

**Feasibility of search  
for nuclear electric dipole moments  
at ion storage rings**

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**Abstract**

The sensitivity much better than  $10^{-24}$  e cm may be expected in the searches for electric dipole moments (EDM) of  $\beta$ -active nuclei at ion storage rings. It would be a serious progress in studies of the CP-violation problem.

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1. Only upper limits have been obtained up to now in the searches for the electric dipole moments of the neutron, atoms and molecules. But these limits are a valuable contribution to elementary particle physics, having strongly constrained theoretical models of CP violation.

Let us note that the bounds on CP violation in nuclei derived from atomic experiments are at least as informative as direct investigations of the neutron EDM. New approaches may arise here due to the progress in the accelerator technique. An experiment was recently proposed to search for the muon EDM with the sensitivity of  $10^{-24} e \text{ cm}$  [1]. The intention is to use the existing muon  $g - 2$  ring. The muons in it have natural longitudinal polarization. An additional spin precession due to the EDM interaction with external field will be monitored by counting the decay electrons, their momentum being correlated with the muon spin due to parity nonconservation in the muon decay.

Just for the coherence of presentation, let us write down few formulae from [1] with simple explanations. The frequency  $\vec{\omega}_m$  of the spin precession in external magnetic and electric fields,  $\vec{B}$  and  $\vec{E}$ , is (see, e.g., book [2], §41)

$$\vec{\omega}_m = -\frac{e}{m} \left\{ \left( a + \frac{1}{\gamma} \right) \vec{B} - a \frac{\gamma}{\gamma + 1} \vec{v} (\vec{v} \vec{B}) - \left( a + \frac{1}{\gamma + 1} \right) \vec{v} \times \vec{E} \right\}. \quad (1)$$

Here the anomalous magnetic moment  $a$  is related to the  $g$ -factor as follows:  $a = g/2 - 1$ , for muon  $a = \alpha/2\pi$ ;  $\vec{v}$  is the particle velocity;  $\gamma = 1/\sqrt{1 - v^2}$ . The units are used where  $\hbar = 1$ ,  $c = 1$ . We supply here  $\vec{\omega}$  by subscript  $m$  to indicate that formula (1) describes the precession due to the magnetic moment (combined with the Thomas effect). As to the precession induced by the EDM, its frequency is

$$\vec{\omega}_e = -\frac{e}{m} \eta \left\{ \vec{E} - \frac{\gamma}{\gamma + 1} \vec{v} (\vec{v} \vec{E}) + \vec{v} \times \vec{B} \right\}. \quad (2)$$

The dimensionless constant  $\eta$  is related to the EDM  $d$  as follows:

$$d = \frac{e}{2m} \eta$$

(here  $\eta$  is 2 times smaller than in [1]). Formula (2) can be obtained from the terms proportional to the anomalous magnetic moment in (1) by substituting  $a \rightarrow \eta$  and changing to dual fields:  $\vec{B} \rightarrow \vec{E}$ ,  $\vec{E} \rightarrow -\vec{B}$ .

However, what is of interest to this muon EDM experiment, is not the frequency of the spin precession with respect to the laboratory frame, i.e., not the sum of expressions (1) and (2). What we need is the frequency of the spin precession with respect to the muon momentum. The precession frequency of the momentum itself  $\vec{\omega}_p$  can be derived easily from the well-known expression for acceleration in external fields (see, e.g., book [3], problem to §18):

$$\dot{\vec{v}} = \frac{e}{m\gamma} \left\{ \vec{v} \times \vec{B} + \vec{E} - \vec{v}(\vec{v}\vec{E}) \right\}. \quad (3)$$

The acceleration component transverse to the velocity is

$$\dot{\vec{v}}_t = \frac{e}{m\gamma} \left\{ \vec{v} \times \vec{B} - \vec{v} \times [\vec{v} \times \vec{E}] \right\}, \quad (4)$$

which corresponds to the precession frequency of momentum (or velocity)

$$\vec{\omega}_p = -\frac{e}{m} \left\{ \frac{1}{\gamma} \vec{B} - \frac{\gamma}{\gamma^2 - 1} \vec{v} \times \vec{E} \right\}. \quad (5)$$

Thus, the frequency of the spin precession with respect to the momentum is

$$\begin{aligned} \vec{\omega} = \vec{\omega}_m + \vec{\omega}_e - \vec{\omega}_p = & -\frac{e}{m} \left\{ a\vec{B} - a \frac{\gamma}{\gamma + 1} \vec{v}(\vec{v}\vec{B}) - \left( a - \frac{1}{\gamma^2 - 1} \right) \vec{v} \times \vec{E} \right. \\ & \left. + \eta \left[ \vec{E} - \frac{\gamma}{\gamma + 1} \vec{v}(\vec{v}\vec{E}) + \vec{v} \times \vec{B} \right] \right\}. \end{aligned} \quad (6)$$

This expression simplifies in the obvious way at  $(\vec{v}\vec{B}) = (\vec{v}\vec{E}) = 0$ . Just this case is considered below.

It is proposed in [1] to compensate for the precession in the vertical magnetic field  $\vec{B}$  by the precession in a radial electric field  $\vec{E}$ , i.e., to choose  $\vec{E}$  in such a way that

$$a\vec{B} - \left( a - \frac{1}{\gamma^2 - 1} \right) \vec{v} \times \vec{E} = 0.$$

In fact, electric fields in a storage ring are much smaller than magnetic ones, and therefore can be neglected in the EDM term. So, with the mentioned

compensation, the spin precession with respect to momentum is due only to the EDM interaction with the vertical magnetic field:

$$\vec{\omega} = \vec{\omega}_e = -\frac{e}{m} \eta \vec{v} \times \vec{B}. \quad (7)$$

In this way the muon spin acquires a vertical component which linearly grows with time. The P-odd correlation of the decay electron momentum with the muon spin leads to the difference between the number of electrons registered above and below the orbit plane.

The statement in [1] on the feasibility of this experiment is: "We are confident that we can improve by six orders of magnitude the current sensitivity to the muon EDM, both in statistics and systematics, bringing it down to  $10^{-24}$  e cm."

**2.** Our point is that in the same way one can search for an EDM of a polarized  $\beta$ -active nucleus in a storage ring. In this case as well, the precession of nuclear spin due to the EDM interaction can be monitored by the direction of the  $\beta$ -electron momentum.

$\beta$ -active nuclei have serious advantages as compared to muon.

The life-time of a  $\beta$ -active nucleus can exceed by many orders of magnitude that of a muon. The characteristic depolarization time of the ion beam is also much larger than  $10^{-6}$  s, the muon life-time. Correspondingly, the angle of the rotation of nuclear spin, which is due to the EDM interaction and which accumulates with time, may be also by orders of magnitude larger than that of a muon. By the same reason of the larger life-time, the quality of an ion beam can be made much better than that of a muon beam.

Then, the typical nuclear magnetic moment is by an order of magnitude smaller than that of a muon. Accordingly, smaller are various spurious effects due to the interaction of a magnetic moment with external fields.

However, necessary conditions here are also quite serious.

First of all, there should be an appreciable P-odd correlation in the  $\beta$ -decay between the electron momentum and spin  $J$  of the decaying nucleus.

Then, to make realistic the mentioned compensation of the EDM-independent spin precession by a relatively small electric field, the effective  $g$ -factor should be close to 2 (as this is the case for the muon). Let us consider this condition

for a nucleus in more detail. The nuclear magnetic moment

$$\frac{e}{2m_p} \mu,$$

when expressed through the total nuclear charge  $Ze$  and mass  $Am_p$ , can be rewritten as

$$\frac{Ze}{2Am_p} \frac{A}{Z} \mu.$$

The effective  $g$ -factor and effective anomalous magnetic moment are now

$$g = \frac{A}{Z} \frac{\mu}{J} \quad \text{and} \quad a = \frac{g}{2} - 1 = \frac{A}{Z} \frac{\mu}{2J} - 1,$$

respectively. Therefore, the condition discussed is

$$\frac{A}{2Z} \frac{\mu}{J} \approx 1, \quad \text{or} \quad \mu \approx \frac{2Z}{A} J \approx 0.8J. \quad (8)$$

Fine-tuning to this condition is possible in many cases by taking, instead of a bare nucleus, an ion with closed electron shells. Then the necessary condition (8) softens to

$$\mu \leq \frac{2Z}{A} J \leq 0.8J. \quad (9)$$

An even number of electrons in an ion does not guarantee by itself that the total electron angular momentum, and therefore the total electron magnetic moment vanish. Its vanishing can be demonstrated at least at the following numbers of electrons

$$Z - z = 2, 4, 6, 10, 12, 14, 18,$$

since up to

$$1s^2 2s^2 2p^6 3s^2 3p^6$$

the filling of atomic shells certainly follows the hydrogen pattern. On the other hand, at  $Z - z = 8, 16$ , for the ground state configurations of the type  $p^4$ , when the  $p$ -subshell is filled more than by half, the total electron angular momentum  $J_e = 2$ . One should exclude also the case  $Z - z = 26$ : neither of the electron configurations conceivable here,  $3d^6 4s^2 {}^5D_4$ ,  $3d^7 4s {}^5F_5$ ,  $3d^8 {}^3F_4$ , has vanishing angular momentum. In other cases of interest vanishing of  $J_e$  should be checked experimentally.

One more comment on the ion charge  $z$  should be made. The nuclear EDM is screened by atomic electrons, and the screening is complete for a neutral atom. It is intuitively clear and can be confirmed by direct calculations that in an ion this screening is partial only, being proportional to the number of electrons  $Z - z$ . So, the smaller is this number, the better for our problem.

Finally, if we take into account the difference  $\Delta m = m_n - m_p$  between the neutron and proton masses, and the finite mass  $m_e$  of the electron, the expression for the effective anomalous magnetic moment of an ion, with the nuclear charge  $Z$  and the total ion charge  $z$ , changes to

$$a = \frac{A}{2z} \frac{\mu}{J} \left( 1 + \frac{A - Z}{A} \frac{\Delta m}{m_p} + \frac{Z - z}{A} \frac{m_e}{m_p} \right) - 1. \quad (10)$$

Typically, due to the correction factor

$$\left( 1 + \frac{A - Z}{A} \frac{\Delta m}{m_p} + \frac{Z - z}{A} \frac{m_e}{m_p} \right),$$

the value of  $a$  increases by  $(8 - 9) \cdot 10^{-4}$ .

The ions which look at the moment promising from the point of view of the EDM searches are presented in the Table. The isotope data are taken from the handbook [4]. The selection is confined to those  $\beta$ -active nuclei for which the positive sign of magnetic moment is established: at negative sign the mentioned fine-tuning is impossible at all. The next demand was that the value of magnetic moment should be known with sufficient precision and should allow reasonable fine-tuning.

The errors in the values of anomalous magnetic moments  $a$  presented in the Table correspond to the experimental errors in values of  $\mu$ . But there is an effect not taken into account in the Table: electron configurations even with vanishing angular momentum  $J_e$  produce a diamagnetic screening of nuclear magnetic moments. The screening should be very small in case of closed electron shells,  $s^2$ ,  $p^6$ . But it is nonnegligible in the configurations of the type  $p^2$ , i.e., at  $Z - z = 6, 14$ . The relative magnitude of the diamagnetic effect can be estimated here roughly as  $1/Z$ . The diamagnetic correction is truly large for  ${}^{24}_{11}\text{Na}$ , changing the  $a$ -value from 0.015 presented in the Table to about  $-0.1$ . The exact value of  $a$  for  ${}^{24}_{11}\text{Na}$  at  $z = 5$  demands rather serious atomic calculations and/or experimental measurements. But most probably

it will stay relatively large. This is quite unfortunate since just  ${}_{11}^{24}\text{Na}^{+5}$  seems to be a good object for the discussed experiments, being available in very large quantities.

We have excluded from the Table isotopes with too short and too long life-time  $t_{1/2}$  (for our purpose approximate values of  $t_{1/2}$  are sufficient, so we present them with 2 digits only). In this respect at least  ${}_{55}^{137}\text{Cs}$  looks already suspicious. By the way, few examples of  $\beta$ -decaying excited states satisfy the above criteria, these isotopes are marked in the Table by \*. The optimum values of life-times depend on too many experimental details and therefore cannot be indicated in general form.

The value  $J = 1/2$  for nuclear spin would allow to avoid the background due to the quadrupole interaction with external fields. Unfortunately, we could not find spin  $J = 1/2$  isotopes satisfying our criteria. Simple estimates demonstrate however, that at reasonable parameters of a storage ring even for relatively large nuclear quadrupole moments  $Q \sim 1$  barn this background is not dangerous at the EDM sensitivity as high as  $10^{-26}$  e cm. The values of  $Q$  (where known) are also presented in the Table with 2 digits only.

All isotopes presented in the Table are  $\beta^-$ -active (their  $\beta^-$  branchings are indicated in the last column). Fortunately, many of them have allowed pure Gamow – Teller transitions ( $|\Delta J| = 1$ ) where the magnitude of the needed correlation between the electron momentum and the initial spin is on the order of unity. Few isotopes in the Table have allowed mixed  $\beta^-$ -transitions ( $|\Delta J| = 0$ ). Here the magnitude of the asymmetry we need may change essentially from nucleus to nucleus. For instance, for the cases presented in book [5] (they do not enter the Table since none of them fits our criteria) this asymmetry varies from 0.016 to 0.33. Obviously, for the allowed mixed transitions, as well as for forbidden transitions which are also represented in the Table, the values of the discussed asymmetry should be found experimentally.

Perhaps, from the point of view of registration, it would be tempting to have isotopes with positron  $\beta$ -decay. Unfortunately, for two isotopes potentially useful in this respect,  ${}_{33}^{70}\text{As}$  and  ${}_{33}^{71}\text{As}$ , the cited 8th edition of handbook [4] contains no quantitative data on their  $\beta^+$  branchings (though these data were present in the 7th edition).

**3.** But how significant would be the discussed experiments with  $\beta$ -active

nuclei for elementary particle physics?

The typical value of a nuclear EDM, as induced by CP-odd nuclear forces, is roughly independent of  $A$  and  $Z$  and can be estimated as [6]

$$d_N \sim 10^{-21} \xi, \quad (11)$$

where  $\xi$  is a dimensionless parameter measuring these forces in the units of the Fermi weak interaction constant  $G$ . The best upper limit on  $\xi$  was obtained in an atomic experiment [7]:

$$\xi < 2 \cdot 10^{-3}. \quad (12)$$

This limit was demonstrated [8] to be at least as significant for elementary particle physics as the best upper limit on the neutron EDM [9, 10]:

$$d_n < 10^{-25} e \text{ cm}. \quad (13)$$

(Detailed discussion of these problems can be found also in book [11].)

So, even at the same sensitivity  $10^{-24} e \text{ cm}$ , as discussed in [1] for muons, the experiments with  $\beta$ -active nuclei would compete with the best EDM studies. Certainly, progress in this direction well deserves serious efforts.

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Table

Ion	$J^\pi \rightarrow J^{\pi'}$	$\mu$	$z$	$a \cdot 10^3$	$t_{1/2}$	$Q$ (barn)	branching
$^{24}_{11}\text{Na}$	$4^+ \rightarrow 4^+$	1.6903(8)	5	15.1(0.5)	15 h		99.944%
$^{60}_{27}\text{Co}$	$5^+ \rightarrow 4^+$	3.799(8)	23	- 8(2)	5.3 y	0.44	99.925%
$^{82}_{35}\text{Br}$	$5^- \rightarrow 4^-$	1.6270(5)	13	27.2(0.3)	35 h	0.75	98.5%
$^{93}_{37}\text{Rb}$	$5/2^- \rightarrow 5/2^+$	1.4095(16)	27	- 28.1(1.1)	5.8 s	0.18	43%
$^{94}_{37}\text{Rb}$	$3^- \rightarrow 3^-$	1.4984(18)	23	21.5(1.2)	2.7 s	0.16	30.6%
$^{110}_{47}\text{Ag}^*$	$6^+ \rightarrow 5^+$	3.607(4)	33	3(1)	250 d	1.4	66.8%
$^{118}_{49}\text{In}^*$	$8^- \rightarrow 7^-$	3.321(11)	25	- 19(3)	8.5 s	0.44	1.4%
$^{120}_{49}\text{In}^*$	$(8^-) \rightarrow 7^-$	3.692(4)	27	26(1)	47 s	0.53	84.1%
$^{121}_{50}\text{Sn}$	$3/2^+ \rightarrow 5/2^+$	0.6978(10)	28	6(1)	27 h	- 0.02(2)	100%
$^{125}_{51}\text{Sb}$	$7/2^+ \rightarrow 5/2^+$	2.630(35)	47	$0 \pm 13$	2.8 y		40.3%
$^{131}_{53}\text{I}$	$7/2^+ \rightarrow 5/2^+$	2.742(1)	51	7.0(0.4)	8.0 d	- 0.40	89.9%
$^{133}_{53}\text{I}$	$7/2^+ \rightarrow 5/2^+$	2.856(5)	53	25(2)	21 h	- 0.27	83%
$^{133}_{54}\text{Xe}$	$3/2^+ \rightarrow 5/2^+$	0.81340(7)	36	2.58(9)	5.2 d	0.14	99%
$^{134}_{55}\text{Cs}$	$4^+ \rightarrow 4^+$	2.9937(9)	51	- 16.0(0.3)	2.0 y	0.39	70.11%
$^{136}_{55}\text{Cs}$	$5^+ \rightarrow 6^+$	3.711(15)	51	- 9(4)	13 d	0.22	70.3%
$^{137}_{55}\text{Cs}$	$7/2^+ \rightarrow 11/2^-$	2.8413(1)	55	11.9(0.1)	30 y	0.051	94.4%

Table (continued)

Ion	$J^\pi \rightarrow J^{\pi'}$	$\mu$	$z$	$a \cdot 10^3$	$t_{1/2}$	$Q$ (barn)	branching
$^{139}_{55}\text{Cs}$	$7/2^+ \rightarrow 7/2^-$	2.696(4)	53	11(1)	9.3 m	- 0.075	82%
$^{141}_{55}\text{Cs}$	$7/2^+ \rightarrow 7/2^-$	2.438(10)	49	3(4)	25 s	- 0.36	57%
$^{143}_{55}\text{Cs}$	$3/2^+ \rightarrow 5/2^-$	0.870(4)	41	12(5)	1.8 s	0.47	24%
$^{140}_{57}\text{La}$	$3^- \rightarrow 3^+$	0.730(15)	17	$3 \pm 21$	1.7 d	0.094	44%
$^{160}_{65}\text{Tb}$	$3^- \rightarrow 2^-$	1.790(7)	47	16(4)	72 d	3.8	44.9%
$^{170}_{69}\text{Tm}$	$1^- \rightarrow 0^+$	0.2476(36)	21	$2.2 \pm 14.5$	129 d	0.74	99.854%
$^{177}_{71}\text{Lu}$	$7/2^+ \rightarrow 7/2^-$	2.239(11)	57	- 6(5)	6.7 d	3.4	78.6%
$^{183}_{73}\text{Ta}$	$7/2^+ \rightarrow 7/2^-$	(+)2.36(3)	61	12(13)	5.1 d		92%
$^{192}_{77}\text{Ir}$	$4(^+) \rightarrow 3^+, 4^+$	1.924(10)	47	- 17(5)	74 d	2.3	42%,54%
$^{196}_{79}\text{Au}$	$2^- \rightarrow 2^+$	0.5906(5)	29	- 1.1(8)	6.2 d	0.81	8%
$^{198}_{79}\text{Au}$	$2^- \rightarrow 2^+$	0.5934(4)	29	13.9(7)	2.7 d	0.68	98.99%
$^{203}_{80}\text{Hg}$	$5/2^- \rightarrow 3/2^+$	0.84895(13)	34	14.71(15)	47 d	0.34	100%
$^{222}_{87}\text{Fr}$	$2^- \rightarrow 3^-$	0.63(1)	35	$0 \pm 20$	14 m	0.51	55%
$^{223}_{87}\text{Fr}$	$3/2(^-) \rightarrow 3/2^-$	1.17(2)	87	$0 \pm 20$	22 m	1.2	67%
$^{224}_{87}\text{Fr}$	$1(^-) \rightarrow 1^-$	0.40(1)	45	- $3 \pm 25$	3.3 m	0.52	42%
$^{242}_{95}\text{Am}$	$1^- \rightarrow 0^+, 2^+$	0.3879(15)	47	- $0.5 \pm 3.9$	16 h	- 2.4	37%,46%

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