ASYMMETRIC ENERGY COLLIDING ION BEAMS IN THE EDM STORAGE RING

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Abstract

In this paper we discuss a possibility to collide co- or counter rotating ion beams, which are circulating in the storage ring with crossed electric and magnetic bending fields. One beam is polarized and its electric dipole moment (EDM) is aimed to be measured by the frozen spin method. The second beam, which has different from the first one the Lorentz factor β and charge to mass ratio Z/A will be used as comagnetometer sensitive to the radial component B_x of the magnetic field. Measuring by pickups the relative vertical orbit shift of two beams, caused by the presence of B_x field, one could cancel the unwanted MDM spin precession, which mimics the EDM signal. By proper choice of the bending electric and magnetic field values one could make rational the ratio of the revolution frequencies of two beams. Examples of parameters for proton-, deuteron-, hellion- and some other ion species EDM storage rings are presented. Finally, a new method of increasing of the spin coherence time (SCT), so called "Spin Wheel", is proposed and its applicability to the EDM search is discussed.

INTRODUCTION

In the frozen spin method [1, 2] the spin precession due to MDM (Magnetic Dipole Moment) is cancel by proper choice of the ratio of the radial electric field E_x to the vertical component B_z of the magnetic field:

$$\frac{E_x}{B_z} = \frac{a}{\beta\left(\frac{1}{\gamma^2-1} - a\right)} \qquad (1)$$

Here a is the particle's anomalous magnetic moment, defined according to:

$$a = \frac{A}{Z}\frac{\mu}{2J} - 1 \qquad (2)$$

where A, Z, μ, J are the nucleus mass number, charge number, full magnetic moment and its spin respectively.

Provided the requirement (1) is fulfilled for the polarized beam, one would like store the second unpolarized beam in the same ring with the same orbit but with different Lorentz factor: $\beta_2 \neq \beta_1$. This beam will become shifted vertically differently with respect to the first one due to difference in their velocities. Measuring small difference in the vertical orbit of two beams one would control the radial component of the magnetic field, which contributes to the spin precession similarly to the EDM:

$$\langle \Omega_x \rangle = -\frac{Z}{Am_p}\frac{1+a}{\gamma^2}\left\{\langle B_x \rangle + \frac{\eta}{a}\langle E_x \rangle\right\} \qquad (3)$$

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In this expression the first term describes precession due to MDM, while the second one presents the EDM effect. That is proportional to the dimensionless parameter η, which is of the order $\eta \approx 10^{-15}$ for $d \approx 10^{-29} e \cdot cm$ [2]. We see, that for proton beam the net radial magnetic field of $B_x \geq 0.15\, pG$ will cause precession equivalent to the EDM one, assuming some realistic electric field value: $E_x = 100 \ \ kV/cm \approx 0.0333\, T$ [3]. Such small magnetic field could be detected via two beams vertical orbit difference, only.

The aim of this paper is to show that for nuclei listed in the Table 1 there are few partner nuclei, which could be used as comagnetometers. These unpolarized beams are co- or counter rotating respect to the first one and its circulation frequency is adjusted to be a rational fraction of the revolution frequency of the polarized beam.

Table 1: Magnetic moments of some light nuclei.

Nuclei	m, GeV/c^2	J	μ	a
p	0.938272	1/2	2.792847351	1.792847
d	1.8756123	1	0.8574376	-0.142988
Li6	5.601518	1	0.8220473	-0.182058
Li7	6.533833	3/2	3.2564268	1.519638
He3	2.808391	1/2	-2.12762485	-4.191437
C13	12.10948	1/2	0.7024118	0.510906

ASYMMETRIC VELOCITIES EDM COLLIDER

The vertical component of the velocity precession angular frequency defines the bending radius ρ of the trajectory, while the radial one cancels in average:

$$\left(\Omega_V\right)_z = -\frac{Z}{\gamma Am_p}\left(B_z + \frac{E_x}{\beta}\right) \equiv \frac{\beta c}{\rho} \qquad (4)$$

$$\langle \Omega_V \rangle_x = -\frac{Z}{\gamma Am_p}\left(\langle B_x \rangle - \frac{\langle E_z \rangle}{\beta}\right) \equiv 0 \qquad (5)$$

Looking on (4) one can see that two beams with different velocities β_1, β_2 could be brought onto the orbit of same radius ρ. Moreover, their velocities could be adjusted to be in rational ratio:

$$\frac{\beta_2}{\beta_1} = \pm\frac{m}{n} \quad \text{with} \quad m, n - \text{integer} \qquad (6)$$

Now, let's assume that vertical focusing is weak and is provided by the constant electric field gradient only, i.e. $\langle E_z \rangle = \langle G_E \rangle \cdot z$. Then from (5) follows:

$$z_{1,2} = \beta_{1,2} \langle B_x \rangle \langle G_E \rangle^{-1} \quad (7)$$

Last equation manifests that the vertical orbit shift due to presence on the orbit of non-zero radial magnetic field is different for ions with different velocities. This paves the way to control this unwanted radial magnetic field component by controlling the vertical orbits difference using ultra-sensitive pickups. We do not intend to discuss here the physical and technical requirements to beam diagnostics, for details see [2, 3].

Obviously, to maximize splitting of vertical orbits one would prefer to have counter-rotating beams, i.e. preferable is the negative sign of β_2. Still, this is not possible in case with the deuteron or Li6 beams, because of large magnetic field which is needed to freeze their spin precession. The last is a consequence of their small anomalous magnetic moments. With large B_z the solutions with negative partner's ion beam velocity β_2 do not exist.

In the Tables 2-7 we present examples of storage ring parameters calculated on the basis of solutions of the equations (4) found for two beams with equal bending radii ρ and taking into account the condition (1) valid for the first one, which EDM is intended to be measured. In all cases the first beam is assumed circulated counter-clockwise and $E \equiv E_x = -100\ kV/cm \approx -0.0333\,T$ except of Li6 and deuteron cases for which E is reversed. The vertical component of the magnetic field B is always negative. The last column in the Tables presents the value of the Lorentz force which defines the rate of a spin precession due to EDM [1, 2]:

$$\langle \Omega_{EDM} \rangle_x = -\frac{Z}{Am_p} \eta \langle E_x + \beta B_z \rangle \quad (8)$$

Table 2: Proton-ion storage ring parameters.

β1, proton	Ion2	β2/ β1	ρ (m)	B (T)	E+ β1·B (T)
0.59838	p	-1/1	41.960	0	-0.0333
0.42862	p	-4/5	15.014	-0.0211	-0.0424
0.36368	p	-3/4	9.863	-0.0322	-0.0451
0.18283	**p**	**-2/3**	**2.120**	**-0.0922**	**-0.0503**
0.17640	**d**	**-1/2**	**1.966**	**-0.0962**	**-0.0502**

Obviously, the nuclei with smaller anomalous magnetic moment (d, Li6) are subjected to larger centripetal force and, hence, their spin precession due to EDM is faster.

For most of the nuclei (p, d, Li6, He3) we have found solutions with the minimal bending radius of 2-3 meters. Only for Li7 and C13 the minimal ρ is increased to 5 m.

Best partner nuclei (see Ion2 columns) are found: proton or deuteron for proton; proton for deuteron, Li6 and He3; He4 for Li7; He3 for C13. In some cases the ratio (6) of the velocities β_2/β_1 and, hence, of the revolution frequencies is integer or half-integer, then single RF-system could be used to bunch both beams. In other cases two RF-systems, each transparent for the opposite partner beam, are needed.

Table 3: Deuteron-proton storage ring parameters.

β1, d	β2, p	β2/β1	ρ (m)	B (T)	E+ β1·B (T)
0.6644	0.8859	4/3	33.12	-0.2181	-0.1116
0.5405	0.8108	3/2	15.36	-0.3233	-0.1414
0.4313	0.7187	5/3	7.93	-0.4544	-0.1626
0.2209	**0.4418**	**2/1**	**1.65**	**-1.0111**	**-0.1900**

Table 4: Li6-proton EDM storage ring parameters.

β1, Li6	β2, p	β2/β1	ρ	B	E+ β1·B
0.6688	0.8859	4/3	45.27	-0.1736	-0.0828
0.5476	0.8215	3/2	21.29	-0.2523	-0.1048
0.4419	0.7365	5/3	11.25	-0.3482	-0.1205
0.2456	**0.4911**	**2/1**	**2.75**	**-0.7088**	**-0.1407**

Table 5: Helion-ion EDM storage ring parameters.

β1, He3	Ion2	β2/ β1	ρ	B	E+ β1·B
0.29764	He3	-7/5	18.79	-0.0343	-0.0231
0.16716	d	-8/7	5.381	-0.0518	-0.0247
0.15424	He4	-8/7	4.552	-0.0555	-0.0248
0.13909	He3	-4/3	3.676	-0.0607	-0.0249
0.10538	**p**	**-5/3**	**2.084**	**-0.0781**	**-0.0251**

Table 6: Li7-ion EDM storage ring parameters.

β1, Li7	Ion2	β2/β1	ρ	B	E+ β1·B
0.63	Li7	-1/1	111.4	0	-0.0333
0.3962	p	-1/1	26.65	-0.0335	-0.0466
0.2934	Li7	-2/3	12.96	-0.0585	-0.0505
0.1905	**He4**	**-2/3**	**5.044**	**-0.1046**	**-0.0533**

Table 7: C13-ion EDM storage ring parameters.

β1, C13	Ion2	β2/β1	ρ	B	E+ β1·B
0.8135	C13	-1/1	230	0	-0.0333
0.542	C13	-1/2	33.8	-0.067	-0.0696
0.3734	C13	-2/5	11.9	-0.1379	-0.0848
0.2611	**He3**	**-2/5**	**5.177**	**-0.2241**	**-0.0918**

Best cases in the Tables 2-7 are marked in bold.

SPIN WHEEL APPROACH

In the frozen spin method (Y.Semertzidis) beam's polarization is directed almost longitudinally. Then small change of its vertical component, which is due to EDM, is recorded. We propose here an approach, which is some modification of the classic frozen spin method.

Idea of the Spin Wheel method is to apply a relatively strong radial magnetic field B_x to provide fast spin rotation in the vertical plane, say about $0.1-1\,Hz$ instead of $10^{-9}\,Hz$, as in the Frozen Spin scenario. If we are able to control with the required accuracy the accompanied beams orbit splitting, then we can extract the EDM contribution to a measured spin precession rate just comparing runs with positive and negative orbit separation:

$$\Omega_{EDM}=\frac{\Omega_x\left(B_x\right)+\Omega_x\left(-B_x\right)}{2} \qquad (9)$$

Large B_x- field is needed to suppress all orthogonal contributions to full precession rate Ω, which are generated, for instance, by poorly controlled longitudinal magnetic field or by small mismatch from the magic energy. With large Ω_x compared to Ω_y, Ω_z the EDM contributes linearly to full precession frequency Ω and we can expand:

$$\Omega=\sqrt{\left(\Omega_x+\Omega_{EDM}\right)^2+\Omega_y{}^2+\Omega_z{}^2}\approx \\ \approx\Omega_x+\Omega_{EDM}+\left(\Omega_y{}^2+\Omega_z{}^2\right)/\,2\Omega_x \qquad (10)$$

Here $\Omega_z$stands for the precession frequency deviation from the wanted zero value. In general:

$$\Omega_z=c1\cdot a_x^2+c2\cdot a_y^2+c3\cdot a_z^2 \qquad (11)$$

where $c1, c2, c3$ are coefficients, which describe dependence of Ω_z on three amplitudes: $a_{x,y,z}$. These coefficients could be minimized to some extent by adjusting the strengths of 2-3 families of sextupoles (VEPP-2M and COSY experiences). Substantial reduction of the spin tune spread could be achieved with $\Omega_x \gg \Omega_z$ ($\Omega_y=0$, for simplicity). Then Ω become less sensitive to Ω_z and the SCT will become increased about $2\Omega_x/\Omega_z$ times. Spins will rotate each around own $\vec{\Omega}$ direction, but all with almost the same frequency. The gain in the spin coherence time could be many orders of magnitude! This is illustrated in the Figure 1.

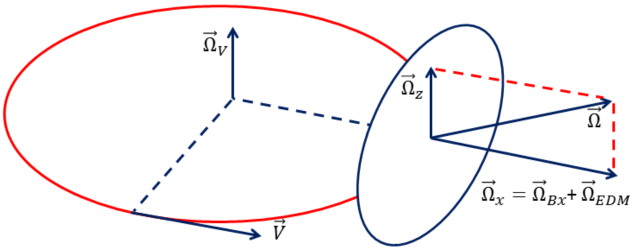

Figure 1: Spin Wheel: spins rotate around the vector sum $\vec{\Omega}$, almost in the vertical plane, if $|\vec{\Omega}_x| \gg |\vec{\Omega}_z|$.

STATISTIC CONSIDERATIONS

Presumably one of the polarization components could be measured using SQUID magnetometers [4]. Let sigma of a noise in units of the signal amplitude is σ_{NS}. Then sigma of the signal frequency determination is equal to:

$$\sigma_\Omega=\sigma_{NS}\frac{\sqrt{24}}{N^{3/2}}\ radian\,/\,turn \qquad (12)$$

Two important things follow from this formula. The first one: independence of σ_Ω on Ω ! This means that Ω could be as small as it is in the frozen spin method: $\Omega=6\cdot 10^{-15}\,s^{-1}$; or as large as about: $\Omega=0.6\,s^{-1}$, as it could be in the spin wheel approach. Accuracy of the precession frequency measurement will be the same! The second feature: high-scaling power law $N^{-3/2}$ for the accuracy of the frequency determination.

Now let see two numerical examples. Let the coherence time is only 1000 s. In one year we would like to reach $\sigma_\Omega=6\cdot 10^{-15}\,s^{-1}$, then in one run $\sigma_\Omega=6\cdot 10^{-13}\,s^{-1}$. Assuming $N=10^9$ turns/run we can reach the required accuracy if noise to signal ratio will not exceed $\sigma_{NS}\leq 4$. But assuming now $N=10^{10}$ turns/run (the coherence time 10000 s) we can reach the required accuracy with 10 times larger ratio of noise to signal $\sigma_{NS}\leq 40$! In reality the signal will not be sinusoidal and processing of a signal will be not as simple. But in any case, it is very helpful to have the spin coherence time as longer as possible.

CONCLUSION

The discussed above approach, based on idea of asymmetric velocities ion-ion colliding beams, opens a possibility to measure EDM of many nuclei. Most important cases are: p, d, $He3$ and $Li6$. Use of crossed electric and magnetic fields helps to reduce the size of a ring by factor 10 to 20. It was shown that bending radius of such all in one EDM storage ring could be made of about 2-3 meters.

The proposed Spin Wheel approach opens a way to increase the spin coherence time by many orders of magnitude.

REFERENCES

[1] F.J.M. Farley et al., Phys. Rev.Lett. 93, 052001(2004). See also AGS proposal: Search for a permanent electric dipole moment of the deuteron nucleus at 10^{-29} e·cm level, April 2008, available at http://www.bnl.gov/edm/

[2] V. Anastassopoulos et al., AGS proposal: A proposal to measure the proton electric dipole moment with 10^{-29} e·cm sensitivity by the storage ring EDM collaboration, June 2011, available at http://www.bnl.gov/edm/

[3] D. Kawall, "Beam Position Monitors with SQUIDs for the pEDM Experiment", Report at 485 WE-Heraeus EDM Seminar, Bad Honnef, 2011.

[4] P.R.Cameron et al., "Squids, snakes and polarimeters: a new Technique for measuring the magnetic moments of polarized beams", 1996. Published in AIP Conf.Proc. 390 (1997) 306-315.