# DETERMINATION OF THE INVARIANT SPIN AXIS IN A COSY MODEL USING BMAD 

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#### Abstract

The matter-antimatter asymmetry might be understood by investigating the EDM (Electric Dipole Moment) of elementary particles. A permanent EDM of a subatomic particle violates time reversal and parity symmetry at the same time and would be, with the currently achievable experimental accuracy, an indication for further CP violation than established in the SM (Standard Model of Particle Physics). The JEDI-Collaboration (Jülich Electric Dipole moment Investigations) in Jülich has performed a direct EDM measurement for deuterons with the so-called precursor experiments at the storage ring COSY (COoler SYnchrotron) by measuring the orientation of the ISA (Invariant Spin Axis). In order to interpret the measured data and to disentangle a potential EDM signal from systematic effects, spin tracking simulations in an accurate simulation model of COSY are needed. Therefore a model of COSY was implemented using the software library Bmad. Systematic effects were considered by including element misalignments, effective dipole shortening, longitudinal fields and steerer kicks. These effects rotate the ISA in addition to the EDM and have to be analyzed and understood. The most recent spin tracking results as well as the methods to find the ISA are presented in this paper.


## INTRODUCTION

In order to explain the matter-antimatter asymmetry in the Universe, CP-violating processes beyond those already known are needed [1]. A permanent non-vanishing EDM of a subatomic particle is a candidate for such a process, since it is a source of P and T violation leading to CP violation, assuming the CPT-theorem holds. An EDM is by far smaller than MDM (Magnetic Dipole Moment), but also predicted by the SM. As there is no direct measurement of the proton or deuteron EDM available so far, the JEDI collaboration has performed a first direct measurement of the deuteron EDM at the COSY accelerator facility in Jülich. Therefore the so-called precursor experiments were carried out in November 2018 and March 2021. A storage ring allows a direct measurement of an EDM, as the interaction of particle's spin with electromagnetic field results in spin rotations defined by EDM and MDM contribution [2, 3]. This paper describes the methodology used to determine the EDM signal and compares the experimental results with the simulation results. This is necessary in order to separate

[^0]systematic effects caused by misaligned elements, steerer contributions, unknown longitudinal fields, etc., from a potential EDM signal [4]. The software tool used to study and benchmark the deuteron EDM effect in a simulation is the Fortran based library Bmad [5].

## METHODOLOGY

The storage ring COSY located at Forschungszentrum Jülich is an ideal starting point for a first EDM measurement of charged particles as it provides phase-space cooled polarized and unpolarized deuteron and proton beams up to a momentum of $3.70 \mathrm{GeV} / \mathrm{c}$. The central idea of the experiment, performed at a momentum of $0.97 \mathrm{GeV} / \mathrm{c}$, is to measure the impact of the EDM on the beam polarization vector. For this purpose, a vertically polarized deuteron beam is injected into the ring, accelerated, bunched and cooled. The beam polarization is measured with a polarimeter by scattering a fraction of the particles on a carbon target. The asymmetry in the scattering rate indicates the polarization orientation [6]. A solenoid is used to rotate the polarization into the radial plane (= accelerator plane), where the polarization vector starts to rotate around the so-called ISA according to the Thomas-BMT equation [2]. In an ideal accelerator without an EDM and no systematic effects, the ISA would be parallel to the vertical magnetic dipole fields around the whole ring. However, the existence of an EDM of magnitude $\eta$ tilts the ISA in radial direction by an angle $\phi_{\mathrm{EDM}}=\arctan (\eta \beta / 2 G)$ as described by Eq. (1) and shown in Fig. 1.

$$
\vec{n}_{\mathrm{ISA}}=\left(\begin{array}{c}
\sin \phi_{\mathrm{EDM}}  \tag{1}\\
\cos \phi_{\mathrm{EDM}} \\
0
\end{array}\right) \approx\left(\begin{array}{c}
\phi_{\mathrm{EDM}} \\
1 \\
0
\end{array}\right) .
$$

Unfortunately ring systematics ( $\phi_{\text {Ring }}, \xi_{\text {Ring }}$ ) like misaligned magnets, etc. tilt the ISA in addition to the EDM. A solenoid can be used to compensate the long. tilt $\xi_{\text {Ring }}$ by an angle $\xi_{\text {Sol }}$. This is indicated in Eq. (2).

$$
\vec{n}_{\mathrm{ISA}} \approx\left(\begin{array}{c}
\phi_{\mathrm{EDM}}+\phi_{\mathrm{Ring}}  \tag{2}\\
1 \\
\xi_{\mathrm{Sol}}+\xi_{\mathrm{Ring}}
\end{array}\right)
$$

As an experiment is not capable to measure the beam polarization vector each turn with sufficient statistical accuracy, another way of determining the ISA was identified. The socalled RF (Radio-Frequency) Wien filter, a RF device with a radial electric $E_{x}^{\mathrm{WF}}$ and vertical magnetic field $B_{y}^{\mathrm{WF}}$ was implemented into COSY. While the orbit in the center of this device is not perturbated, as the fields are set up so that


Figure 1: A permanent EDM of magnitude $\eta$ tilts the ISA in radial direction $n_{x}$ by an angle $\phi_{\text {EDM }}$. The longitudinal direction $n_{z}$ is not affected by a permanent EDM. Here $n_{z}$ points in the moving direction of the particle beam, while $n_{x}$ is perpendicular to $n_{z}$ and shows into the accelerator plane.
the Lorentz force is zero, the particles receive a spin kick in the same direction each turn as the Wien filter is changing its fields on one of the harmonics $k$ of the spin precession frequency $v_{s}$. This way, the EDM signal can accumulate over time, resulting in a build-up of vertical polarization $P_{y}$ if the Wien filter runs on resonance $[7,8]$. Equation (3) and Eq. (4) are showing the adjustments of the Wien filter fields needed, to achieve this build-up.

$$
\begin{align*}
& E_{x}^{\mathrm{WF}}=E_{0} \cos \left(2 \pi f_{\mathrm{COSY}}\left|k+v_{s}\right|+\phi_{\text {rel }}\right)  \tag{3}\\
& B_{y}^{\mathrm{WF}}=B_{0} \cos \left(2 \pi f_{\mathrm{COSY}}\left|k+v_{s}\right|+\phi_{\text {rel }}\right) \tag{4}
\end{align*}
$$

The build-up $\epsilon \propto \frac{d}{d t} P_{y}(t)$ depends on the orientation of ISA $\vec{n}_{\text {ISA }}$ to Wien filter fields $\vec{n}_{\mathrm{WF}}$ and the compensation of long. fields via solenoidal fields. To compensate the EDM signal and radial systematics $\phi_{\text {Ring }}$ the Wien filter is rotated by an angle $\phi_{\mathrm{WF}}$ around beam. This is described by Eq. (5).

$$
\begin{align*}
\epsilon^{2} & \propto\left|\vec{n}_{\mathrm{WF}} \times \vec{n}_{\mathrm{ISA}}\right|^{2}=\left|\left(\begin{array}{c}
\phi_{\mathrm{WF}} \\
1 \\
0
\end{array}\right) \times\left(\begin{array}{c}
\phi_{\mathrm{EDM}}+\phi_{\text {Ring }} \\
1 \\
\xi_{\mathrm{Sol}}+\xi_{\text {Ring }}
\end{array}\right)\right|^{2} \\
& =\left(\left(\phi_{\mathrm{EDM}}+\phi_{\mathrm{Ring}}\right)-\phi_{\mathrm{WF}}\right)^{2}+\left(\xi_{\mathrm{Sol}}+\xi_{\mathrm{Ring}}\right)^{2} . \tag{5}
\end{align*}
$$

By plotting the build-up $\epsilon$ in dependency of $\phi_{\mathrm{WF}}$ and $\xi_{\text {Sol }}$ one receives an EDM resonance map. Its minimum indicates the point where all systematic effects in the ring as well as a potential EDM signal are compensated by the solenoid and the Wien filter. In addition, it is also possible to determine the long. ISA by observing the spin tune changes $\Delta v_{s}$ in dependency of the strength $B_{\text {Sol }}$ of the solenoid [9]. This way one can determine the so-called solenoid calibration factor $k_{\text {Sol }}$, giving the relation between the solenoid field $B_{\text {Sol }}$ and the spin kick $\xi_{\text {Sol }}=k_{\text {Sol }} \cdot B_{\text {Sol }}$, as well as the projection of the long. ISA $c_{\text {Sol }}$. This is sketched by Eq. 6. It has to be mentioned that $c_{\text {Sol }}$ is different from $\xi_{\text {Sol }}+\xi_{\text {Ring }}$. This will be shown and discussed later.

$$
\begin{equation*}
\Delta v_{s}=-\frac{1}{\pi}\left(\cot \left(\pi v_{s}\right)\left(\cos \frac{k B}{2}-1\right)\right)-c \sin \frac{k B}{2} \tag{6}
\end{equation*}
$$

Both methods have been used in an experiment and in the simulation to determine the ISA for a lattice containing all known systematic effects.

## RESULTS OF THE SIMULATION

## EDM Resonance Map

A Bmad model of COSY was created according to the parameters given in Table 1.

Table 1: Relevant parameters for the deuteron EDM experiment at COSY.

| Quantity | Unit | Size |
| :--- | :--- | :--- |
| Momentum | $p$ | $0.970 \mathrm{GeV} / \mathrm{c}$ |
| Velocity | $\beta$ | 0.459 |
| Lorentz Factor | $\gamma$ | 1.126 |
| Gyromagnetic Anomaly | G | -0.143 |
| Rev. Frequency | $f_{\mathrm{COSY}}$ | 752 kHz |
| Spin Prec. Frequency | $f_{\mathrm{COSY}} \cdot v_{s}$ | 121 kHz |
| Wien Filter Frequency | $f_{\mathrm{WF}}$ | 873 kHz |

To simulate the experiment used to determine the orientation of the ISA all known magnet strengths, misalignments, multipol fields and other systematic effects were included in the model. Using the resonant Wien filter method describe above, the point of no build-up of vertical polarization was identified. The result is displayed in Fig. 2 and compared to the experimental results in Table 2.


Figure 2: Simulated EDM resonance map including all known systematic effects and assuming no EDM.

Table 2: Results of the experiment compared with the corresponding simulation results.

| Method | $\phi_{\mathrm{WF}}$ | $\xi_{\text {Sol }}$ |
| :--- | :--- | :--- |
| Experiment [10] | $-2.91(8) \mathrm{mrad}$ | $-5.22(7) \mathrm{mrad}$ |
| Simulation | $-0.1119(3) \mathrm{mrad}$ | $-0.3697(3) \mathrm{mrad}$ |

Even though all known parameters and systematic effects have been included the results of experiment and simulation
differ by one order of magnitude. So far it is unclear where this difference exactly comes from. One reason could be the inaccurate model, as it does not directly reflect the orbit and tune, even though all parameters and systematic effects are included. Another reason could be the need of correction factors as Eq. (5) and Eq. (6) are only valid for the beam passing through the devices without any angle. This will be discussed in the next section.

## Solenoid Calibration and Correction Factors

As explained earlier the spin tune changes due to the solenoidal strength can be used to determine the long. ISA $n_{\text {Sol, } z}$ at the position of the solenoid. This method is shown in Fig 3.


Figure 3: Calibration of the 2 MeV Solenoid for a lattice with all known systematic effects included.

For an ideal lattice, the fit parameter $c_{\text {Sol }}$ coincides with the long. ISA at the position of the solenoid $n_{\text {Sol, } z}$. In case the lattice is distorted, a correction to the projection has to be applied. It can be shown that this correction is dominated by the vertical momentum when passing the solenoid. This is shown in Fig 4. There is evidence that a similar correction has to be applied to the EDM resonance map data. This is currently under investigation.

## CONCLUSION

In this paper it was shown how the EDM and ring imperfections affects the ISA of a polarized particle beam. A measurement technique was introduced which uses a socalled RF Wien filter to accumulate an EDM signal over time to measure a build-up of vertical polarization. A solenoid was used to compensate the long. tilt of the ISA due to ring imperfection. This way an EDM resonance map can be created. Measured and simulated data have been compared and show a difference of one order of magnitude. There is evidence that improvements of the model, as well as identifying correction factor when passing through the Wien filter and the solenoid off axis will lead to a much better agreement between experimental results and simulated results. These objectives will be addressed in an upcoming beam time within the next months.


Figure 4: Relation between difference of the projection of longitudinal ISA and the actual longitudinal ISA as a function of the vertical momentum when the beam passes the solenoid.

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