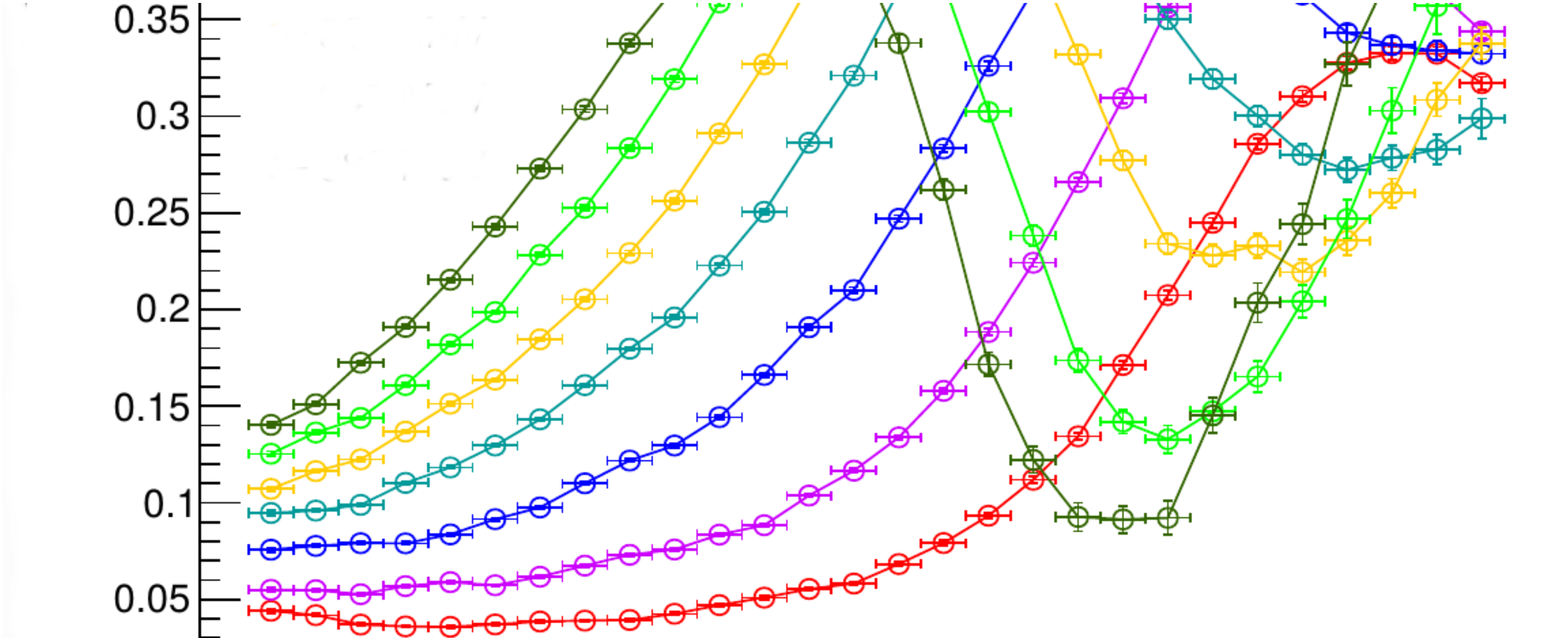

Extraction of Azimuthal Asymmetries Using Optimal Observables

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OUTLINE

- Introduction
  - Cross Section
  - Asymmetries
- Getting Asymmetries by Counting
- New Method: Asymmetries from χ^2-Fit
- Summary / Conclusion

INTRODUCTION

Unpolarized Cross Section

Definition: Cross Section → $\dfrac{\text{Detected number of scattered particles per solid angle}}{\text{Number of incoming beam particles}}$

Formal:
Differential Cross Section

$$\frac{d\sigma}{d\Omega} = \frac{n}{\alpha \mathcal{L}} \qquad \mathcal{L} = f \rho_N L$$

- n: Rate of scattered particles
- α: Detector acceptance
- $\mathcal{L}$: Luminosity
- f: Flux of incoming particles
- ρ_N: Number density (Number of scattering centers per unit volume)
- L: Target length

Total Cross Section

$$\sigma = \int \left(\frac{d\sigma}{d\Omega} \right) \partial\Omega = \frac{N}{\alpha \mathcal{L}^{int}}$$

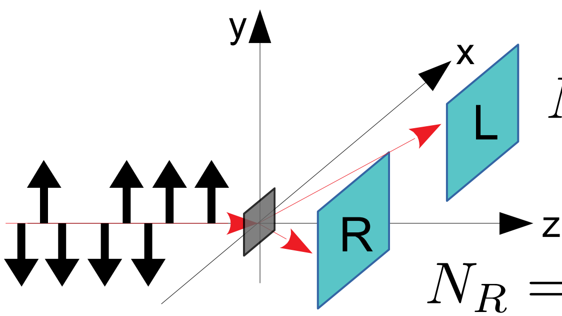
INTRODUCTION

Asymmetries

From previous definition:

$$N(\Theta, \Phi) = n\rho_N L\alpha(\Theta, \Phi)\sigma(\Theta, \Phi)$$

n: # of incoming particles



$$N_L = \rho_N L\alpha(n^{\uparrow}\sigma_L^{\uparrow} + n^{\downarrow}\sigma_L^{\downarrow}) \quad \Phi - sym : \sigma_L^{\downarrow} = \sigma_R^{\uparrow} := \sigma_R$$
$$N_L = \rho_N L\alpha(n^{\uparrow}\sigma_L^{\uparrow} + n^{\downarrow}\sigma_R^{\uparrow})$$

$$N_R = \rho_N L\alpha(n^{\uparrow}\sigma_R^{\uparrow} + n^{\downarrow}\sigma_R^{\downarrow}) \quad \Phi - sym. : \sigma_R^{\downarrow} = \sigma_L^{\uparrow} := \sigma_L$$
$$N_R = \rho_N L\alpha(n^{\uparrow}\sigma_R^{\uparrow} + n^{\downarrow}\sigma_L^{\uparrow})$$

$$\epsilon = \frac{N_L - N_R}{N_L + N_R} = \left(\frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R}\right)\left(\frac{n^{\uparrow} - n^{\downarrow}}{n^{\uparrow} + n^{\downarrow}}\right) := A_y P_y$$

ϵ: Asymmetry

A_y: Vector Analyzing Power

P_y: Vector Polarization

INTRODUCTION

Elastic Cross Section for Deuterons

Using definitions for A_y and P_y

$$\sigma = \frac{1}{2}(\sigma_R + \sigma_L)$$

$$\Rightarrow \quad \begin{aligned} N_L &= \alpha \rho_N L \sigma (1 + A_y P_y) \\ N_R &= \alpha \rho_N L \sigma (1 - A_y P_y) \end{aligned}$$

This factor accounts for the fact that the asymmetry is largest along the X-axis

This can be generalized to the *Elastic Cross Section*

$$\sigma_{pol}(\Theta, \Phi) = \sigma_{unpol}(\Theta)[1 + P_y A_y(\Theta) \cos(\Phi)]$$

Polarized elastic cross section

Unpolarized elastic cross section

Vector polarization

Vector analyzing power

GETTING ASYMMETRIES BY COUNTING

Full Cross Ratio Method

First Example was oversimplified..

- Exact number of $n^{\uparrow}$ and $n^{\downarrow}$ is usually not known
  $\rightarrow$ Polarization P_y might be known..
- Acceptance was assumed to be the same for both sides
  $\rightarrow$ This is might not be the case

Better: Subsequent measurement of beams with two opposing polarization states!

$$P_y^{\uparrow} = -P_y^{\downarrow} \quad \text{and} \quad |P_y^{\uparrow}| = |P_y^{\downarrow}|$$

Full Cross Ratio:

$$\epsilon_{CR} = \frac{1-r}{1+r} \quad \text{with} \quad r^2 = \frac{\sigma_L^{\uparrow}\sigma_R^{\downarrow}}{\sigma_L^{\downarrow}\sigma_R^{\uparrow}}$$

Do we know the cross ratios?
No!

But..

$$r^2 = \frac{N_L^{\uparrow} \cdot N_R^{\downarrow} \cdot \alpha^L \mathcal{L}_{int}^{\downarrow} \cdot \alpha^R \mathcal{L}_{int}^{\uparrow}}{\alpha^L \mathcal{L}_{int}^{\uparrow} \cdot \alpha^R \mathcal{L}_{int}^{\downarrow} \cdot N_L^{\downarrow} \cdot N_R^{\uparrow}} = \frac{N_L^{\uparrow} N_R^{\downarrow}}{N_L^{\downarrow} N_R^{\uparrow}}$$

GETTING ASYMMETRIES BY COUNTING

Half Cross Ratio

What if $|P_y^{\uparrow}| \neq |P_y^{\downarrow}|$?

→ Measure a cycle of unpolarized beam with the same detector

Half Cross Ratios:

$$\epsilon_{HCR}^{\uparrow} = \frac{1-r^{\uparrow}}{1+r^{\uparrow}} \qquad \epsilon_{HCR}^{\downarrow} = \frac{1-r^{\downarrow}}{1+r^{\downarrow}}$$

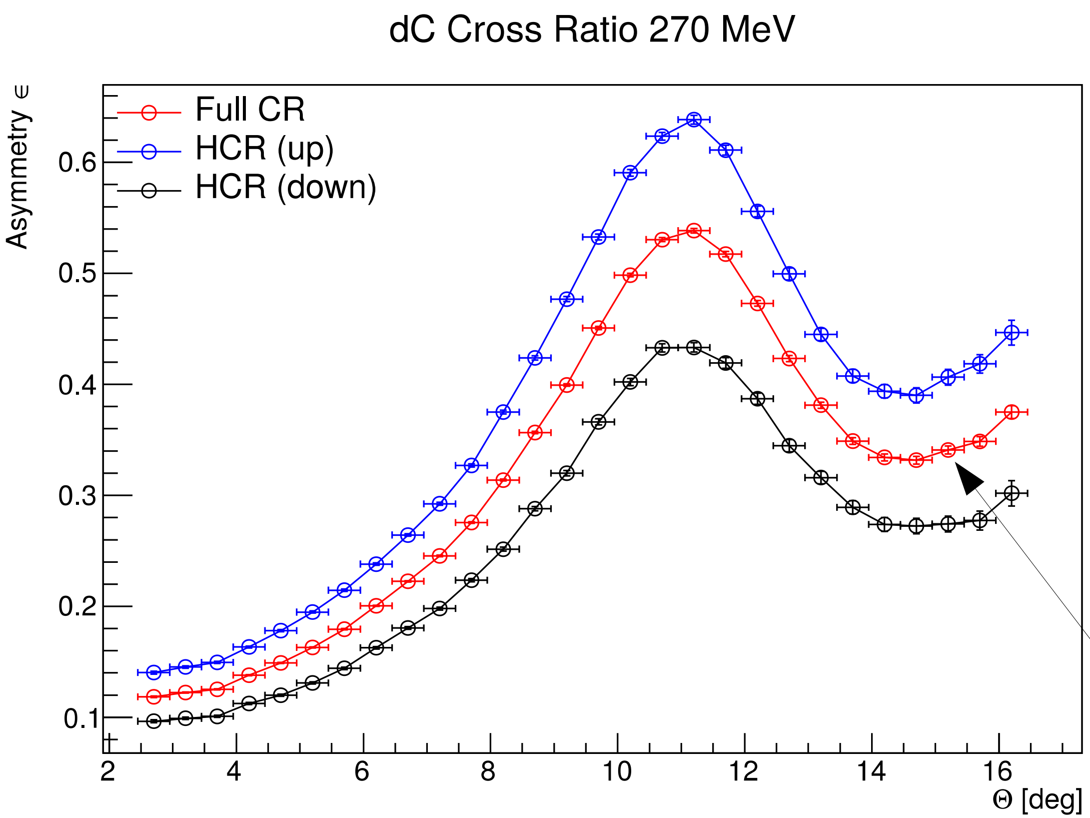
with

$$r^{\uparrow} = \frac{\sigma_L^{\uparrow}\sigma_R^0}{\sigma_R^{\uparrow}\sigma_L^0} = \frac{N_L^{\uparrow}N_R^0}{N_R^{\uparrow}N_L^0} \qquad r^{\downarrow} = \frac{\sigma_R^{\downarrow}\sigma_L^0}{\sigma_L^{\downarrow}\sigma_R^0} = \frac{N_R^{\downarrow}N_L^0}{N_L^{\downarrow}N_R^0}$$

Trick: include unpolarized cross sections σ_L^0 and σ_R^0. Because $\sigma_L^0 = \sigma_R^0$, acceptance and luminosity cancels as in the *Full Cross Ratio*

GETTING ASYMMETRIES BY COUNTING

Some Results for 270 MeV Deuteron Carbon Scattering



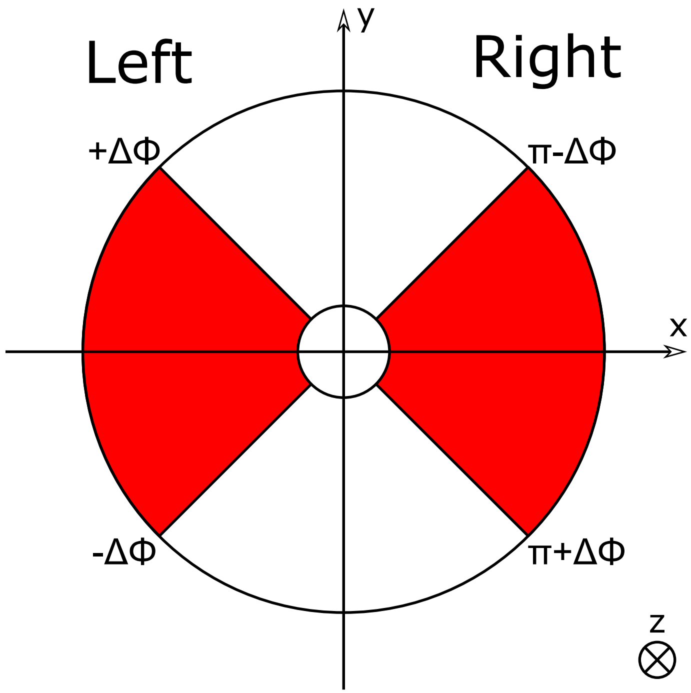
- Formulas for the cross rations applied for each Θ-bin with a width of 0.5°
- Results show: $|P_y^{\uparrow}| \neq |P_y^{\downarrow}|$
- Error: Statistical Error from error propagation

Not a good idea to use *Full Cross Ratio*

GETTING ASYMMETRIES BY COUNTING

Integration Range

So far:
- N_L and N_R were defined for an infinitesimal small Φ-range → Not feasible
- Asymmetry calculated for a given Θ-bin; assumed constant here → OK



Solution: Integrate over a range in Φ, and go from N to $\langle N \rangle$

$$\langle N_{L,R}^{\uparrow,\downarrow} \rangle = \mathcal{L}_{\uparrow,\downarrow}\sigma^0 \int_{-\Delta\Phi}^{+\Delta\Phi} \alpha_{L,R}(1 \pm \epsilon^{\uparrow,\downarrow}\cos(\Phi))d\Phi.$$

Still assuming constant acceptance within one side, previous formula have to be modified to:

$$\epsilon_{CR} = \frac{1}{\langle\cos\rangle}\frac{1-r}{1+r} \quad \text{with} \quad r^2 = \frac{\langle N_L^{\uparrow}\rangle\langle N_R^{\downarrow}\rangle}{\langle N_L^{\downarrow}\rangle\langle N_R^{\uparrow}\rangle}.$$

$$\epsilon_{HCR}^{\downarrow,\uparrow} = \frac{1}{\langle\cos\rangle}\frac{1-r^{\downarrow,\uparrow}}{1+r^{\downarrow,\uparrow}} \quad \text{with} \quad r^{\downarrow,\uparrow} = \frac{\langle N_R^{\downarrow,\uparrow}\rangle\langle N_L^{0}\rangle}{\langle N_L^{\downarrow,\uparrow}\rangle\langle N_R^{0}\rangle}$$

Only counting in Φ-range

GETTING ASYMMETRIES BY COUNTING

Integration Range

What is the best range for integration?

Counting experiment → Error $\sim \sqrt{N}$

Naive assumption:
Bigger range → More Counts → Smaller Error

Let's see..

Introducing *Figure of Merit (FoM)*

$$\mathrm{FoM}(\epsilon) \equiv \frac{1}{(\Delta\epsilon)^2}$$

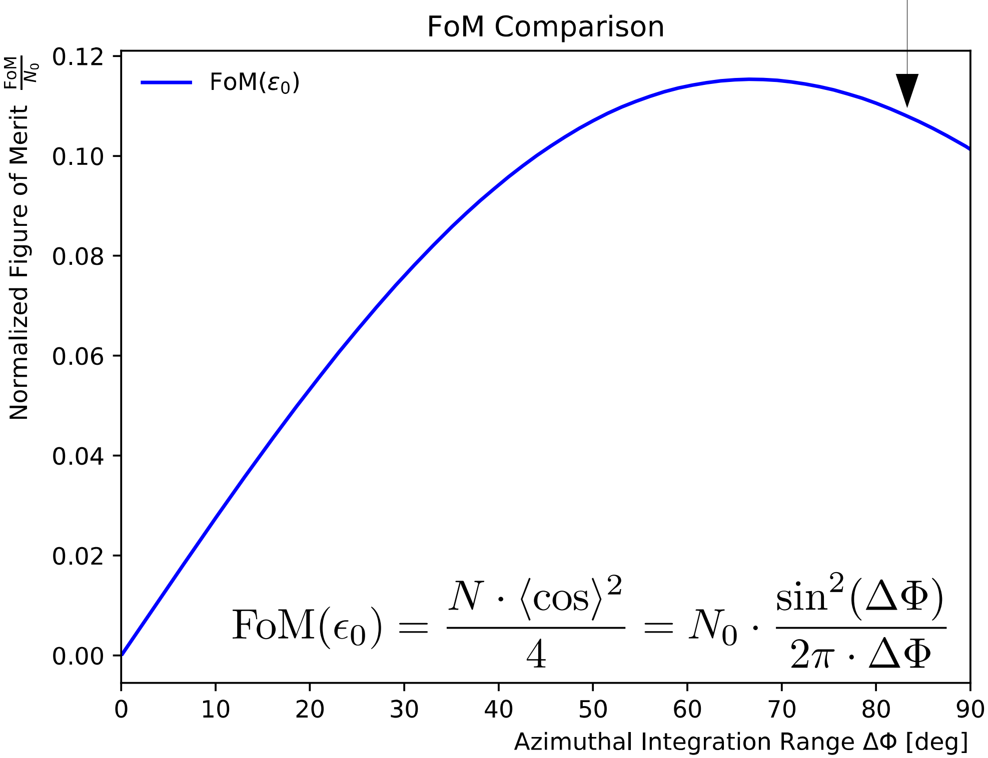
We want it to be large!

GETTING ASYMMETRIES BY COUNTING

Integration Range

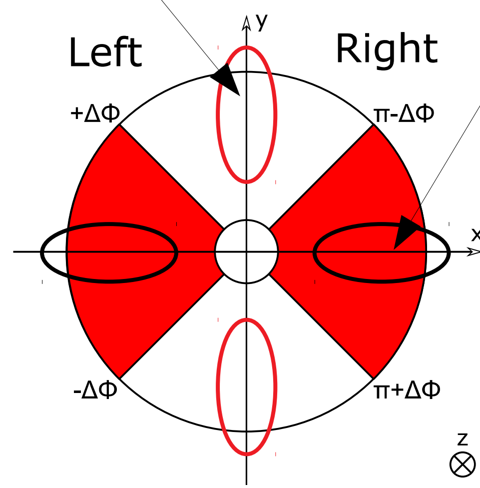
Error increases again for large range!

But why?



Little asymmetry information here

Lots of asymmetry information here



Adding more events from the top/bottom region dilutes the FoM!

GETTING ASYMMETRIES BY COUNTING

Weighting Method

Can we do better? Yes! → Adding a weight to each event

Clever weight should account for the amount of information "carried" by each event. → Good choice: $\cos(\Phi)$

$$\langle N^{\uparrow,\downarrow}_{L,R} \rangle \rightarrow \langle N^{\uparrow,\downarrow}_{L,R} \cos \rangle = \sum_{i}^{ev(\uparrow,\downarrow,L,R)} \cos(\Phi_i)$$

Not only counting anymore

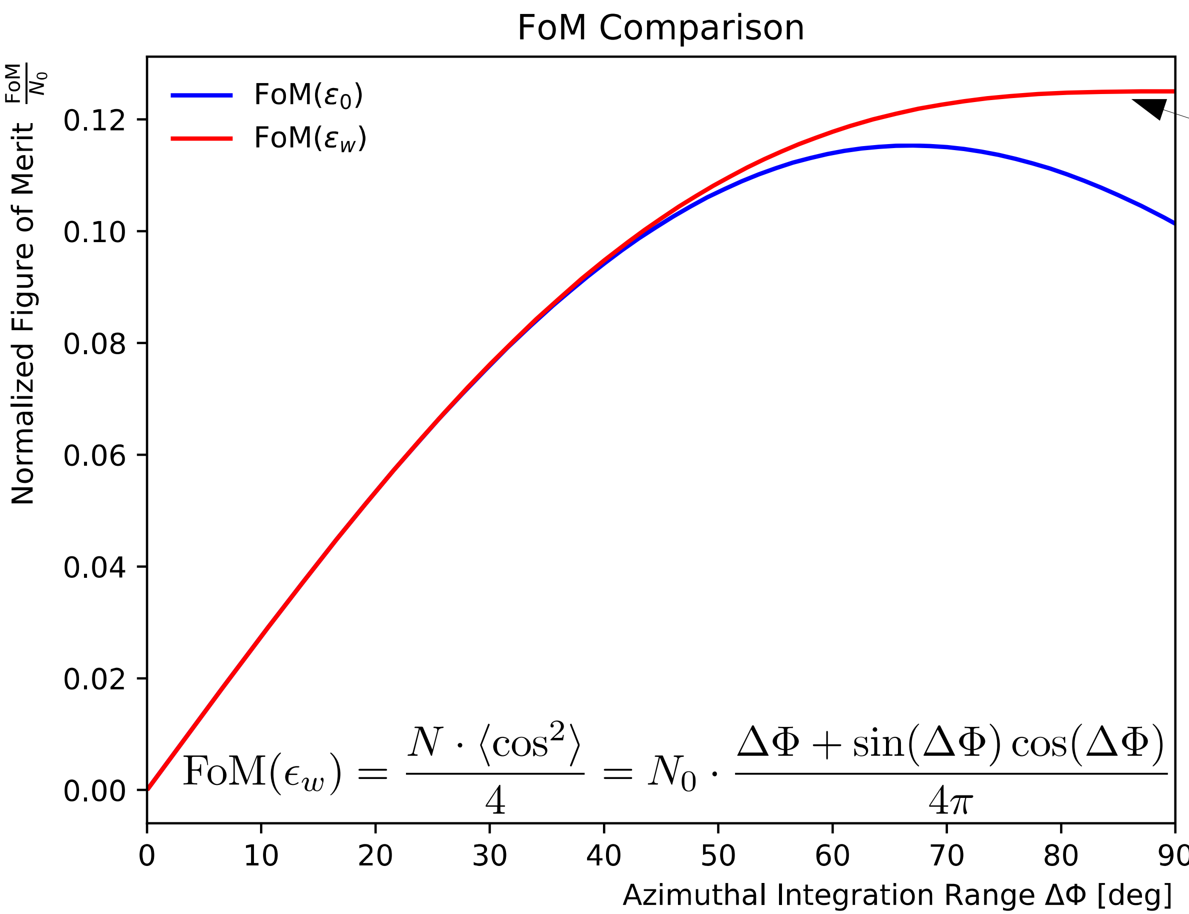
Cross ratio equations get modified:

$$\epsilon_{CR} = \frac{\langle \cos \rangle}{\langle \cos^2 \rangle} \frac{1-r}{1+r} \quad \text{with} \quad r^2 = \frac{\langle N^{\uparrow}_{L} \cos \rangle \langle N^{\downarrow}_{R} \cos \rangle}{\langle N^{\downarrow}_{L} \cos \rangle \langle N^{\uparrow}_{R} \cos \rangle}$$

$$\epsilon^{\downarrow,\uparrow}_{HCR} = \frac{\langle \cos \rangle}{\langle \cos^2 \rangle} \frac{1-r^{\downarrow,\uparrow}}{1+r^{\downarrow,\uparrow}} \quad \text{with} \quad r^{\downarrow,\uparrow} = \frac{\langle N^{\downarrow,\uparrow}_{R} \cos \rangle \langle N^{0}_{L} \cos \rangle}{\langle N^{\downarrow,\uparrow}_{L} \cos \rangle \langle N^{0}_{R} \cos \rangle}$$

GETTING ASYMMETRIES BY COUNTING

Weighting Method – Integration Range



Error keeps decreasing

Events with low asymmetry information content contribute less to the final result and can not dilute it anymore!

Can we do better?
→ Yes!

ASYMMETRIES FROM X²-FIT

Introduction / Motivation

Main problem with Cross Ratio Methods → acceptance assumed to be constant:

$$\alpha(\Phi) = \text{const.}$$

This is not necessarily the case! Lets try to model acceptance as a Fourier series:

$$\alpha(\Phi) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n \cdot \Phi) + b_n \sin(n \cdot \Phi)$$

Question: Can we find a model with parameters α_n, β_n and $\epsilon^{\uparrow,\downarrow}$ that can be determined by fitting this model to the measured data? → Yes!

ASYMMETRIES FROM X²-FIT

Extraction of Azimuthal Asymmetries Using Optimal Observables

Weights for model and data

n = 0, 1, 2

$$\left\langle \sum_i^{ev(\uparrow,\downarrow)} \cos(\Phi)^n \right\rangle = \frac{\mathcal{L}^{\uparrow,\downarrow}\sigma^0}{2\pi} \int_0^{2\pi} \cos(\Phi)^n \cdot$$

$$\left[a_0 + \sum_{k=1}^{\infty} a_k \cos(k\cdot\Phi) + b_k \sin(k\cdot\Phi)\right] \cdot$$

$$\left(1 + P_y^{\uparrow,\downarrow} A_y \cos(\Phi)\right) \partial\Phi$$

Integration over 2π cancel all parameters but $\alpha_0, \alpha_1, \alpha_2$ and α_3

Left/Right ratio is considered for, by the sign of $\cos(\Phi)$

Is known, a beam cycling through two polarization states can be used $\rightarrow A_y$
Is not known, additional unpolarized cycles are needed $\rightarrow \epsilon^{\uparrow}$and $\epsilon^{\downarrow}$

ASYMMETRIES FROM χ^2-FIT

Extraction of Azimuthal Asymmetries Using Optimal Observables

Data

$$y_{data} = \Big[N^{\uparrow}, \textstyle\sum_i^{\uparrow} \cos(\Phi_i), \sum_i^{\uparrow} \cos(\Phi_i)^2, N^{\downarrow}, \sum_i^{\downarrow} \cos(\Phi_i), \sum_i^{\downarrow} \cos(\Phi_i)^2\Big]$$

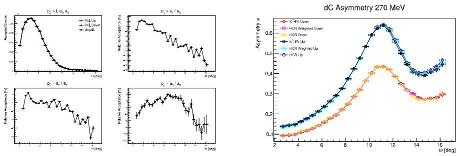
$$\mathbb{C} = \begin{bmatrix} C_{\uparrow} & 0 \\ 0 & C_{\downarrow} \end{bmatrix}$$ with 3x3 matrices $C_{\uparrow,\downarrow}$

Model

$$y_{model} = \overrightarrow{f}\left(\mathcal{L}^{\uparrow}\sigma^0\alpha_0, \mathcal{L}^{\downarrow}\sigma^0\alpha_0, \frac{\alpha_1}{\alpha_0}, \frac{\alpha_1}{\alpha_0}, \frac{\alpha_3}{\alpha_0}, A_y\right)$$

$$\chi^2 = \left(y_{data} - y_{model}\right)\mathbb{C}^{-1}\left(y_{data} - y_{model}\right)^T$$

χ^2 for each Θ -bin → ROOT : TMinuit →

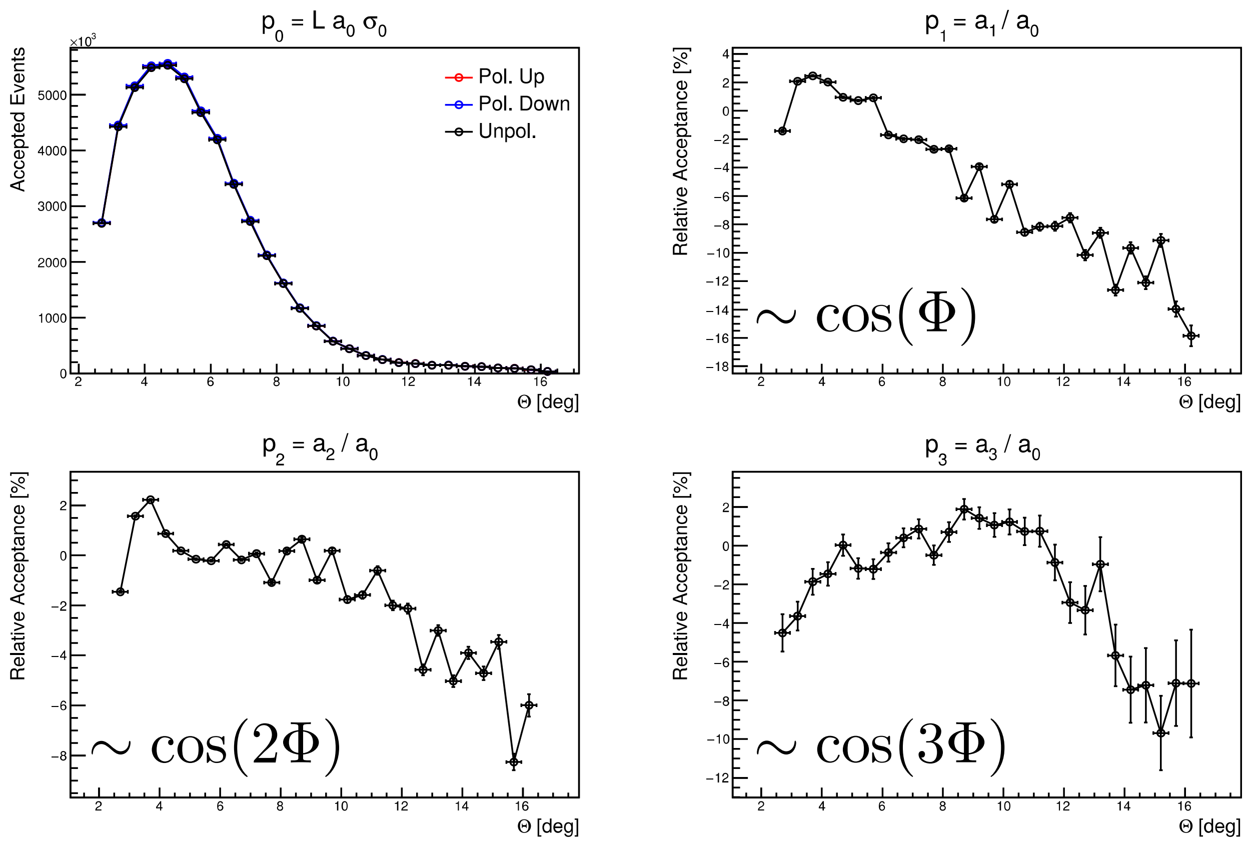


ASYMMETRIES FROM X²-FIT

Extraction of Azimuthal Asymmetries Using Optimal Observables

Proportional to Cross Sections

Acceptance is not flat at all!

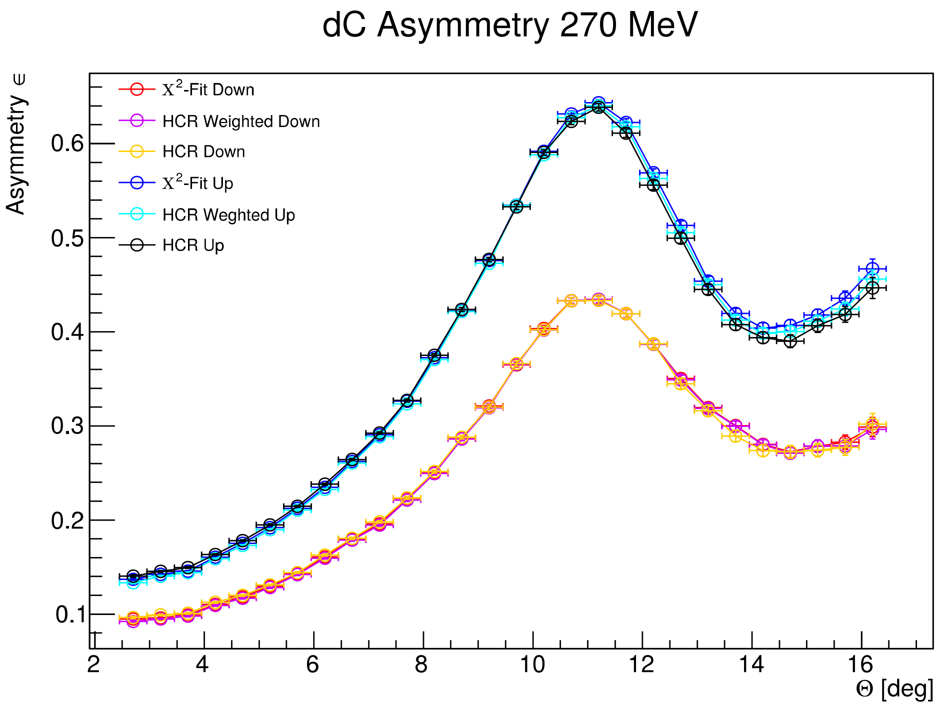


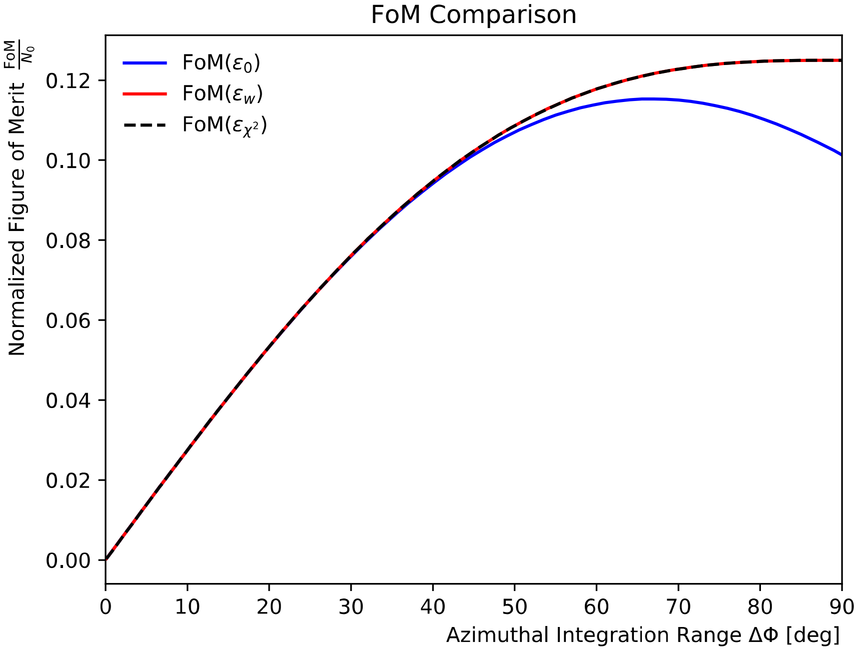
SUMMARY / CONCLUSION

Comparison of the Three Methods

Non-flat acceptance has an effect especially on larger angles

χ^2- fit method is able to maintain maximal FoM → Errors get minimized





SUMMARY / CONCLUSION

Summary:

- Asymmetries can be extracted using simple *Cross Ratio* method
- If the magnitude of the polarization states in not equal, it is better to use the *Half Cross Ratio* rather than the *Full Cross Ratio*
- Adding a $\cos(\Phi)$ weight to each event helps to avoid dilution of the asymmetry even when the full Φ- range of the detector is used → Errors get minimized
- Using the method of *Extraction of Azimuthal Asymmetries Using Optimal Observables* helps to overcome the limitations of the requirement of a flat acceptance for the Cross Ratio methods.

More detail on the calculation of FoM using different weights

Paper for this talk

References:

- Pretz, J. & Müller, F. Eur. Phys. J. C (2019) 79: 47. *Extraction of Azimuthal Asymmetries Using Optimal Observables*
- Jörg Pretz, Nucl.Instrum.Meth. A659 (2011) 456-461 arXiv:1104.1038 [physics.data-an] *Comparison of methods to extract an asymmetry parameter from data*