Spin Dynamics in Electrostatic Lattices

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EDM search methods in storage rings

- Resonant method with vertical initial spin orientation in magnetic ring with RF flipper.
- "Magic" method with initial spin orientation along the momentum.

"Magic" method in purely electrostatic ring

In purely electrostatic ring the spin of "magic" particle rotates with the same angular frequency as the momentum and the vertical spin component S_y grows up due to the EDM with angular rate

$$d\overrightarrow{S} = -\frac{e\eta}{2m}\overrightarrow{E} \times \overrightarrow{S}\,dt$$

Our goal

Track particle with initial horizontal spin polarization for a very large number of orbits, say 10^9, to detect the appearance of a vertical spin component that will indicate the presence of an electric dipole moment.
So we need a storage ring which will conserve horizontal spin polarization for a long time.

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Spin coherence time

SCT

Spin coherence time (SCT) — time when RMS spin orientation of the bunch particles reaches one radian.

Requirements of planned SrEDM experiment: the SCT should be more than 1000 seconds.
During this time each particle performs about 10^9 turns in storage ring moving on different trajectories through the optics elements.

Spin-orbital motion analysis with COSY Infinity

All the numerical results presented were obtained using COSY Infinity.
COSY Infinity features:

- spin motion calculation;
- contains electrostatic elements to study purely electrostatic ring;
- symplectic tracking;
- allows long term evolution studies (it is possible to track billions of turns in a reasonable time);
- parallel version (thousands of particles can be tracked simultaneously, if you have a supercomputer);
- results coincide with analytical estimations.

T-BMT equation

$$\frac{d\vec{S}}{dt} = \mu\vec{S} \times \left(\vec{B} - c\beta \times \vec{E}\right) + d_{EDM}\vec{S} \times \left(\vec{E} + \vec{\beta}c \times \vec{B}\right)$$

$\mu,\ d_{EDM}$ — magnetic and electric dipole moments, c is the speed of light, β is the relative velocity and $E,\ B$ — the electric and the magnetic field vectors.

$$\begin{aligned} \frac{d\vec{S}}{dt} &= \vec{\omega}_G \times \vec{S} \\ \vec{\omega}_G &= -\frac{e}{m_0\gamma c}\left\{G\vec{B} - \left(G - \frac{1}{\gamma^2 - 1}\right)\left(\vec{\beta} \times \vec{E}\right)\right\} \\ G &= \frac{g-2}{2}, \end{aligned}$$

G — the anomalous magnetic moment, g is the gyromagnetic ratio and ω_G is the spin precession frequency.

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SrEDM

Purely electrostatic ring

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Frozen spin method

Consider $\gamma = \gamma_{mag}$:

$$\frac{1}{\gamma_{mag}^2 - 1} - G = 0,$$

and $\omega_G = 0$. Proton "magic" energy is 232 MeV.

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Spin oscillation of a non-magic particle

If $\gamma \neq \gamma_{mag}$:

$$\vec{\omega}_G = -\frac{e}{m_0 \gamma c} \left\{ -2G \frac{\Delta p}{p} \left(\vec{\beta} \times \vec{E} \right) \right\}.$$

Spin oscillation tune ν_{sz} satisfies:

$$S_z = S_{z_0} \cos 2\pi \nu_{sz} n, \; \nu_{sz} = \frac{e \overline{E}_x L_{cir}}{2\pi m_0 c^2 \gamma} G \frac{\Delta p}{p},$$

where $\overline{E}_x$ is the average value of the deflecting electric field.
If $(\Delta p/p)_{\max} = 10^{-4}$, then $\nu_{sz} = 1.588 \cdot 10^{-4}$, or
SCT = 6300 turns ≈ 1msec.

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Using of RF cavity to increase SCT

With $(\Delta p/p) = (\Delta p/p)_{\max} \cos(\nu_z \varphi)$ equation describing the oscillation of the spin:

$$\frac{d^2 S_z}{d\varphi^2} + \left\{ \frac{e\overline{E}_x L_{cir}}{2\pi m_0 c^2 \gamma} 2G \left(\frac{\Delta p}{p}\right)_{\max} \cos(\nu_z \varphi) \right\}^2 S_z = 0.$$

The spin oscillates within a very narrow angle $\Phi_{\max}$ with longitudinal tune $\Phi \sim \Phi_{\max} \sin(\nu_z \varphi)$. The value $\Phi_{\max} \sim (\nu_{sz}/\nu_z)^2$ depends on the frequency ratio.

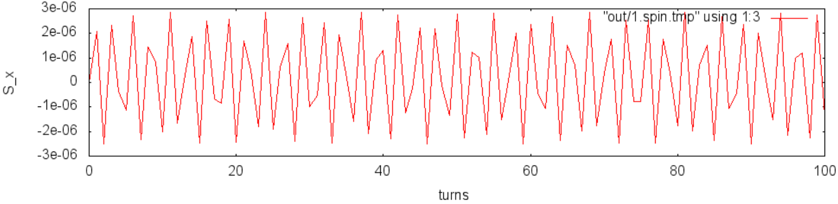

Figure: Spin oscillations with RF cavity.

Off-axial particles

Particles with different initial deviations oscillate with respect to different energy levels:

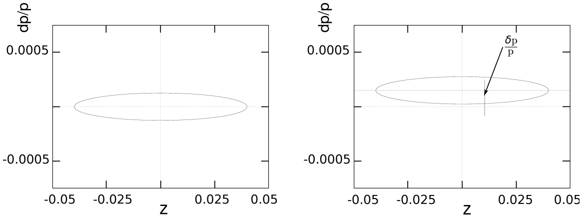

Figure: Phase trajectory in longtitudinal plane for initial coordinates $x = 0, y = 0$ (a) and $x = 3\text{mm}, y = 0$ (b).

RF cavity is not able to reduce the spin oscillation for off-axial particles:

$$\nu_{sz} = \frac{e\overline{E}_x L_{cir}}{\pi m_0 c^2 \gamma} G \frac{\delta p}{p}$$

Equilibrium energy level modulation

The only solution to increase SCT is the modulation of the energy level itself relative to the magic level.

For this purpose we have reduced the tunes to the values $\nu_x = 1.31, \ \nu_y = 0.64$ and increased coupling between longitudinal and transverse motion.

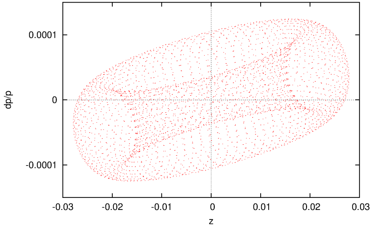

Figure: Phase trajectory in longtitudinal plane for initial coordinate $x = 3\text{mm}, y = 0$.

If $(\Delta p/p)_{\text{max}} = 10^{-4}$ and the beam emmitance $2\text{mm} \cdot \text{mrad}$, SCT will be ~ 400 sec.

The second order influence on spin

The spin tune in the second approach versus momentum:

$$\frac{d^2 S_z}{d\varphi^2} + \left\{ \frac{e\overline{E}_x L_{cir}}{2\pi m_0 c^2 \gamma} \left[-2G \frac{\Delta p}{p} + \frac{1+3\gamma^2}{\gamma^2} G \left(\frac{\Delta p}{p} \right)^2 \right] \right\}^2 S_z = 0$$

and

$$\begin{aligned} \nu_{sz} &= \frac{e\overline{E}_x L_{cir}}{2\pi m_0 c^2 \gamma} \left\langle -2G \left(\frac{\Delta p}{p} \right) \cos(\nu_z \varphi) + \frac{1+3\gamma^2}{\gamma^2} G \left(\frac{\Delta p}{p} \right)^2 \cdot \right. \\ & \left. \cdot \cos^2(\nu_z \varphi) \right\rangle = \frac{e\overline{E}_x L_{cir}}{2\pi m_0 c^2 \gamma} \frac{1+3\gamma^2}{\gamma^2} \frac{G}{2} \left(\frac{\Delta p}{p} \right)^2 \end{aligned}$$

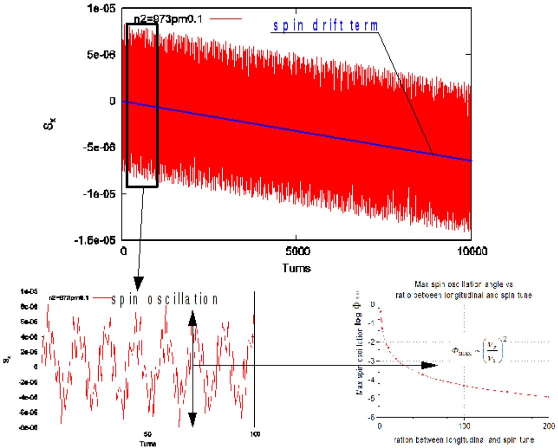

Figure: Spin drift due to $\frac{\Delta p}{p}$.

Variation of spin tune

Assuming "magic" condition we define a variation of the spin tune through the finite differences up to second order:

$$\delta\nu_S = \frac{e}{2\pi m_0 c^2}\delta\left(\frac{1}{\gamma^2-1} - G\right) L_{\text{orb}} E_x \frac{1}{\gamma}\left[1 + \frac{\delta L_{\text{orb}}}{L_{\text{orb}}} + \frac{\delta E_x}{E_x} + \gamma\delta\left(\frac{1}{\gamma}\right)\right],$$

$$\delta\left(\frac{1}{\gamma^2-1} - G\right) = -2G\frac{\Delta p}{p} + \frac{1+3\gamma^2}{\gamma^2}G\left(\frac{\Delta p}{p}\right)^2 + \dots$$

$$\frac{\delta L_{\text{orb}}}{L_{\text{orb}}} = \alpha_1\frac{\Delta p}{p} + \alpha_2\left(\frac{\Delta p}{p}\right)^2 + \dots$$

$$\frac{\delta E_x}{E_x} = -k_1\frac{x}{R} + k_2\left(\frac{x}{R}\right)^2 + \dots$$

$$\gamma\delta\left(\frac{1}{\gamma}\right) = -\frac{\gamma^2-1}{\gamma^3}\left(\frac{\Delta p}{p}\right) + \frac{(\gamma^2-1)^2}{2\gamma^5}\left(\frac{\Delta p}{p}\right)^2 + \dots$$

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Variation of spin tune

Grouping the coefficients we have:

$$\delta\nu_S = \frac{eL_{\text{orb}}E_x}{2\pi m_0\gamma c^2}G\left[F_2\left(\alpha_1, k_1, k_2, \frac{x}{R}\right)\left(\frac{\Delta p}{p}\right)^2 + 2F_1\left(k_1, k_2, \frac{x}{R}\right)\frac{\Delta p}{p}\right]$$

where $F_1(k_1, k_2, \frac{x}{R})$, $F_2(\alpha_1, k_1, k_2, \frac{x}{R})$ have quadratic dependence on axial deviation.

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Or the spin tune equation can be represented in another form:

$$\delta\nu_S = \frac{eL_{\text{orb}}E_x}{2\pi m_0\gamma c^2}G\left[\tilde{F}_2\left(k_2, \frac{\Delta p}{p}\right)\left(\frac{x}{R}\right)^2 + 2\tilde{F}_1\left(k_1, \frac{\Delta p}{p}\right)\frac{x}{R} + \tilde{F}_0\left(\alpha_1, \frac{\Delta p}{p}\right)\right]$$

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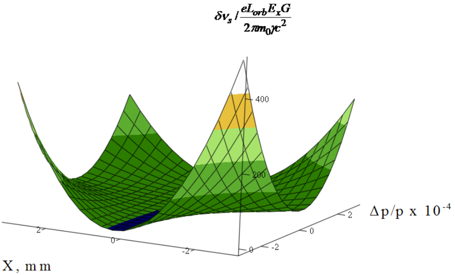

Figure: Two-dimensional parabolic dependence of spin tune aberration on $\left(\frac{\Delta p}{p}\right)^2$ and x, $k_2 = 0.973$.

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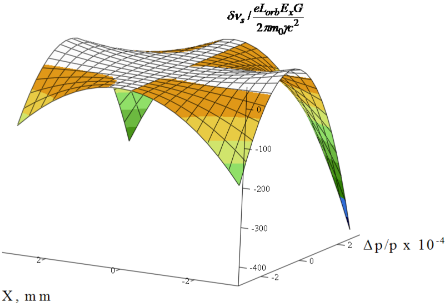

Figure: Two-dimensional parabolic dependence of spin tune aberration on $\left(\frac{\Delta p}{p}\right)^2$ and x, $k_2 = 0.975$.

Methods to increase SCT

Our goal is to achieve maximum flatness in the working range of the beam parameters.

- Fit the parameters of electrical deflector and ring lattice in order to reduce this dependence that is to choose the lattice with a compensation of the mutual influence of all parameters.
  In other words, we need to make the surface maximally flat in the workspace of $(\Delta p/p)^2$ and $(x/R)^2$.
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Alternating spin aberration lattice

The ring is equipped with two types of deflector with $k_1 = \text{const}$, and $k_2 = k_{av} \pm \delta k$ changing from one deflector to another.

- In such optics is easier to achieve minimum spin aberration
- Raising the field strength between the plates in even deflectors and reducing in the odd deflectors it effectively adjust the required coefficients k_1 and k_2. It allows to adjust the spin of aberration to minimum.

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Cylindrical deflectors and alternating aberation lattices

- Cylindrical deflectors: after 10^6 turns $S_{x_{RMS}} \approx 0.002$ or $SCT \approx 500 sec$
- Alternating k_2 deflectors: after 10^6 turns $S_{x_{RMS}} \approx 0.0002$ or $SCT \approx 5000 sec$

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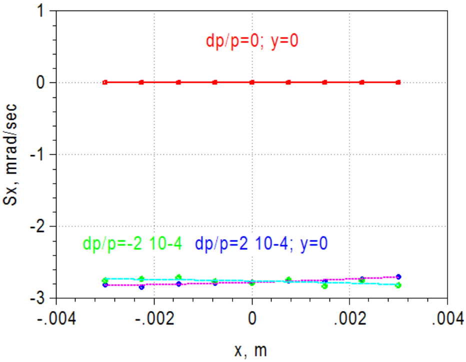
Simulation results



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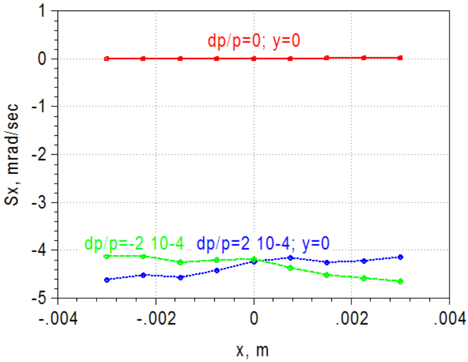

Figure: Cylindrical deflectors with sextupoles.

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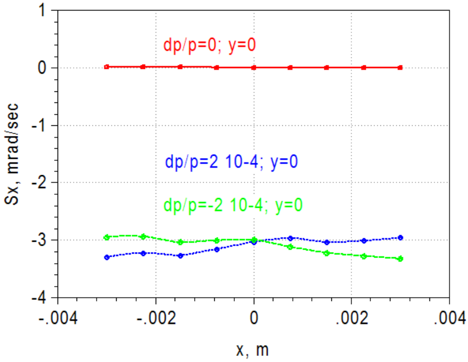

Figure: Smooth lattice (without quadrupoles).

Summary

- Using optimized electrode shapes, SCT can be increased up to thousands of seconds.
- It can be done either with high precision shapes, or with using rough shapes with different field indices.
- SCT in a lattice with cylindrical condensers can be increased using sextupoles.
- All of the methods give approximately the same value of SCT.

Thanks

Thank you for your attention.