# SIMULATION MODEL IMPROVEMENTS AT THE COOLER SYNCHROTRON COSY USING THE LOCO ALGORITHM* 

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#### Abstract

The JEDI (Jülich Electric Dipole moment Investigations) collaboration is searching for Electric Dipol Moments (EDMs) of charged particles in storage rings. In a step-wise approach, a first direct deuteron EDM measurement was performed at the Cooler Synchrotron COSY and design studies for a dedicated proton EDM storage ring are underway. In an experiment with a polarized beam in a storage ring, an EDM leads to a vertical polarization buildup. However, the vertical polarization component is also induced by systematic effects such as magnet misalignments. To investigate systematic effects individually and to support data analysis, a realistic simulation model of the storage ring is needed. In this paper, the development of such a model based on the Bmad software library is presented. Furthermore, various systematic effects and their impact on the spin motion in COSY are investigated and quantified by means of beam and spin tracking simulations.


## INTRODUCTION

In order to explain the observed matter-antimatter asymmetry in our Universe, the magnitude of $C P$ violation provided by the Standard Model is not sufficient. Therefore, the existence of permanent EDMs of subatomic particles could explain the matter antimatter asymmetry [1], because they are violating $C P$ transformations. EDMs of charged particles can be studied by observing the evolution of the beam polarization in storage rings.

The JEDI collaboration performed a direct EDM measurement of deuterons at the COoler SYnchrotron (COSY). In order to analyze the measured data, spin tracking simulations are needed to separate the EDM signal from systematic effects. Therefore, the underlying model of COSY must describe the storage ring as accurately as possible. For beam optics and spin tracking simulations, the software library Bmad [2] is used. The model contains all types of magnets including their measured displacements. A comparison between the model and the real optics was made by measuring the orbit response matrix at COSY. To improve the simulation model and to minimize the discrepancies between model and the real storage ring, the Linear Optics from Closed Orbit (LOCO) technique is applied. It was originally used as a calibration and correction tool for light sources [3,4]. The al-

[^0]gorithm was implemented and benchmarked with simulated data and could then be tested with measurements.

## ALGORITHM FOR DATA ANALYSIS

The main principle of an EDM measurement using a storage ring is based on an initial polarization in the horizontal plane [5]. An EDM leads to a vertical polarization buildup which is directly proportional to the size of the EDM. To prevent a complete averaging out of the signal in a pure magnetic ring [6-8], an RF Wien filter was implemented in COSY [9]. For a more realistic description of the COSY experiment, algorithms are implemented to fit the simulation model to the conditions in the storage ring by varying selected machine parameters.

## Spin Dynamics and Invariant Spin Axis

The spin motion in presence of electromagnetic fields is described by the Thomas-BMT equation [10]. It is critical for data analysis to know the orientation of the spin rotation axis (invariant spin axis) for the reference particle with vanishing EDM at the position of the RF Wien filter. Especially its radial component is unknown and spin-tracking simulations can be used to determine this missing value [11].


Figure 1: Spin tracking simulations using the reference particle moving on the closed orbit. The invariant spin axis is the vector perpendicular to the plane that is defined by the spin wheel. For each turn $i$, the normal vector $\vec{n}_{i}$ is calculated. The average normal vector $\langle\vec{n}\rangle$ is then considered as the invariant spin axis.

To identify the location of the invariant spin axis and the spin tune, the reference particle is tracked for $n$ turns and the spin orientation is recorded at the position of interest after each turn in the storage ring. Since the particle moves exactly on the closed orbit, a well-defined spin wheel is the result of the tracking simulation, as shown in Fig. 1. The
tilt of the invariant spin axis depends on the EDM value and other systematic effects such as magnet misalignments. The spin tune describes the spin precession per turn in the plane perpendicular to the invariant spin axis.

## Model Fitting

Due to unavoidable misalignments of magnets in storage rings, the closed orbit deviates from the ideal trajectory defined by all magnetic centers. If field imperfections, magnet misalignments and other external influences on the beam are considered, the closed orbit changes and its deviations become larger. To ensure an orbit as close as possible to the desired target orbit, an orbit correction system is needed. This requires adding additional correction dipoles (steerers) to control the beam in the vertical and horizontal direction. To further improve the model while varying all parameter of interest simultaneously, LOCO is used. The method is based on a measured ORM, which is compared to the ORM of the simulation model. The measured ORM contains any information about the machine optics and the focusing structure of the storage ring. It is therefore a suitable tool to determine the level of agreement between model and the real machine and, moreover, to improve the simulation model by adjusting the ORM.

In general, the list of parameters can be of any size. The machine parameters contain unknown errors relative to their design values and the ORM analysis algorithm is supposed to find them. Under the assumption of small influences of higher order, the Taylor series of the orbit response vector in linear order reads [12]:

$$
\begin{equation*}
\vec{R}(\vec{V}) \approx \vec{R}\left(\vec{V}_{0}\right)+R^{\prime}\left(\vec{V}_{0}\right)\left(\vec{V}-\vec{V}_{0}\right) . \tag{1}
\end{equation*}
$$

Here $\vec{V}_{0}$ denotes the initial estimate of the machine parameters, usually given by the simulation model settings, and $\vec{R}(\vec{V})$ describes the measured orbit response vector. Thus, $\vec{R}\left(\vec{V}_{0}\right)$ is the model orbit response vector based on the initial estimate of the machine parameters. The difference of the simulated orbit response vector and the measured one can be described by a linear Jacobian matrix $R^{\prime}\left(\vec{V}_{0}\right)$, also denoted as $\mathbf{J}$ :

$$
\begin{equation*}
\Delta \vec{R}=\vec{R}(\vec{V})^{\text {meas. }}-\vec{R}\left(\vec{V}_{0}\right)^{\text {model }}=\mathrm{J}\left(\vec{V}-\vec{V}_{0}\right) \tag{2}
\end{equation*}
$$

The goal of the LOCO algorithm is to decrease the difference of the orbit response vectors $\Delta \vec{R}$. Therefore, a $\chi^{2-}$ minimization is performed where the $\chi^{2}$ function is defined as the squared sum of the difference of the ORM entries:

$$
\begin{equation*}
\chi^{2}=\frac{\left|R_{i, j}^{\text {meas. }}-R_{i, j}^{\text {model }}\right|^{2}}{\sigma_{i, j}} \tag{3}
\end{equation*}
$$

The implemented algorithm to fit the model to the measured data works as follows [12]:

- Set the first estimate for the real machine parameters $\vec{V}_{0}$ and apply them to the simulation model.
- Determine the ORM of the simulation model $\mathrm{R}^{\text {model }}$ and reformat it into an orbit response vector $\vec{R}^{\text {model }}$.
- Measure the ORM in the real machine by varying the steerer strengths and observing the changes of the BPM readings. This has to be done only once.
- Compute the difference of the orbit response vectors $\Delta \vec{R}$.
- Compute the Jacobian matrix $\mathbf{J}$ : Vary the machine parameters in two directions and observe the changes in $\Delta \vec{R}$. Perform a linear $\Delta \vec{R}$ for each entry of $\Delta \vec{R}$ and each machine parameter.
- Determine the pseudo-inverse of the Jacobian matrix by using SVD (Single Value Decomposition).
- Calculate a new set of machine parameter estimates using $\Delta \vec{R}^{\text {new }}=\Delta \vec{R}^{\text {old }}+\Delta \vec{R}$.

After one cycle, the result of the current iteration is used to set the new machine parameter estimate for the next iteration. Performing several iterations can lead to a better fitting result, since a single iteration cannot account for non-linear effects.


Figure 2: Working principle of the ORM analysis algorithm based on the LOCO method. All parts that are purely related to the simulation are marked with Bmad. The ORM has to be measured only once independently of the number of fitting iterations that are performed.

For better understanding, the procedure is sketched in Fig. 2. Several iterations of the algorithm can further improve the fitting results, since the Jacobian matrix entries are based on linear fits. Therefore, non-linear effects cannot be resolved within one iteration. After each iteration, the value of $\chi^{2}$ is calculated and the algorithm stops when a chosen threshold value is reached.

## APPLYING THE ALGORITHM TO MEASUREMENTS

To test the algorithm and to ensure accuracy of the simulation, the quadrupole family settings were randomly distorted using a Gaussian distribution around zero with a width of $1 \%$. The distorted lattice takes the role of the measured lattice and the undisturbed model is the starting point of the algorithm. The algorithm needs three iterations to reproduce the undisturbed lattice with an accuracy of $\chi^{2}=0.0009$ [13].

For a first application of the LOCO algorithm within the Bmad framework, the orbit response matrix was measured in

October 2019. For this purpose, the settings of the steerers were changed one after another and the corresponding orbit was measured. The set values of the quadrupole and sextupole currents were translated into magnetic field strength and applied to the Bmad model. The difference between the simulated and measured ORM is also shown in Fig. 3 at the beginning and the end of the fitting procedure [13].


Figure 3: The initial (left) and final (right) orbit reponse matrix is shown. The magnet settings during the measurement were applied to the Bmad model and the initial difference matrix serves as the starting point for the LOCO algorithm.

The LOCO algorithm was set up to change the quadrupole gradients as well as the positions in each direction. In addition to the LOCO fitting, the simulated orbits are adjusted. Five iterations were performed and the final orbits are displayed in Fig. 4.


Figure 4: Closed orbits in horizontal and vertical direction after fitting the simulated to the measured orbits by changing the steerer kicks using the orbit matching method.

As can be seen, very good agreement was obtained after applying the algorithm. In addition to the horizontal and vertical orbits, betatron function measurements were also performed at specific quadrupoles to validate the results of the model fitting. The strength of one quadrupole magnet was changed and the resulting tune change was measured. Both the simulated and measured betatron function values are shown in Fig. 5 for the utilized quadrupoles.

The error bars for the measured values are relatively large, since each quadrupole strength was changed only once upwards and downwards. A more precise value could be obtained if more and smaller steps were taken during a longer beam time. Nevertheless, the comparison of simulation and measurement shows a large overlap, validating the LOCO result. The simulated and measured betatron $\left(Q_{x}, Q_{y}\right)$ tunes agree within a $2 \sigma$ range (see Table 1).

This table also compares simulated and measure spin tunes $v_{s}$ and horizontal invariant spin axis components
$v_{x}, v_{s}$. One must be cautious when comparing the measured invariant spin axis and spin tune with the presented simulation results, as the model optimization was performed based on data taken in October 2019, when no polarized beam was available. The spin-related data were taken prior to the beam-based alignment procedure in an earlier beam time when slightly different magnet settings of COSY were use and the orbit response matrix was not measured.


Figure 5: Comparison of simulated and measured horizontal (left) and vertical (right) betatron functions. The betatron functions are measured at the position of quadrupoles by varying the quadrupole strength and observing the corresponding betatron tune change.

Table 1: Comparison of Simulation and Measurement.

|  | Simulation | Measurement |
| :---: | :---: | :---: |
| $Q_{x}$ | 3,58210 | $3,57119 \pm 0,00603$ |
| $Q_{y}$ | 3,59430 | $3,58641 \pm 0,00396$ |
| $v_{s}$ | 0,16143665 | 0,16099023 |
| $n_{x}$ | $-0,003122$ | $-0,00348$ |
| $n_{s}$ | 0,0009970 | 0,00557 |

## CONCLUSION AND OUTLOOK

The simulation model of COSY was successfully extended towards a more realistic description of the machine by implementing a sophisticated fitting algorithm based on orbit response matrix measurements and adding several systematic effects. The fitting procedure can be used to achieve a deeper understanding of measurements and can easily be extended to other machine parameters. The consideration of measured magnet misalignments and their measurement errors results in a minimum resolvable EDM of roughly $1.5 \cdot 10^{-19} e \cdot \mathrm{~cm}$ [13]. The results confirm additional magnet displacements and lead overall to a significantly increased agreement between simulation model and measurement.

In the future, beam and polarization measurements will have to be performed with the same setup of COSY in order to actually be able to compare the results of the simulation model with measurements.

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