Contributions from classical electrodynamics and from the Thomas effect to the spin precession of a particle with the electric dipole moment

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(Dated: October 28, 2014)

Abstract

The fulfilled derivation of equation of spin precession of a particle possessing magnetic and electric dipole moments uses a fully covariant approach and explicitly separates contributions from classical electrodynamics and from the Thomas effect. The expression of the final equation in terms of the fields in the instantly accompanying frame presents it in a very simple form. The Lorentz transformations of the magnetic and electric dipole moments and the spin are derived from basic equations of classical electrodynamics, namely, from the equation connecting the angular momentum and the magnetic moment and from the Maxwell equations in matter. An antisymmetric four-tensor is constructed from the electric and magnetic dipole moments.

PACS numbers: 03.50.De, 03.30.+p, 13.40.Em

I. INTRODUCTION

A spin motion of a particle with the anomalous magnetic moment (AMM) and the electric dipole moment (EDM) in electric and magnetic fields is an important problem of classical spin physics. A search for the EDM [1] is a part of exploration of new physics beyond the Standard Model. For charged particles in storage rings, this search is based on the relativistic equation of spin motion. The corresponding equation for a particle without the EDM has been derived by Thomas [2] (also by Frenkel [3]) and, in a more general form, by Bargmann, Michel and Telegdi [4]. This is so-called Thomas-Bargmann-Michel-Telegdi (T-BMT) equation. There are two main methods of derivation of this equation. The Thomas method [2] (clearly explained in Ref. [5]) is based on separated calculations of the spin precession in the instantly accompanying frame and of the contribution from the Thomas effect. Addition of this contribution to the angular velocity of the spin precession obtained with a Lorentz transformation from the instantly accompanying frame leads to the needed equation. The Bargmann-Michel-Telegdi method [2] (transparently clarified in Ref. [6]) consists in the use of covariant equations of motions for four-vectors of spin and velocity, a^μ and u^μ, and the orthogonality condition $a_\mu u^\mu = 0$. The transition to the rest frame spin, $\boldsymbol{\zeta}$, allows one to derive the T-BMT equation. This method does not explicitly use the equation for the angular velocity of the Thomas precession.

An extension of the T-BMT equation due to the EDM has already been discussed in the original paper of Bargmann, Michel and Telegdi [4]. Then, the equation of spin motion of the particle with the AMM, μ', and the EDM, d, has been obtained in Refs. [7, 8] by the dual transformation $\mu' \to d,\ \boldsymbol{B} \to \boldsymbol{E},\ \boldsymbol{E} \to -\boldsymbol{B}$. The rigorous derivation of this equation has been presented in Ref. [9]. The resulting equation of spin motion coincides with that presented in Refs. [7, 8]. However, the derivation fulfilled in Ref. [9] has not used the supplementary assumption of dual symmetry.

We demonstrate in the present work that including the EDM into a consideration opens new possibilities to relate the particle spin motion with basic equations of classical electrodynamics, namely, the equation connecting the angular momentum and the magnetic moment and the Maxwell equations in matter. We also extract a contribution from the Thomas effect to the resulting spin motion with the use of a fully covariant approach.

II. ELECTROMAGNETIC INTERACTIONS OF A MOVING PARTICLE

We consider an extended charged particle in electric and magnetic fields. In fact, the fields may be nonuniform and nonstationary if we neglect terms proportional to their derivatives. In the framework of classical electrodynamics, we can divide the particle into point-like charges q and currents $\boldsymbol{J}$. Let $\boldsymbol{R}$ be the radius-vector of the center of mass of the particle:

$$\boldsymbol{R}=\frac{\sum \mathfrak{E}\boldsymbol{r}}{\sum \mathfrak{E}}, \tag{1}$$

where $\mathfrak{E}$ and $\boldsymbol{r}$ are the total energy and the radius-vector of a constituent part of the particle.

One defines an interaction of the electric and magnetic dipole moments, $\boldsymbol{d}$ and $\boldsymbol{\mu}$, with the external fields by the Hamiltonian

$$H=-\boldsymbol{d}\cdot\boldsymbol{E}-\boldsymbol{\mu}\cdot\boldsymbol{B}, \tag{2}$$

$$\boldsymbol{d}=\sum q\boldsymbol{r}=\int \rho(\boldsymbol{r})\boldsymbol{r}dV, \tag{3}$$

$$\boldsymbol{\mu}=\frac{1}{c}\sum[\boldsymbol{r}\times\boldsymbol{J}]=\frac{1}{c}\int[\boldsymbol{r}\times\boldsymbol{j}]dV. \tag{4}$$

Here $\rho(\boldsymbol{r})$ and $\boldsymbol{j}(\boldsymbol{r})$ are the charge and current densities. The sums are replaced with the integrals. This conventional definition becomes inexact for the moving particle. The rigorous definition should take into account a motion of the center of mass $\boldsymbol{R}$. When its velocity is $\boldsymbol{V}$, the EDM takes the form

$$\boldsymbol{d}=\sum e(\boldsymbol{r}-\boldsymbol{V}t). \tag{5}$$

Similar correction should be made for the magnetic moment. The center-of-mass velocity is given by

$$\boldsymbol{V}=\frac{c^2\sum\boldsymbol{\pi}}{\sum\mathfrak{E}}=\frac{c\sum\boldsymbol{\pi}}{\sum\sqrt{\mathfrak{M}^2c^2+\boldsymbol{\pi}^2}}, \tag{6}$$

where $\mathfrak{M}$ is the mass and $\boldsymbol{\pi}$ is the kinetic momentum of a constituent part of the particle.

Electric and magnetic dipole moments of the particle depend on a reference frame. In this section, we find a connection between the dipole moments in the lab frame and in the instantly accompanying one. The connection between the latter frame and the rest frame (which is noninertial) has been found by Thomas [2].

To express the electromagnetic interactions of the moving particle in terms of the intrinsic dipole moments, we can use the covariant form of the well-known expression for the relativistic transformation of lengths:

$$x_i = x_i^{(0)} - \frac{\gamma}{\gamma+1}\beta_i\beta_k x_k^{(0)}, \quad \beta_i = \frac{V_i}{c} = \frac{1}{c}\cdot\frac{dX_i}{dt}, \tag{7}$$

where $x_i^{(0)}$ are the rest frame coordinates and X_i are components of $\boldsymbol{R}$.

A motion of the magnetic moment leads to the appearance of the EDM and other way round. Since the charge and current densities form a four-vector, the charge density is influenced by the motion of currents constituting a magnetic dipole: $\rho = \gamma\boldsymbol{\beta}\cdot\boldsymbol{j}^{(0)}/c$. The current EDM [10] appearing due to a motion of the magnetic dipole is given by $\boldsymbol{d} = \boldsymbol{\beta}\times\boldsymbol{\mu}^{(0)}$, where $\boldsymbol{\mu}^{(0)}$ is the intrinsic magnetic moment.

To derive general equations for the dipole moments and for the Hamiltonian of the moving particle, it is convenient to compare the definitions of the dipole moments and the antisymmetric four-(pseudo)tensor of angular momentum. In the definition of the latter, the four-momentum, p^μ, should be replaced with the kinetic four-momentum $\pi^\mu = \left(\sqrt{m^2c^2+\boldsymbol{\pi}^2}, \boldsymbol{\pi}\right)$:

$$L^{\mu\nu} = \sum (x^\mu\pi^\nu - x^\nu\pi^\mu), \quad \pi^\mu = p^\mu - \frac{e}{c}A^\mu, \tag{8}$$

where A^μ is the four-potential. Classical mechanics presents this antisymmetric tensor in the form (see Ref. [11], § 14)

$$\begin{aligned} L^{\mu\nu} = (-\boldsymbol{K}, -\boldsymbol{L}), \quad \boldsymbol{K} = (-L^{01}, -L^{02}, -L^{03}) = \sum\left(\sqrt{m^2c^2+\boldsymbol{\pi}^2}\,\boldsymbol{r} - ct\boldsymbol{\pi}\right) \\ = \sum\sqrt{m^2c^2+\boldsymbol{\pi}^2}\,(\boldsymbol{r} - \boldsymbol{V}t), \quad \boldsymbol{L} = \sum\boldsymbol{r}\times\boldsymbol{\pi}. \end{aligned} \tag{9}$$

Let us consider single electric and magnetic dipoles. A comparison of Eq. (9) with the definitions (5) and (4) of the electric and magnetic dipole moments shows that

$$\boldsymbol{d} = \frac{e}{mc\gamma}\boldsymbol{K}, \quad \boldsymbol{\mu} = \frac{e}{mc\gamma}\boldsymbol{L}. \tag{10}$$

Therefore, the electric and magnetic dipole moments do not form a four-tensor and the related four-tensor is given by

$$\mathcal{D}^{\mu\nu} = (\gamma\boldsymbol{d}, \gamma\boldsymbol{\mu}). \tag{11}$$

The use of the well-known transformation law for four-tensors leads to the following equations ($\gamma_0 = 1$):

$$\boldsymbol{d} = \boldsymbol{d}^{(0)} - \frac{\gamma}{\gamma+1}\boldsymbol{\beta}(\boldsymbol{\beta}\cdot\boldsymbol{d}^{(0)}) + \boldsymbol{\beta}\times\boldsymbol{\mu}^{(0)}, \tag{12}$$

$$\boldsymbol{\mu} = \boldsymbol{\mu}^{(0)} - \frac{\gamma}{\gamma+1}\boldsymbol{\beta}(\boldsymbol{\beta}\cdot\boldsymbol{\mu}^{(0)}) - \boldsymbol{\beta}\times\boldsymbol{d}^{(0)}. \tag{13}$$

It is possible to determine physical quantities which are based on the electric and magnetic dipole moments and form an antisymmetric four-tensor. The electric and magnetic dipole moment densities

$$\boldsymbol{P} = \frac{d\boldsymbol{d}}{dV}, \quad \boldsymbol{M} = \frac{d\boldsymbol{\mu}}{dV}$$

form the antisymmetric four-tensor $\mathcal{P}^{\mu\nu} = (\boldsymbol{P}, \boldsymbol{M})$ because the Lorentz contraction results in $dV = dV_0/\gamma$. This is quite natural since $\boldsymbol{P}$ and $\boldsymbol{M}$ enter into the Maxwell equations in matter:

$$\begin{gathered}
\nabla\times\boldsymbol{E} = -\frac{1}{c}\cdot\frac{\partial\boldsymbol{B}}{\partial t}, \quad \nabla\times\boldsymbol{H} = \frac{4\pi}{c}\boldsymbol{j}^{(ext)} + \frac{1}{c}\cdot\frac{\partial\boldsymbol{D}}{\partial t}, \\
\nabla\cdot\boldsymbol{D} = 4\pi\rho^{(ext)}, \quad \nabla\cdot\boldsymbol{B} = 0, \quad \boldsymbol{D} = \boldsymbol{E} + 4\pi\boldsymbol{P}, \quad \boldsymbol{B} = \boldsymbol{H} + 4\pi\boldsymbol{M},
\end{gathered} \tag{14}$$

where $\rho^{(ext)}$ and $\boldsymbol{j}^{(ext)}$ are the densities of external charges and currents. As a result, $\boldsymbol{P}$ and $\boldsymbol{M}$ transform like the electric and magnetic fields, $\boldsymbol{E}$ and $\boldsymbol{B}$. It is important that the Lorentz transformation of the dipole moments can be connected with the Maxwell equations. Equations (12) and (13) can be obtained when $\boldsymbol{P}$ and $\boldsymbol{M}$ are small as compared with $\boldsymbol{E}$ and $\boldsymbol{B}$, respectively.

A possibility to construct a four-tensor from the electric and magnetic dipole moments was first mentioned by Frenkel [12]. However, his assumption that this four-tensor has the form $\mathcal{D}^{\mu\nu} = (\boldsymbol{d}, \boldsymbol{\mu})$ [cf. Eq. (11)] had resulted in incorrect transformation laws of the quantities $\boldsymbol{d}$ and $\boldsymbol{\mu}$. Similar error has been made by Nyborg [13]. Thus, a correct analysis has not be done in Refs. [12, 13].

To describe spin effects, we need to express the intrinsic dipole moments in terms of the rest frame spin (pseudo)vector. In this case, $\boldsymbol{d}^{(0)} = d\boldsymbol{\zeta}/s$, $\boldsymbol{\mu}^{(0)} = \mu\boldsymbol{\zeta}/s$, where $s = |\boldsymbol{\zeta}|$. The quantities $\boldsymbol{\zeta}$ and s have the dimensionality of the angular momentum. The quantity s in classical physics corresponds to $\hbar s$ (s is here the spin quantum number) in quantum mechanics. As a result, the Hamiltonian (2) takes the form

$$\begin{aligned}
H = -\frac{\mu}{s}&\left[\boldsymbol{B}\cdot\boldsymbol{\zeta} - \frac{\gamma}{\gamma+1}(\boldsymbol{\beta}\cdot\boldsymbol{B})(\boldsymbol{\beta}\cdot\boldsymbol{\zeta}) - (\boldsymbol{\beta}\times\boldsymbol{E})\cdot\boldsymbol{\zeta}\right] \\
-\frac{d}{s}&\left[\boldsymbol{E}\cdot\boldsymbol{\zeta} - \frac{\gamma}{\gamma+1}(\boldsymbol{\beta}\cdot\boldsymbol{E})(\boldsymbol{\beta}\cdot\boldsymbol{\zeta}) + (\boldsymbol{\beta}\times\boldsymbol{B})\cdot\boldsymbol{\zeta}\right].
\end{aligned} \tag{15}$$

It is important that this expression for the Hamiltonian can be reduced with the use of the fields in the instantly accompanying frame, $\boldsymbol{E}^{(0)}$ and $\boldsymbol{B}^{(0)}$, satisfying the relations (see

Ref. [14])

$$\begin{aligned}\boldsymbol{E}^{(0)} &= \gamma\left[\boldsymbol{E}-\frac{\gamma}{\gamma+1}\boldsymbol{\beta}(\boldsymbol{\beta}\cdot\boldsymbol{E})+\boldsymbol{\beta}\times\boldsymbol{B}\right],\\ \boldsymbol{B}^{(0)} &= \gamma\left[\boldsymbol{B}-\frac{\gamma}{\gamma+1}\boldsymbol{\beta}(\boldsymbol{\beta}\cdot\boldsymbol{B})-\boldsymbol{\beta}\times\boldsymbol{E}\right].\end{aligned} \tag{16}$$

Therefore, the Hamiltonian can be presented as follows:

$$H = -\frac{d\boldsymbol{E}^{(0)}\cdot\boldsymbol{\zeta}}{s\gamma}-\frac{\mu\boldsymbol{B}^{(0)}\cdot\boldsymbol{\zeta}}{s\gamma}. \tag{17}$$

The use of the Poisson brackets allows one to obtain the corresponding angular velocity of spin precession:

$$\boldsymbol{\omega} = -\frac{d\boldsymbol{E}^{(0)}}{s\gamma}-\frac{\mu\boldsymbol{B}^{(0)}}{s\gamma}. \tag{18}$$

Equations (15) and (17) for the Hamiltonian and Eq. (18) for the angular velocity of spin precession *in the lab frame* are general. The angular velocity of spin precession *in the instantly accompanying frame* can be obtained from Eq. (18) with $\gamma \to 1$.

However, we have started from the instantly accompanying frame while the spin (pseudo)vector $\boldsymbol{\zeta}$ is defined in the *noninertial* particle rest frame. Angular velocities of spin precession in the two frames differ due to the famous Thomas effect which should also be taken into consideration.

III. GENERAL DERIVATION WITH ALLOWANCE FOR THE THOMAS EFFECT

The Thomas effect [2] consists in a change of the angular velocity of spin precession due to a rotation of the particle rest frame. Thomas has shown [2] that the difference between the spin precession in the nonrotating instantly accompanying frame and in the rest frame is defined by

$$\left(\frac{\partial\boldsymbol{\zeta}}{\partial t}\right)_{nonrot} = \left(\frac{\partial\boldsymbol{\zeta}}{\partial t}\right)_{rest\,frame}+\boldsymbol{\omega}_T\times\boldsymbol{\zeta}, \tag{19}$$

where ω_T is the angular velocity of the Thomas precession:

$$\boldsymbol{\omega}_T = -\frac{\gamma^2}{\gamma+1}\left(\boldsymbol{\beta}\times\frac{d\boldsymbol{\beta}}{dt}\right). \tag{20}$$

A very short and clear derivation of Eq. (20) has been recently given in Ref. [15].

We can immediately find the total angular velocity of spin precession of the particle with the AMM and EDM with Eqs. (19) and (20). However, it is more consistent to keep the fully covariant approach.

An inclusion of the EDM brings the covariant equation of spin motion to the general form [9]

$$\frac{da^\mu}{d\tau}=A_1F^{\mu\nu}a_\nu+A_2\beta u^\mu F^{\nu\lambda}u_\nu a_\lambda+A_3G^{\mu\nu}a_\nu+A_4u^\mu G^{\nu\lambda}u_\nu a_\lambda, \tag{21}$$

where the four-vectors of spin and velocity are defined by

$$a^\mu=(a^0,\ \boldsymbol{a}),\quad \boldsymbol{a}=\boldsymbol{\zeta}+\frac{\gamma^2\boldsymbol{\beta}(\boldsymbol{\beta}\cdot\boldsymbol{\zeta})}{\gamma+1},\quad a^0=\boldsymbol{\beta}\cdot\boldsymbol{a}=\gamma\boldsymbol{\beta}\cdot\boldsymbol{\zeta},\quad u^\mu=(\gamma,\gamma\boldsymbol{\beta}) \tag{22}$$

and $G^{\mu\nu}=\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}/2$ is the antisymmetric four-tensor dual to the electromagnetic field tensor $F_{\alpha\beta}$. The translational motion of the particle is given by

$$\frac{du^\mu}{d\tau}=\frac{e}{mc}F^{\mu\nu}u_\nu. \tag{23}$$

The transition to the instantly accompanying frame and the use of the orthogonality condition $a_\mu u^\mu=0$ result in (see Ref. [9])

$$A_1=\frac{\mu}{s},\quad A_2=-\frac{1}{s}\left(\mu-\frac{es}{mc}\right)=-\frac{\mu'}{s},\quad A_3=-\frac{d}{s},\quad A_4=\frac{d}{s}. \tag{24}$$

With the use of Eq. (23), the obtained equation can be presented in the form

$$\frac{da^\mu}{d\tau}=\frac{\mu}{s}\left(F^{\mu\nu}a_\nu-u^\mu F^{\nu\lambda}u_\nu a_\lambda\right)-\frac{d}{s}\left(G^{\mu\nu}a_\nu-u^\mu G^{\nu\lambda}u_\nu a_\lambda\right)-u^\mu\frac{du^\lambda}{d\tau}a_\lambda. \tag{25}$$

Next derivations can be made similarly to Ref. [14]. It is convenient to denote

$$\Phi^\mu=\frac{\mu}{s}\left(F^{\mu\nu}a_\nu-u^\mu F^{\nu\lambda}u_\nu a_\lambda\right)-\frac{d}{s}\left(G^{\mu\nu}a_\nu-u^\mu G^{\nu\lambda}u_\nu a_\lambda\right). \tag{26}$$

Evidently, $\Phi^\mu=(\Phi^0,\boldsymbol{\Phi})$ is a four-vector. Since $u_\mu\Phi^\mu=\gamma(\Phi^0-\boldsymbol{\beta}\cdot\boldsymbol{\Phi})=0$, it satisfies the relation $\Phi^0=\boldsymbol{\beta}\cdot\boldsymbol{\Phi}$. The last term in Eq. (25) can be transformed as follows [14]:

$$u^\mu\frac{du^\lambda}{d\tau}a_\lambda=-u^\mu\gamma\boldsymbol{a}\cdot\frac{d\boldsymbol{\beta}}{d\tau}. \tag{27}$$

Thus, Eq. (25) leads to

$$\frac{da^0}{d\tau}=\Phi^0+\gamma^2\boldsymbol{a}\cdot\frac{d\boldsymbol{\beta}}{d\tau},\quad \frac{d\boldsymbol{a}}{d\tau}=\boldsymbol{\Phi}+\gamma^2\boldsymbol{\beta}\left(\boldsymbol{a}\cdot\frac{d\boldsymbol{\beta}}{d\tau}\right). \tag{28}$$

Now we can calculate the equation of motion for the rest frame spin $\boldsymbol{\zeta}$ with the use of the relations

$$\boldsymbol{\zeta}=\boldsymbol{a}-\frac{\gamma}{\gamma+1}\boldsymbol{\beta}(\boldsymbol{\beta}\cdot\boldsymbol{a}),\quad \frac{d}{d\tau}\left(\frac{\gamma}{\gamma+1}\boldsymbol{\beta}\right)=\frac{\gamma}{\gamma+1}\frac{d\boldsymbol{\beta}}{d\tau}+\frac{\gamma^3}{(\gamma+1)^2}\boldsymbol{\beta}\left(\boldsymbol{\beta}\cdot\frac{d\boldsymbol{\beta}}{d\tau}\right).$$

The needed equation has the form (cf. Ref. [14])

$$\frac{d\boldsymbol{\zeta}}{d\tau}=\boldsymbol{\Phi}-\frac{\gamma\boldsymbol{\beta}}{\gamma+1}\Phi^0+\frac{\gamma^2}{\gamma+1}\boldsymbol{\zeta}\times\left(\boldsymbol{\beta}\times\frac{d\boldsymbol{\beta}}{d\tau}\right). \tag{29}$$

The transformation of the given four-vector Φ^μ to the instantly accompanying frame results in $\left(\Phi^{(0)}\right)^\mu=\left(0,\boldsymbol{\Phi}^{(0)}\right)$, where

$$\boldsymbol{\Phi}^{(0)}=\boldsymbol{\Phi}-\frac{\gamma}{\gamma+1}\boldsymbol{\beta}(\boldsymbol{\beta}\cdot\boldsymbol{\Phi})=\boldsymbol{\Phi}-\frac{\gamma\boldsymbol{\beta}}{\gamma+1}\Phi^0.$$

Since $dt=\gamma\, d\tau$, the derivation of $\boldsymbol{\Phi}^{(0)}$ from Eq. (26) brings the equation of spin motion to the form

$$\frac{d\boldsymbol{\zeta}}{dt}=-\left(\frac{d\boldsymbol{E}^{(0)}}{s\gamma}+\frac{\mu\boldsymbol{B}^{(0)}}{s\gamma}\right)\times\boldsymbol{\zeta}-\frac{\gamma^2}{\gamma+1}\left(\boldsymbol{\beta}\times\frac{d\boldsymbol{\beta}}{dt}\right)\times\boldsymbol{\zeta}. \tag{30}$$

The angular velocity of spin precession is given by

$$\boldsymbol{\Omega}=-\left(\frac{d\boldsymbol{E}^{(0)}}{s\gamma}+\frac{\mu\boldsymbol{B}^{(0)}}{s\gamma}\right)-\frac{\gamma^2}{\gamma+1}\left(\boldsymbol{\beta}\times\frac{d\boldsymbol{\beta}}{dt}\right)=\boldsymbol{\omega}+\boldsymbol{\omega}_T, \tag{31}$$

where $\boldsymbol{\omega}$ and $\boldsymbol{\omega}_T$ are given by Eqs. (18) and (20), respectively.

Equations (30) and (31) show that the total angular velocity of spin precession is the sum of two parts. The first part is given by the Lorentz transformation between the instantly accompanying frame and the lab frame. The second one is the contribution from the Thomas precession. This part defines the additional spin precession caused by a purely kinematical effect of a rotation of the particle rest frame (see, e.g., Ref. [14]). The presented derivation of Eqs. (30) and (31) is fully covariant but it does not specify the two contributions to the total effect. The origins of these contributions are considered in detail in Sec. II and in the theory of the Thomas effect [2, 14–16].

The particle acceleration is expressed in terms of the lab frame fields as follows:

$$\frac{d\boldsymbol{\beta}}{dt}=\frac{e}{mc\gamma}\left[\boldsymbol{E}+\boldsymbol{\beta}\times\boldsymbol{B}-\boldsymbol{\beta}(\boldsymbol{\beta}\cdot\boldsymbol{E})\right]. \tag{32}$$

With the use of Eqs. (16), (32), one can bring Eq. (31) to the form

$$\begin{aligned}\boldsymbol{\Omega}=-\frac{e}{mc}\Biggl[\left(G+\frac{1}{\gamma}\right)\boldsymbol{B}-\frac{\gamma G}{\gamma+1}(\boldsymbol{\beta}\cdot\boldsymbol{B})\boldsymbol{\beta}-\left(G+\frac{1}{\gamma+1}\right)\boldsymbol{\beta}\times\boldsymbol{E}\\+\frac{\eta}{2}\left(\boldsymbol{E}-\frac{\gamma}{\gamma+1}(\boldsymbol{\beta}\cdot\boldsymbol{E})\boldsymbol{\beta}+\boldsymbol{\beta}\times\boldsymbol{B}\right)\Biggr],\end{aligned} \tag{33}$$

where $G=(g-2)/2,\ g=2mc\mu/(es)$, and $\eta=2mcd/(es)$. This equation has been previously derived in Ref. [9].

The equation of spin motion takes a pretty simple form after an expression of the Thomas precession in terms of the fields in the the instantly accompanying frame. While

$$\boldsymbol{E}+\boldsymbol{\beta}\times\boldsymbol{B}-\boldsymbol{\beta}(\boldsymbol{\beta}\cdot\boldsymbol{E})\neq\boldsymbol{E}^{(0)},$$

the angular velocity of the Thomas precession is equal to

$$\boldsymbol{\omega}_T=-\frac{e}{mc(\gamma+1)}\left(\boldsymbol{\beta}\times\boldsymbol{E}^{(0)}\right). \tag{34}$$

Therefore,

$$\frac{d\boldsymbol{\zeta}}{dt}=-\frac{e}{mc}\left(\frac{g\boldsymbol{B}^{(0)}}{2\gamma}+\frac{\eta\boldsymbol{E}^{(0)}}{2\gamma}+\frac{\boldsymbol{\beta}\times\boldsymbol{E}^{(0)}}{\gamma+1}\right)\times\boldsymbol{\zeta}. \tag{35}$$

This final equation explicitly shows the contributions from the magnetic and electric dipole moments and from the Thomas precession.

IV. DISCUSSION AND SUMMARY

The earlier derivations of the equation of spin motion of a particle with the AMM and EDM [7, 8] used the dual transformation of terms proportional to the AMM in the T-BMT equation describing a particle without the EDM. The rigorous derivation of the equation for a particle with the AMM and EDM has been first performed in Ref. [9]. In the present work, we have made a next step and have deduced this equation with the explicit separation of contributions from the Lorentz transformation between the instantly accompanying frame and the lab frame and from the Thomas effect. This deduction is fully covariant. The transition to the fields in the instantly accompanying frame has allowed us to present the final equation in the very simple form. Amazingly, one need not to divide the magnetic moment into the normal and anomalous parts. This division is a result of expression of the equation of spin motion in terms of the lab frame fields.

In fact, the obtained equation of motion (35) cannot exhaustively specify origins of the two contributions. However, a needed specification of the Thomas term is presented by the theory of the Thomas effect [2, 14–16]. This theory shows that the Thomas effect has a purely kinematical origin and is caused by a rotation of the particle rest frame. The origin of the contribution of the magnetic and electric dipole moments to Eq. (35) has been cleared in Sec. II. It has been demonstrated that the form of this contribution is conditioned by the Lorentz transformation from the instantly accompanying frame to the lab frame.

The key moment of the analysis fulfilled in Sec. II is the Lorentz transformation of the magnetic and electric dipole moments. We have determined the connection between the dipole moments in the lab frame and in the instantly accompanying one and have corrected errors made in previous investigations [12, 13]. We have derived this connection from basic equations of classical electrodynamics, namely, from the equation connecting the angular momentum and the magnetic moment and from the Maxwell equations in matter. We have also constructed a four-tensor from the electric and magnetic dipole moments. These new results have been obtained thank to the inclusion of the EDM into the consideration. The existence of the direct relation between the particle spin motion and the above mentioned basic equations is an important fact.

The results obtained show a deep self-consistency of classical electrodynamics.

Acknowledgements

The author is grateful to the reviewer of Physica Scripta for valuable remarks and propositions. The author also acknowledges the support by the Belarusian Republican Foundation for Fundamental Research (Grant No. Φ14D-007).

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