AN EXPERIMENT TO DETECT ELECTRON STERN-GERLACH DEFLECTION AND "REFUTE" A BOHR-PAULI CLAIM

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Abstract. A well-known, century-old, claim of Bohr and Pauli states that the Stern-Gerlach (S-G) experiment cannot be performed using electrons. We describe an experiment which, if successful, will "refute" this claim—the quotation marks here acknowledge that the B-P claim was predicated on technical capabilities of their day—they had no way of anticipating future technical advances. Furthermore, the most essential aspect of their claim is not disputed.

An apparatus capable of detecting and measuring Stern-Gerlach deflection of polarized electrons at Jefferson Lab is described. Quite inexpensive, it will use only well understood apparatus.

A more futuristic, "S-G resonant storage ring", based on the same principles, and intended for measuring electric dipole moments of the electron or the proton, is also outlined.

Introduction. This paper describes a first step of an ambitious plan culminating in the measurement of the electric dipole moment (EDM) of the proton. The first step amounts to establishing a "base camp" from which later steps can be staged. A following step will be to improve on the already impressively-small upper limit on the electron's EDM[1] by exploiting a storage ring as a "Penning-like trap" for "frozen spin" electrons. This will require[2][3] the non-destructive, phase-locked spin control that Stern-Gerlach polarimetry proposed in this paper promises to provide. The final step will be significantly more expensive, because the 234 MeV frozen spin proton kinetic energy is much higher than the 15 MeV electron frozen spin energy.

To be tested at Jefferson Lab is the performance of 2 cells of the short beam line section shown in Figure 1 and also (as insert) in Figure 3. With multiple passages, this S-G accumulating insertion will be especially effective in a frozen spin storage ring. The unit cell amounts to being the intersection region optics for a colliding beam storage ring. As a periodic structure the cell has phase advance of π, placing it on the boundary between stable and unstable. In the eventual storage ring application stability would have to be restored by weak focusing, limiting the resonant build-up to, for example, one thousand times the single passage value (with octupole vertical focusing to prevent actual beam loss).

This optics is especially insensitive to beam focusing but is especially sensitive to beam-steering, as is needed to detect S-G steering. This is brought about by locating the S-G quadrupole at a point focus (or, more correctly, at a beta function minimum in both transverse coordinates) with meter-long drift spaces, long enough for the S-G deflection to produce measureably-large transverse betatron displacement.

Here the experiment to demonstrate performance of the S-G insertion section is mainly emphasized, with futuristic EDM measurement capability outlined only to provide motivation. The test will use a 500 KeV polarized electron beam line at the TJNAF lab. Limited by single passage of any single linac electron through the device, extremely sensitive (though well within the state of the art) deflection detection will be needed.

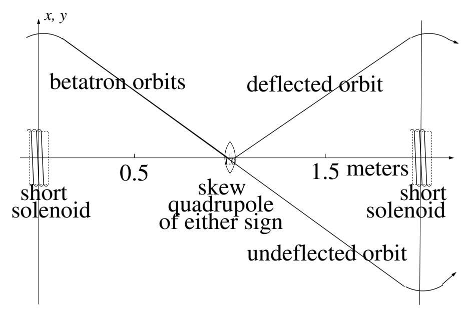

Figure 1: One cell of a two-cell Stern-Gerlach sensitive beamline to be tested at Jefferson Lab, for eventual polarimetry application in EDM storage rings. In geometric optics an ideal (zero-length) quadrupole situated at a point focus (here at the cell mid point) is "inert" and does not alter particle trajectories. However, physical optics and finite element length cause orbit deflections such as those shown (greatly exaggerated), but with acceptably small emittance growth for sufficiently paraxial orbits. As at colliding beam interaction points, the purpose of this configuration is to minimize the focusing effect of the quadrupole. The S-G quadrupole is referred to as "skew" only for the convenience of referring to the beam polarization as "up-down", rather than at 45 degrees.

The defining relations, in the electron rest frame, of electric and Stern-Gerlach forces are

$$\mathbf{F}^{\rm elec} = -e\mathbf{E}, \tag{1}$$

$$\mathbf{F}^{SG} = \frac{\mu_e^*}{c}\,(\hat{\mathbf{s}}^*\cdot\boldsymbol{\nabla})c\mathbf{B} = \mu_e^*\,\boldsymbol{\nabla}(\hat{\mathbf{s}}^*\cdot\mathbf{B}). \tag{2}$$

Here, $\hat{\mathbf{s}}^*$ is a unit spatial 3-vector specifying the orientation of the spin angular momentum in the rest frame. Experimentally, transverse $\hat{\mathbf{s}}^*$ is expected to produce a Stern-Gerlach signal, that will vanish with $\hat{\mathbf{s}}^*$ longitudinal—this dependence on the beam polarization state will provide the experimental evidence for S-G deflection.

Only short, longitudinally-uniform elements, for which $\hat{\mathbf{s}}^*$ can be treated as constant, will be considered. Then, since $\nabla \times \mathbf{B} = 0$, it is valid to move $\hat{\mathbf{s}}^*$ inside the ∇ operator in the second of Eqs. (2). Factors of c introduced, then cancelled, in Eq. (2), are an artifact of MKS units, the result of E and cB having the same units. To the extent that it is "natural" for E and cB magnitudes to be comparable, the ratio of Stern-Gerlach to electromagnetic force is determined by the ratio

$$\frac{\mu_B/c}{e} \equiv \frac{1}{4\pi}\,\frac{hc}{m_e c^2} = 1.930796 \times 10^{-13}\,\text{m}, \qquad (3)$$

where, except for sign and fractionally-small anomalous magnetic moment, Bohr magneton μ_B is the electron magnetic moment. This ratio (the Compton wavelength divided by 4π) has the dimension of length, needed to compensate the inverse length coming from the spatial derivative in Eq. (2).

The Stern-Gerlach experiment was highly influential in the development of quantum mechanics. It is curious, then, that Eq. (1) has been confirmed to high accuracy, while Eq. (2) has never been confirmed directly, to much better than the accuracy of the original Stern-Gerlach experiment[4]. The poor quality (even after a century) of experimental checks of Stern-Gerlach deflection can probably be ascribed to the smallness of ratio (3). The present paper is motivated partly by the desire to improve this experimental determination.

This paper is concerned with just a single aspect of the Stern-Gerlach phenomenon; namely spin-dependent particle deflection. Unlike the original S-G experiment, existence of this deflection does not imply any ability to separate electrons, based on their quantum mechanical spin states. In fact our analysis supports the claim that any S-G orbit shift (typically 1Å) will always be several orders of magnitude less than achievable electron beam sizes at the same location. The proposed experiment *starts* with a polarized beam, prepared upstream, which passes on-axis through a quadrupole representing the non-uniform magnetic field needed to cause S-G deflection.

The Bohr-Pauli claim proving the impossibility of replicating the original Stern-Gerlach experiment with electrons is clearly explained by Mott and Massey[5]. For the non-zero bunch width (required, in their day, by Heisenberg uncertainty principle and, even more so today, by minimum achievable emittance) the Lorentz force deflection of off-axis electrons overwhelms the Stern-Gerlach deflection of on-axis electrons. The argument applies to the deflection of single electrons, for their eventual downstream separation. It does not apply directly to the downstream centroid shift of an intense beam that has been pre-polarized upstream of the S-G deflecting magnet. Nowadays, with beam centroid shifts small compared to beam size being observed routinely (for example in stochastic cooling apparatus) the B-P argument has (in this sense) become outdated.

It is shown here that modern storage ring developments have made it possible to measure the extremely small orbit centroid shifts caused by the S-G deflection of a pre-polarized beam of electrons, thereby making Eq. (2) more precise.

Stern-Gerlach Deflection of a Relativistic Particle. We are primarily interested in the Stern-Gerlach deflection caused by the on-axis passage of an electron with velocity $v\hat{\mathbf{z}}$ and rest frame, transversely-polarized magnetic dipole moment vector $\mu_x^*\hat{\mathbf{x}}$ or $\mu_y^*\hat{\mathbf{y}}$, through a DC quadrupole, of (short) length L_q, that is stationary in the laboratory frame.

As derived in reference [6], Stern-Gerlach deflections in a quadrupole are strictly proportional to the (equal in magnitude, opposite in sign) inverse focal lengths q_x and q_y of the quadrupole;

$$\Delta\theta_x^{SG} = -\frac{\mu_x^*}{ec\beta} q_x, \quad \text{and} \quad \Delta\theta_y^{SG} = \frac{\mu_y^*}{ec\beta} q_y. \qquad (4)$$

The historical Stern-Gerlach apparatus: used a uniform magnetic field (to orient the spins) with quadrupole magnetic field superimposed (to deflect opposite spins oppositely) and a neutral, somewhat mono-energetic, unpolarized, neutral atomic beam of spin 1/2 silver atoms. For highly-monochromatic, already-polarized beams produced from an electron gun cathode, the uniform magnetic field has become superfluous, and a quadrupole produces polarization-dependent S-G deflection. The absence of constant magnetic field on the unperturbed electron orbit, along a quadrupole axis, has the further effect of guaranteeing zero electric field at the electron's instantaneous position in its own (unperturbed) rest frame. (Unlike in the original Stern-Gerlach configuration) any non-zero rest frame electric field can be neglected.

Starting from neutral silver atoms, with approximate velocity 500 m/s in the original experiment, for which the angular deflections were roughly $\Delta\theta^{Ag} \approx 0.002$ radians, we can estimate the Stern-Gerlach deflections of 500 KeV electrons in a quadrupole of a modern-day accelerator to be in the Å ngstrom range. Modern magnets resemble the original (1923) Stern-Gerlach magnets, though the original magnetic field gradient$\times$length product was obsessively larger[4] than for typical quadrupoles in the CEBAF injection line[1].

Proposed Detection Apparatus. Dual CEBAF electron sources produce superimposed 0.25 GHz (bunch separation 4 ns) electron beams for which the polarization states and the arrival times can be adjusted independently. For example, the (linear) polarizations can be opposite and the arrival times adjusted so that (once superimposed) the bunch spacings are 2 ns and the bunch polarizations alternate between plus and minus. The effect of this beam preparation is to produce a bunch charge repetition frequency of 0.5 GHz different from the bunch polarization frequency of 0.25 GHz. This difference will make it possible to distinguish Stern-Gerlach-induced bunch deflections from spurious charge-induced deflections.

[1]Some parameters for the original Stern-Gerlach experiment were: central field 0.1 T, peak field gradient 100 T/m, $T = 1350°$K, magnet length 0.035 m, distance from magnet center to detecting film 0.02 m.

Transverse bunch displacements produce narrow band BPM signals proportional to the f_r Fourier frequency components of transverse beam centroid displacement. Because linac bunches are short there can be significant resonator response at numerous strong low order harmonics of the 0.25 GHz bunch polarization frequency. The proposed S-G responses are centered at odd harmonics, $f_r = 0.25, 0.75, 1.25\,$GHz. The absence of beam-induced detector response at these frequencies greatly improves the rejection of spurious "background" bunch displacement correlated with bunch charge. For further background rejection the polarization amplitudes are modulated at a low, kHz, frequency, which shifts the S-G response to sidebands of the central S-G frequencies.

S-G Specific Beam Preparation. The smallness of the S-G signal, especially relative to spurious charge-sensitive cavity responses, makes it critical for the polarized beam to be prepared for maximum rejection of spurious background.

In a ring the polarization of each bunch can be altered before its next passage; in a linac each bunch passes an S-G sensitive BPM only once. But, as already explained, by superposing staggered bunch trains having different polarizations the beam polarization can change at high frequency. Figure 2 illustrates our planned, superimposed bunch train. Bunches are labeled A in one of two pre-superimposed bunch trains and B in the other. Time domain plots are on the left, frequency spectra on the right. The *foreground* S-G betatron signal oscillates at (odd harmonics of) 0.25 GHz, while the *background* charge signal oscillates at (harmonics of) 0.5 GHz.

The superimposed A and B beams are also modulated with ≈kHz frequency ω_m. The time domain, $i\,p(t)$ current-polarization products of the separate A and B beams are plotted on the left in Figure 2. There are two essential differences between the A and B beams. The beam pulses are shifted in time by one half cycle and the sign of the modulation is reversed. The (low) modulation frequency ω_m is exaggerated by many orders of magnitude in this figure. The frequency spectra, shown on the right in Figure 2, are derived in reference [6].

Beam Line Optics. The beamline layout is shown in Figure 1 and in the insert of Figure 3, which shows how, in a circular ring, each particle passes repeatedly through the same S-G magnifying section, allowing the resonant accumulation of betatron amplitude. A fuller discussion of the figure would explain how apparatus like this can act as a "Penning-like trap", resembling the "geonium" apparatus used by Gabrielse and others[7] to measure the electron's magnetic moment with orders of magnitude higher precision than any other fundamental constant of nature.[2]

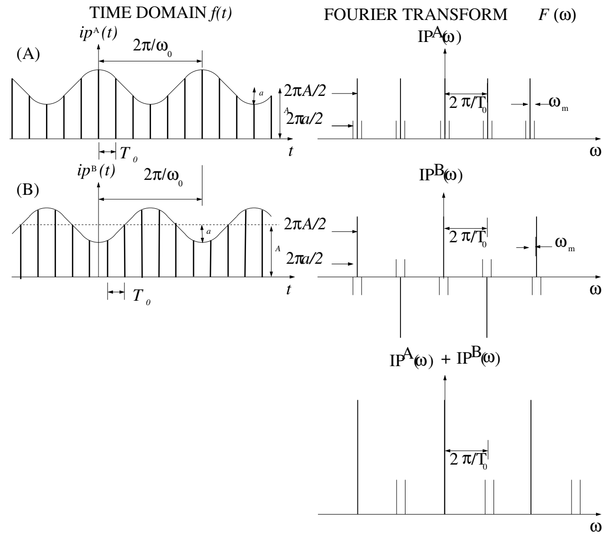

Figure 2: Time domain and frequency domain beam pulses for the A and B staggered, modulated-polarization beams. It is current-weighted polarization spectra that are plotted in these figures. The current spectra itself can be obtained by suppressing the modulation sidebands in the upper right hand figure. In the A+B spectra the odd harmonics of beam current cancel, effectively doubling the fundamental current frequency from 0.25 GHz to 0.5 GHz. But the current-weighted polarization side bands survive as odd harmonics of 0.25 GHz.

The storage ring Penning-like trap is quantum-mechanically quite similar to the geonium trap, with the (important) exception that the zero velocity limit is excluded. Though this prevents the study of quantum-mechanical effects, it makes a semi-classical treatment valid in all degrees of freedom. The large number of stored electrons then makes it feasible to externally monitor their mean polarization non-destructively. Discussion in the present paper is limited to single passage in the linac test.

To assure the design is practically achievable, the strong S-G quadrupole is copied from permanent magnet quadrupoles described in Table III of a paper by Li and Musumeci[8]. The magnetic field gradient is 440 T/m, and the quad length is $L_Q = 4.8\,$cm.

Signal Levels and Noise Suppression. The anticipated initial reaction, especially by accelerator experts, is that Ångstrom-level betatron oscillations are unmeasureably small. For the J-Lab test to succeed this adverse reaction has to be refuted. Based on common experience, we

[2]For various reasons the configuration shown in Figure 3 would not be directly applicable to measuring the electron EDM. Because solenoidal fields would provide unacceptably large systematic error effects, they would have to be replaced by quadrupole doublets. Furthermore the S-G apertures would be too small, and larger apertures with their weaker gradients would give reduced S-G deflection. Nevertheless, radiation damping is so weak that SG-correlated betatron oscillations present during injection persist indefinitely, thereby "remembering" the spin state while the betatron/spin correlation is being resonantly refreshed to a steady state.

Any particle EDM will then be made manifest by the difference of forward and backward "Koop wheel" rotation frequency[2]. The most important source of systematic error is suppressed by "self-magnetometry"[3].

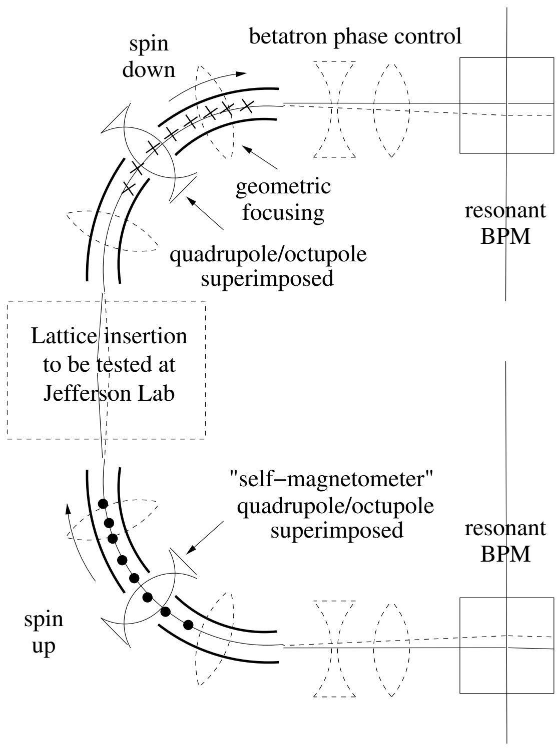

Figure 3: Cartoon of all-electric frozen spin ring (or "Penning-like trap") for storing alternating-polarization bunches of 15 MeV electrons, in preparation for measuring the electron EDM[2][3]. The lattice insertion is to be copied from Figure 1. The sector bends are provided by DC electric fields between the cylindrical electrodes shown. The broken and solid lines in orbits through the resonant BPM's represent vertical displacement (displayed as if horizontal) of spin-up and spin-down bunches. In the frequency domain, alternating bunch polarization separates the S-G signal from spurious direct charge signals, as in the Jefferson Lab test.

assume the best choice is to use resonant beam position monitors (BPMs). Quite good documentation of achievable beam positioning accuracy exists. Data on achievable betatron amplitude detection sensitivity is more sparse. We (conservatively) assume that the minimum detectable betatron oscillation amplitude is not greater than the rms beam position reproducibility—it is the vanishing of the averaged BPM signal that establishes the r.m.s. beam position uncertainty. Surely the response at the BPM resonant frequency, to individual bunch passage at this displacement, exceeds this uncertainty.

Resonant beam position detection relies on two TM cavities. A charge-sensitive cavity (needed to normalize the charge) is tuned to resonate in a transversely symmetric mode at the bunch frequency. In standard beam positioning operation the anti-symmetric position sensitive mode of the position-sensitive cavity is tuned to the same frequency. To achieve the required extra selectivity our configuration separates these two frequencies.

There are three essentially-different effects limiting the detectability of our small transverse S-G displacements. The most fundamental, assuming that everything else behaves perfectly, is for the signal power induced in the position-sensitive cavity by the Å ngstrom scale amplitude betatron oscillation to be larger than the inherent thermal noise "floor". This noise floor could, if necessary, be reduced by some three orders of magnitude, by using liquid Helium temperature apparatus, but we have strived for a beam line design making such an extreme measure unnecessary.

Pusch et al.[9] report that beam positioning in their optimized resonant BPM fails at the 0.1 mm level for a 250 picoampere beam. The J-Lab current can be 250 microampere, a million times greater. The off-axis shunt impedance of a resonant cavity is proportional to the square of the (beam-current$\times$beam-displacement) product. By this estimate, the resonator excitation at 1 Å will be close to the noise floor. Multi-second runs will be necessary to detect betatron displacement of sub-Å ngstrom magnitude. The S-G displacement predicted for the beamline shown in Figure 1 is approximately 10 Å.

Secondly, in practice, other unknown noise sources are typically more important than thermal noise. The same Pusch et al. limitation just mentioned was actually caused by unknown noise sources. These noise sources will not necessarily be reduced by long time averaging. They should, however, be reduced by the beam processing described next.

Recent International Collider (ILC)-motivated BPM performance investigations are relevant to our proposed S-G detection experiment. Design studies have shown that the ± 20 Å beam position pulse-to-pulse reproduceability planned for effective ILC operation will be achievable. We need better position resolution than this by a factor of about one hundred.

A CEBAF beam is CW, with average current about five orders of magnitude higher than for the BPM test at the ATF Test Facility at the KEK laboratory [10][11]. Averaging over longer times can reduce some noise sources. For these, the increased average beam current can improve the signal to noise by the square root of the current ratio. Also the ILC cavity discharging time is far shorter than the ATF repetition period, which makes it necessary to treat their BPM resonant response on a pulse-by-pulse basis. The high bunch frequency permits (statistically superior) CW treatment in our linac beam.

Rejection of Spurious Charge-Induced Excitation. The third, and by far the most serious, impediment to S-G detection is spurious cavity response to bunch charge rather than bunch polarization. We assume this is the dominant limitation on the minimum detectable betatron amplitude. The beam preparation to provide the extra rejection of this source of spurious betatron excitation has already been described. We now estimate the improved selectivity this provides.

The polarized beam has been tailored so that the bunch polarization and bunch charge frequencies are different. In this condition the BPM cavity is sensitive to polarization at one frequency (0.75 GHz) and to charge at a different frequency (say 0.5 GHz). Ideally, the resulting frequency domain filtering will suppress the spurious background response proportional to the inverse cavity Q-factor, or about 10^{-4}. More realistically, there will still be background response, for example due to the small Fourier component of charge excitation due to not-quite-cancelling beam A and beam B currents. Empirical beam steering to null "common mode" BPM responses at both even and odd harmonics of 0.25 MHz (which would all vanish for ideal beam preparation) is especially useful for rejecting this spurious background excitation. One can expect significant background/foreground suppression from these measures—perhaps three orders of magnitude.

The effect of low frequency modulation of the beam polarizations is to shift the S-G response to sidebands of the central cavity resonance. To the extent the beam currents are unaffected by this modulation, the sideband response will provide a pure S-G signal. In practice the beam currents will, in fact, also be weakly modulated which will allow some background signal to leak out to the side-band frequencies. Still one can expect significant background/foreground suppression—perhaps two orders of magnitude.

It is also true that the resonator responses will be coherent with the beam bunch frequency. By phase-sensitive and lock-in detection, the in-phase and out-of-phase S-G sideband deflections can be determined individually. This is where long term averaging is especially helpful. Perhaps another order of magnitude selectivity improvement can be achieved.

After all these measures have been taken, we expect the leading spurious signal sources to derive from initial conditions at the electron gun cathode. Different A beam and B beam initial betatron amplitudes correlated with the low frequency modulation can produce signals indistinguishable from S-G deflection. Such signals can be detected and suppressed upstream of the S-G apparatus. Furthermore, being independent of beam polarization, they can be nulled using unpolarized A and B beams.

The possibility of many orders of magnitude rejection of spurious background has been described. This seems conservatively greater than the factor of one hundred needed, compared to current BPM capabilities. We are confident, therefore, that the Stern-Gerlach signal will be readily visible in the Jefferson Lab test.

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