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General dynamics of tensor polarization of particles and nuclei in external fields

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Abstract

The tensor polarization of particles and nuclei becomes constant in a coordinate system rotating with the same angular velocity as the spin, and it rotates in the laboratory frame with the above angular velocity. The general equation defining the time dependence of the tensor polarization is derived. An explicit form of the dynamics of this polarization is found in the case when the initial polarization is axially symmetric.

Keywords: spin, tensor polarization, deuteron

1. Introduction

Polarized beams are often used as a research tool to study fundamental interactions and to search for new physics. Unlike spin-1/2 fermions, particles and nuclei with spin $s \geqslant 1$ possess tensor polarization. This polarization is an important property of particles and nuclei and can be measured with good accuracy. Investigations of the dynamics of the tensor polarization of light nuclei (e.g., the deuteron with spin 1) have made it possible to discover [1, 2], predict [3, 4], and describe [5–7] some new effects. However, these effects are conditioned either by quadratics in the spin interactions of deuterons [1–6] or by quantum beats in positronium [7]. To get a basic understanding of polarization effects, it is necessary to consider linear-in-spin interactions of *tensor-polarized* particles and nuclei. While this problem is rather important, it is not often discussed. The information available on the dynamics of tensor polarization in external fields has been considered in [8]. In this work, some examples of the evolution of tensor polarization have been investigated and some properties of the 5×5 spin transfer matrix describing this polarization have been considered. Evidently, the use of the 5×5 matrix is much more difficult than that of the corresponding 2×2 matrix for vector polarization.

In the present work, we develop a simple general approach to describe the dynamics of the tensor polarization of particles and nuclei when this dynamics is caused by linear-in-spin interactions with external fields. The proposed approach allows us to couple the dynamics of the tensor and vector polarizations and to derive general formulas defining an evolution of particles/nuclei polarization in external fields.

2. General properties of spin dynamics

As it is well known, spin rotation is exhaustively described with the polarization vector, $\boldsymbol{P}$, defined by

$$P_i = \frac{\langle S_i \rangle}{s}, \quad i = x, y, z. \tag{1}$$

Here, S_i are the corresponding spin matrices and s is the spin quantum number. The averages of the spin operators are expressed by their convolutions with the wave function, $\Psi(t)$. Particles with spin $s \geqslant 1$ also possess a tensor polarization. The main characteristics of such a polarization are specified by the polarization tensor P_{ij}, which is given by

$$P_{ij} = \frac{3\left\langle S_i S_j + S_j S_i \right\rangle - 2s(s+1)\delta_{ij}}{2s(2s-1)}, \quad i, j = x, y, z. \tag{2}$$

The polarization tensor satisfies the conditions $P_{ij} = P_{ji}$ and $P_{xx} + P_{yy} + P_{zz} = 0$, and therefore has five independent components. In the general case, the polarization vector and the polarization tensor are time dependent. Additional tensors composed of products of three or more spin matrices are needed only for the exhaustive description of the polarization of particles/nuclei with spin $s \geqslant 3/2$.

The eight parameters defined by the polarization vector and the polarization tensor are independent. In particular, $\langle S_i S_j \rangle \neq \langle S_i \rangle \langle S_j \rangle$.

The quantum-mechanical Hamiltonian describing the interaction of the spin with external fields contains linear and bilinear terms on the spin:

$$\mathcal{H} = \boldsymbol{\Omega} \cdot \boldsymbol{S} + Q_{jk} S_j S_k. \tag{3}$$

Here, $\boldsymbol{S}$ is the matrix operator describing the rest-frame spin. Since the spin matrices satisfy the commutation relation

$$\left[S_i, S_j\right] = i e_{ijk} S_k \quad (i, j, k = x, y, z), \tag{4}$$

the first term in equation (3) defines the spin rotation with the angular velocity, $\boldsymbol{\Omega}$. The absolute value and the direction of $\boldsymbol{\Omega}$ may arbitrarily depend on time.

Even when the second term in equation (3) is omitted, the dynamics of the polarization tensor is not trivial. We cannot characterize the particle spin wave function by only a single frequency. For a spin-1 particle, there are two frequencies, $+\Omega$ and $-\Omega$, defining three equidistant levels. For a particle with the integer spin $s = N$, there are $2N$ frequencies, $\pm\Omega$, $\pm 2\Omega$, ..., $\pm N\Omega$, defining $2N + 1$ equidistant levels. The single frequency of the spin rotation, Ω, originates from the specific commutation relations (4) for the spin operators, S_i. For a spin-1 particle, this property of the spin operators manifests in $(S_i)_{13} = (S_i)_{31} = 0$ $(i = x, y, z)$. Therefore, the spin operators mix only two neighboring levels, but that is not the case for the tensor polarization operators. Three of them mix levels 1 and 3 because the components $(S_x^2)_{13}$, $(S_x^2)_{31}$, $(S_y^2)_{13}$, $(S_y^2)_{31}$, $(S_x S_y + S_y S_x)_{13}$, and $(S_x S_y + S_y S_x)_{31}$ are nonzero. As a result, the frequency 2Ω also appears. For particles with greater spins ($s \geqslant 1$), the situation is even more complicated.

We consider electromagnetic interactions, namely the coupling of spins to electric and magnetic fields. The second term in equation (3) characterizes the tensor electric and magnetic polarizabilities and the quadrupole interaction. It influences the dynamics of both the polarization vector and the polarization tensor. As a rule, this term is much lower than the first term. Its order of magnitude can be easily determined. We can calculate the commutator $[\mathcal{H}, S_i]$, taking into account that $s \sim 1$. The term under consideration brings a correction of the order of $|Q_{jk}S_jS_k|$ to the angular frequency of spin precession, Ω. Let us evaluate this correction, whose effect on spin dynamics is rather complicated and does not reduce to a change of $\boldsymbol{\Omega}$ (see [3–5]). The angular frequency of the spin precession of the deuteron in the magnetic field $B = 3$ T is equal to 1.2×10^8 s^{-1}. If the tensor magnetic polarizability of the deuteron is close to the value $\beta_T = 1.95 \times 10^{-40}$ cm^3 as predicted by theory [15, 16], the related *effective* change of the angular frequency of spin precession is about 2×10^{-4} s^{-1}. The predicted values of the tensor electric polarizability of the deuteron [15–17] are slightly less and cannot lead to a greater effect. On the other hand, spin–tensor effects can be caused by intrinsic electric quadrupole moments of the nuclei. For example, a spatial derivative of an electric field strength in Penning traps is usually of the order of 10^6 V m^{-2}. The quadrupole moment of the deuteron Q_d is equal to 0.286 fm^2. A simple estimate shows that the related *effective* change of the angular frequency of spin precession of the deuteron is of the order of 10^{-8} s^{-1}. For polarized deuteron beams in accelerators and storage rings, the intrinsic electric quadrupole moment also causes spin–tensor interactions. However, its effect on the spin evolution is a few orders of magnitude lower than that of the tensor magnetic and electric polarizabilities. When $B = 3$ T, the ring radius $R \sim 10$ m, and the field index $n = -(R/B)(\partial B/\partial r) \sim 0.1$, and the *effective* change of the angular frequency of spin precession caused by the electric quadrupole moment is of the order of 10^{-9} s^{-1}, even for relativistic deuterons.

Thus, one can usually neglect the influence of the spin–tensor interactions on spin dynamics. Such an influence can be significant only in special cases like hyperfine interactions in atoms or in experiments specially designed to measure the tensor magnetic and electric polarizabilities. Hereafter, we will ignore the spin–tensor interactions and will take into account only the first term in the interaction Hamiltonian (3).

The equation of spin motion is given by

$$\frac{\mathrm{d}\boldsymbol{S}}{\mathrm{d}t} = \frac{i}{\hbar}[\boldsymbol{\mathcal{H}}, \boldsymbol{S}] = \boldsymbol{\Omega} \times \boldsymbol{S}. \tag{5}$$

When the particle/nucleus moves in electromagnetic fields, $\boldsymbol{\Omega}$ is defined by the Thomas–Bargmann–Michel–Telegdi (T–BMT) equation [9] extended on the electric dipole moment (EDM):

$$\begin{aligned}\boldsymbol{\Omega} = -\frac{e}{mc}\Bigg[&\left(G + \frac{1}{\gamma}\right)\boldsymbol{B} - \frac{\gamma G}{\gamma + 1}(\boldsymbol{\beta}\cdot\boldsymbol{B})\boldsymbol{\beta} - \left(G + \frac{1}{\gamma + 1}\right)\boldsymbol{\beta}\times\boldsymbol{E}\\ &+ \frac{\eta}{2}\left(\boldsymbol{E} - \frac{\gamma}{\gamma + 1}(\boldsymbol{\beta}\cdot\boldsymbol{E})\boldsymbol{\beta} + \boldsymbol{\beta}\times\boldsymbol{B}\right)\Bigg].\end{aligned} \tag{6}$$

Here $G = (g - 2)/2$, $g = 2mc\mu/(e\hbar s)$, $\eta = 2mcd/(e\hbar s)$, $\boldsymbol{\beta} = \mathbf{v}/c$, γ is the Lorentz factor, and μ and d are the magnetic and electric dipole moments, respectively. An extension of the T-BMT equation due to the EDM has already been discussed in the original paper by Bargmann, Michel, and Telegdi [9]. Then, the equation of the spin motion of the particle with the anomalous magnetic moment μ' and the EDM has been obtained in [10, 11] by the dual transformation, $\mu' \to d$, $\boldsymbol{B} \to \boldsymbol{E}$, $\boldsymbol{E} \to -\boldsymbol{B}$. The rigorous derivation of this equation has been presented in [12]. The resulting equation of spin motion coincides with that presented in [10, 11]. However, the derivation fulfilled in [12] did not use the supplementary assumption of dual symmetry.

The Pauli matrices, along with the unit matrix, describe the spin of a Dirac particle and generate an irreducible representation of the $SU(2)$ group. Algebraically, the $SU(2)$ group is a double cover of the three-dimensional rotation group, $SO(3)$. As a result, the spin dynamics defined by the Dirac equation fully corresponds to the classical picture of the rotation of an intrinsic angular momentum (spin) in external fields. The angular velocity of the spin rotation does not depend on the spin quantum number. Therefore, both the classical description and the formalism based on the Pauli matrices are applicable to particles/nuclei with an arbitrary spin if the effect of spin rotation is analyzed. Certainly, this formalism becomes insufficient if one considers the spin–tensor effects mentioned in section 1.

In equation (6), we can estimate the terms dependent on the EDM. Any experimental constraints on the deuteron's EDM are not established yet. For the proton, the current indirect bound of $|d_p| < 7.9 \times 10^{-25}$ e·cm was obtained using Hg atoms [18]. If the deuteron's EDM was equal to this value, its contribution to the angular frequency of the spin precession of 1 GeV/c deuterons in the magnetic field $B = 3$ T is equal to 5×10^{-3} s^{-1}. This means that one can often neglect the influence of the spin–tensor terms in the Hamiltonian (3) on the spin dynamics, but can keep the EDM-dependent terms in equation (6). Certainly, a possible future ascertainment of very strong restrictions on the deuteron's EDM can change this situation.

3. Huang–Lee–Ratner approach

Let us consider the approach used by Huang, Lee, and Ratner [8] to describe an evolution of the tensor polarization. Instead of the polarization tensor (2), the following spin–tensor operators can be used [8]:

$$T_0 = \frac{1}{\sqrt{2}}\left(3S_z^2 - 2\right), \quad T_{\pm 1} = \pm\frac{\sqrt{3}}{2}\left(S_\pm S_3 + S_3 S_\pm\right), \quad T_{\pm 2} = \frac{\sqrt{3}}{2}S_\pm^2. \tag{7}$$

To obtain observable quantities, these operators should be averaged.

The approach proposed in [8] was based on the T-BMT equation, and it disregarded the spin–tensor interactions in the Hamiltonian (3). This approach used the the Frenet-Serret curvilinear coordinates. Expressing the spin vector in terms of its components,

$$\boldsymbol{S} = S_1\boldsymbol{e}_1 + S_2\boldsymbol{e}_2 + S_3\boldsymbol{e}_3, \tag{8}$$

and defining $S_\pm = S_1 \pm iS_2$, one obtains the following equation of spin motion:

$$\frac{\mathrm{d}S_\pm}{\mathrm{d}\phi} = \pm iG\gamma S_\pm \pm iF_\pm S_3, \quad \frac{\mathrm{d}S_3}{\mathrm{d}\phi} = \frac{i}{2}\left(F_- S_+ - F_+ S_-\right), \tag{9}$$

where $F_\pm$ characterizes the spin depolarization kick and ϕ is the azimuthal orbit rotation angle. The spin tune, $G\gamma$, is the number of spin revolutions per orbit turn.

Equations (7)–(9) allow one to obtain the set of equations defining the evolution of the tensor polarization [8]:

$$
\begin{aligned}
\frac{\mathrm{d}T_0}{\mathrm{d}\phi} &= \frac{3}{\sqrt{2}}\left(\frac{\mathrm{d}S_3}{\mathrm{d}\phi}S_3 + S_3\frac{\mathrm{d}S_3}{\mathrm{d}\phi}\right) = -i\frac{\sqrt{6}}{2}\left(F_-T_{+1} + F_+T_{-1}\right),\\
\frac{\mathrm{d}T_{\pm1}}{\mathrm{d}\phi} &= -\frac{\sqrt{3}}{2}\left(\frac{\mathrm{d}S_\pm}{\mathrm{d}\phi}S_3 + S_3\frac{\mathrm{d}S_\pm}{\mathrm{d}\phi} + \frac{\mathrm{d}S_3}{\mathrm{d}\phi}S_\pm + S_\pm\frac{\mathrm{d}S_3}{\mathrm{d}\phi}\right) = \pm iG\gamma T_{\pm1} - i\frac{\sqrt{6}}{2}F_\pm T_0 - iF_\mp T_{\pm2},\\
\frac{\mathrm{d}T_{\pm2}}{\mathrm{d}\phi} &= \frac{\sqrt{3}}{2}\left(\frac{\mathrm{d}S_\pm}{\mathrm{d}\phi}S_\pm + S_\pm\frac{\mathrm{d}S_\pm}{\mathrm{d}\phi}\right) = \pm 2iG\gamma T_{\pm2} - iF_\pm T_{\pm1}.
\end{aligned}
\tag{10}
$$

These equations can be presented in the matrix form:

$$
\frac{\mathrm{d}T}{\mathrm{d}\phi} = AT,
$$

$$
T = \begin{pmatrix} T_{+2}\\ T_{+1}\\ T_0\\ T_{-1}\\ T_{-2}\end{pmatrix},\quad A = \begin{pmatrix}
2iG\gamma & -iF_+ & 0 & 0 & 0\\
-iF_- & iG\gamma & -i\frac{\sqrt{6}}{2}F_+ & 0 & 0\\
0 & -i\frac{\sqrt{6}}{2}F_- & 0 & -i\frac{\sqrt{6}}{2}F_+ & 0\\
0 & 0 & -i\frac{\sqrt{6}}{2}F_- & -iG\gamma & -iF_+\\
0 & 0 & 0 & -iF_- & -2iG\gamma
\end{pmatrix}. \tag{11}
$$

In [8], some applications of this approach have been considered.

While the approach of Huang, Lee, and Ratner is absolutely correct, we can make some obvious remarks. First of all, the use of 5×5 matrices instead of the standard 2×2 accelerator matrices necessitates much more cumbersome derivations. Moreover, even the investigations of spin–tensor effects performed in [5] used three-component wave functions and 3×3 matrices. Also, note the absence of a direct connection between evolutions of the polarization vector and the polarization tensor, while spin–tensor interactions are not taken into consideration in [8].

In the present work, we propose a different approach, which couples the dynamics of the tensor and vector polarizations and simplifies a description of the evolution of the tensor polarization of partcles/nuclei in external fields.

4. Evolution of the polarization tensor

It is not evident whether the evolution of the polarization tensor also reduces to rotation. To clear up this problem, we use the following approach. Let us consider the Cartesian coordinate system rotating around the z-axis with the same angular velocity, $\boldsymbol{\Omega}(t)$, as the spin. In the general case, this angular velocity depends on time. We may superpose the rotating and nonrotating coordinate systems at the initial moment of time, $t = 0$ ($\boldsymbol{e}_i'(0) = \boldsymbol{e}_i$). The rotating coordinate system is denoted by primes.

The spin components in the rotating coordinate system remain unchanged:

$$
\frac{\mathrm{d}S_i'}{\mathrm{d}t} = 0 \quad (i = x, y, z). \tag{12}
$$

As a result, all tensor polarization operators and all components of the polarization tensor are also unchanged in this coordinate system ($S_i'S_j' + S_j'S_i' = \text{const}$, $P_{ij}' = \text{const}$). This important property shows that the tensor polarization of particles/nuclei with spin $s \geqslant 1$

rotates in external fields, similar to the vector polarization. This is valid not only for electromagnetic interaction, but also for other (weak, gravitational) interactions. Other products of the spin operators, $S_i'S_j'...S_k'$, are also conserved in the rotating coordinate system.

To determine the particle polarization in the nonrotating coordinate system (laboratory frame), we need to connect the directions of the basic vectors of the primed and unprimed coordinate systems. This problem can be easily solved. We choose the initial moment of time, $t = 0$. The evolution of any basic vector of the primed coordinate system coincides with that of the spin when it is initially directed along the considered basic vector. Therefore, the final directions of the three primed basic vectors are defined by the three final directions of the spin when its corresponding initial directions are parallel to the three Cartesian axes of the unprimed coordinate system. As a result, the dynamics of the polarization tensor reduces to that of the polarization vector. In particular, the evolution of the polarization tensor in accelerators and storage rings can be unambiguously defined with the usual 2×2 spin transfer matrices. Thus, one need not apply the more cumbersome 5×5 matrices.

The time dependence of the polarization tensor defined in the laboratory frame can be easily expressed in terms of the basic vectors, $\boldsymbol{e}_i'$. Since $\boldsymbol{e}_i'(0) = \boldsymbol{e}_i$ and

$$S_i'(t) = \sum_k \left(\boldsymbol{e}_i'(t) \cdot \boldsymbol{e}_k\right) S_k, \tag{13}$$

the polarization tensor is given by

$$P_{ij}(t) = \frac{3}{2s(2s-1)} \sum_{k,l} \left[\left(\boldsymbol{e}_i'(t) \cdot \boldsymbol{e}_k\right)\left(\boldsymbol{e}_j'(t) \cdot \boldsymbol{e}_l\right)\left\langle S_k S_l + S_l S_k \right\rangle \right] - \frac{s+1}{2s-1}\delta_{ij}. \tag{14}$$

This simple equation defines the general dynamics of the tensor polarization of particles and nuclei in external fields.

5. Evolution of axially symmetric polarization

A calculation of the polarization tensor can usually be simplified even further. In particular, this is possible when the initial polarization is axially symmetric. It is convenient to direct the z-axis along the symmetry axis. For the considered initial polarization, $P_{xx}' = P_{yy}'$ and $P_{ij}' = 0$ when $i \neq j$. It has been ascertained in the preceding section that the primed polarization tensor remains unchanged.

Let us suppose that the direction of $\boldsymbol{e}_z'(t)$ in the laboratory frame is defined by the time-dependent spherical angles, θ and ϕ. In the considered case, the laboratory frame polarization tensor does not depend on directions of the x'- and y'-axes in the plane orthogonal to z', and we can choose the primed basic vectors as follows:

$$\begin{aligned}&\boldsymbol{e}_x' = \boldsymbol{e}_x \cos\theta\cos\phi + \boldsymbol{e}_y \cos\theta\sin\phi - \boldsymbol{e}_z \sin\theta, \quad \boldsymbol{e}_y' = -\boldsymbol{e}_x \sin\phi + \boldsymbol{e}_y \cos\phi,\\ &\boldsymbol{e}_z' = \boldsymbol{e}_x \sin\theta\cos\phi + \boldsymbol{e}_y \sin\theta\sin\phi + \boldsymbol{e}_z \cos\theta.\end{aligned} \tag{15}$$

The connection between the spin operators in the unprimed and primed coordinate systems is defined by equations (13) and (15) and is given by

$$\begin{aligned}S_x &= S_x'\cos\theta\cos\phi - S_y'\sin\phi + S_z'\sin\theta\cos\phi,\\ S_y &= S_x'\cos\theta\sin\phi + S_y'\cos\phi + S_z'\sin\theta\sin\phi,\\ S_z &= -S_x'\sin\theta + S_z'\cos\theta.\end{aligned} \tag{16}$$

Due to the axial symmetry, averaging vanishes $\langle S_x' \rangle$ and $\langle S_y' \rangle$. The polarization vector, $\boldsymbol{P} = \langle \boldsymbol{S} \rangle / s$, is given by

$$P_x(t) = P_z(0)\sin\theta\cos\phi, \quad P_y(t) = P_z(0)\sin\theta\sin\phi,$$
$$P_z(t) = P_z(0)\cos\theta. \tag{17}$$

The use of equations (14) and (16) leads to the following equation defining the polarization tensor:

$$P_{xx}(t) = A\left(\sin^2\theta\cos^2\phi - \frac{1}{3}\right), \quad P_{yy}(t) = A\left(\sin^2\theta\sin^2\phi - \frac{1}{3}\right),$$
$$P_{zz}(t) = A\left(\cos^2\theta - \frac{1}{3}\right), \quad P_{xy}(t) = A\sin^2\theta\sin\phi\cos\phi,$$
$$P_{xz}(t) = A\sin\theta\cos\theta\cos\phi, \quad P_{yz}(t) = A\sin\theta\cos\theta\sin\phi, \quad A = \frac{3}{2}P_{zz}(0). \tag{18}$$

This general equation exhaustively describes the dynamics of the tensor polarization of particles and nuclei in external fields. The time dependence of the angles, θ and ϕ, can be determined with the use of the spin motion equations (5) and (6). For particles/nuclei in accelerators and storage rings, 2×2 spin transfer matrices are commonly applied [13, 14]. To take into account spin–tensor interactions transforming the tensor polarization into the vector polarization, and vice versa, 3×3 spin matrices should be used (see [5]).

6. Summary

In the present work, we have considered the evolution of the tensor polarization of particles and nuclei when this evolution is caused by linear-in-spin interactions with external fields. The tensor polarization becomes constant in the coordinate system rotating with the same angular velocity as the spin. This means that the tensor polarization rotates in the laboratory frame with the above angular velocity. We have derived the general equation (14) defining the time dependence of this polarization. The calculations becomes appreciably simpler when the initial polarization is axially symmetric. In this case, the explicit form of the dynamics of the tensor polarization is given by equation (18).

The approach used has allowed us to couple the dynamics of the tensor and vector polarizations. This result considerably simplifies the determination of the tensor polarization because the vector polarization is exhaustively described by 2×2 spin transfer matrices. As a result, one need not apply 5×5 matrices to investigate the evolution of the five independent components of the polarization tensor. The obtained general equations can be widely used (e.g., for polarized beams in accelerators and storage rings).

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