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Method to evaluate systematic uncertainties due to magnet misalignments in electric dipole moment measurements using a storage ring

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Abstract

Various scenarios of measurements of electric dipole moment (EDM) of light hadrons with the use of a storage ring were proposed. Most of these methods are based on the measurement of the vertical spin component for an initially horizontal polarized beam. Since the expected EDM effect is very small, one has to pay attention to various sources of systematic uncertainties. One of the most important sources are misalignments of the magnets forming the storage ring lattice, which may produce an effect that mimics an EDM. This false signal could be much larger than the expected EDM signal, even for very small magnet misalignments. This paper describes a novel method for the determination of the contribution of magnets misalignments to the expected EDM signal. It is shown that the magnitude of this effect could be estimated via a Fourier analysis of the time-dependent vertical polarization. This could be achieved by sampling the vertical polarization with a frequency larger than the beam revolution frequency, which corresponds to polarization measurements in at least two positions in the storage ring. The presented method can be applied to any scenario proposed for EDM measurements using a storage ring.

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Keywords: EDM, spin precession, storage ring, systematic uncertainties

(Some figures may appear in colour only in the online journal)

1. Introduction

A search for an electric dipole moment (EDM) of charged particles with the use of a storage ring was proposed [1]. The idea of such a measurement is the observation of a vertical polarization component being induced by an EDM for an initially horizontally polarized beam. Various considered experimental scenarios [2–7] are based on measurement of the vertical polarization build-up with the aimed EDM limit of 10^{-29} e · cm for protons and deuterons. Future experiments searching for proton and deuteron EDM are planned at CERN within CPEDM collaboration [8].

The required accuracy can be achieved when systematic uncertainties are kept under control. In [1] the systematic uncertainty due to spurious electromagnetic fields was considered. To reduce this uncertainty, it has been proposed to use two beams moving clockwise and counterclockwise, than the difference of observed signals cancels many systematic uncertainties. Recently the importance of some selected lattice imperfections was discussed for the storage ring frozen spin concept, and the formalism describing spin evolution was developed [9]. In the present paper, the method of experimental determination of lattice imperfections effects on spin evolution is discussed. The presented method could be applied in the first direct EDM measurement at COSY and also in other scenarios of EDM limit determination as e.g. proposed prototype ring [10, 11], 'Spin Whell' [6] and 'Quasi Frozen' methods [7].

The JEDI collaboration (Jülich electric dipole moments investigations) [12, 13] has been formed to demonstrate the feasibility of the EDM measurement using a storage ring and to perform the necessary developments towards the design of a dedicated storage ring. More comprehensive information on the physics case, precursor experiment and future plans is presented in [11]. A set of rich information was already collected by JEDI collaboration [14–21], further developments are ongoing [11, 22] and a precursor experiment is planned. While the statistical accuracy to achieve sensitivity of 10^{-21} e · cm for the EDM could be reached within a quite short measurement, the major issue is a control on the systematic uncertainties. The most crucial source of systematic uncertainties are the storage ring imperfections due to magnet misalignments (displacements and tilts). They introduce unwanted horizontal magnetic fields, and the magnetic dipole moment interaction with these fields induce exactly the same effect as expected due to EDM interaction with the main vertical magnetic guiding field. An additional uncertainty originates from the positioning accuracy of the beam correction magnets used to control the beam orbit. Therefore, a method allowing the separation of these two effects is obligatory for the determination of the sensitivity in EDM measurements. An attempt to determine the angular orientation of the stable spin axis was carried out by the JEDI collaboration [18]. This method is based on the spin tune response to an artificial longitudinal magnetic field and is valid under the assumption of ideally placed beam position monitors (BPMs) and steerer magnets. Here a novel method (originally first presented in [11]) of controlling the systematics in the EDM measurement is proposed, which allows simultaneous measurement of magnet misalignments and EDM induced effects. This new method could be applied only while two polarimeters are available, which should be located within a half distance of the storage ring length. Up to now, the former WASA forward detector [23] located in one straight section was used for polarization measurements in COSY. Recently, a new polarimeter [24, 25] was installed in the second opposite straight section. These two polarimeters will allow to apply the proposed method to determine the effects of magnets misalignment in precursor EDM experiment at COSY storage ring.

A simple analytical model is presented in section 2, while numerical calculations with use of the BMAD software library [26] are presented in section 3. In this section, detailed analysis of dipole and quadrupole misalignment effects on vertical spin component $s_y(t)$ is presented. Orbit correction method minimizing misalignment effects and the beam phase space influence on the results are also discussed. Final results of the proposed method are discussed and conclusions are presented in section 4.

2. Simple analytical model

Analytical calculations were performed with the Mathematica software [27] with the use of the method outlined in [28]. The standard coordinate system is used with $\hat{x}$ pointing radially, $\hat{y}$ perpendicular to the trajectory plane and $\hat{z}$ along with the particle momentum. A storage ring with drift spaces and magnetic dipoles with field B_0 and bending radius equal to ρ was considered. A shape of the vertical $B_{\mathrm{V}}(t)$ and horizontal $B_{\mathrm{H}}(t)$ magnetic field components (in the time domain) along the particle trajectory are represented by the Fourier series

$$B_{\mathrm{H}}(t) = \frac{H_0^c}{2} + \sum_{j=1}^{\infty} H_j^c \cos\, j\omega_o t + \sum_{j=1}^{\infty} H_j^s \sin\, j\omega_o t, \tag{1}$$

$$B_{\mathrm{V}}(t) = \frac{V_0^c}{2} + \sum_{j=1}^{\infty} V_j^c \cos\, j\omega_o t + \sum_{j=1}^{\infty} V_j^s \sin\, j\omega_o t, \tag{2}$$

where $\omega_o = \beta c V_0^c/(2\rho)$ is the orbital frequency for the particle moving on closed orbit, βc is the particle velocity and H_i^c, H_i^s, V_i^c and V_i^s are the Fourier coefficients describing horizontal and vertical magnetic field distribution. The spin precession frequency is $\omega_s = \gamma\omega_o(g-2)/2$, where γ is the Lorentz factor and g is the particle g-factor.

First the Bargmann–Michel–Telegdi (BMT) equation [29] was solved neglecting small effects due to the EDM and horizontal magnetic fields induced by magnets misalignments. Then the solution for horizontal spin component along with the particle momentum is described by:

$$s_z(t) = \sum_{k=0}^{\infty} \cos\left(\omega_s t + k\frac{\pi}{2}\right)\frac{\Phi^k}{k!}, \tag{3}$$

$$\Phi = \frac{2\omega_s}{\omega_o V_0^c}\sum_{n=1}^{\infty}\frac{1}{n}(V_n^s - V_n^s \cos\, n\omega_o t + V_n^c \sin\, n\omega_o t). \tag{4}$$

Using this solution the time dependence of the vertical spin component is calculated for misalignment $s_y^m(t)$ and EDM $s_y^e(t)$ effects

$$s_y^m(t) = \omega_s \int_0^t B_{\mathrm{H}}(t')s_z(t')\mathrm{d}t', \tag{5}$$

$$s_y^e(t) = \omega_e \int_0^t B_{\mathrm{V}}(t')s_z(t')\mathrm{d}t', \tag{6}$$

where $\omega_e = D\beta c B_0/\hbar$ and D denotes the EDM value.

The leading-order term for the vertical spin component due to magnets misalignments and EDM are obtained for $k = 0$ in equation (3), and their time dependence is described by:

$$s_y^m(t) = \frac{\sin\,\omega_s t}{2} H_0^c + \frac{\omega_s}{2} \sum_{i=1}^{\infty} \left[\left(\frac{\sin(i\omega_o - \omega_s)t}{i\omega_o - \omega_s} + \frac{\sin(i\omega_o + \omega_s)t}{i\omega_o + \omega_s} \right) H_i^c \right.$$
$$\left. - \left(\frac{\cos(i\omega_o - \omega_s)t}{i\omega_o - \omega_s} + \frac{\cos(i\omega_o + \omega_s)t}{i\omega_o + \omega_s} \right) H_i^s \right], \tag{7}$$

$$s_y^e(t) = \frac{\omega_e\,\sin\,\omega_s t}{2\omega_s} V_0^c + \frac{\omega_e}{2} \sum_{i=1}^{\infty} \left[\left(\frac{\sin(i\omega_o - \omega_s)t}{i\omega_o - \omega_s} + \frac{\sin(i\omega_o + \omega_s)t}{i\omega_o + \omega_s} \right) V_i^c \right.$$
$$\left. - \left(\frac{\cos(i\omega_o - \omega_s)t}{i\omega_o - \omega_s} + \frac{\cos(i\omega_o + \omega_s)t}{i\omega_o + \omega_s} \right) V_i^s \right]. \tag{8}$$

The time dependence of vertical spin component for magnets misalignments and EDM induced vertical polarization is very similar and differ only in the normalizing factor and Fourier coefficients describing the horizontal and vertical magnetic field. Therefore the misalignment and EDM effects are indistinguishable and their contributions add up coherently, giving a common vertical spin component.

From the presented model, it is easy to deduce that the Fourier analysis of the harmonic time dependence of the vertical spin components $s_y^e(t)$ and $s_y^m(t)$ should give maxima of the Fourier amplitudes $F(\omega_s)$ at a frequency of $\omega = \omega_s$ and for $F(\omega_o \pm \omega_s)$ at a frequency of $\omega = \omega_o \pm \omega_s$ (and at higher frequencies $\omega = 2\omega_o \pm \omega_s$, etc). These amplitudes are proportional to Fourier coefficients describing the horizontal and vertical components of the magnetic field.

In order to find the differences between effects induced by magnet misalignments and the EDM, it is worth checking the first terms in equations (7) and (8). The storage rings usually have a rotational symmetry (at least of order 2), therefore for undisturbed vertical fields odd Fourier coefficients are equal to zero. For small rotation angles δ_m the vertical field of each dipole magnet scales with $\cos\,\delta_m \approx 1 - \delta_m^2/2$, while the horizontal field scales with $\sin\delta_m \approx \delta_m$. Therefore Fourier coefficients for vertical fields are only slightly influenced by magnets rotation, then only even coefficients are large, while odd coefficients are by a large factor smaller. In case of horizontal fields the Fourier coefficients take random values being a factor of a few orders of magnitude smaller than the vertical coefficients.

As an example, the Fourier coefficients were calculated for the case of COSY storage ring with 24 dipole magnets being randomly rotated (Gaussian distribution with mean value equal to zero and standard deviation $\sigma = 5$ mrad) around the beam axis. The Fourier coefficient V_0^c is dominating, while all V_i^s, H_0^c and H_0^s coefficients are small.

This general behaviour of Fourier coefficients determines the time dependence of the vertical spin component induced by EDM and misalignment effects as shown in figure 1. It is seen that $s_y^e(t)$ is completely dominated by the term containing the V_0^c coefficient and is mainly described by the function $\sin\omega_s t$. On other hand, harmonic functions with larger frequencies contribute strongly to the time dependence of $s_y^m(t)$. Therefore, a Fourier analysis of the vertical spin component should deliver information about the magnets misalignments effect contribution to a vertical spin component. Experimentally this information could be extracted by sampling of the vertical spin component with a frequency of at least $2\omega_o$. Such sampling could be achieved with the use of two polarimeters located in the storage ring at some distance.

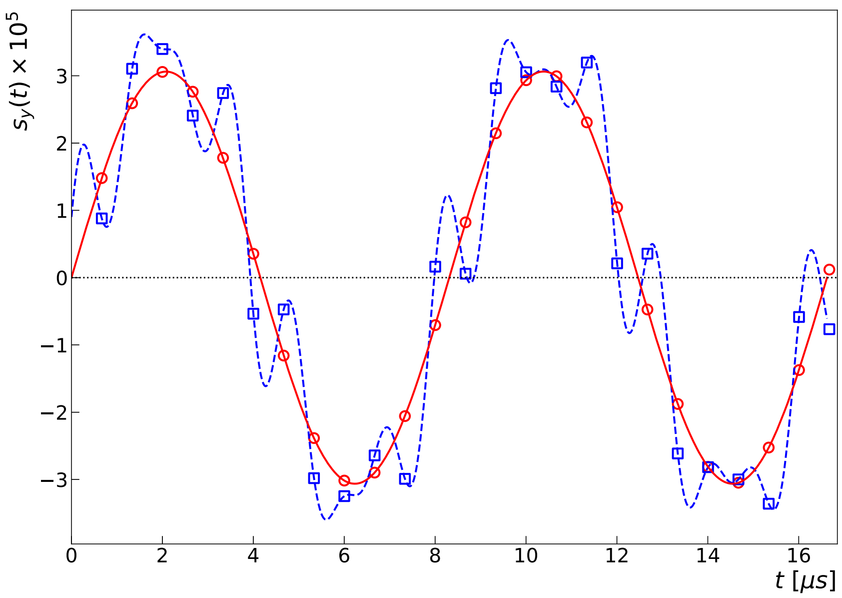

Figure 1. Vertical spin component time dependence: $s_y^m(t)$ induced by magnet misalignments (dashed blue line) and $s_y^e(t)$ induced by an EDM of $D = 10^{-21}$ e · cm (solid red line). Symbols denote the sampling results of the vertical spin component with the frequency set to $2\omega_o$—blue squares for misalignment and red circles for EDM effects. The amplitude of the misalignment effect was artificially scaled for easy comparison with the EDM effect.

3. Numerical calculations of EDM and misalignment effects

A detailed analysis of the spin evolution was performed with use of the BMAD software library [26]. It was possible to include misalignments for all magnets, studying the phase space effect and implement an orbit correction system based on kicker magnets. All these extensions discussed in the present section are important for understanding their influence on the finally obtained EDM value limit.

3.1. Quadrupole and dipole magnets misalignment

First, it was necessary to check which of the misalignments gives a significant effect on the vertical spin component. BMAD simulations were performed varying individually dipole and quadrupole magnet translations and rotations. In all cases, the magnet positions (separately for x, y and z directions) were changed randomly with a Gaussian distribution with a mean value equal to zero and a standard deviation of $\sigma = 1$ mm. Similarly, magnet rotations (separately around x, y and z axes) were changed randomly with a Gaussian distribution with a mean value equal to zero and a standard deviation of $\sigma = 1$ mrad. Additionally, combinations of translations and rotations were analysed separately for dipole and quadrupole magnets, as well as for all magnets at a time. For each setting, the Fourier analysis of the induced vertical spin component $s_y(t)$ was performed. For each case, the Fourier amplitude spectra exhibit pattern shown in figure 2. For comparison the Fourier amplitude spectrum for nonzero EDM only is also shown in this figure. For magnets misalignments the Fourier amplitude is peaked at ω_s—spin precession frequency and at $\omega_o - \omega_s$ frequency (ω_o is beam revolution frequency).

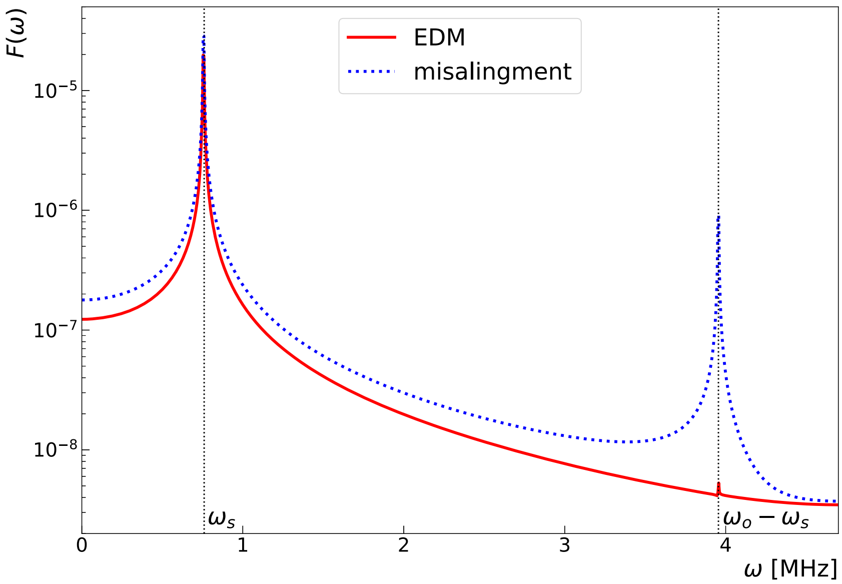

Figure 2. Fourier amplitude $F(\omega)$ for an EDM of $D = 10^{-21}$ e · cm (solid red line) and misalignment (dotted blue line) induced vertical polarization for a sampling frequency of $2\omega_o$. The amplitude of the misalignment effect was artificially scaled for easy comparison with the EDM effect. Vertical dotted lines mark the Fourier amplitudes maxima at frequencies ω_s and $\omega_o - \omega_s$.

In case of nonzero EDM without misalignments effect the Fourier amplitude exhibit only one peak at ω_s frequency.

The resulting Fourier amplitudes $F(\omega_s)$ and $F(\omega_o - \omega_s)$ are presented in table 1. Clearly, quadrupole magnets misalignments play the most important role, since the total effect caused by all misalignments at a time is similar to that induced by quadrupole magnets only. In the case of dipole magnets, the largest contribution to $s_y(t)$ is due to rotations around y axis, while for quadrupole magnet misalignments the most important part are translations along the y axis. Misalignments effects contribute to $s_y(t)$ coherently since they are indistinguishable, therefore it is impossible to deconvolve $s_y(t)$ and obtain information about each misalignment type and each element. Moreover, misalignment and EDM effects are also indistinguishable, therefore it is possible to measure only their coherent sum.

3.2. Phase space effect

Every particle moves on individual phase-space ellipses. Therefore, particles occupying the whole six-dimensional phase space are moving along various trajectories experiencing therefore different magnetic fields. Hence, it is mandatory to check the effect of the emittance on $s_y(t)$ when misalignments are present. The calculations were performed for an initial transverse phase space covering a grid with $x = (0, \pm 1$ mm), $x' = (0, \pm 1$ mrad), $y = (0, \pm 1$ mm), $y' = (0, \pm 1$ mrad). For each nonzero phase space parameter, an additional peak emerges in the Fourier amplitude spectrum. They are due to betatron oscillations of the particles not moving on the central trajectory. The largest side peaks occur for non-zero y', however the Fourier amplitudes $F(\omega_s)$ and $F(\omega_o - \omega_s)$ change by about 1% when compared to amplitudes for the central trajectory. Sum of all grid phase space points reduce the amplitudes of side peaks, and

Table 1. Fourier amplitudes for frequencies ω_s and $\omega_o - \omega_s$ and for various magnets misalignments. The results are median values for 100 randomly distributed deviations for each type of misalignment.

	Fourier amplitudes			
	Dipole		Quadrupole	
Misalignment	$F(\omega_s)$	$F(\omega_o - \omega_s)$	$F(\omega_s)$	$F(\omega_o - \omega_s)$
Translation x	5×10^{-14}	5×10^{-15}	3×10^{-14}	5×10^{-15}
Translation y	2×10^{-14}	2×10^{-15}	2×10^{-3}	4×10^{-4}
Translation z	5×10^{-14}	4×10^{-15}	2×10^{-14}	4×10^{-16}
Rotation x	3×10^{-14}	2×10^{-15}	5×10^{-14}	2×10^{-15}
Rotation y	2×10^{-4}	5×10^{-5}	1×10^{-5}	4×10^{-6}
Rotation z	3×10^{-5}	1×10^{-5}	5×10^{-14}	2×10^{-16}
All	2×10^{-4}	5×10^{-5}	2×10^{-3}	4×10^{-4}

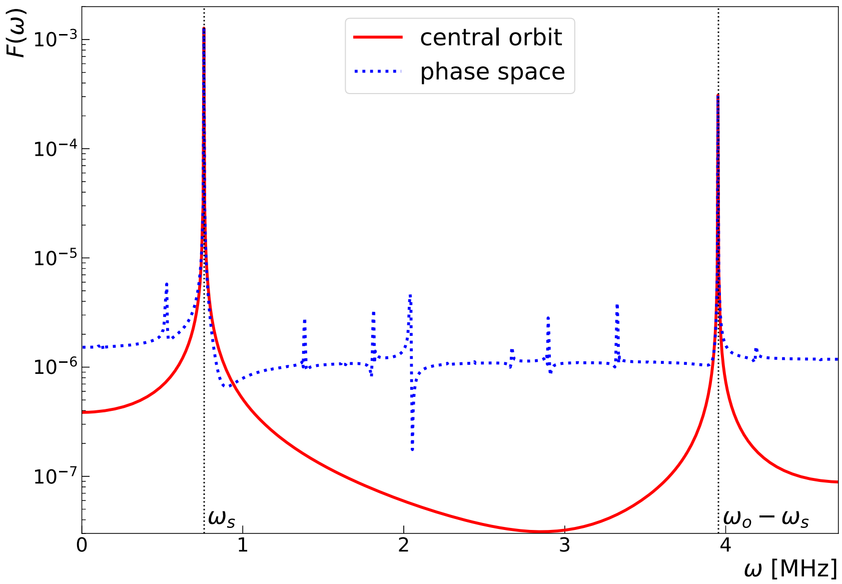

Figure 3. Comparison of the Fourier amplitude spectrum for the central trajectory (red solid line) and the sum of 81 points for the grid in the phase space (blue dotted line).

the Fourier amplitudes $F(\omega_s)$ and $F(\omega_o - \omega_s)$ are very similar to that for central trajectory, what is shown in figure 3.

3.3. Orbit correction

Due to magnet misalignment, the closed orbit deviates from the trajectory through all magnet centres. Therefore, a correction system is necessary to ensure an orbit that is as close as possible to the central (design) orbit. Such a system consists of additional dipole magnets (steerers) used to steer the beam in the vertical and horizontal direction. The BPMs located in many various places are necessary to measure the beam position. The beam correction relies on minimizing the RMS of the closed orbit by varying the steerer magnet strengths. The beam correction

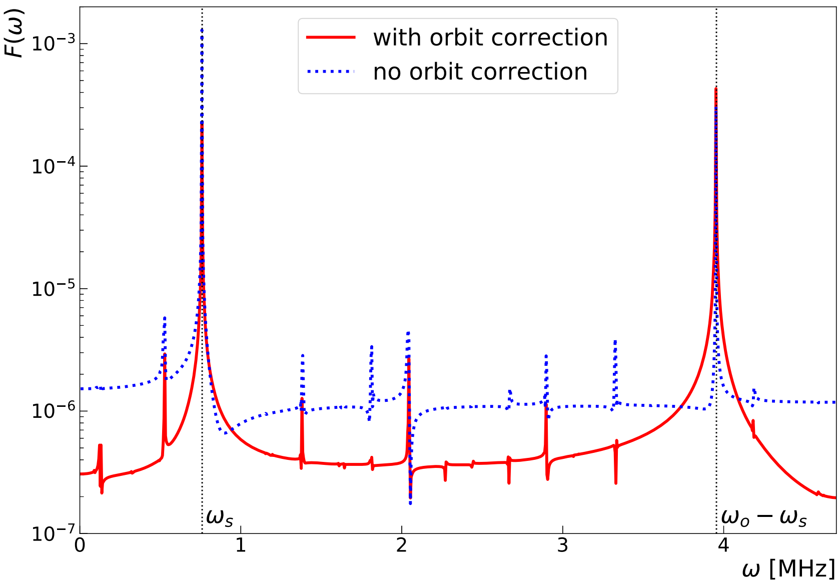

Figure 4. Comparison of the Fourier amplitude spectrum with applied orbit correction (red solid line) and without orbit correction (blue dotted line) for the misalignment effect only and for the full phase space.

system works with a certain accuracy because the steerers and BPM's are placed with some accuracy and some large parts of the orbit are not monitored. Nevertheless, application of such an orbit correction method leads to a reduction of the orbit deviation from the central orbit.

The orbit correction method [30] applied in the BMAD simulations for COSY is described in details in [31]. The whole COSY orbit correction system consists of 22 steerers and 30 BPM's in the horizontal plane and 19 steerers and 29 BPMs in the vertical plane. The orbit response due to perturbations induced by steerers is related to the steerer's magnetic field strength and can be summarized in the so-called orbit response matrix [30]. This matrix holds the information about how the closed orbit changes due to an individual steerer change and it can be used to determine the set of steerer strengths that lead to an improved orbit.

In addition to this orbit correction, a beam-based alignment procedure [32, 33] enabling to align all 56 magnetic centers of the quadrupole magnets with the use of the 31 BPM's was recently performed at COSY. With this procedure, the beam was aligned with respect to the centre of the quadrupole magnets with a precision of 40 μm, while the quadrupole magnets are aligned to a precision of 200 μm to the design beam axis.

The influence of the orbit correction on the Fourier amplitudes was investigated using kicker elements defined as dipole magnets. Therefore, kickers change not only the particle orbit, but also influence the spin precession. In this way, the orbit becomes more central and is less influenced by magnet misalignments. On the other hand, kickers introduce additional horizontal and vertical dipole fields what has an impact on spin rotation. It was found that dipole kickers also cure unwanted spin rotation due to misalignments. The influence of orbit correction on the vertical spin component $s_y(t)$ induced by misalignments is presented in figure 4 where Fourier amplitudes are compared. It can be seen that orbit correction reduces the Fourier amplitude $F(\omega_s)$ by a factor of about six, while the Fourier amplitude at $F(\omega_o - \omega_s)$ remains unchanged. Therefore, the orbit correction improves the precision of the EDM effect limit

Table 2. Fourier amplitudes for frequencies ω_s and $\omega_o - \omega_s$ and for selected EDM values. The measured COSY quadrupole magnets misalignments [33] were used and an orbit correction was applied.

EDM value [e · cm]	$10^4 \cdot F(\omega_s)$	$10^4 \cdot F(\omega_o - \omega_s)$
0	0.65	1.73
10^{-20}	0.84	1.73
5×10^{-20}	1.92	1.73
10^{-19}	3.41	1.73
5×10^{-19}	15.58	1.73
10^{-18}	30.83	1.73
5×10^{-18}	152.80	1.72
10^{-17}	305.45	1.71

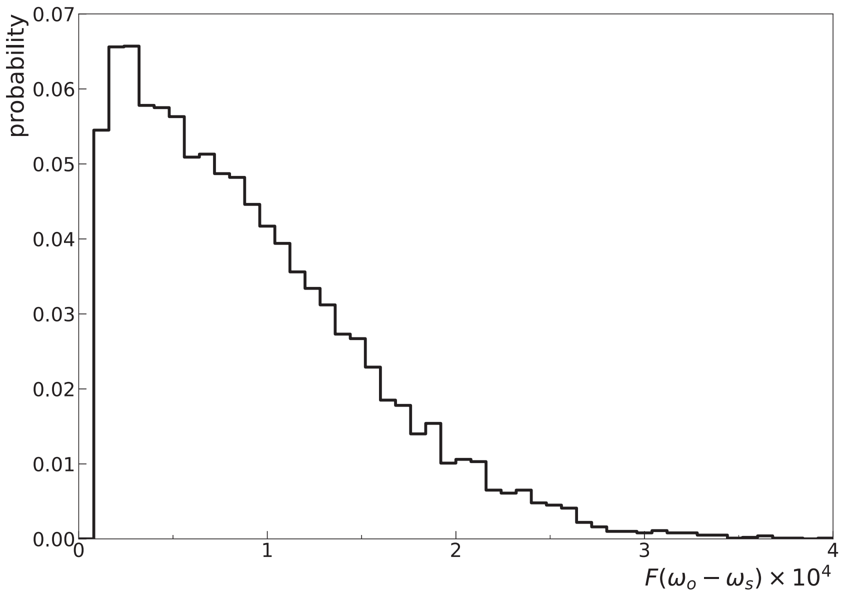

Figure 5. Probability distribution of Fourier amplitude $F(\omega_o - \omega_s)$. This amplitude is independent on EDM values.

determination, since the EDM effect contributes only to the Fourier amplitude $F(\omega_s)$, while the misalignment effect could be deduced from the Fourier amplitude $F(\omega_o - \omega_s)$.

3.4. Determination of the EDM limit by Fourier analysis

The sensitivity of the proposed method to the determination of the experimental limit for the EDM value was investigated via BMAD simulations. Calculations were performed for 10^4 randomly chosen sets of COSY magnet misalignments. For each quadrupole and dipole magnet all misalignments were considered: x, y and z offsets and tilts around the x, y and z axis. The misalignments were randomly generated with Gaussian distributions with mean value equal zero and standard deviations of $\sigma_x = \sigma_y = \sigma_y = 0.2$ mm for translations and $\sigma_{x'} = \sigma_{y'} = \sigma_{z'} = 0.2$ mrad for rotations. In subsection 3.2 it was shown that Fourier amplitudes are not affected by the beam phase space. Therefore calculations were performed on

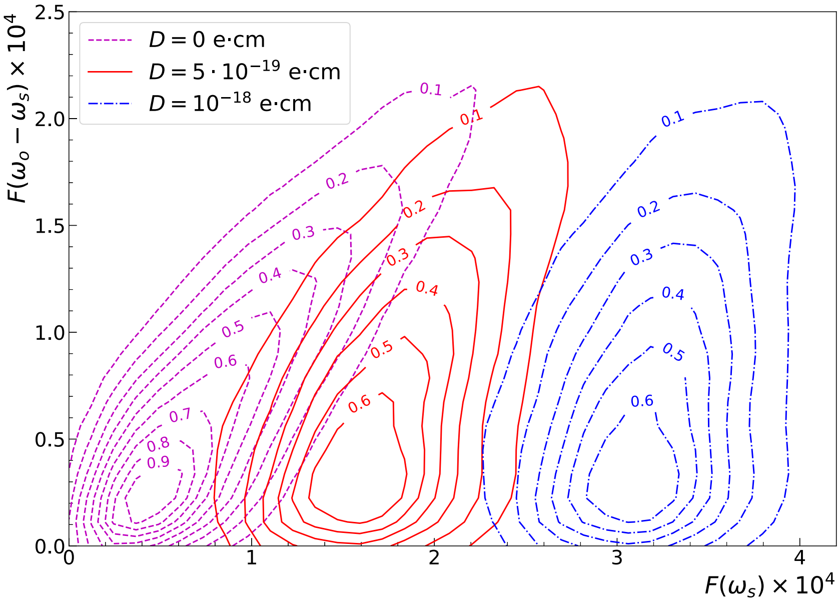

Figure 6. Correlation of Fourier amplitudes $F(\omega_s)$ and $F(\omega_o - \omega_s)$ for EDM values of $D = 0$ e · cm (magenta dashed lines), $D = 5 \times 10^{-19}$ e · cm (red solid lines) and $D = 10^{-18}$ e · cm (blue dot-dashed lines).

the closed orbit only. In each case, the orbit correction described in subsection 3.3 was applied. For each set of the misalignments, the Fourier analysis of the calculated vertical spin $s_y(t)$ time dependence was performed. The calculations were performed for EDM values of $D = 0, 10^{-20}, 10^{-19}, 5 \times 10^{-19} 10^{-18}$ e · cm for each misalignments setting.

Additionally, similar calculations were performed for quadrupole magnet misalignments (only translation in all directions) being measured recently by the Vermessungsbüro Stollenwerk & Burghof [33]. These results for Fourier amplitudes are presented in table 2 for few EDM values.

It is seen that the magnitude of the Fourier amplitude $F(\omega_o - \omega_s)$ delivers information on the misalignment effect only. As the misalignments are known with limited accuracy one may only determine probability distributions of the Fourier amplitude $F(\omega_o - \omega_s)$. The probability distribution for Fourier amplitude $F(\omega_o - \omega_s)$ obtained with the use of BMAD calculations for 10^4 magnet misalignments is shown in figure 5. This probability distribution is independent of EDM value, thus it provides information on the misalignment effect contributing to Fourier amplitude $F(\omega_s)$.

The Fourier amplitude $F(\omega_s)$ depends on the coherence sum of EDM and misalignment effect, since both these effects are indistinguishable. Therefore the discussed method of experimental determination of EDM value limit base on the correlation of the obtained Fourier amplitudes $F(\omega_s)$ and $F(\omega_o - \omega_s)$. This correlation is presented in figure 6 for EDM values $D = 0, 5 \times 10^{-19}, 10^{-18}$ e · cm. The results for $D = 10^{-20}$ e · cm and $D = 10^{-19}$ e · cm are not shown since they almost overlap with distribution for $D = 0$ e · cm.

The distinct separation of Fourier amplitudes $F(\omega_s)$ for various EDM values is clearly visible, but it depends on the magnitude of the Fourier amplitude $F(\omega_o - \omega_s)$. Therefore it is necessary to check distributions of $F(\omega_s)$ amplitude for different values of $F(\omega_o - \omega_s)$ amplitude. In figure 7 the probability distributions of $F(\omega_s)$ amplitude for selected ranges

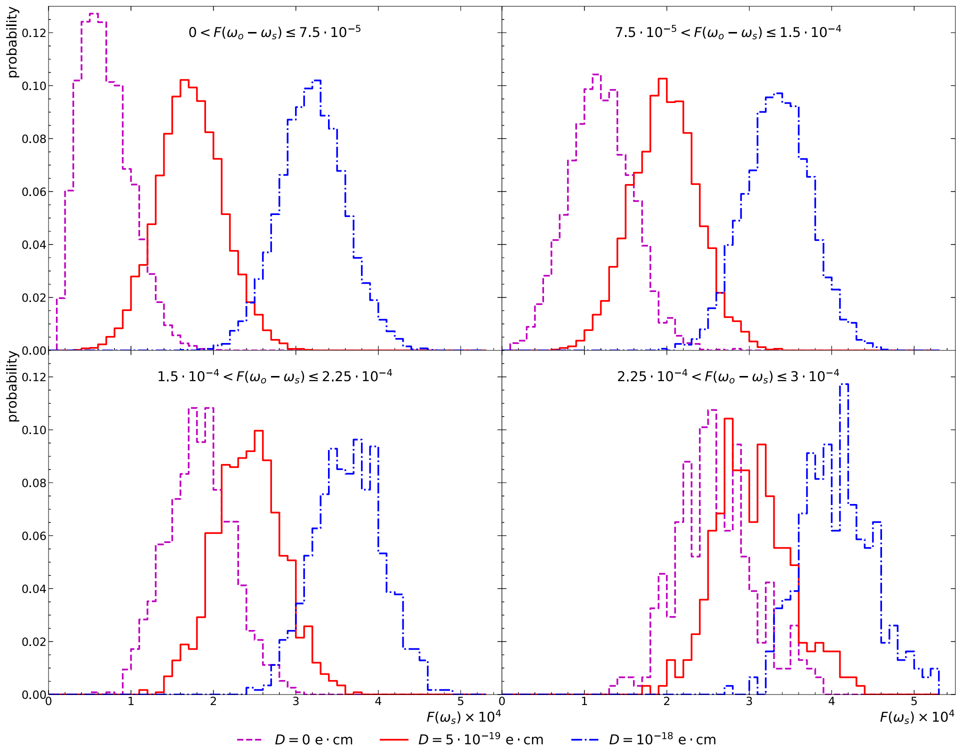

Figure 7. Probability distribution of Fourier amplitude $F(\omega_s)$ for selected intervals of Fourier amplitude $F(\omega_o - \omega_s)$ for EDM values of $D = 0$ e · cm (magenta dashed lines), $D = 5 \times 10^{-19}$ e · cm (red solid lines) and $D = 10^{-18}$ e · cm (blue dot-dashed lines).

of $F(\omega_o - \omega_s)$ amplitude is shown. It is seen that for small values of $F(\omega_o - \omega_s)$ amplitude it is possible to distinguish between all shown probability distributions for EDM values $D = 0, 5 \times 10^{-19}, 10^{-18}$ e · cm. With increasing $F(\omega_o - \omega_s)$ amplitude value the separation of $F(\omega_s)$ values for different EDM values becomes worse.

The experimental limit for EDM value in presence of magnet misalignments could be well determined with the use of the discussed method. The BMAD calculations have to be performed with the large statistic for known misalignments and their accuracy and for various EDM values. For the measured $F(\omega_o - \omega_s)$ amplitude within its accuracy limit the probability distributions for $F(\omega_s)$ amplitude could be obtained similar to that presented in figure 7. With these distributions for measured $F(\omega_s)$ amplitude value and assuming some confidence level it will be possible to determine the upper limit for measured EDM value.

4. Conclusion

Determination of a charged particles EDM with the use of a storage ring is based on the measurement of the vertical polarization for initially horizontally polarized beam. Since the expected EDM value is very small, it is essential to control all systematic uncertainties and pin them down to the smallest possible level. The dominating systematic uncertainty could be induced by unavoidable horizontal magnetic fields. Such fields occur due to misalignment of storage ring magnets. Such effects could be simulated with appropriate particle tracking codes, but the accuracy of such a method is limited.

In the present paper, a simple model was developed, which allows for the analytical determination of the contributions due to a horizontal magnetic field on the vertical polarization. The model may be used for any field distribution which is described by Fourier series coefficients. The analytical formulae are given up to first order. Therefore, the time dependence of the horizontal spin component for a vertically polarized beam moving in the given magnetic field could be precisely calculated. With the same method, it is possible to calculate the horizontal spin component induced by the EDM.

Using this analytical model, the time evolution of the vertical spin component was calculated for a horizontal field induced by dipole magnet rotations and by EDM interactions with the vertical field. The calculations were performed for the COSY storage ring for particles moving on the closed orbit. Similar time dependence of the vertical spin was observed in both cases, with small differences due to different distributions of the vertical and horizontal field components. The Fourier analysis of the time dependence of the vertical spin was performed. It was shown that Fourier amplitudes due to the EDM have only one maximum at a frequency ω_s, while for misalignments an additional peak at a frequency $\omega_o - \omega_s$ was observed. This observation made it possible to develop a method allowing to determine the limit of the measured value of EDM.

More detailed calculations including dipole and quadrupole fields and their misalignments were performed with the use of the BMAD software library. The developed method of the Fourier analysis was applied for results of simulations and the Fourier amplitudes for ω_s and $\omega_o - \omega_s$ frequencies were obtained. All further analysis is based on the comparison of these Fourier amplitudes. The effects due to all possible dipole and quadrupole translations and rotations were studied. It was found that the major misalignment effect is due to quadrupole magnet displacements in the vertical direction, while the next important effect due to dipole magnet rotation around the vertical axis is by one order of magnitude smaller. Next, the phase space influence on the Fourier amplitudes was studied. It was shown that for particles not moving on the closed orbit additional peaks in the Fourier amplitude spectrum occur. However, considering the full phase space these side peaks are very strongly reduced and the averaged amplitudes at ω_s and $\omega_o - \omega_s$ frequencies are the same as for the closed orbit. An orbit correction method was discussed and calculations were performed with the use of this method. It was shown that the orbit correction reduces the Fourier amplitude at ω_s frequency but does not change the Fourier amplitude at $\omega_o - \omega_s$. This leads to improvement of the EDM effect determination since the EDM effect contributes only to the Fourier amplitude at ω_s.

Finally, calculations were performed for many randomly distributed dipole and quadrupole magnet misalignments for a few EDM values. It was shown that with the present precision for magnet positioning at COSY it is possible to achieve a lower limit of $D < 10^{-19}$ e $\cdot$ cm for the deuteron EDM value. The discussed Fourier analysis features enable us to distinguish between the EDM and the misalignment effect and demonstrate the power of the proposed method. Presently, the discussed method is the only one that enables an experimental verification of the systematic uncertainties due to magnets imperfections in the EDM measurements with the use of a storage ring.

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Data availability statement

No new data were created or analysed in this study.

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