LASER COMPTON POLARIMETRY OF PROTON BEAMS

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Abstract

A need exists for non-destructive polarization measurements of the polarized proton beams in the AGS and, in the future, in RHIC. One way to make such measurements is to scatter photons from the polarized beams. Until now, such measurements were impossible because of the extremely low Compton scattering cross section from protons. Modern lasers now can provide enough photons per laser pulse not only to scatter from proton beams but also, at least in RHIC, to analyze their polarization.

I. THEORY

Compton scattering, the elastic scattering of photons from charged particles, has proven useful in polarimeters for electron beams. Laser polarimeters are common at the worlds $e^{\pm}$ storage rings.[1], [2], [3]. In this method, a laser provides circularly polarized photons which then scatter from the polarized electrons or positrons. Spin-dependent terms in the scattering cross-section cause an asymmetry in the rate of back-scattered photons. This assymetry depends on the relative orientations of the incoming photon's wave vector and the electron's spin, and so provides the signal proportional to the beam polarization. It would be of great benefit to extend this utility to polarized proton beams, but the cross-section for Compton scattering from protons is small. For polarimetry, it is smaller than the analogous electron cross section by a factor of $(m_e/m_p)^3$, where m_e and m_p are the masses of the electron and proton, respectively. Modern lasers, with their high power densities and pulse energies, stand a chance of making up this difference in rate.

For polarized laser light scattering from polarized electron beams (following the development in [3]), the cross section is [4]

$$d\sigma = \frac{1}{2}r_e^2 \left(\frac{\omega'}{\omega'_0}\right)^2 (\Phi_0 + \Phi_1 + \Phi_2),\qquad(1)$$

where

$$\Phi_0 = (1 + \cos^2\theta') + \frac{1}{m_e}(\omega'_0 - \omega')(1 - \cos\theta')$$

$$\Phi_1 = (\xi_1\cos 2\phi' + \xi_2\sin 2\phi')sin^2\theta'$$

$$\Phi_2 = -\xi_3\frac{1}{m_e}(1 - \cos\theta')\vec{\zeta}\cdot(\vec{k'}_0\cos\theta' + \vec{k'}).$$

Here, $\vec{k'_0}$, $\vec{k'}$ and ω'_0, ω' are the wave vectors and frequencies of the initial and final photons, $\vec{\zeta}$ is the electron spin vector, θ' is the scattering angle of the outgoing photon, and ϕ' is the polar angle of the scattered photon with respect to the incoming photon. The primes indicate electron rest frame values, and all expressions are in natural units, so that $\hbar = c = 1$ and $e^2 \approx 1/137$. The laser polarization vector is $\vec{\xi}$, a vector whose three components

are the Stokes parameters ξ_1, ξ_2, ξ_3. Note that the polarization vector is independent of the spatial axes. This distinction causes the terms that appear in the cross section which are products of a Stokes parameter and a particle spin to be non-invariant. They depend on the spatial orientation of the laser polarization vector. That is, these terms depend on the choice of the scattering plane. When $\xi_3 = \pm 1$, the laser is right or left circularly polarized; ξ_1 and ξ_2 determine the linear polarization. Eq.1 shows that only circularly polarized photons enter into the spin-dependent scattering.

II. The Scattering Rate

To determine the cross section for scattering by protons, it is sufficient merely to substitute m_p for m_e in eq.1. The magnitude of the Compton scattering from protons is much weaker than the scattering from electrons. Indeed, it is lower by a factor of $(m_e/m_p)^3$ since $r_e = e^2/m_e \rightarrow e^2/m_p$ and the term dependent on circular polarization involves a power of $1/m_p$. This is a large reduction, on the order of 10^{-10}. Getting useful statistics from the scattering requires a laser with high energy in a pulse on the order of a bunch length. Such a pulse length, (e.g. 50 ns) is long by modern laser standards, and the energy and power densities necessary are also within reason.

The flux of photons striking a detector due to spin dependent scattering is

$$\dot{n}_\gamma = \mathcal{L}d\sigma(\vec{\zeta}, \vec{\xi})\qquad(2)$$

where $\mathcal{L}$ is the luminosity of the proton-photon interaction area and $d\sigma$ includes only terms which depend on the product of particle spin, $\vec{\xi}$ and laser polarization, $\vec{\zeta}$. It is important to remember that this flux is in addition to the flux of photons from the unpolarized scattering, and that subtracting the large unpolarized background is necessary for successful polarization measurements. The luminosity of the interaction region depends on the overlap of the photon bunch with the charged particle bunch. For laser pulses that are in phase with and smaller than the particle beam bunch length, this luminosity is

$$\mathcal{L} = \frac{NN_\gamma f}{hA} = \frac{N\rho_\gamma \tau_\gamma f}{h}\qquad(3)$$

where ρ_γ is the number density of photons in the interaction region, N is the number of protons in the beam, h is the harmonic number, f is the interaction rate, and τ_γ is the laser pulse length. Note that the number of protons per bunch is N/h. The relationship between photon density and photon flux density (i.e. photons/sec/cm²) is $2\pi\rho_\gamma = \lambda'W_\gamma$. Thus, in terms of proton beam and laser parameters, the detector sees a flux of photons

$$\dot{n}_\gamma = \frac{N\tau_\gamma f\lambda W_\gamma d\sigma}{2\pi h},\qquad(4)$$

In laser polarimetry of particle beams, there are two distinct modes of operation[3]. In the *single photon method*, the recurrence of the particle bunches in a storage ring allows a low

power, high repetition rate laser to generate signal at the rate of about one photon/sec. In the *multi-photon method*, a high power laser generates several thousand signal photons per laser pulse. In the proton beam situation, the scattering is so weak that the multi-photon option requires a laser with energy and power densities that are state-of-the-art. The single-photon method is not an option for proton polarimetry, for reasons which become clear in section III.

To estimate the laser requirements, assume a laser pulse width of 50 ns, a laser energy of 1 eV, a harmonic number h of 1, an interaction rate f (beam revolution freq.) of 100 kHz, and a beam intensity N of 6×10^{13} (the current AGS unpolarized intensity). For a back-scattering rate of 1 kHz, the laser power density must be, in cgs units,

$$W_\gamma \approx 8.8 \times 10^{11} [\text{W/m}^2], \qquad (5)$$

deliverable as pulses in phase with the proton bunches. Lasers that generate pulses of this energy density are common, even ones that can sustain kilohertz pulse rates. Much higher power densities are not unusual, but at the expense of lower pulse rates[5], [6], [7]. Note also that, for a beam interaction region of 1 cm^2 area, the polarimeter requirement is for about 4.4 J per pulse. This is not the final word on the laser power requirement, though. Although a laser with an output of 10^{12}W/m^2 would cause count rates of a few kHz, there is a more stringent requirement on the count rate, that of the accuracy of the asymmetry measurement.

III. The Asymmetry Measurement

The relative polarization of the proton beam is proportional to the vertical asymmetry in rates of back scattering by the two distinct photon polarizations, $\xi_3 = \pm 1$. What happens is that photons scatter from the protons in the vertical plane, and these scattering rates differ above and below the horizontal plane. The relative difference between the scattering rates above the plane and below the plane is the asymmetry. For vertically polarized protons denote this asymmetry as [3]

$$A_\uparrow = \frac{d\sigma_+ - d\sigma_-}{d\sigma_+ + d\sigma_-} = \frac{\Phi_2}{\Phi_0} = P_p P_\gamma \cos\phi' F(\theta', \omega_0'), \quad (6)$$

where P_p and P_γ are the polarization components of the proton and photon respectively, and $d\sigma_+$ and $d\sigma_-$ are the right and left helicity photon scattering cross sections. In RHIC, the polarization of the proton bunches alternates with each bunch. This provides an opportunity for reducing systematic errors by making measurements of the total asymmetry,

$$A = (A_\uparrow - A_\downarrow)/2 = \frac{\Phi_2}{\Phi_0}. \qquad (7)$$

This measurement, although it achieves a reduction of systematic error, requires the laser to switch polarizations on every other bunch. Automatic polarization switching is possible in the design of the laser. The spacing between the pulses of alternately polarized laser light depends on the sampling scheme for the similarly alternating polarizations of the proton bunches. However, pulse separation times for the laser polarizations are naturally on the order of 4.5 ns [6].

The function $F(\theta', \omega_0')$ in equation 6 determines the ultimate detectability of the asymmetry. Scaling the photon energy in units of the proton mass, this asymmetry is, from equation 1,

$$\frac{\Phi_2}{\Phi_0} = -\frac{\xi_3(1 - \cos\theta')\vec{\zeta}\cdot(k_0'\cos\theta' + k')}{(1 + \cos^2\theta') + (\omega_0' - \omega')(1 - \cos\theta')}. \quad (8)$$

In what follows, it will be convenient to retain these units for the photon energy. Now, since the polarization components of the incoming photon and the proton are $\vec{\xi} = (0, 0, \pm P_\gamma)$ and $\vec{\zeta} = (0, 0, \pm P_p)$ and since, by introducing the azimuth angle, ϕ, $\vec{\zeta} \cdot \vec{k'} = \zeta_3 \omega' \sin\theta' \cos\phi$, then

$$F(\theta', \omega_0') = \frac{\omega' \sin\theta'(1 - \cos\theta')}{(1 + \cos^2\theta) + (\omega_0' - \omega')(1 - \cos\theta')}. \quad (9)$$

The angular dependance of this function is at a maximum for scattering angles close to $\theta' = \pi/2$. Assuming placement of detectors at this angle, the energy dependence of F is then

$$F(\pi/2, \omega_0') = \frac{\omega'}{1 + \omega_0' - \omega'}. \qquad (10)$$

Using the Compton formula,

$$1/\omega' - 1/\omega_0' = 1 - \cos\theta', \qquad (11)$$

yields

$$F(\pi/2, \omega_0') = \frac{\omega_0'}{1 + \omega_0' + \omega_0'^2}, \qquad (12)$$

which has a maximum at

$$\omega_0' = 1 \qquad (13)$$

if photon energies are measured in units of proton mass. Note that at its maximum, $F = 1/3$, implying that the relative polarization is not simply the asymmetry, A. In fact the absolute polarization of the proton beam is

$$P_p = A/\Pi P_\gamma, \qquad (14)$$

where Π is the analyzing power and is defined for beams purely polarized.

The condition $\omega_0' = 1$ also leads to a relationship between the energy of the particle beam and the input photon wavelength. This relationship exists because the detector is fixed at the optimum scattering angle, and the Compton formula again provides the connection between ω_0' and E_{beam}. In terms of the rest frame incoming photon energy, ω_0', the energy of the lab frame photon, ω_0 is

$$\omega_0' = 2\gamma\omega_0. \qquad (15)$$

Now, explicitly entering the mass scale into equation 13 gives

$$1 = 2\gamma E_\gamma/m = 2E_{beam}E_\gamma/m_p^2 \qquad (16)$$

or

$$E_\gamma E_{beam} = m_p^2/2 \qquad (17)$$

Using the relationship between the energy of a photon and its wavelength,

$$E_\gamma \lambda = 1.24 [\text{eV}\mu\text{m}] \qquad (18)$$

the optimal scattering wavelength, in terms of particle beam parameters, is

$$\lambda = \frac{2.48\gamma}{m_p}[\text{eV}\,\mu\text{m}]. \qquad (19)$$

For RHIC, this wavelength is $\approx 0.5 \times 10^{-3}$nm. Such photons are too energetic for creation by lasers, so any measurement of polarization must use a less than optimal wavelength. To determine just what the implications on the scattering are of using practical wavelengths, it is necessary to look at the time it takes to make satisfactory polarization measurements.

Suppose the polarization measurement uses high energy visible photons of 300 nm to analyze RHIC beams, then $F(\pi/2, \lambda) \approx 1.7 \times 10^{-6}$. Since $F(\pi/2, \lambda)$ directly determines the degree of asymmetry, this low value requires a large number of interactions to build up the statistics necessary for reasonable precision. In terms of the number of back-scattered photons, the relative accuracy in the asymmetry is [3]

$$\frac{\delta A}{A} = \frac{1}{A\sqrt{2\langle n_A \rangle}}, \qquad (20)$$

where $\langle n_A \rangle$ is the number of counts necessary to produce a given accuracy. This number, divided by the production rate, gives the time necessary for a given measurement accuracy. The measurement time for 20% accuracy is,

$$T = \frac{\langle n_A \rangle}{\dot{n}} = \frac{25}{A^2}. \qquad (21)$$

For a scattering rate of 1 kHz, the measurement time in Eq. 21 is too long to be workable. It is on the order of 10^4 yr. Some means of enhancing the effectiveness of the counting is necessary. Fortunately, there are real possibilties. The immediate thought that comes to mind is to increase the photon energy, since the dependence of A on λ' goes as $1/\lambda'$, thus, the dependence of $\langle n_A \rangle$ on λ' goes as λ'^2. Meanwhile, the count rate $\dot{n}$ goes as λ', so the measurement time for a particular accuracy decreases as λ' decreases. It does not decrease enough though. Optimal photons have a wavelength so much shorter than any possible laser-generated photons that the decrease in measurement time is quite minimal. This goes even for Compton-scattered photons from e-beams or x-rays from undulators[8]. At the present state of the art, there are no photon sources that would provide the necessary intensity of sub-nanometer radiation.

The only reasonable alternative is to use the *sampling method*. This method samples the polarization of the proton beam in a very small area. Focusing the laser spot to sub-millimeter dimensions increases the power density by several orders of magnitude without changing the energy. The laser in [7] delivers 1/3 of the required energy, but 10^{11} the power density. The laser output is then not a limiting factor. However, the count rate of back-scattered photons now goes up substantially, to the GHz range. At a count rate of 100 GHz, and a wavelength of 300 nm, the measurement time to 20% accuracy is now ≈ 165 hrs.

IV. Conclusion

Laser polarimetry of proton beams is difficult, but not impossible. The limitations to accurate measurement of beam polarization is not, surprisingly, the laser output, but the ability to count back-scattered photons at high rates. This limit arises from the accuracy requirement of the measurement, and the need to spatially sample the polarization of the proton beam. The function $F(\theta', \omega_0')$ in Eq. 6 has a natural scale set by the mass of the scattering particle. Since the proton mass is so much heavier than the electron, this scale changes by a factor of 2000. This factor decreases even further the already small cross section for Compton scattering from protons. If the interest were only in unpolarized scattering, for a profile monitor, for example, the rates would be quite good. It is the polarimetry requirement, *i.e.* the requirement of a reasonable analyzing power, that makes this measurement difficult. This dependence of the analyzing power on the mass implies a few other things as well. First, laser polarimetry of heavy ions is unfeasible. Second, laser polarimetry of muons is certainly possible, and should give very good results.

Finally, laser polarimetry is inherently non-destructive of the beam and of its polarization. Running a polarimeter for 200 hrs. can be a non-invasive part of the standard operation of an experimental program.

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