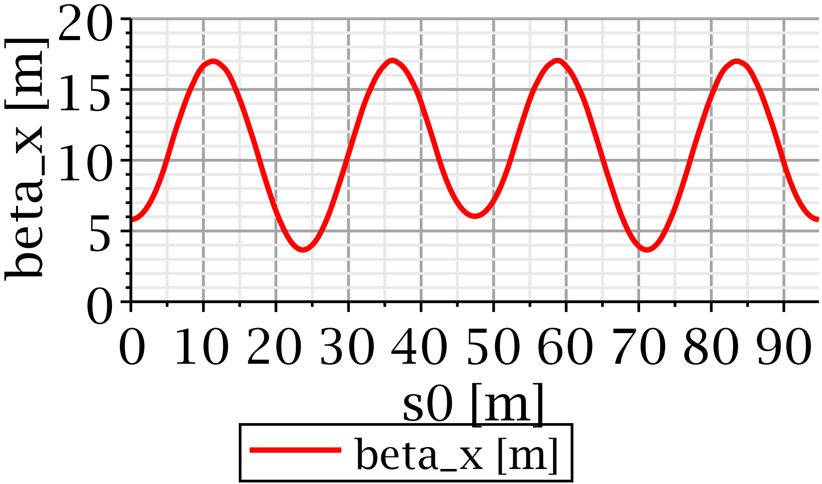


Figure 4: β_x is plotted against longitudinal coordinate s, yielding, for example, a maximum value of $\beta_X^{\max} =$ m.

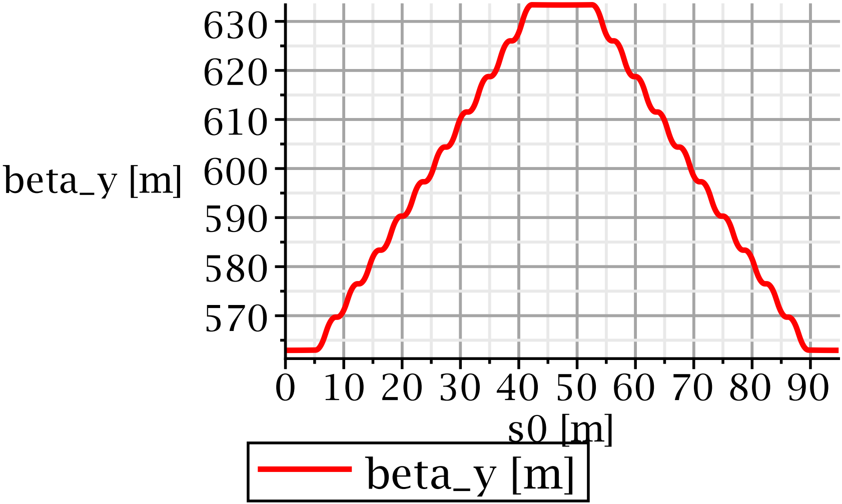


Figure 5: $\beta_y \approx$ m.

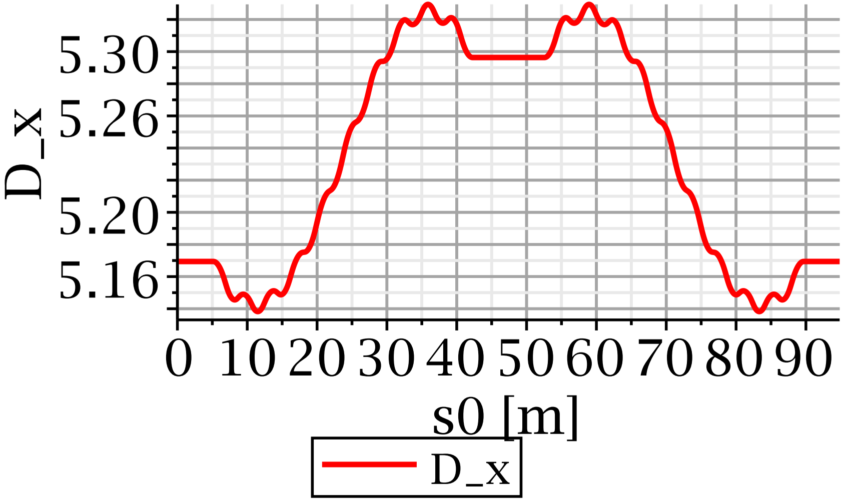


Figure 6: $D \approx$ m.

The "spin tune" Q_E in an electric field is given by

$$Q_E = G\beta^2\gamma - \frac{1}{\gamma} = G\gamma - \frac{G+1}{\gamma}. \tag{6}$$

The "spin tune" Q_M in an magnetic field is given by

$$Q_M = G\gamma. \tag{7}$$

For the proton, G = 1.792847356. Notice that

$$Q_E = Q_M - \frac{G+1}{\gamma}. \tag{8}$$

For the electron, $|G| \approx 0.001$ and $Q_E \approx Q_M = G\gamma$ for any realistically high energy electron storage ring.

13 Superimposed electric and magnetic fields

For circular motion at radius r_0 in superimposed electric and magnetic field the centripetal force is $eE + e\beta cB$. By Newton's law

$$\frac{(pc/e)\beta}{r_0} = E + \beta cB. \tag{9}$$

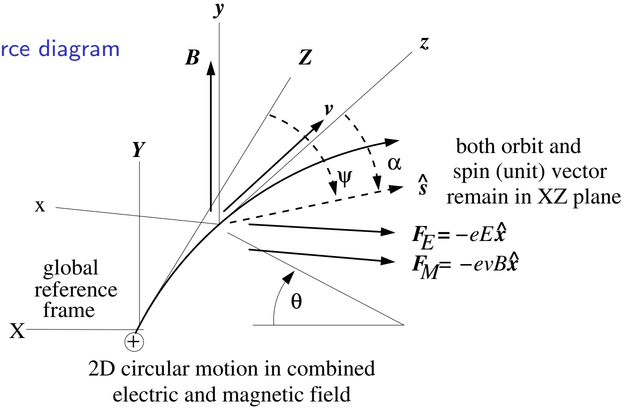
Dividing out a common factor, the centripetal force can be expressed as electric and magnetic bending fractions $\eta_{_E}$ and $\eta_{_M}$;

$$\eta_{_E} = \frac{r_0}{pc/e}\frac{E}{\beta}, \quad \eta_{_M} = \frac{r_0}{pc/e}\,cB, \quad \text{where } \eta_{_E} + \eta_{_M} = 1. \tag{10}$$

- We assume $E > 0$ and $\eta_{_E} > 0$, but without necessarily requiring $\eta_{_M}$ to also be positive. We also assume $G > 0$ (which includes electron and proton, but not deuteron and helion.)
- But, together, the η's must sum to 1; i.e. B can be negative, providing centrifugal rather than centripetal force.
- Expressed in terms of the eta's, the fields are given by

$$E = \frac{pc/e}{r_0}\,\beta\,\eta_{_E}, \quad cB = \frac{pc/e}{r_0}\,\eta_{_M}. \tag{11}$$

14 Vector force diagram



- For a positive particle moving away, along the positive-z axis, with increasing global angle ψ, for electric field $\mathbf{E} = -E\hat{\mathbf{x}}$ and magnetic field $\mathbf{B} = B\hat{\mathbf{y}}$ to sum constructively, causing the particle to veer to the right (in the negative-x direction), requires both E and B to be positive.
- For positive spin tune Q_s the spin precession angle α increases with increasing θ; i.e.

$$\frac{d\alpha}{d\theta} = Q_s. \tag{12}$$

15 Superimposed electric and magnetic bending—protons

We require the resulting spin tune Q_{EM} to vanish;

$$Q_{EM} = \eta_{_E} Q_E + (1 - \eta_{_E}) Q_M = 0. \tag{13}$$

Solving for $\eta_{_E}$,

$$\eta_{_E} = \frac{G}{G+1}\,\gamma^2. \tag{14}$$

For example, with $G_p = 1.7928474$, try $\gamma = 1.25$;

$$\eta_{_E} = \frac{1.7926}{2.7926} \times 1.25^2 = 1.000, \tag{15}$$

which agrees with the "magic" proton value, for which no magnetic bending is required.

In the non-relativistic limit $\gamma = 1$ and

$$\eta_E^{\rm NR} = \frac{1.7926}{2.7926} = 0.6419 \approx \frac{2}{3}. \tag{16}$$

16 Magnetic field in current-carrying stripline

A (fairly weak) uniform magnetic field B can be produced by current I_B flowing in a stripline of width w. To produce magnetic bending fraction $\eta_{_M}$ (using Ampère's law) the current is

$$I_B = \frac{B}{\mu_0}\, w = \frac{pc/e}{r_0}\, \frac{w}{\mu_0 c}\, \eta_{_M}, \tag{17}$$

where $\mu_0 c = Z_0 = 377\,\Omega$ is the free space impedance. The I_B/E ratio then, for example with about $1/3$ of the bending being magnetic, for $K = 45\,\text{MeV}$ protons, is

$$\frac{I_B}{E} = \frac{w}{377\,\Omega}\, \frac{1}{\beta}\, \frac{\eta_{_M}}{\eta_{_E}} \overset{\text{e.g.}}{=} \frac{0.19}{377}\, \frac{1}{0.39}\, \frac{1}{2} = 0.65 \times 10^{-3}. \tag{18}$$

- To turn 45 MeV protons on a 10 m radius requires electric field $E = 8.79 \times 10^6\,\text{V/m}$.
- The bending produced by current $(0.65 \times 10^{-3}) \times (8 \times 10^6) \overset{\text{actually}}{=} 5661\,\text{A}$, the magnetic bending would be roughly half as great as this electric bending, and the ring radius would be about 10 m.
- and the proton spins would be approximately frozen.
- See previous figure.

17 Q_E and Q_M spin tune plots

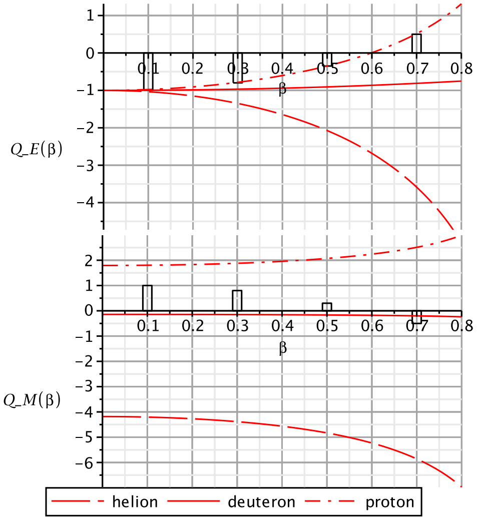


Figure 7: The bar heights roughly indicate, depending on β, how much magnetic bending, relative to electric bending, is needed to "freeze" proton spins.

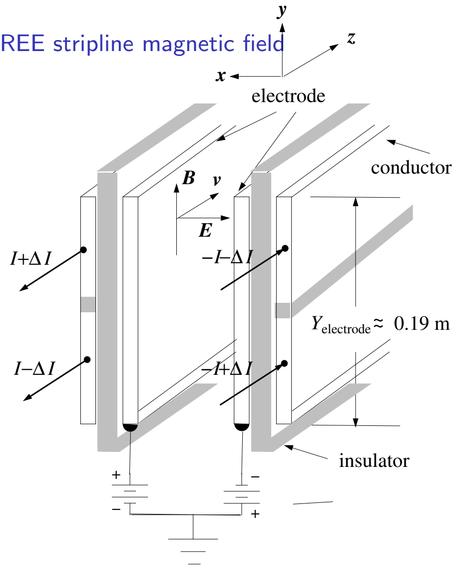
18 Reduced energy proton EDM with IRON-FREE stripline magnetic field

At least in principle, the required magnetic field can be produced by stripline currents shown in the figure. For not very relativistic protons the magnetic force needs to be approximately half the electric force. For magnetic field strength

$$B = \frac{E/c}{2\beta_p} \qquad (19)$$

The stripline current producing this magnetic field is

$$I = \frac{B}{\mu_0}\, Y_{\mathrm{electrode}}. \qquad (20)$$



Superimposed electric and magnetic fields.

Weakest-possible vertical focusing can be provided by ΔI current imbalance (as shown). Up/down current (milliamp scale) imbalance can provide radial magnetic field compensation.

- 233 MeV ($\beta = 0.6$) proton spins are frozen in an electrostatic storage ring. But a purely electrostatic storage ring may be subject to regenerative vacuum degradation causing the beam lifetime to be too short for sensitive EDM measurement.
- Steering ions in a direction perpendicular to the electric field by superimposing a weak vertical magnetic field ΔB might help to suppress this loss mechanism.
- By Eq. (14), a change $\Delta\gamma$ in beam energy associated with a non-vanishing magnetic fraction $\Delta\eta_{_M}$ needs to be compensated by a change $\Delta\eta_{_E} = -\eta_{_M}$, such that

$$-\eta_{_M} = \frac{G}{G+1}\,(\gamma_0 + \Delta\gamma)^2 - \frac{G}{G+1}\,\gamma_0^2 \approx \frac{2G\gamma_0^2}{G+1}\,\frac{\Delta\gamma}{\gamma_0} = \frac{2\Delta\gamma}{\gamma_0}. \qquad (21)$$

20 Frozen spin 233 MeV proton operation with weak magnetic field (continued)

For example, with magic beta value at its nominal (full energy) value of $\beta_0 = 0.6$, suppose the electric field is increased from 5×10^6 to $6 \times 10^6\,\mathrm{V/m}$. This is a twenty percent change that would increase the magic gamma value by ten percent. Re-arranging Eq. (19), the magnetic field required to cancel the steering change is

$$B = -\frac{\Delta E/c}{\beta_p} = -\frac{10^6}{0.6 \times 3 \times 10^8} = -0.0055\,\mathrm{T}. \tag{22}$$

The required longitudinal current would then be given by Eq. (20);

$$I = \frac{B}{\mu_0}\, Y_{\mathrm{electrode}} = \frac{0.0055}{4\pi \times 10^{-7}}\, 0.19 = 851\, A. \tag{23}$$

- This current is as small as it is both because of the nearness to the all-electric magic parameter value.
- However, the given magnetic field might not be strong enough to influence beam dynamics significantly.

21 Proton EDM measurement in small ring

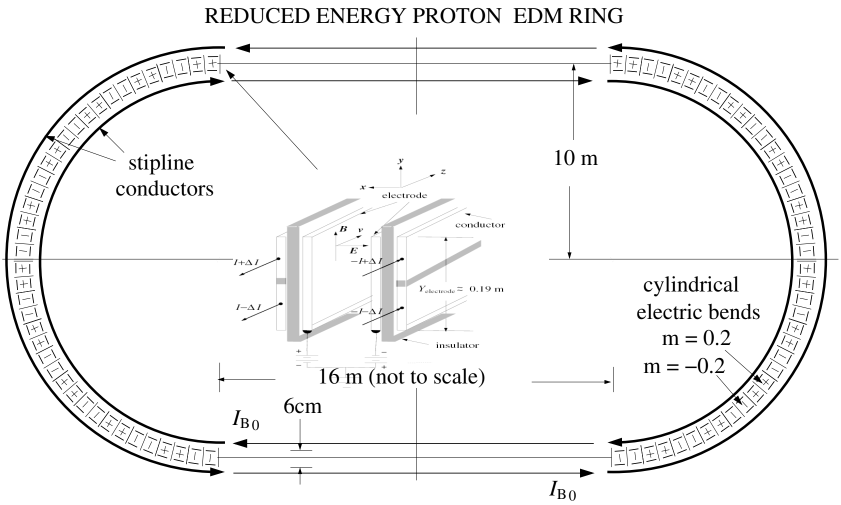


Figure 8: (Reduced energy) proton EDM ring. Superimposed magnetic field (0.03743 T) is required because the proton 45 MeV energy is less than the 233 MeV magic energy required to freeze the spins in an all-electric ring.

22 Proton EDM prototype options parameter table

The table below gives parameters for possible proton EDM prototype rings described in the slides. The final column gives parameters for the 2011 Brookhaven proton EDM proposal[4].

Table 2: Some values are only crude because the short drift lengths are being neglected.

parameter	symbol	unit	proposed ring p-all-electric	minimal p-PROTO	pEDM-BNL
circumference	C	m	94.83	40	500
bend radius	r_p	m	10	3	40
momentum $\times c$	$p_0 c$	GeV	0.2392		0.70074
kinetic energy	K	GeV	0.030	7.5	233
proton beta	β_0		0.2470		0.6
proton velocity	v_p	m/s	0.74047e8	3.77e7	1.8e8
proton gamma	γ_0		1.0320		1.25
revolution period	T_1	μs	1.2807		2.78
elec. bend frac.	η_E		1.0	1.0	1.0
electric field	E	MV/m	8.794	5	10
electrode gap	gap	cm	6	3	3
electrode voltage	V_0	KV			$\pm$157
magn. bend frac.	η_M			0.0	0.0
"magic" magn. field	B_0	T			
"magic" current	I_{B0}	A			

23 Magic spin electric/magnetic combinations

The table below gives parameters for electric and magnetic frozen spin values for different proton energies

Table 3: Some values are only crude because the short drift lengths are being neglected.

parameter	symbol	unit	value		
circumference	C	m		94.83	
bend radius	r_p	m		10	
momentum$\times c$	$p_0 c$	GeV	0.2586	0.2940	0.3259
kinetic energy	K	MeV	35	45	55
proton beta	β_0		0.2657	0.2991	0.3281
proton velocity	v_p	m/s	0.7967e8	0.897e8	0.9838e8
proton gamma	γ_0		1.0373	1.0480	1.0586
revolution period	T_1	μs	0.8402	1.0577	0.9640
elec. bend frac.	η_E		0.6907	0.7050	0.7194
electric field	E	MV/m	4.748	6.200	7.694
electrode gap	gap	cm		6	
electrode voltage	V_0	KV	±142	±186	±231
magn. bend frac.	η_M		0.30927	0.2950	0.2806
"magic" magn. field	B_0	T	0.00709	0.00865	0.01001
"magic" current	I_{B0}	A	1072.2	1308.4	1513.5

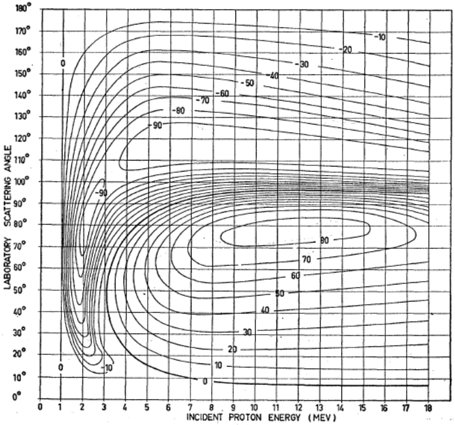


FIG. 2. Contour plot of percent polarization in proton-helium scattering as a function of incident proton energy and laboratory scattering angle.

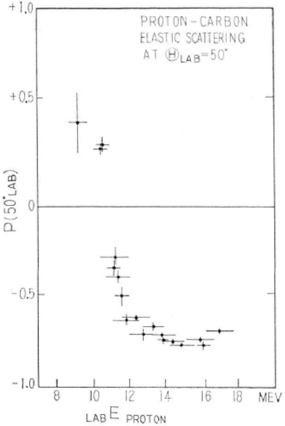
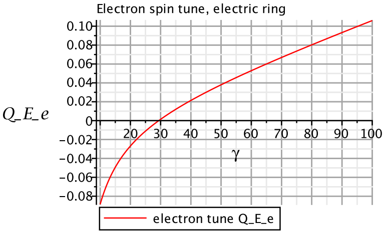


Figure 3

The energy dependence of $P(50^{\circ}_{LAB})$.

25 Electron EDM measurement in small ring

- (As Bill Morse first emphasized) superimposed magnetic bending permits the electron spins to be frozen over a large parameter range, permitting controlled investigation of systematic errors.
- Above $\gamma_e = 30$ one can increase the electric field more or less arbitrarily and cancel most of the bending magnetically to preserve frozen spins. In effect the magnetic contribution to the spin tune is then negative.



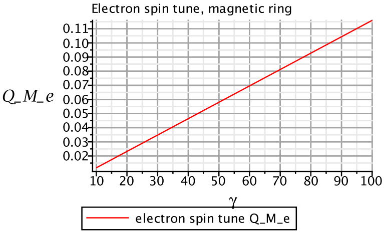


Figure 9: The "magic" value is $\gamma_e \approx 30$, but this can be changed by a large factor by superimposing magnetic field on the electric bending field.

Superimposed electric and magnetic bending—electrons

- Spin tunes in electric and magnetic fields are related by

$$Q_E = Q_M - \frac{G+1}{\gamma}. \tag{24}$$

- For the electron, $|G| \approx 0.001$ and $Q_E \approx Q_M = G\gamma$ for any realistically high energy electron storage ring.
- With $\eta_{_E}$ the electric bending fraction, and $\eta_{_M}$ the magnetic bending fraction, we require the resulting spin tune Q_{EM} to vanish;

$$Q_{EM} = \eta_{_E} Q_E + (1-\eta_{_E}) Q_M = 0. \tag{25}$$

- For electrons $G = 0.001159652$. Solving for $\eta_{_E}$,

$$\eta_{_E} = \frac{G}{G+1}\,\gamma^2 \approx 0.001159\,\gamma^2; \quad \eta_{_M} = 1 - G\gamma^2/(G+1) \approx 1 - 0.001159\,\gamma^2. \tag{26}$$

- For purely electric bending $\eta_{_M} = 0$ and

$$\gamma_{\rm magic} = \sqrt{\frac{G+1}{G}} = \sqrt{\frac{1.001159652}{0.001159652}} = 29.382. \tag{27}$$

27 Electron spin tunes in electric and magnetic rings

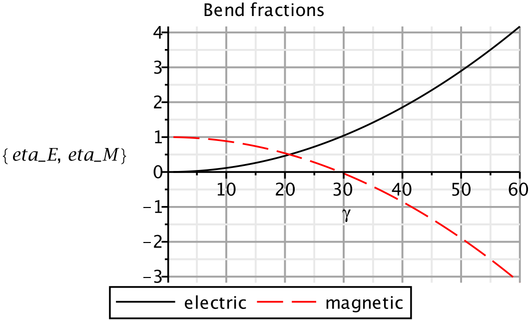


Figure 10: Electron spin tunes in electric and magnetic rings. By superimposing electric and magnetic bending fields the frozen spin condition can be satisfied for arbitrary electron energy.

- For $\gamma < 30$ both $\eta_{_E}$ and $\eta_{_M}$ are positive, meaning that both electric and magnetic forces are centripetal.
- But for $\gamma > 30$ both $\eta_{_M}$ and B are negative, as is required for the spins to remain frozen.
- Note, though, that the electric field in the electron rest frame continues to increase with increasing γ, as required to provide the increased bending force to keep the particle on a 16 m radius circle.

28 Parameters for frozen spin electron EDM scan

Table 4: Some values are only crude because the short drift lengths are being neglected.

parameter	symbol	unit	small proton prototype ring		
circumference	C	m	94.83		
arc bend radius	r_p	m	10		
short drift length	L_D	m	0.3		
accum. drift length		m	32.0		
momentum $\times\, c$	$p_0 c$	GeV	0.01550	0.0205	0.02550
kinetic energy	K	GeV	0.0150	0.0200	0.0250
electron beta	β_e		0.99945	0.99970	0.99980
electron gamma	γ_e		30.354	40.139	49.924
elec. bend frac.	η_E		1.06724	1.86619	2.8870
electric field	E	MV/m	1.6536	3.825	7.3619
electrode gap	gap	cm	6		
magn. bend frac.	η_M		-0.06724	-0.8661	-1.8870
"magic" magn. field	B_0	T	-0.000347	0.005922	-0.160e-1
"magic" current	I_{B0}	A	-52.54	-895.4	-2426.8

- For electron EDM measurement, with magic energy 14.5 MeV, bend radius $r_0 = 10$ m seems unnecessarily large, since the electric field is unnecessarily small.
- One probably prefers to keep the electron energy low to reduce synchrotron radiation

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