Application of Nonlinear Matrix Integration Method in Electrostatic Ring Simulation

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Outline

1. Mathematical modeling of spin-orbit dynamics in electrostatic fiels:
    - using of generalized coordinate system along design orbit allows to model arbitrary field distribution;
    - low of energy conservation accounting (particle changes its kinetic energy during motion inside a field);
2. Nonlinear matrix integration as a mapping approach for long-term dynamics simulation
3. Application of given techniques to spin dynamics investigation for EDM search in electrostatic storage ring
    - horizontal decoherence effects;
    - vertical fake EDM signal due to the errors in vertical fields;

Derivation of nonlinear equation of spin-orbit dynamics

Newton – Lorentz and T – BMT equations are presented in curvilinear coordinate system in generalized form

$$x' = \frac{p_x h_s}{\sqrt{(m_0\gamma v)^2 - p_x^2 - p_y^2}},\ y' = \frac{p_y h_s}{\sqrt{(m_0\gamma v)^2 - p_x^2 - p_y^2}},\ t' = \frac{H}{v},$$

$$p_x' = p_x\left(\frac{v'}{v} - \frac{\gamma'}{\gamma}\right) + m_0\gamma v\frac{x''}{H} - p_x\left(\frac{p_x}{m_0\gamma v}\frac{x''}{H} + \frac{p_y}{m_0\gamma v}\frac{y''}{H} + \frac{h_s h_s'}{H^2}\right),$$

$$p_y' = p_y\left(\frac{v'}{v} - \frac{\gamma'}{\gamma}\right) + m_0\gamma v\frac{y''}{H} - p_y\left(\frac{p_x}{m_0\gamma v}\frac{x''}{H} + \frac{p_y}{m_0\gamma v}\frac{y''}{H} + \frac{h_s h_s'}{H^2}\right)$$

$$W' = -qu'(x, y, s) = q(E_x x' + E_y y' + E_s),$$

$$S_x' = \kappa S_s + H\Big((k_1(B_y S_s - B_s S_y) + k_2(p_y S_s - p_s S_y)/v +$$
$$+ k_3\big((p_s E_x - p_x E_s)S_s - (p_x E_y - p_y E_x)S_y\big)\Big),$$

$$S_y' = H\Big((k_1(B_s S_x - B_x S_s) + k_2(p_s S_x - p_x S_s)/v +$$
$$+ k_3\big((p_x E_y - p_y E_x)S_x - (p_y E_s - p_s E_y)S_s\big)\Big)$$

Low of energy conservation

Motion of charged particle in an electrostatic field has more complicated behavior than in magnetic one. During this motion the Kinetic energy of a particle is changed. This leads to following statement.

Theorem

In the electrostatic fields lower order of multipoles produce higher ones in effected force.
E.g. for a field $E_x = E_0 + E_1 x$ we can claim that "Lorentz" force is proportional to

$$\frac{E_0}{v_0} + \frac{E_1 - kE_0^2}{v_0} x - \frac{3kE_0E_1}{2v_0} x^2 + \dots$$

Consequence: second-order aberrations can be controlled by both sextupole and quadrupole electrostatic fields (both deflector and FODO optimization)

Mapping approach for beam dynamics simulation

Simulation of spin-orbit dynamics requires a numerical approach:

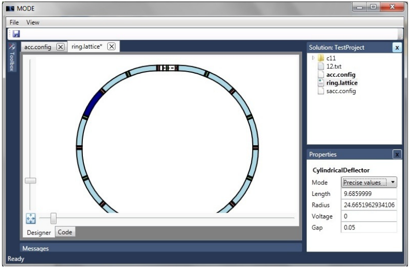
- step-by-step integration methods
  - 3.6 sec per one turn for 1 particle (Runge – Kutta method of 4th order)
  - 4 days for 10^4 turns
- mapping approach
  - 0.001 sec per one turn for 1 particle (matrix map of 4th order)
  - 1.7 min for 10^4 turns

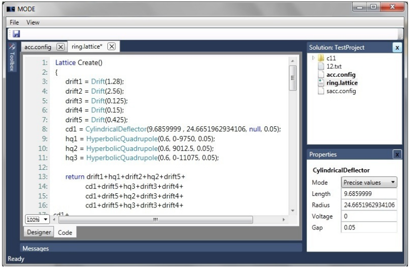
Such performance is achieved by using simple matrix operations:

$$X = R_0 + R_1 X_0 + R_2 X_0^{[2]} + \dots,$$

where R_i is a numerical 2 dimensional matrix

MODE: nonlinear matrix map building





Program MODE allows building numerical matrix maps for spin-orbit dynamics in electro-magnetic fields:

1. high-order maps;
2. symplectic numerical maps;
3. arbitrary fields distribution (in frame of Taylor series expansion)

MODE: verification

Program MODE was developed and verified on:

1. standard models of nonlinear dynamics (comparison with step-by-step integration);
2. COSY Infinity simulation in ideal electrostatic rings (comparison with differential algebra technique);
3. Twiss parameters estimation in OptiM (comparison with linear transfer matrices)

Electrostatic storage ring for searching of EDM

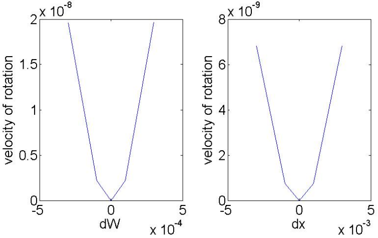
Magic ring with frozen spin is considered. Reference particle in ideal ring:

- has no rotation in horizontal plane with respect to its momentum;
- has spin rotation in vertical plane in conformity with EDM

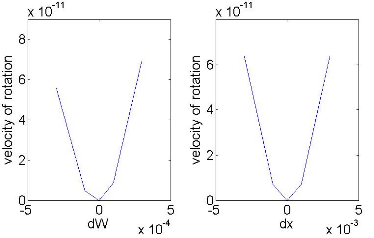
$\Longrightarrow$ need to study the aberrations in horizontal plane and arising of fake EDM signal in vertical one

Aberrations of spin rotation in horizontal plane

Incoherent spin rotation in horizontal plane leads to decreasing of EDM signal. First order aberrations can be correct by RF field. Second orders require sextupole corrections.



FODO (54 sec)

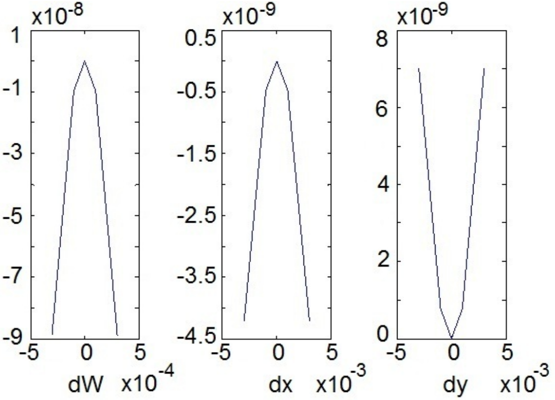


optimal FODO (15000 sec)

For 3D beam we still need sextupoles.

Aberrations of spin rotation in horizontal plane

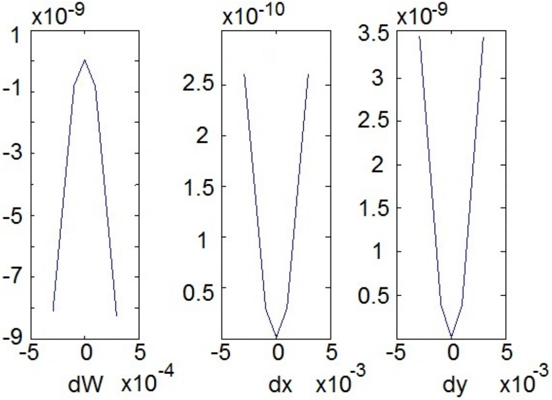
One can start an optimization procedure and find optimal values of sextupoles.



gf = 0.721, gd = - 0.886, **Sext1 = 400**, **Sext2 = - 300**

Aberrations of spin rotation in horizontal plane

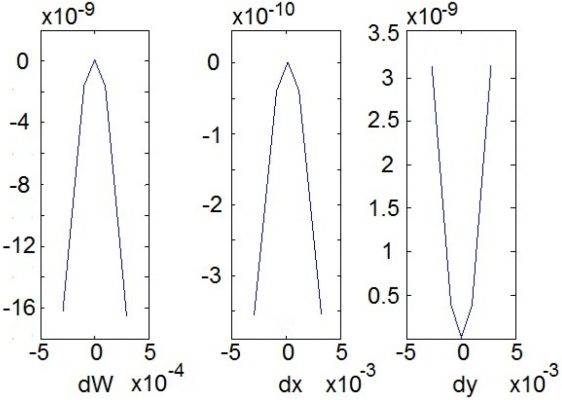
One can start an optimization procedure and find optimal values of sextupoles.



gf = 0.721, gd = - 0.886, Sext1 = 400, **Sext2 = - 335**

Aberrations of spin rotation in horizontal plane

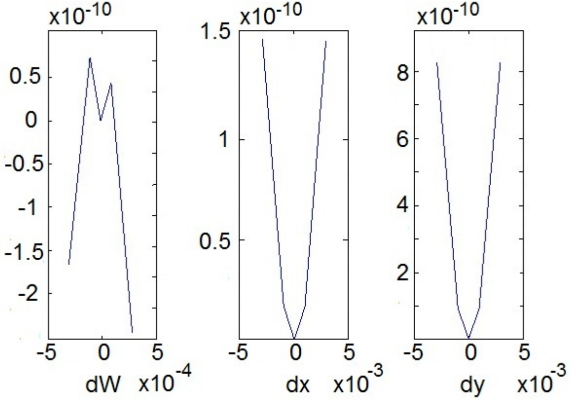
One can start an optimization procedure and find optimal values of sextupoles.



gf = 0.710, gd = - 0.886, Sext1 = 400, Sext2 = - 335

Aberrations of spin rotation in horizontal plane

One can start an optimization procedure and find optimal values of sextupoles.



gf = 0.710, gd = - 0.886, Sext1 = 400, Sext2 = - 335

Errors of radial field

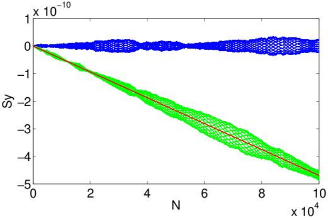
Radial field errors arise from such effects as fringe fields and instability of voltage on tips of deflectors or shifts of plates in horizontal plane.

Length of fringe field, m	0.000	0.001	0.010	0.050
Magic energy, MeV	232.79	-0.00004%	-0.00013%	-0.0003%
Radial field E_0, kV/cm^2	-170	-0.001%	-0.01%	-0.05%

For all cases magic energy is changed (as well as rigidity). For a new value of magic energy new frozen spin condition can be found and sextupole correction can be also performed.

Fake EDM signal

Vertical field induces additional spin rotation in vertical plane. Homogeneous fields (e.g. gravity consideration) leads to predefined spin rotation and can be estimated either by CW/CCW or by statistical data collection.



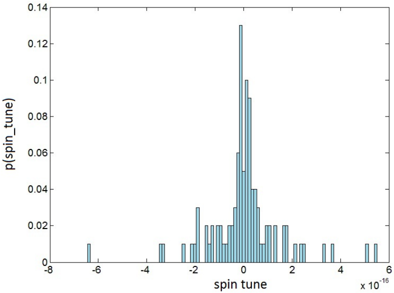
Random errors in vertical field distribution lead to difficulties. Assume that errors in vertical field $E_y \in \mathcal{N}(0, \sigma)$ then the frequency ω of spin rotation in vertical field

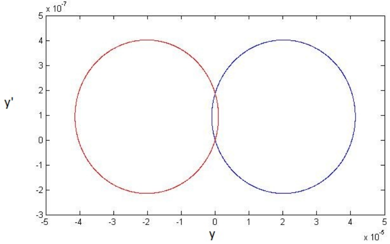
1. $\omega(E_y) \notin \mathcal{N}$
2. $< \omega(E_y) > \neq 0$, despite the fact that $< E_y >= 0$
3. $\omega_{CW} \neq \omega_{CCW}$ for predefined error distribution

All this statements are properties of nonlinearity of spin dynamics.

Fake EDM signal

If $E_y \in \mathcal{N}(0, \sigma)$, where σ corresponds to the deflector rotation along its axis on 10^{-6} rad $\Longrightarrow$

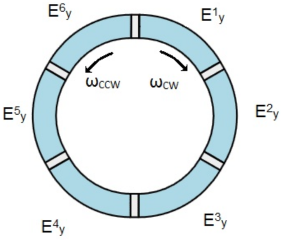


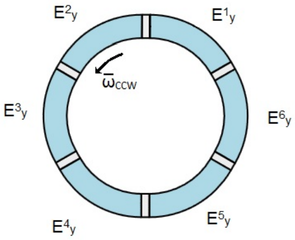


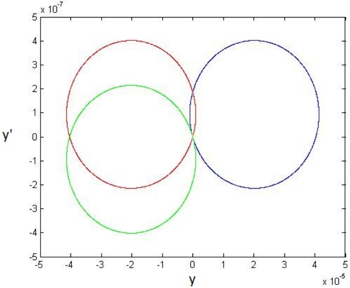
Lets define some error distribution $E_y^1, E_y^2, \ldots, E_y^n$ in deflectors, then

$\omega_{CW} = -1.224 \cdot 10^{-14}$
$\omega_{CCW} = 1.419 \cdot 10^{-15}$

Fake EDM signal



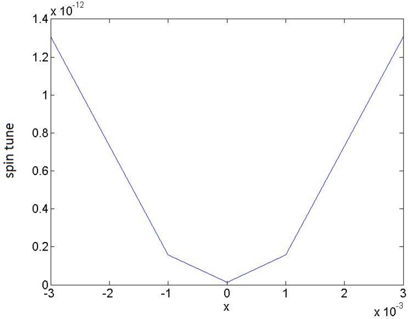




$$\omega_{CW} = -1.224 \cdot 10^{-14} \text{ (blue)}$$
$$\omega_{CCW} = \ 1.419 \cdot 10^{-15} \text{ (red)}$$
$$\overline{\omega}_{CCW} = \ 1.224 \cdot 10^{-14} \text{ (green)}$$

Fake EDM signal

In deflectors with E_y field spin tune in vertical field does not depends on particle coordinates ($\omega \sim 10^{-8} = \text{const}$). But introducing quadrupoles in the ring leads to the fact that the frequency of fake EDM signal depends on particle coordinates.



for reference particle $\omega = 1.82 \cdot 10^{-17} \ll \omega_{EDM} = 4.69 \cdot 10^{-15}$
for a beam ($\pm 3mm$) $\omega = 4.25 \cdot 10^{-11} \gg \omega_{EDM} = 4.69 \cdot 10^{-15}$

Conclusion

1. Nonlinear equations of spin-orbit dynamics considering the low of conservation of energy are derived
   - the theorem of "mixed multipoles" in electrostatic field is proved;

     [1] Doroshko A., Ivanov A. Influence of electrostatic multipoles on spin-orbit dynamics// BDO Workshop, St. Petersburg, 2014.

     [2] Ivanov A. Particle tracking in electrostatic fields with energy conservation // Proc. of ICAP2012. — 2012. — P. 149–151.

2. Nonlinear matrix mapping method for ODEs solving is developed
   - comparison with Runge – Kutta method shows good coincidence in standard models of nonlinear dynamics (oscillators, stable/unstable focuses, limit cycles, dynamical chaos);

     [3] Ivanov A. High-Performance Matrix Integration of Nonlinear Ordinary Differential Equations // Trans. on Comp. Science. — 2014.

     [4] Ivanov A., Kuznetcov P. Dynamical systems identification based on nonlinear matrix Lie presentation. // HPC Conf., 2013, Russia.

Conclusion

3. MODE program for spin-orbit dynamics simulation is developed
   - allows building of high-order matrix map for arbitrary electromagnetic field distribution;

[4] Ivanov A. MODE software for nonlinear spin-orbit dynamics simulation in electromagnetic fields // BDO Workshop, St. Petersburg, 2014.

[5] Ivanov A. Comparison of Matrix Formalism ans step-by-step integration for the long-term 4dynamics simulation in electrostatic fields // Proc. of RuPAC2012. — 2012. — P. 370–372.

[6] Ivanov A., Andrianov S. Matrix Formalism for long-term evolution of charged particle and spin dynamics in electrostatic fields // Proc. of ICAP2012. — 2012. — P. 187–189.

   - spin-orbit dynamics is compared with COSY Infinity;
   - Twiss parameters are compared with OptiM;

[7] Ivanov A., Andrianov S., Kulabukhova N., Maier R., Senichev Yu., Zyuzin D. Testing of symplectic integrator of spin-orbit motion based on matrix formalism // Proc. of IPAC2013. — 2013. — P. 2582–2584.

[8] D. Zyuzin, R. Maier, Yu. Senichev, A. Ivanov, S. Andrianov, M. Berz. Comparison of different numerical modelling methods for beam dynamics in electrostatic rings // Proc. of IPAC2012. — P. 1335–1337.

Conclusion

4. Regarding to study of the spin dynamics in the electrostatic fields the following results were achieved
   - optimization procedure for decoherence minimization in horizontal plane by sextupoles;
   - fringe fields as well as errors in radial fields were investigated;
   - random errors in vertical field induce CW/CWW asymmetry with respect to the spin tune $\Rightarrow$ for EDM search it is necessary to achieve an appropriate error spread ($E_y = 0.01 V/m \sim 10^{-9} rad$).
5. Further development
   - investigation of influence of vertical field on spin tune (fake EDM signal) for beam $(\Delta x, \Delta y, \delta\gamma)$: long-term dynamics simulation for a set of particles can be performed based on MODE matrices but requires high-performance computing;
   - necessity to investigate and optimize electrostatic lattice (both deflectors and quadrupoles) for fake EDM signal minimization.

Thank you for your attention