# Spin Tune Analysis for Electric Dipole Moment Searches

## Master's Colloquium

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# Outline

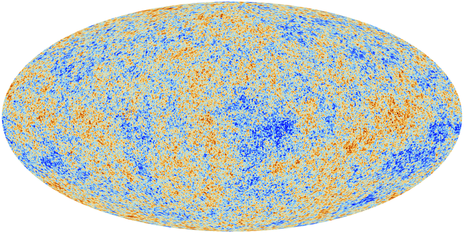
# Introduction

# Motivation

- What is the cause of the matter-antimatter asymmetry observed in the universe today?
- The asymmetry is quantified by $\eta = \frac{n_B - n_{\bar{B}}}{n_\gamma}$,
    - $n_B$ - baryon number density
    - $n_{\bar{B}}$ - antibaryon number density
    - $n_\gamma$ - CMB photon number density

The three conditions identified by Sakharov to explain the asymmetry -

- Processes violating baryon number conservation,
- Charge inversion (C) and charge-parity inversion (CP) symmetries must be violated,
- Interactions outside the thermal equilibrium must occur.



Latest data from Planck's measurement of CMB gives a baryon assymetry of $\eta \approx 10^{-10}$, while current predictions are of the order of $10^{-18}$

# Electric Dipole Moments (EDMs)

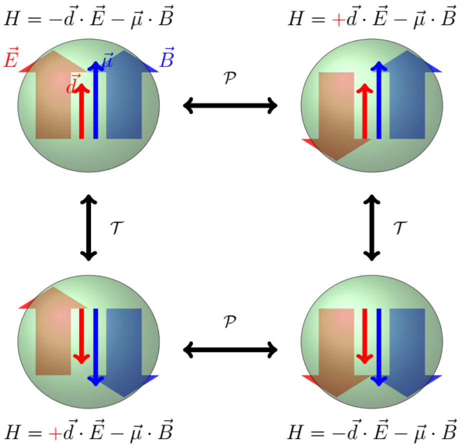
- Classically, an EDM is created by a separation of opposite charges by a distance

$$\mathbf{d}_{EDM} = \int_V \mathbf{x} \cdot \rho(\mathbf{x})\ d^3\mathbf{x}. \tag{1}$$

- For particles, EDMs are collinear with MDMs, and are defined as

$$\mathbf{d}_{EDM} = \eta_{EDM} \frac{q}{2mc} \mathbf{S}. \tag{2}$$

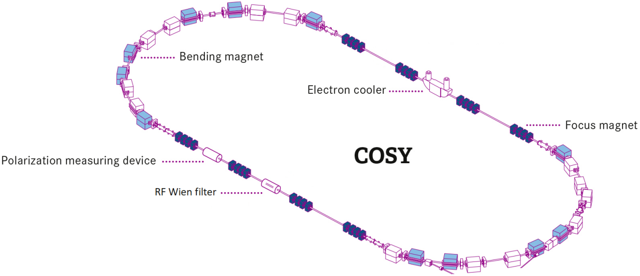
- EDMs violate P, T, and according to the CPT theorem, CP symmetry.




Behaviour of EDM and MDM under parity and time inversion transformations, H is the Hamiltonian of the system

# The Cooler Synchrotron (COSY)

- Magnetic storage ring of circumference 184 m
- Beam energy range of 0.3 to 3.7 GeV
- Stores polarized or unpolarized deuterons or protons



# Spin Dynamics and Polarimetry

# Spin Dynamics

- Spin polarization is defined as

$$\mathbf{P} = \frac{1}{N}\sum_{i=1}^{N}\mathbf{S}_i \qquad (3)$$

- Spin precesses about vertical axis once in the storage ring (direction of **B**)
- Dynamics described by the Thomas-BMT equation

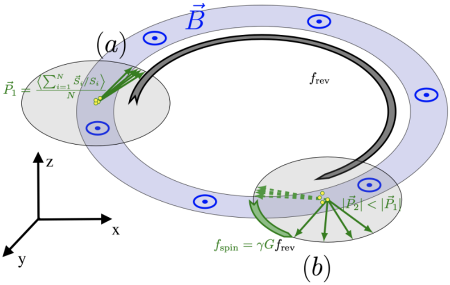
$$\frac{d\mathbf{S}}{dt} = (\boldsymbol{\Omega}_{MDM} + \boldsymbol{\Omega}_{EDM}) \times \mathbf{S} \qquad (4)$$

- Spin tune -

$$\nu_s = \frac{f_{spin}}{f_{rev}} = \gamma G, \qquad (5)$$

  - $\gamma$ - Lorentz factor
  - $G$ - the anomalous magnetic moment

- $\nu_{s,COSY} \approx -0.16$, for deuterons with a momentum of $p = 970$ MeV/c

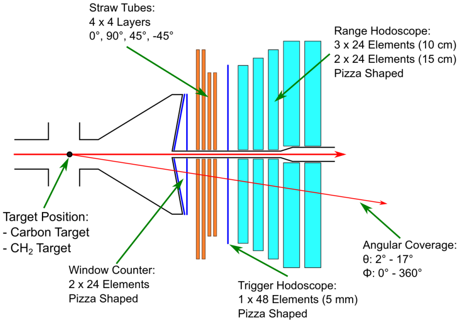


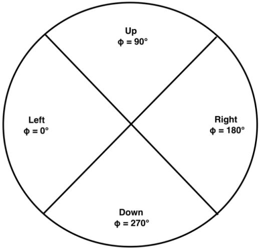
# Polarimetry

- **W**ide **A**ngle **S**hower **A**pparatus (WASA) Polarimeter is used
- Detector consists of four quadrants - up, down, left, and right
- Count rates in each detector quadrant is given by

$$\dot{N}_X = \alpha\sigma(\phi; p_H, p_V)\mathcal{L}. \qquad (6)$$

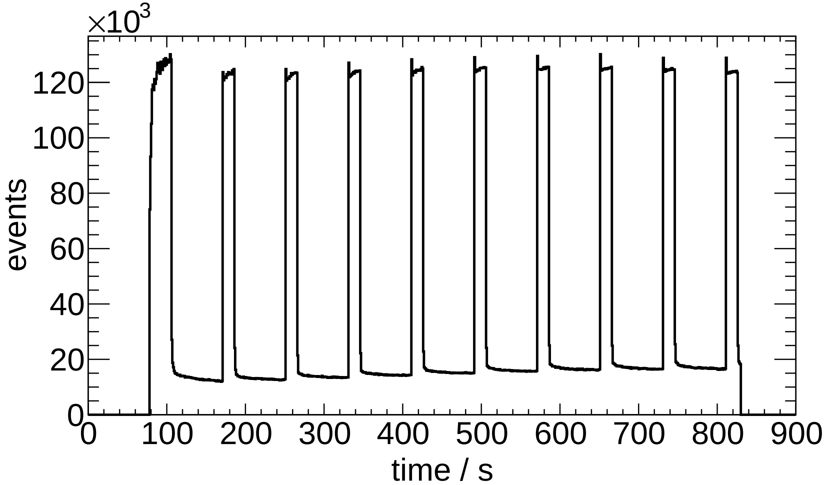
  - $p_H$ - horizontal spin polarization
  - $p_V$ - vertical spin polarization
  - $\phi$ - angle of deflection
  - $\mathcal{L}$ - luminosity



# Data Analysis

# Run 51180, April 2019

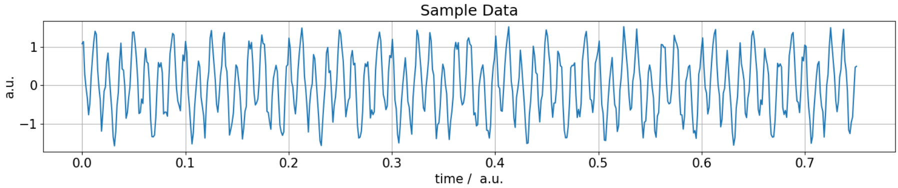


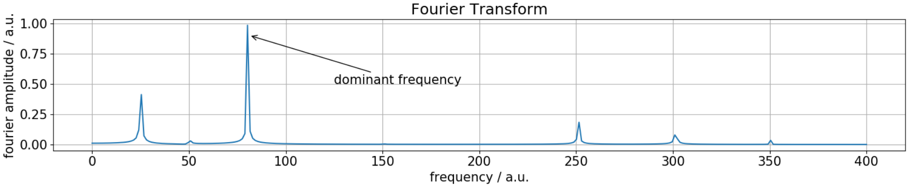

- Polarized deuteron beam consisting of four particle bunches was used
- Extraction of the beam was not continuous, occurred in ten extraction intervals
- Run consisted of two polarized, and two unpolarized cycles

# Overview of Experimental Procedure

- A polarized beam is injected into COSY, with initial vertical polarization.
- The beam is accelerated to a momentum of $p = 970$ MeV/c, after which the beam can be cooled using the electron cooler.
- The spin is flipped onto the accelerator (horizontal) plane.
- The scattering process is initiated. The beam is guided onto the carbon target and slowly extracted with an efficiency of approximately one particle out of a thousand being scattered and recorded as an event.

# Fourier Transform




$$\tilde{f}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt \quad (7)$$

# Discrete Turn Fourier Transform

- Since the data taken consists of discrete turn numbers, the Discrete Turn Fourier Transform is used -

$$g_{\nu_k} = \sum_{n=0}^{N-1} g[n] \left(\cos(2\pi n \nu_k) - i \sin(2\pi n \nu_k)\right), \tag{8}$$

$$g[n] = \begin{cases} 1 & n = n(n_{ev}) \\ 0 & \text{else} \end{cases} . \tag{9}$$

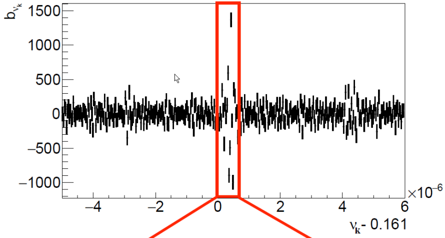
- The Fourier coefficients are -

$$\begin{aligned} a_{\nu_k} &= \mathbb{R}(g_{\nu_k}) = \frac{2}{N_{ev}} \sum_{n_{ev}=1}^{N_{ev}} \cos(2\pi \nu_k n(n_{ev})), \\ b_{\nu_k} &= \mathbb{I}(g_{\nu_k}) = \frac{2}{N_{ev}} \sum_{n_{ev}=1}^{N_{ev}} - \sin(2\pi \nu_k n(n_{ev})). \end{aligned} \tag{10}$$

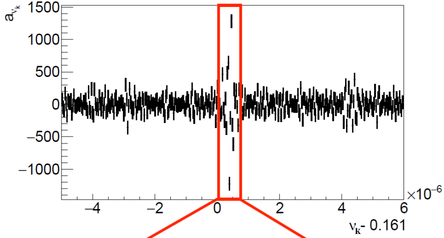
- The corresponding statistical uncertainties are -

$$\sigma_{a_{\nu_k}} = \frac{2}{N_{ev}} \sqrt{\sum_{n_{ev}=1}^{N_{ev}} \cos^2(2\pi \nu_k n(n_{ev}))}, \ \sigma_{b_{\nu_k}} = \frac{2}{N_{ev}} \sqrt{\sum_{n_{ev}=1}^{N_{ev}} \sin^2(2\pi \nu_k n(n_{ev}))} . \tag{11}$$

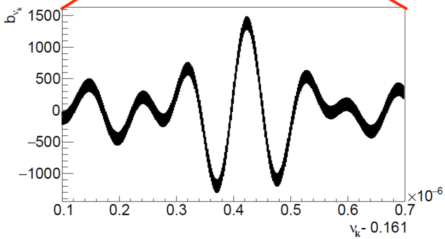
# Fourier Spectrum



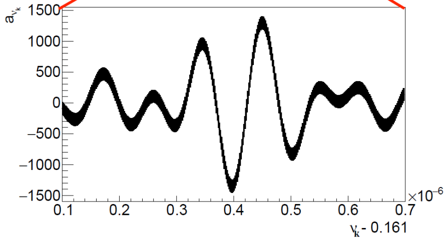

(a) $b_{\nu_k}$




(b) $a_{\nu_k}$




(c) $b_{\nu_k}$



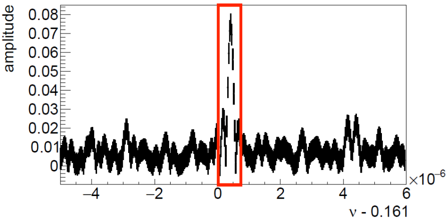

(d) $a_{\nu_k}$

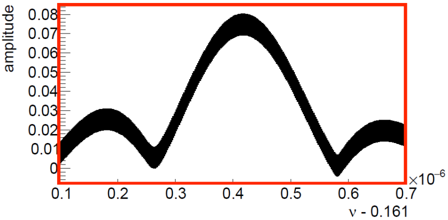
Fourier spectrum in the first macroscopic turn bin seen in two frequency ranges of different orders.

# Amplitude of the Fourier Spectrum

$$\epsilon_{\nu_k} = |g_{\nu_k}| = \sqrt{\mathbb{R}(g_{\nu_k})^2 + \mathbb{I}(g_{\nu_k})^2} = \sqrt{a_{\nu_k}^2 + b_{\nu_k}^2}. \tag{12}$$

$$\sigma_{\epsilon_{\nu_k}} = \sqrt{\frac{a_{\nu_k}^2 \sigma_{a_{\nu_k}}^2 + b_{\nu_k}^2 \sigma_{b_{\nu_k}}^2}{a_{\nu_k}^2 + b_{\nu_k}^2}} \tag{13}$$



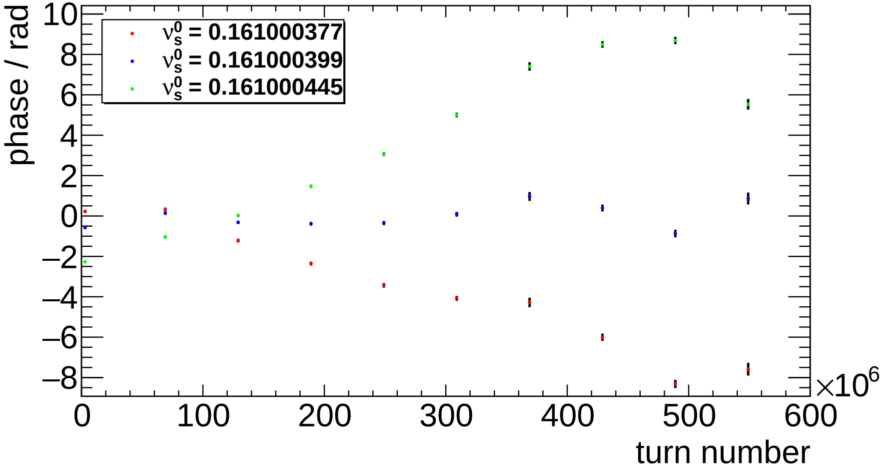




Amplitude of the Fourier transform in the 2nd turn bin in different frequency ranges. Note that both $x$-axes are offset by 0.161.

# Phase of the Fourier Spectrum

$$\phi_{\nu_k} = \arg(g_{\nu_k}) = \operatorname{atan2}(\mathbb{I}(g_{\nu_k}), \mathbb{R}(g_{\nu_k})) = \operatorname{atan2}(b_{\nu_k}, a_{\nu_k}) \quad (14)$$

$$\sigma_{\phi_{\nu_k}} = \sqrt{\frac{a^2_{\nu_k}\sigma^2_{b_{\nu_k}} + b^2_{\nu_k}\sigma^2_{a_{\nu_k}}}{(a^2_{\nu_k} + b^2_{\nu_k})^2}} \quad (15)$$



# Spin Tune Measurement

- Turn dependent spin tune -

$$\nu_s(n) = \nu_s^0 + \frac{1}{2\pi}\frac{\partial \phi_s}{\partial n} \qquad (16)$$

- Phase plot at a fixed frequency $\nu_s^0$ is fitted with a second degree polynomial -

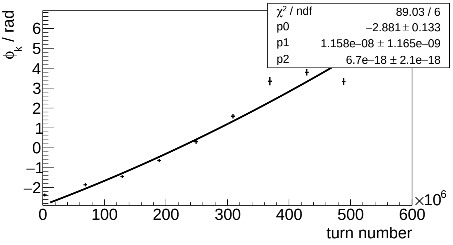
$$\phi_s(n) = an^2 + bn + c \qquad (17)$$

- The slope is then given by

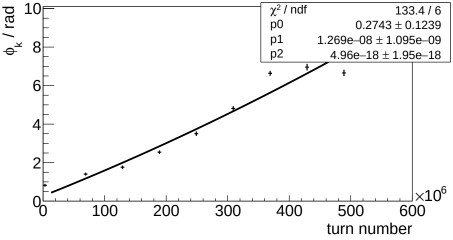
$$\frac{\partial \phi_s(n)}{\partial n} = 2an + b \qquad (18)$$

$$\Rightarrow \nu_s(n) = \nu_s^0 + \frac{1}{2\pi}(2an + b) \qquad (19)$$

- In the figure
  - $\nu_s^0 = 0.161\,000\,425$
  - p0 = c
  - p1 = b
  - p2 = a




(a) Up detector




(b) Down detector

# Spin Tune Measurement

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$$\nu_s(n) = \nu_s^0 + \frac{1}{2\pi}\frac{\partial \phi_s}{\partial n} \tag{20}$$

- Phase plot at a fixed frequency $\nu_s^0$ is fitted with a second degree polynomial -

$$\phi_s(n) = an^2 + bn + c \tag{21}$$

- The slope is then given by

$$\frac{\partial \phi_s(n)}{\partial n} = 2an + b \tag{22}$$

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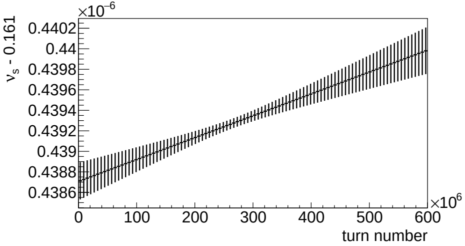


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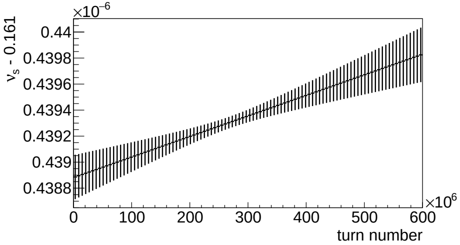



(b) Down detector

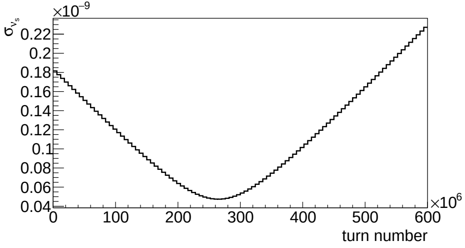
Note the offset in the frequencies in the $y$-axes.

# Uncertainty in the Spin Tune

- Uncertainty in the spin tune $\sigma_{\nu_s}$ calculated using standard error propagation
-

$$\sigma^2_{\nu_s} = \left(\frac{1}{2\pi}\sigma_{slope}\right)^2$$
$$= \frac{1}{4\pi^2}\left(2\sigma_a^2 n + \sigma_b^2 + 4\mathrm{cov}(a,b)\right)$$
$$\Rightarrow \sigma_{\nu_s} = \frac{1}{2\pi}\sqrt{2\sigma_a^2 n + \sigma_b^2 + 4\mathrm{cov}(a,b)}. \quad (24)$$

- $\sigma_{\nu_s,\mathrm{min}} \sim \mathcal{O}(10^{-11})$

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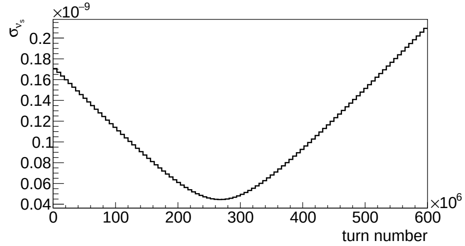


(b) Down detector

# Analytical Uncertainty in the Spin Tune

- Count rates in the up and down detectors is given by

$$N_{\uparrow,\downarrow}(t) = N_0 \left(1 \pm P \sin(\omega t + \phi)\right), \tag{25}$$

  - $N_0$ - unpolarized cross section
  - $\uparrow$ / $\downarrow$ - up / down detectors
  - $\omega = 2\pi\nu f_{rev}$ , $f_{rev}$ - beam revolution frequency
  - $P$ - product of the analyzing power and horizontal polarization

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- An asymmetry can be formed using the count rates

$$A(t) = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow} = P\sin(\omega t + \phi) \tag{26}$$

and fitted with a function $f(t)$. However, due to the low number of events recorded per second the fit would not be feasible.

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- The extended log-likelihood method is used for calculating the uncertainties in the parameters $P$, $\omega$, and $\phi$.
- Log-likelihood function is given as -

$$\begin{aligned} \ell = \log\mathcal{L} = &\sum_{\uparrow}\log\left[N_0\left(1 + P\sin(\omega t_i + \phi)\right)\right] \\ &+ \sum_{\downarrow}\log\left[N_0\left(1 - P\sin(\omega t_i + \phi)\right)\right]. \\ &- \left[N_\uparrow(\omega,\phi,P) + N_\downarrow(\omega,\phi,P)\right] \end{aligned} \tag{27}$$

# Analytical Uncertainty in the Spin Tune

- Elements of the (inverse) covariance matrix -

$$(\mathrm{cov}^{-1})_{ij} = -\left\langle \frac{\partial^2 \ell}{\partial a_i \partial a_j} \right\rangle = -\int_0^T N(t) \cdot \frac{\partial^2 \ell}{\partial a_i \partial a_j} \cdot dt, \tag{28}$$

$(a_1, a_2, a_3) = (\omega, \phi, P)$

[1] D. Eversmann et al. "New Method for a Continuous Determination of the Spin Tune in Storage Rings and Implications for Precision Experiments".

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$$\sigma_\omega^2 = \frac{24}{N(PT)^2},\ \sigma_\phi^2 = \frac{8}{NP^2},\ \sigma_P^2 = \frac{2}{N}. \tag{30}$$

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- $\Rightarrow \sigma_\omega \propto T^{-1} \Rightarrow \sigma_\nu \propto T^{-1}$
- An earlier study[1] found that for a cycle length of $10^2$ s, $\sigma_\nu \sim \mathcal{O}(10^{-10})$.

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- An earlier study[1] found that for a cycle length of $10^2$ s, $\sigma_\nu \sim \mathcal{O}(10^{-10})$.
- For the current cycle of length $10^3$ s, $\sigma_\nu \sim \mathcal{O}(10^{-11})$, which agrees with measurements.

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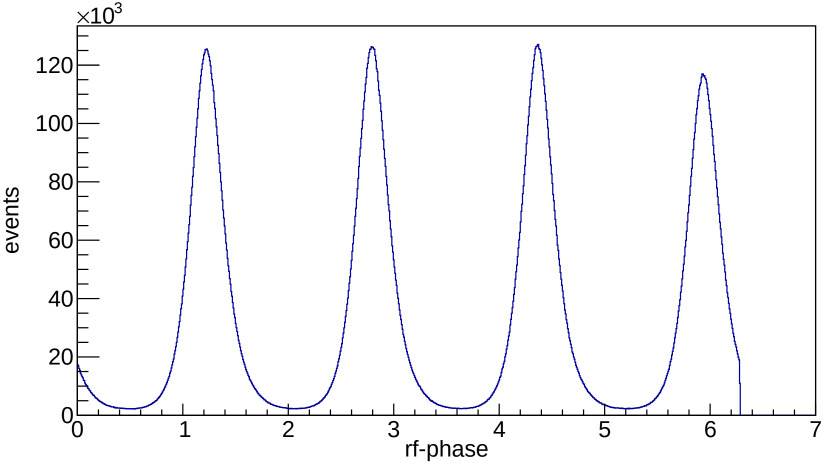
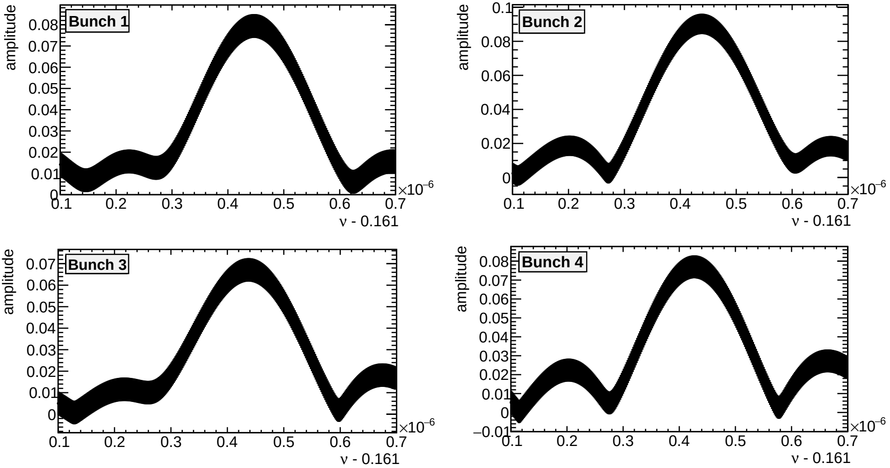
# Results from Individual Bunches

# Bunching




- The particle beam is bunched by passing it through an RF-cavity before it is accelerated to relaticistic velocities.
- The phase of the RF-cavity is used to distinguish particles in separate bunches.

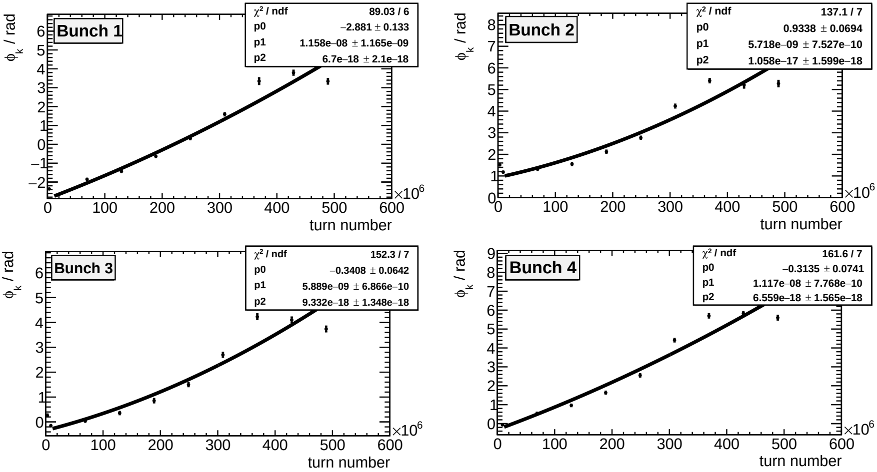
# Fourier Amplitude




$$\epsilon_{\nu_k} = |g_{\nu_k}| = \sqrt{\mathbb{R}(g_{\nu_k})^2 + \mathbb{I}(g_{\nu_k})^2} = \sqrt{a_{\nu_k}^2 + b_{\nu_k}^2}. \qquad (31)$$

$$\sigma_{\epsilon_{\nu_k}} = \sqrt{\frac{a_{\nu_k}^2 \sigma_{a_{\nu_k}}^2 + b_{\nu_k}^2 \sigma_{b_{\nu_k}}^2}{a_{\nu_k}^2 + b_{\nu_k}^2}} \qquad (32)$$

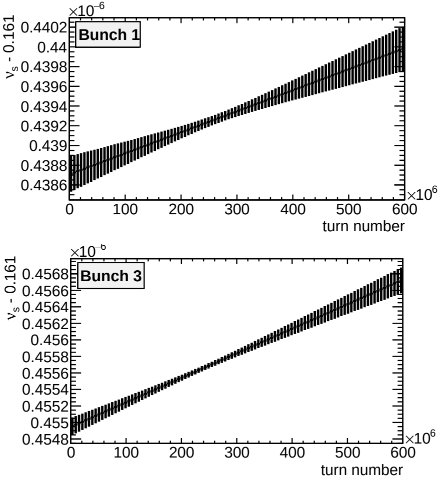
# Fourier Phase




$$\phi_{\nu_k} = \arg(g_{\nu_k}) = \text{atan}\,2(\mathbb{I}(g_{\nu_k}), \mathbb{R}(g_{\nu_k})) = \text{atan}\,2(b_{\nu_k}, a_{\nu_k}) \tag{33}$$

$$\sigma_{\phi_{\nu_k}} = \sqrt{\frac{a^2_{\nu_k}\sigma^2_{b_{\nu_k}} + b^2_{\nu_k}\sigma^2_{a_{\nu_k}}}{(a^2_{\nu_k} + b^2_{\nu_k})^2}} \tag{34}$$

# Spin Tune




$$\nu_s(n) = \nu_s^0 + \frac{1}{2\pi}\frac{\partial \phi_s}{\partial n} \tag{35}$$

# Consistency Check

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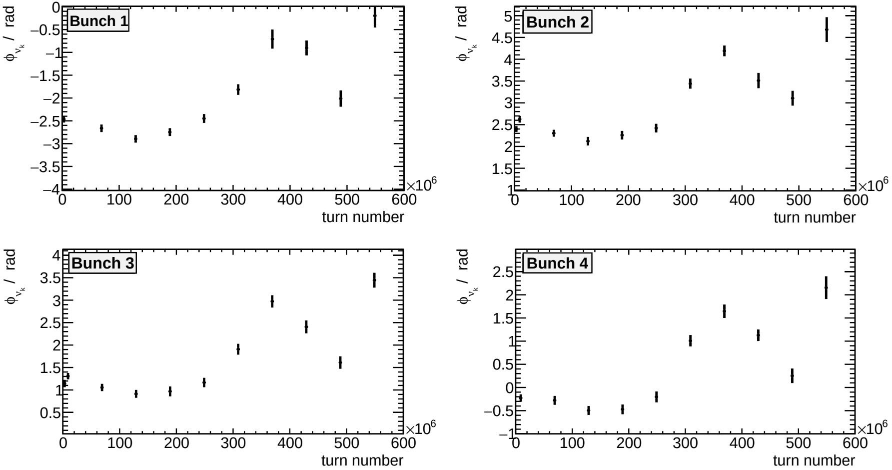
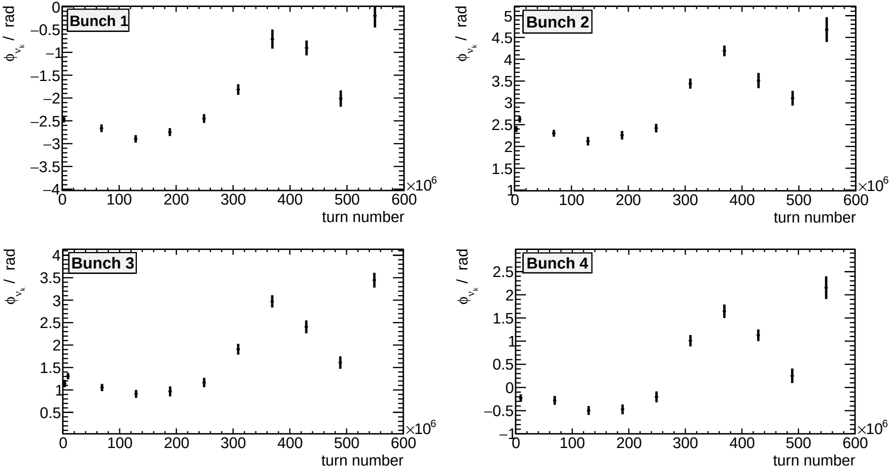


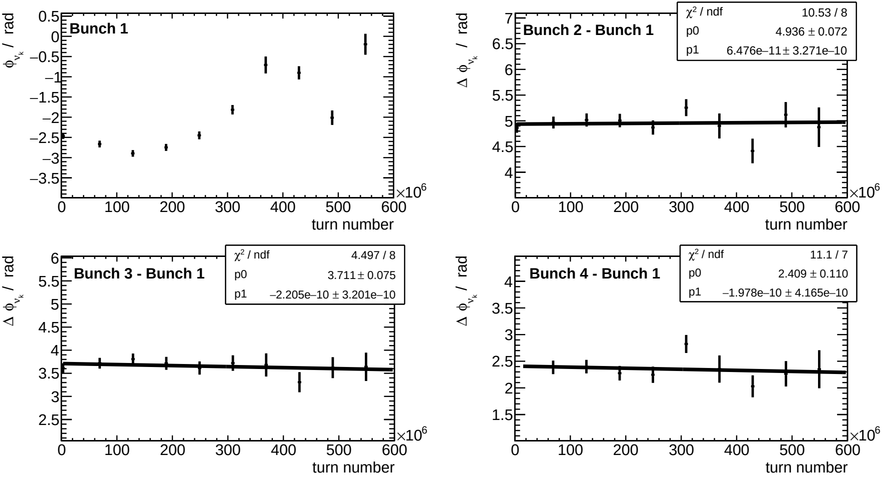

- Phase plotted at a fixed frequency for all 4 bunches, here $\nu_s^0 = 0.161\,000\,439$

# Consistency Check



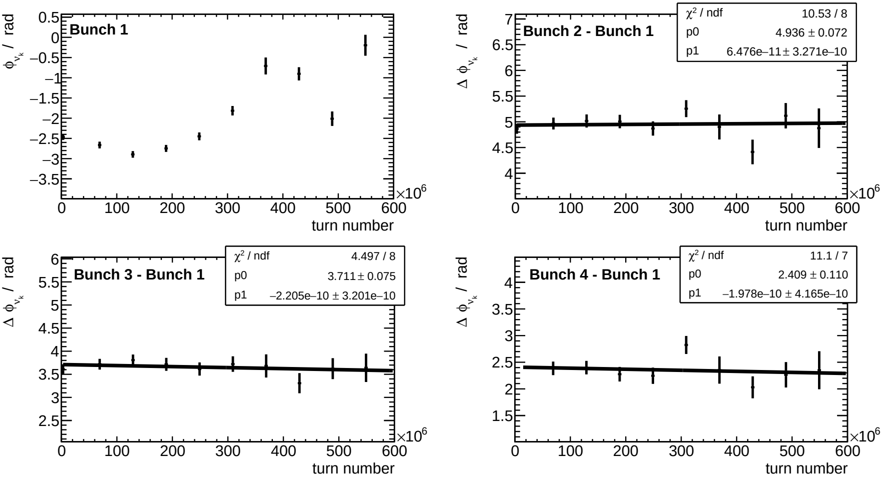

- Phase plotted at a fixed frequency for all 4 bunches, here $\nu_s^0 = 0.161\,000\,439$
- Taking one bunch as a reference, difference between phases of other three bunches is plotted

# Consistency Check



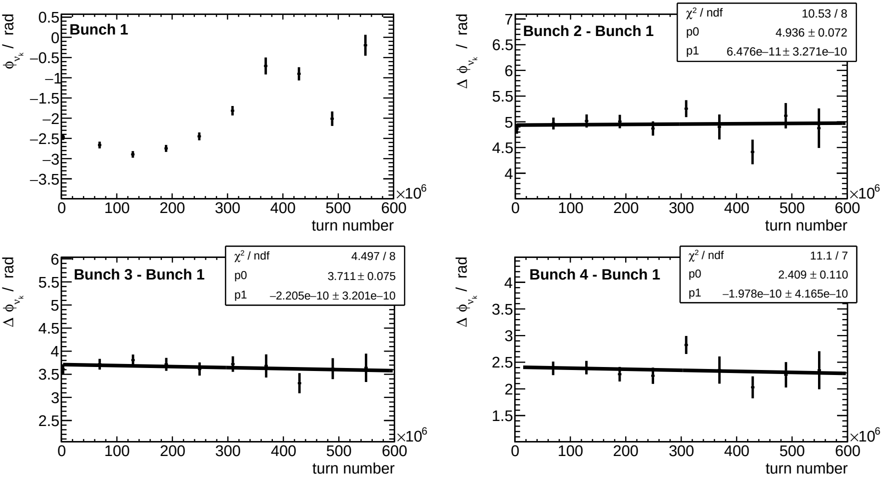

- Bunch 1 taken as a reference, difference between phases of other three bunches is plotted

# Consistency Check




- Bunch 1 taken as a reference, difference between phases of other three bunches is plotted
- A straight line fit is performed on the differences

# Consistency Check




- Bunch 1 taken as a reference, difference between phases of other three bunches is plotted
- A straight line fit is performed on the differences
- The difference between phases in the four bunches must remain constant within measurement errors and slope of a straight line fit must be consistent with zero

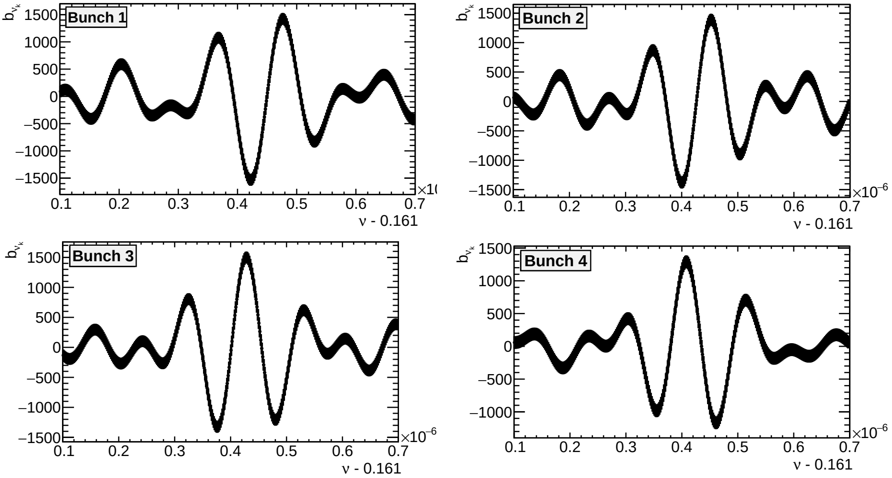
# Summary

- The run from April 2019 was analyzed. The beam consisted of four bunches and the cycle lengths were of the order of $10^3$s.
- The uncertainty in the spin tune, which is inversely related to the length of the cycle, is seen to be in the order of $10^{-11}$ for all the bunches.
- The spin tune in the four bunches was analyzed separately for the first time and the consistency of the results was verified.

Thank You

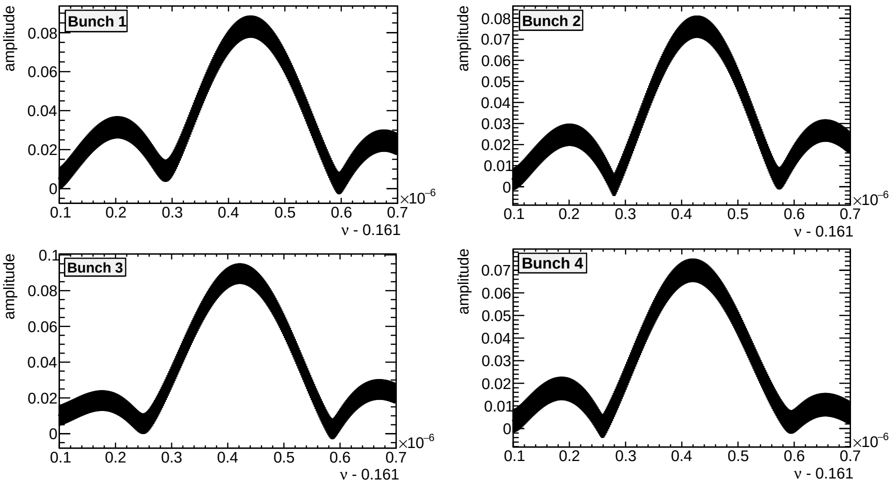
# Backup Slides

# Results from Cycle 3

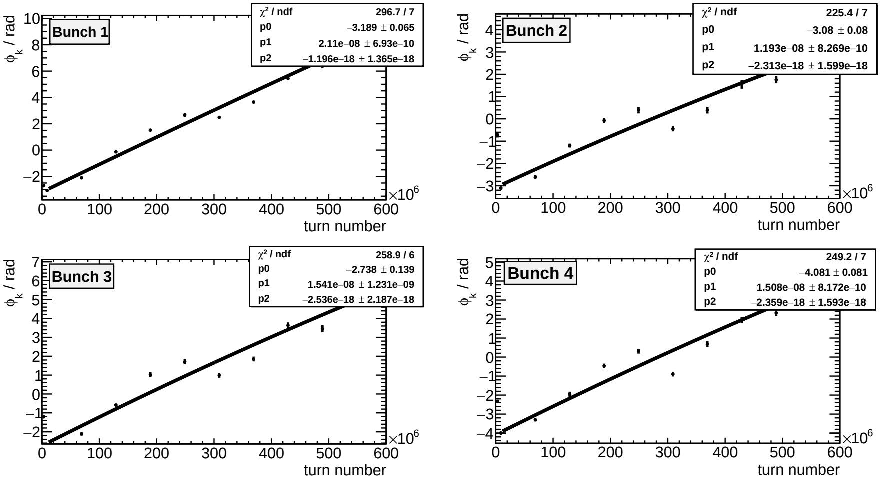



The fourier coefficient $b_{\nu_k}$ for the up detector in the first macrosocopic time bin in the four bunches in cycle 3. Note the $x$-axis offset.

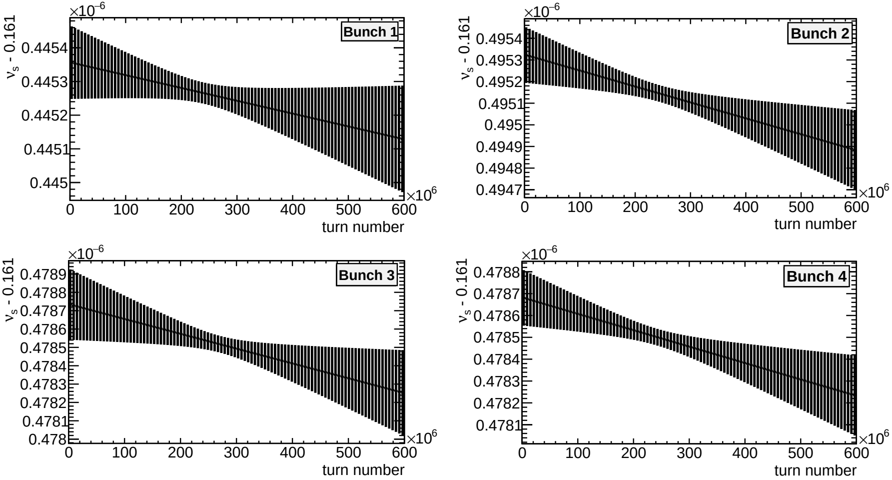
# Amplitude of the Fourier Spectrum



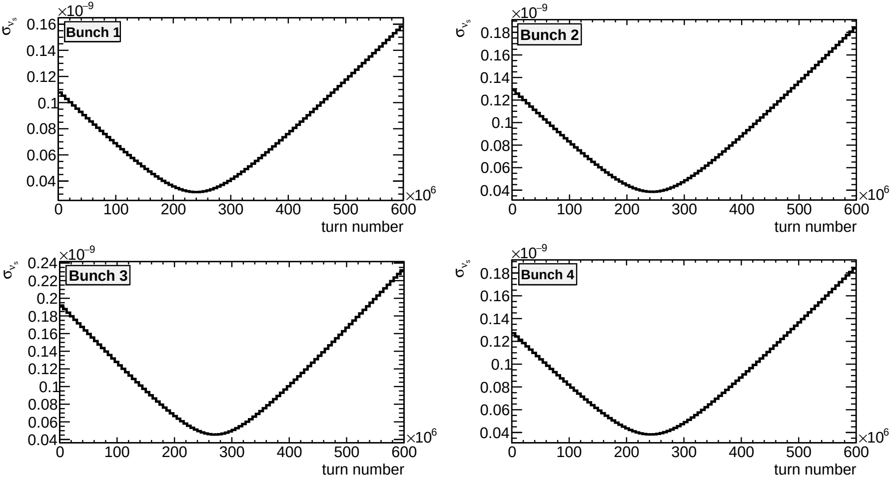
# Phase of the Fourier Spectrum



# Spin Tune


Bunch 1
Bunch 2
Bunch 3
Bunch 4
$\nu_s$ - 0.161
$\times10^{-6}$
turn number
$\times10^{6}$
0
100
200
300
400
500
600

# Uncertainty in the Spin Tune



# Consistency Check