Full-speed ahead all-electric proton EDM ring
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- Day 1: Full-speed ahead all-electric proton EDM ring
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- Day 5: Review
- Day 6: Spin evolution and coherence
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- Day 8: TBD
- Day 9: TBD
- Day 10: TBD
Extending historical force field symmetry studies

The measurement of electric dipole moments (EDMs) of elementary particles would provide a modest extension to our understanding of force field symmetries. The most important of these historical milestones can be encapsulated in the following list:

- Newton: Gravitational field, (inverse square law) central force
- Coulomb: By analogy, electric force is the same (i.e. central, $1/r^2$)
- Ampere: How can a compass needle near a current figure out which way to turn? A right hand rule is somehow built into E&M and into the compass needle. Mathematically this requires the magnetic field to be a pseudo-vector.
- The upshot: by introducing pseudo-vector magnetic field, E&M respects reflection symmetry. This was the first step toward the grand unification of all forces, which culminated eventually in Maxwell’s completion of electromagnetic theory.
Lee, Yang, etc: A particle with spin (pseudo-vector), say “up”, can decay more up than down (vector);
  ▶ i.e. the decay vector is parallel (not anti-parallel) to the spin pseudo-vector,
  ▶ viewed in a mirror, this statement is reversed.
  ▶ i.e. weak decay force violates reflection symmetry (P).

Fitch, Cronin, etc: standard model violates both parity (P) and time reversal (T), (so protons, etc. must, at some level, have non-vanishing EDM).

Current task: How to exploit the implied symmetry violation to measure the EDM of proton, electron, etc?
Proton is "magic" with all three spin components "frozen" (relative to orbit)

Two issues:

- Can the tipping angle be measurably large for plausibly large EDM, such as $10^{-30}$ e-cm? With modern technology, yes
- Can the symmetry be adequately preserved when the idealized configuration above is approximated in the laboratory? This is the main issue

The very smallness of EDMs that makes measuring them so important, makes the measurement difficult, or even impossible?
Two experiments that “could not be done”

![Graph](image)

**FIG. 2.** Measured asymmetry \( \epsilon(\phi_s) \) of... feedback systems and other means to minimize them further. Several systematic effects that may affect the spin tune.

Two experiments that “could not be done”... deuterons, and Bonn, Paul et al.: neutron storage ring... neutron storage ring under construction at Preliminary results from the Bonn neutron... superconducting magnet gives a peak field... minutes, the level of neutrons begins to... the University of Bonn. Its 1.2 m diameter storage ring... Bonn: neutron storage ring... supected to provide stability below... because it was not designed to provide stability below... COSY is stable to such a precision, because it...

**Figure 1:** COSY, Juelich, Eversmann et al.: (Pseudo-)frozen spin deuterons, and Bonn, Paul et al.: neutron storage ring
Remarkable coincidence

We can also include two experiments that “were not even thinkable” at the time they were performed.

- Frankfurt: Stern-Gerlach experiment—1923, beginning of quantum mechanics (shortly after Hans Bethe had transferred from there to Munich to complete his PhD, and before he returned in 1928—Rose (Ewald) Bethe knew Gerlach)

Remarkable coincidence!

- All four impossible experiments were performed in the same general area—central Rhine
- Should be designated “Cultural heritage treasure”
- Science is “culture”
- Politicians can understand this
- Even scientists should be able to understand it

The challenge is to succeed in performing another “impossible” experiment
9 Why all-electric ring?

- “Frozen spin” operation in all-electric storage ring is only possible with electrons or protons—by chance their anomalous magnetic moment values are appropriate. The “magic” kinetic energies are 14.5 MeV for e, 233 MeV for p.

- Beam direction reversal is possible in all-electric storage ring, with all parameters except injection direction held fixed. This is crucial for reducing systematic errors.
10 Precision limit—space domain method

- Measure difference of beam polarization orientation at end of run minus at beginning of run.
- p-Carbon left/right scattering asymmetry polarimetry.
- This polarimetry is well-tested, “guaranteed” to work,
- but also “destructive” (measurement consumes beam)

| particle | $|d_{elec}|$ current upper limit e-cm | error after $10^4$ pairs of runs e-cm |
|----------|-------------------------------------|---------------------------------------|
| neutron  | $3 \times 10^{-26}$                | $\pm 10^{-29}$                        |
| proton   | $8 \times 10^{-25}$                |                                       |
| electron | $10^{-28}$                         | $\pm 10^{-29}$                        |
Resonant polarimetry—more detail next week

- Planned Stern-Gerlach electron polarimetry test(s)
- R. Talman, LEPP, Cornell University; B. Roberts, University of New Mexico; J. Grames, A. Hofler, R. Kazimi, M. Poelker, R. Suleiman; Thomas Jefferson National Laboratory

2017 International Workshop on Polarized Sources, Targets & Polarimetry, Oct 16-20, 2017,
12 Precision limit—frequency domain method

- Frequency domain—"Fourier", "interferometry", "fringe counting", "resonant" etc.
- Measure the spin tune shift when EDM precession is reversed
- Relies on phase-locked Stern-Gerlach polarimetry
- Like the Ramsey neutron EDM method.
- This polarimetry has not yet been proven to work.
- **This method cannot be counted on until resonant polarimetry has been shown to be practical.**

| particle    | $|d_{elec}|$ current upper limit e-cm | excess fractional cycles per pair of 1000 s runs | error after $10^4$ pairs of runs e-cm | roll reversal error e-cm |
|-------------|-------------------------------------|-------------------------------------------------|---------------------------------------|--------------------------|
| neutron     | $3 \times 10^{-26}$                 | $8 \times 10^{-25}$                             | $\pm 8 \times 10^3$                   | $\pm 10^{-30}$           | $\pm 10^{-30}$           |
| proton      | $8 \times 10^{-25}$                 | $10^{-28}$                                      | $\pm 1$                               | $\pm 10^{-30}$           | $\pm 10^{-30}$           |
| electron    | $10^{-28}$                          |                                                 |                                       |                          |                          |
13 Achievable precision (assuming perfect phase-lock)

- To make estimates more concrete, measure EDM in units of (nominal value) $10^{-29}$ e-cm $\equiv \tilde{d}$
- The challenge is to measure an EDM value less than 1 (in units of $10^{-29}$ e-cm).
- $2 \times \frac{\text{EDM(nominal)}}{\text{MDM precession rate ratio}}$:
  $$2\eta^{(e)} = 0.92 \times 10^{-15} \approx 10^{-15}$$
- about the same as Pound-Rebka “falling” photon gravitational Mossbauer shift experiment
- “Frozen spin method” recovers “off the top” about 6 out of these 15 orders of magnitude
Achievable precision (continued)

- duration of each one of a pair of runs $= T_{\text{run}}$
- smallest detectable fraction of a cycle $= \eta_{\text{fringe}} = 0.001$
- small, but achieved in Pound-Rebka experiment

Using this terminology, the smallest meaningful non-zero detection is one fractional fringe. Then the EDM signal detected in a single run can be expressed as a number of fractional fringes $N_{\text{FF}}$. The result is

$$N_{\text{FF}} = \frac{\eta^{(p)} \tilde{d}}{\eta_{\text{fringe}}} h_r f_0 T_{\text{run}} \left( \frac{10^{-15}}{0.001} \cdot 100 \cdot (0.4 \times 10^6) \cdot 10^5 \tilde{d} \approx 0.4 \tilde{d} \right).$$  

By this estimate, for $\tilde{d} = 1$, i.e. an EDM of $10^{-29}$ e-cm, a meaningful measurement can be obtained in a few days.

But this assumes the existence of resonant polarimetry.

Though under development, as discussed later, resonant polarimetry has never been shown to be practical.
15 Design requirements for proton EDM storage ring

- Measuring the proton electric dipole moment (EDM) requires an electrostatic storage ring in which 233 MeV, frozen spin polarized protons can be stored for an hour or longer without depolarization.
- The design orbit consists of multiple electrostatic circular arcs
  - Electric breakdown limits bending radius, e.g. $r_0 > 40$ m
  - For longest spin coherence time (SCT) and for best systematic error reduction the focusing needs to be as weak as possible
  - This is a “worst case” condition for electric and magnetic storage rings to differ (because kinetic energy depends on electric potential energy)
  - To reduce emittance dilution by intrabeam scattering (IBS) the ring needs to operate “below transition”
- Ring must be accurately clockwise/counter-clockwise symmetric
  - Accurately symmetric injection lines are required.
  - Initially single beams would be stored, with run-to-run alternation of circulation directions.
  - Ultimate reduction of systematic error will require simultaneously counter-circulating beams.
“Magic” central design parameters for frozen spin proton operation:

\[
c = 2.99792458 \times 10^8 \text{ m/s}
\]

\[
m_p c^2 = 0.93827231 \text{ GeV}
\]

\[
G = 1.7928474 \text{ anomalous magnetic moment}
\]

\[
g = 2G + 2 = 5.5856948
\]

\[
\gamma_0 = 1.248107349
\]

\[
\mathcal{E} = \gamma_0 m_p c^2 = 1.171064565 \text{ GeV}
\]

\[
K_0 = \mathcal{E} - m_p c^2 = 0.232792255 \text{ GeV}
\]

\[
p_0 c = 0.7007405278 \text{ GeV}
\]

\[
\beta_0 = 0.5983790721
\]

For mnemonic purposes it is enough to remember \( \beta_0 = 0.6 \), \( \gamma_0 = 1.25 \), and \( p_0 c = 0.7 \) MeV.
An ultraweak focusing, “weak/weaker, alternating-gradient, combined-function” (WW-AG-CF) electric storage ring is described. All-electric bending fields exist in the tall slender gaps between inner and outer, vertically-plane, horizontally-curved electrodes.
Figure 2: Above: Electrode edge shaping to maximize uniform field volume; Below left: bulb-corrected field uniformity; Below right: uncorrected field intensity. Only the top 5 cm is shown. The electrode height can be increased arbitrarily without altering the electric field.
The radial electric field dependence is

\[ E = E_r \sim \frac{1}{r^{1+m}}, \]

where, ideally for spin decoherence, the field index \( m \) would be exactly \( m = 0 \).

\( m = 0 \) (pure-cylindrical) field produces horizontal bending as well as horizontal “geometric” focusing, but no vertical force.

(Not quite parallel) electrodes, with \( m \) alternating between \( m = -0.2 \) and \( m = +0.2 \) provides net vertical focusing.

Not “strong focusing”, this is “weak-weaker” WW-AG-CF focusing, just barely strong enough to keep particles captured vertically.

Beam distributions are highly asymmetric, much higher than wide, matching the good field storage ring aperture.
(Not counting trims, nor slanted poles) there are no quadrupoles
This is favorable for systematic electric dipole moment (EDM) error reduction. *There is no spin decoherence (for frozen spins) in a pure $m = 0$ field* — explained later
The average particle speeds in drift sections do not need to be magic—because there is no spin precession in drift sections.
Still, the dependence of revolution period on momentum offset is very small, making the synchrotron oscillation frequency small, and not necessarily favorable as regards being above or below transition.
IBS stability requires below-transition operation, which requires quite long total drift length.
21 Total drift length condition for below-transition operation

- As with race horses, faster particles can lose ground in the curves but still catch up in the straightaways.
- To run “below transition”, the sum of all drift lengths has to exceed $L_{D}^{\text{trans}}$, given in terms of dispersion $D^{O}$ by
  \[
  L_{D}^{\text{trans}} = 2\pi D^{O} \beta_{0}\gamma_{0} \approx 1.5\pi D^{O} \approx 115 \text{ m}.
  \]

- On 17 December, 2017, I suddenly realized that there is a serious disagreement between my formalism and Valeri Lebedev’s (and all other Wollnik 6x6 linearized transfer matrix user’s) formalism concerning longitudinal dynamics.
- (Naturally) I assume I am correct, but perhaps not.
- The disagreement has a huge impact on the detailed lattice design. But it does not seriously effect strategic EDM planning.
- **The disagreement has to be resolved.**
- I propose deferring this until the weak-weaker/weak/strong focusing discussion on Day 3.
Longitudinal $\gamma$ variation on off-momentum orbits

Figure 3: Dependence of deviation from “magic” $\Delta \gamma(s) = \gamma(s) - \gamma_0$ on longitudinal position $s$, for off-momentum closed orbits (circular arcs within bends) just touching inner or outer electrodes at $x = \pm 0.015$ m. Notice the anomalous cross-overs in $m > 0$ bends.

The dispersion is essentially positive everywhere, and the speed within bends is essentially the same for all particles. If the circumference fraction allotted to bends is close to 1, the revolution period will be dominated by momentum offset $\delta$ (rather than velocity offset). This implies “above transition” operation.
For central radius $r_0$ the off-momentum radius is determined by Newton’s centripetal force law

$$eE_0 \left( \frac{r_0}{r} \right)^{1+m} = \frac{\beta pc}{r} = \frac{m_pc^2}{r} \left( \gamma - \frac{1}{\gamma} \right),$$

where $r = r_0 + x_D$ is the radius of an off-momentum arc of a circle with the same center.

For $m \neq 0$, $r$ cancels, and the radius is indeterminant.

A powerful coordinate transformation is:

$$\xi = \frac{x}{r} = \frac{x}{r_0 + x}$$

For our typical values ($x = 1$ cm, $r_0 = 40$ m), for all practical purposes, $\xi$ can simply be thought of as $x$ in units of $r_0$. 
The electric field is then

\[ \mathbf{E}(\xi) = -E_0 (1 - \xi)^{1+m} \hat{r}, \]

Off-momentum closed orbits are “parallel” arcs of radius \( r = r_0 + x_D \) inside a bend, entering and exiting at right angles to straight line orbits displaced also by \( x_D \).

The relativistic gamma factor on the orbit (inside) is \( \gamma^I \), which satisfies

\[ eE_0 r_0 (1 - \xi)^m = \beta^I p^I c = m_p c^2 \left( \gamma^I - \frac{1}{\gamma^I} \right), \]

This is a quadratic equation for \( \gamma^I \) inside bend.

For \( r \neq r_0 \), because of the change in electric potential at the ends of a bend element, the gamma factor outside has a different value, \( \gamma^O \).
For $m \neq 0$ the orbit determination is no longer degenerate.

Solving the quadratic equation for $\gamma^I$, the gamma factor is given by the positive root;

$$
\gamma^I(\xi) = \frac{E_0 r_0 (1 - \xi)^m}{2 m_p c^2 / e} + \sqrt{\left(\frac{E_0 r_0 (1 - \xi)^m}{2 m_p c^2 / e}\right)^2 + 1}.
$$

This function is plotted next for $m = \pm 0.2$. 
Figure 4: This figure shows a “dispersion plot” of “inside” gamma value $\gamma^I$ plotted vs $\xi$. The curves intersect at the magic value $\gamma^I = 1.248107$. Because $d\gamma/d\beta = \beta\gamma^3$ is equal to about 1.17 at the magic proton momentum, the fractional spreads in velocity, momentum, and gamma are all comparable in value—in this case about ±$2 \times 10^{-5}$. This figure may be confusing, since it is rotated by 90 degrees relative to conventional dispersion plots. For this reason one should also study the following plot, which is identical except for being rotated, and is annotated as an aid to comprehension. Subsequent plots have the present orientation, however.
Figure 2: Identical "Dispersion plots", but with the upper rotated into customary orientation and annotated as an aid to comprehension (though momentum then decreases from left to right). Subsequent plots will have the lower orientation.

Dependence of "inside" gamma value $\gamma$ on $\xi = x/r$ for $m = -0.2$ and $m = 0.2$. The curves intersect at the magic value $\gamma = 1.248107$. Because $d\gamma/d\beta = \beta \gamma^3$ is equal to about 1.17 at the magic proton momentum, the fractional spreads in velocity, momentum, and gamma are all comparable in value—in this case about $\pm 2 \times 10^{-5}$.

Figure 5: This plot is identical to the previous one except for being rotated by 90 degrees into conventional orientation (except momentum increases from right to left). It shows the dependence of $\xi = x/r$ vs "inside" gamma value $\gamma^I$, for $m = -0.2$ and $m = 0.2$. Note that, for $m < 0$ larger momentum causes larger radius while, for $m > 0$ the opposite is true. What is striking is that the slope is opposite for $m > 0$ and $m < 0$. This is "anomalous".
Potential energy

Electric potential is defined to vanish on the design orbit. Expressed as power series in $\xi$, the electric potential is

$$V(r) = -\frac{E_0 r_0}{m} \left( (1 - \xi)^m - 1 \right)$$

$$= E_0 r_0 \left( \xi + \frac{1 - m}{2} \xi^2 + \frac{(1 - m)(2 - m)}{6} \xi^3 \ldots \right). \tag{2}$$

This simplifies spectacularly for the Kepler $m=1$ case. But we are concerned with the small $|m| \ll 1$ case.

As a proton orbit passes at right angles from outside to inside a bend element, its total energy is conserved;

$$\gamma^O(\xi) = \frac{\mathcal{E}^O}{m_p c^2} = \frac{\mathcal{E}^l}{m_p c^2}$$

$$= \gamma^l(\xi) + \frac{E_0 r_0}{m_p c^2/e} \left( \xi + \frac{1 - m}{2} \xi^2 + \frac{(1 - m)(2 - m)}{6} \xi^3 \ldots \right).$$

Plots of $\gamma^O(\xi)$ for $m = \pm 0.2$ are shown next.
Figure 6: “Outside” dispersion plots. Note that dispersion slopes are the same for \( m < 0 \) and \( m > 0 \). Dependence of “outside” gamma value \( \gamma^O \) on \( \xi = x/r \) for \( m = -0.2 \) and \( m = 0.2 \). Because \( d\gamma/d\beta = \beta \gamma^3 \) is equal to about 1.17 at the magic proton momentum, the fractional spreads in velocity, momentum, and gamma are all comparable in value—in this case about \( 2 \times 10^{-4} \). The fractional spreads are an order of magnitude greater outside than inside. This is helpful.
## Parameter table

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Figure 7: Horizontal beta function $\beta_x(s)$, plotted for full ring. For this case the total circumference is 411.3 m and the total drift length is $L_D = 160.0$ m. Since this total drift length exceeds $L_D^{\text{trans}}$, the ring will be “below transition”, as regards synchrotron oscillations.
Figure 8: Vertical beta function $\beta_y(s)$, plotted for full ring. For this case the total circumference is 411.3 m and the total drift length is $L_D=160.0$ m. Since this total drift length exceeds $L_D^{\text{trans}}$, the ring will be “below transition”, as regards synchrotron oscillations.
Figure 9: Outside dispersion function $D^O(s)$, plotted for full ring. For this case the total circumference is 411.3 m and the total drift length is 160.0 m.
Figure 10: Transverse tune advances. The full lattice tunes are $Q_x = 1.640$ and $Q_y = 0.032$. Even smaller horizontal tune (for improved self-magnetometry) can be provided by trim quadrupoles, rather than by electrode shape or voltage adjustment, even consistent with zero net quadrupole focusing, but with octupole focusing for net vertical stability.
The leading source of systematic error in the EDM measurement is unintentional, unknown, radial magnetic fields.

Acting on MDM, they cause spurious precession mimicking EDM-induced precession.

(Apart from eliminating radial magnetic field) the only protection is to measure the differential beam displacement of counter-circulating beams.

Greatest sensitivity requires weakest vertical focusing.

i.e. extremely large value for $\beta_y$.

or even octupole-only vertical focusing.
Many significant advances:

- highly polarized beam
- electron cooling
- stochastic cooling
- spin tune determination accurate to 10 digits
- phase locked beam polarization
- long spin coherence time (in strong-focusing ring far from optimal for SCT)
- machine position and powering stability over long times far superior to their absolute uncertainty

still needed is a 450 m circumference electric ring (etc.)

or low energy prototype proton EDM storage ring


N. Hempelmann et al., *Phase-locking the spin precession in a storage ring*, P.R.L. **119**, 119401, 2017


R. Talman, LEPP, Cornell University; B. Roberts, University of New Mexico; J. Grames, A. Hofler, R. Kazimi, M. Poelker, R. Suleiman; Thomas Jefferson National Laboratory; *Resonant (Logitudinal and Transverse) Electron Polarimetry*, 2017 International Workshop on Polarized Sources, Targets and Polarimetry, KAIST, Republic of Korea, 2017


S. Møller and U. Pedersen, *Operational experience with the electrostatic ring, ELISA*, PAC, New York, 1999
Loss of protons by single scattering from residual gas is discussed in detail in a paper Frank Rathmann drew to my attention: C. Weidemann et al., *Toward polarized anti-protons: Machine development for spin-filtering experiments*, PRST-AB **18**, 0201, 2015