Stabilization of the Deuteron Spin Tune in a Storage Ring Using Active Feedback

Nils Hempelmann*
Institut für Kernphysik, Forschungszentrum Jülich†
(JEDI Collaboration)

(Dated: December 11, 2016)

Permanent electric dipole moments (EDM) in elementary particles would violate CP-symmetry. The JEDI (Jülich Electric Dipole moment Investigations) collaboration will measure the EDM of charged hadrons using a storage ring. To keep the spin oscillation in phase with an external frequency, which is a requirement for EDM measurements in magnetic storage rings, an active feedback system was developed and tested with a polarized deuteron beam at the Cooler Synchrotron (COSY).

The feedback system determines the spin polarization using information from the polarimeter EDDA. Data measured over a time of about one second are analyzed to determine the necessary correction. The phase of the spin rotation is continuously adjusted by modifying the accelerator frequency, which changes the beam velocity and hence the rate of spin precession.

The effects of the EDM in a storage ring were mimicked using an RF solenoid whose frequency was locked with respect to the spin oscillation. Like an EDM, the solenoid gradually tilts the spin from a horizontal to a vertical direction.

The results of the feedback system tests demonstrate that the method is suitable for a proof-of-principle experiment of EDM measurements at COSY.

I. INTRODUCTION

Permanent electric dipole moments (EDM) in elementary particles would be a violation of CP symmetry. Various methods have been proposed for measuring the electric dipole moment of charged hadrons such as protons or deuterons in storage rings [3]. These measurements work by injecting a horizontally polarized beam into the ring and observing a gradual build-up of vertical polarization caused by the interaction of the electric dipole moment of the particle with the electric or magnetic guiding field.

To measure the electric dipole moment in a purely magnetic ring without any electrostatic guiding fields, such as COSY [1], a Wien filter is required [2, 4]. This device creates a resonant RF **E** and **B** field, which advances the spin precession but does not affect the orbit. This breaks the symmetry between upward and downward spin motion in a magnetic ring, which makes an accumulation of vertical polarization over time possible.

The spin precession must be kept in phase with the Wien filter. A feedback system was developed and tested at COSY that can match the precession to an external frequency and meets the requirements for a future EDM measurement.

II. FEEDBACK SYSTEM AT COSY

The feedback system [5] keeps the phase between the horizontal spin precession and an external frequency constant over time. While the system was designed with an RF Wien filter in mind, such a filter is not currently installed at COSY. Therefore an RF solenoid is used as a substitute.

The COSY revolution frequency f_{COSY}, the solenoid frequency f_{sol}, and deuteron-carbon scattering events in the polarimeter EDDA are measured using the same time reference. The polarimeter data can be used to determine the spin tune to a high precision [6]. Using the common timing standard, the relative phase between the spin precession and the external frequency can be determined. The COSY frequency is adjusted to control the phase.

A change in the accelerator frequency changes the beam velocity, which has two distinct effects: the spin tune ν_s changes and the particles arrive at the detector at a different time. The relative phase ϕ between the solenoid frequency and the spin precession can be expressed as

$$\phi = 2\pi f_{\mathrm{sol}} T - 2\pi \nu_s f_{\mathrm{COSY}} T. \qquad (1)$$

The particles arrive at the detector at a fixed time $T = n/f_{\mathrm{COSY}}$, where n is the turn number. Inserting that into the equation (1) and taking the derivative yields:

$$\frac{d\phi}{df_{\mathrm{COSY}}} = 2\pi n \left(-\frac{f_{\mathrm{sol}}}{f_{\mathrm{COSY}}^2} - \frac{d\nu_s}{df_{\mathrm{COSY}}} \right). \qquad (2)$$

The first term in (2) corresponds to the change in time at which the particles arrive at the detector, the second term corresponds to the change in spin tune, which can be calculated using

$$\frac{\Delta\nu_s}{\nu_s} = \frac{\Delta\gamma}{\gamma} = \beta^2 \frac{\Delta p}{p} = \frac{\beta^2}{\eta} \frac{\Delta f_{\mathrm{COSY}}}{f_{\mathrm{COSY}}}. \qquad (3)$$

At frequencies of $f_{\mathrm{COSY}} \approx 750\,\mathrm{kHz}$ and $f_{\mathrm{sol}} \approx 871\,\mathrm{kHz}$ this leads to $\frac{|\Delta\phi|}{\Delta T} \approx 7.7\,\frac{\mathrm{rad}}{\mathrm{Hz\,s}} \Delta f_{\mathrm{COSY}}$. The COSY frequency can be adjusted in steps of 3.7 mHz, which corresponds to $\frac{|\Delta\phi|}{\Delta T} \approx \pm 30\,\mathrm{mrad/s}$.

---

* n.hempelmann@fz-juelich.de
† http://collaborations.fz-juelich.de/ikp/jedi/

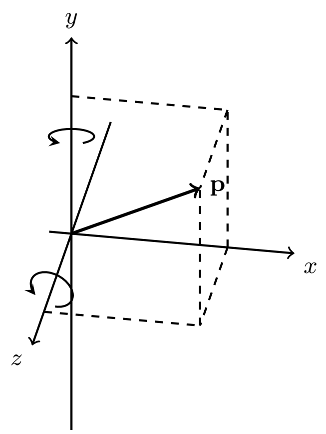

FIG. 1. Rotation of the polarization vector **p** in RF devices. The z-axis is defined by the beam, the y-axis is vertical. A Wien filter rotates the spin about the y-axis, a solenoid about the z-axis.

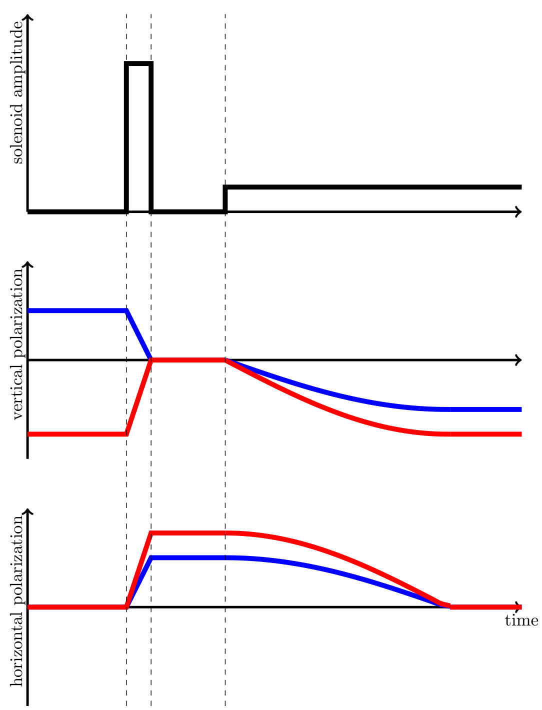

FIG. 2. Rough polarization over time during the experiment

III. EXPERIMENTS IN FALL 2015

During the beam time in fall 2015, the feedback system was tested using an RF solenoid as a substitute for a Wien filter. The main difference between the solenoid and a Wien filter is the axis of the spin rotation (fig. 1). An RF Wien filter causes a rotation about the vertical axis, which is also the stable spin axis. A solenoid rotates the spin about the beam axis.

Fig. 2 shows the principle of the measurement. The polarization is initially vertical, either positive or negative at a negligible tensor polarization. The magnitude of

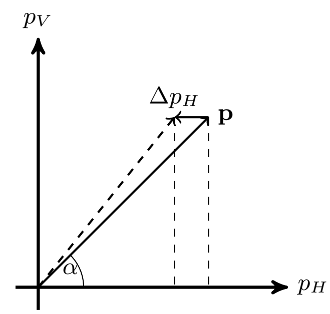

FIG. 3. Definition of polarization variables. A decrease in p_H leads to an increase in α.

the negative initial polarization is about 1.7 times greater than that of the positive initial state (see also fig. 4).

The solenoid is switched on briefly to rotate the spins into the horizontal plane where they precess around the vertical axis. After the feedback is switched on to keep the phase between the spin and the solenoid stable, the solenoid is switched on again as well, at a lower amplitude, which gradually rotates the spin up or down.

Once the spin approaches the vertical axis the feedback system fails because a spin pointing along the stable axis does not precess and has no well-defined phase to be fixed. This means that the spin will remain vertical and not rotate further. The speed of the rotation towards the vertical axis is proportional to $\sin\phi$, the final polarization state can be up or down.

IV. ANALYSIS

A. Model

The goal of the analysis is to determine the rate of spin rotation for measurements at different values of ϕ. The spin rotation caused by the solenoid is analogous to the effect of an EDM, which also leads to a slow build-up of vertical polarization over time. The analysis discussed here is in principle applicable to future EDM data as well.

The polarization is parametrized by the angle α between the polarization vector and the horizontal plane and the magnitude of the polarization:

$$\tan\alpha = \frac{p_V}{p_H}, \qquad p^2 = p_V^2 + p_H^2. \tag{4}$$

The horizontal polarization is determined using the algorithm described in [6], while the vertical polarization can be determined from the rates in the left and right polarimeter detector. Fig. 3 gives an overview of the variable definitions.

Under ideal circumstances the rate of rotation is $\frac{d\alpha}{dt} = k\sin\phi$, where k is a constant depending on the amplitude of the solenoid. In a real experiment there is an additional effect due to the decoherence of the spin rotations. While the vertical spin component is stable over long

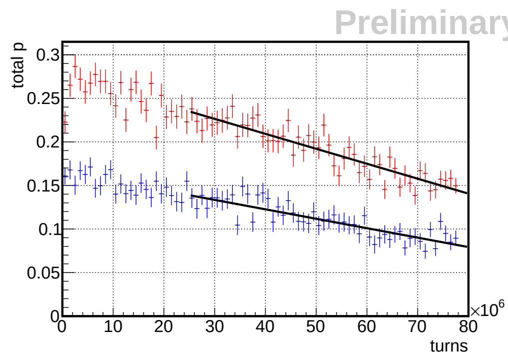

FIG. 4. Magnitude of the polarization over time for positive (blue) and negative (red) initial polarization.

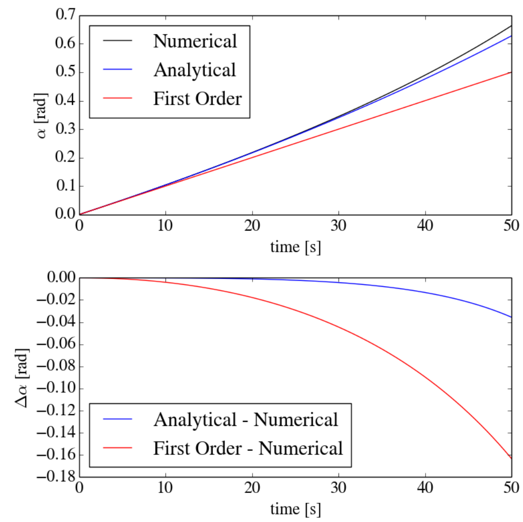

FIG. 5. The upper plot shows the analytical approximation from (8), the numerical solution, and a first order calculation that neglects $\frac{dp}{dt}$. The bottom plot shows the difference from the numerical solution.

times, the horizontal component decays over the course of the measurement because the spins of different particles precess at slightly different rates. This effectively accelerates the rotation towards the vertical axis, as shown in fig 3.

The rotation caused by the solenoid - or the EDM in a future experiment - can be distinguished from this effect by taking the magnitude of the polarization into account. The solenoid does not change the total magnitude, a decay of the vertical component decreases it. Empirically, a near linear decrease of total polarization over time can be found in the data (fig. 4). The polarization at any given time can therefore be approximated as $p = p_0 - p't$.

The equation of motion including decoherence is

$$\frac{d\alpha}{dt} = k\sin\phi + \frac{\partial\alpha}{\partial p_H}\frac{\partial p_H}{\partial p}\frac{dp}{dt}. \tag{5}$$

The first term in (5) corresponds to the effect of the solenoid, the second term is the effect of spin decoherence. The remaining derivatives are determined by trigonometry and the definitions in fig. 3. $p' = \frac{\partial p}{\partial t}$ is determined empirically from a fit as shown in fig. 4:

$$\frac{\partial\alpha}{\partial p_H} = \frac{-p_V}{p^2}, \qquad \frac{\partial p_H}{\partial p} = \frac{p}{p_H}. \tag{6}$$

Inserting (6) in (5) and expressing p_H and p_V as a function of α and p yields a differential equation for the spin under the influence of a solenoid:

$$\frac{d\alpha}{dt} = k\sin(\phi) - \frac{\tan\alpha}{p_0 + p't}p'. \tag{7}$$

This equation does not have a simple analytical solution. An approximate solution can be found by treating the effect of spin decoherence as a small perturbation of the ideal solution $\alpha = kt\sin\phi$. Defining $A = k\sin(\phi)$ and $B = \frac{p'}{p_0}$ the solution for a spin rotation starting at $\alpha(0) = 0$ is approximately

$$\alpha(t) = At - \frac{ABt^2}{2} + \frac{AB^2t^3}{3} + \mathcal{O}(t^4). \tag{8}$$

The solution was verified using numerical integration. Fig. 5 shows the difference between the numerical solution and the analytical approximation (8) for parameters near the worst case. The approximation is accurate within 2°. If $\alpha_0 = \alpha(0)$ is allowed to be nonzero, (8) becomes:

$$\begin{aligned}\alpha(t) = {} & \alpha_0 + At - Bt\tan\alpha_0 \\ & + \frac{t^2}{2}\left[-AB\left(1+\tan^2\alpha_0\right) + B^2\tan\alpha_0\right] \\ & + \frac{Bt^3}{3}\left[-A^2\tan^3\alpha_0 - A^2\tan\alpha_0\right. \\ & \left. + AB\left(1+\tan^2\alpha_0\right) - B^2\tan\alpha_0\right].\end{aligned} \tag{9}$$

This equation is used to fit the data.

B. Systematic uncertainties

There are two main sources of systematic error in the measurement:

- The left and right detector of the polarimeter do not have the same acceptance. This causes a systematic shift of both polarization states.
- The spin flip that rotates the initial vertical polarization into the horizontal plane is not complete. This means that measurements with positive and negative initial polarization start at different values of α.

Fig. 6 shows the asymmetry between the left and right events rates, which is roughly proportional to p_V. Assuming that both the positive and negative initial polarization state are rotated by the same angle, the detector

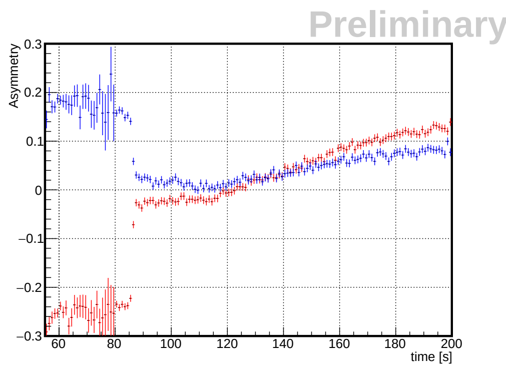

FIG. 6. Asymmetry between the left and right detector $\frac{N_L - N_R}{N_L + N_R}$ for positive (blue) and negative (red) initial polarization. The initial spin flip occurs at about 85 s.

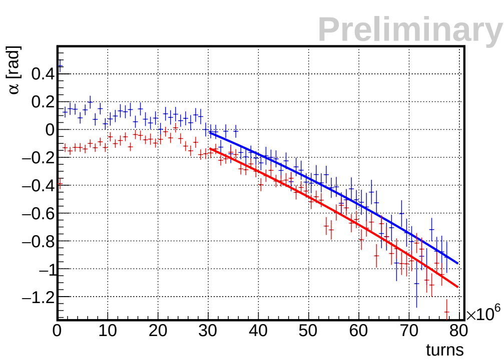

FIG. 7. Fit of one cycle for positive (blue) and negative (red) initial polarization. The decrease in α is slightly faster than linear.

asymmetry can be calculated from the asymmetries before and after the spin flip. The detector asymmetry ϵ_{det} is

$$\epsilon_{\mathrm{det}} = \frac{\epsilon_+^i \cdot \epsilon_-^f - \epsilon_+^f \cdot \epsilon_-^i}{\epsilon_+^i + \epsilon_+^f - \epsilon_+^f - \epsilon_-^i}. \qquad (10)$$

Where $\epsilon_\pm^{i,f}$ are the initial and final asymmetries for the positive and negative polarization states. There is also a solution for the angle of the initial spin flip:

$$\cos(\alpha_{\mathrm{flip}}) = \frac{\epsilon_+^f - \epsilon_-^f}{\epsilon_+^i - \epsilon_-^i}. \qquad (11)$$

All vertical polarizations are corrected for ϵ_{det}. The initial angle between the polarization and the horizontal plane could be calculated using $\alpha_0 = \pi/2 - \alpha_{\mathrm{flip}}$. However, α_0 is used as a parameter in the fit of (9) instead.

C. Results

Fig. 7 shows an example of a fit for one cycle. The general shape is reproduced well by the model, however

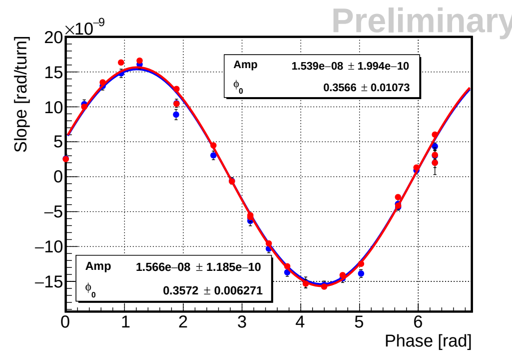

FIG. 8. The term $A = k\sin(\phi)$ of (7) as a function of the nominal phase ϕ. The top fit parameter box corresponds to positive polarization, the bottom box to negative polarization.

the statistical precision is not sufficient to distinguish the prediction of (7) from an ideal linear decrease without spin decoherence using α alone. The effect of decoherence can only be estimated using fits of the magnitude of the polarization such as in fig. 4.

Fig. 8 shows the fit parameter $A = k\sin(\phi)$ as a function of the value of ϕ set for the feedback system. A sine function is fitted to the data obtained with a positive and a negative initial polarization. The fit parameters agree within the statistical uncertainty, proving that the method is robust to changes in the initial polarization sign and magnitude.

There is an offset of 0.38 rad between the phase set for the feedback system and the phase observed in the data. The correction terms in (7) have the opposite sign of the leading order term and about 20% of its magnitude. The observed sinusoidal shape proves that ϕ is indeed kept stable at a given value.

V. CONCLUSION

A system to fix the phase between the in plane precession of a spin and an external frequency was developed and tested at COSY. The correct operation of the system was verified by fixing the phase between the spin and an RF solenoid and observing the rate of spin rotation towards the vertical axis. A method was developed to correct the measured data for the effects of a limited spin lifetime in the detector. This scheme is in principle applicable to future measurements of electric dipole moments in storage rings as well.

[1] R. Maier, Nucl. Instr. and Meth. A 390 (1997) 1

[2] J. Slim et al., *Electromagnetic Simulation and Design of a Novel Waveguide RF Wien Filter for Electric Dipole Moment Measurements of Protons and Deuterons*, Nucl. Instr. Meth. Phys. Res. A 828, 116-124 (2016), ISSN 0168-9002, http://dx.doi.org/10.1016/j.nima.2016.05.012.

[3] J. Pretz on behalf of the JEDI collaboration, *Measurement of permanent electric dipole moments of charged hadrons in storage rings*, Hyperfine Interact (2013) 214:111–117, DOI 10.1007/s10751-013-0799-4

[4] W. Morse, Y. Orlov, Y. Semertzidis, *rf Wien filter in an electric dipole moment storage ring: The "partially frozen spin" effect*, Phys. Rev. ST Accel. Beams 16, 114001 – Published 27 November 2013, DOI 10.1103/PhysRevSTAB.16.114001

[5] E. Stephenson, *Long polarization lifetimes and feedback control of polarization direction*, 22nd International Spin Symposium (2016)

[6] D. Eversmann et al., *New Method for a Continuous Determination of the Spin Tune in Storage Rings and Implications for Precision Experiments*, Phys. Rev. Lett. 115, 094801 (2015)