Triton beta-decay from Nuclear Lattice Effective Field Theory

Ulf-G. Meißner, Univ. Bonn & FZ Jülich

by CAS, PIFI

by DFG, SFB 1639

by ERC, EXOTIC

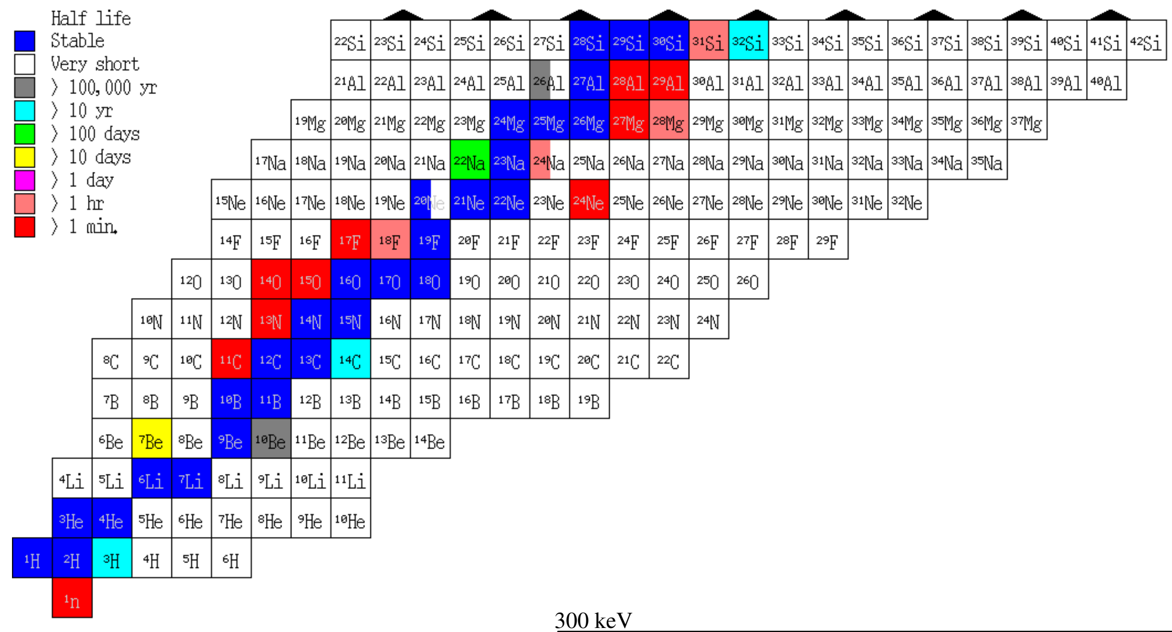
by NRW-FAIR

Contents

Brief Introduction

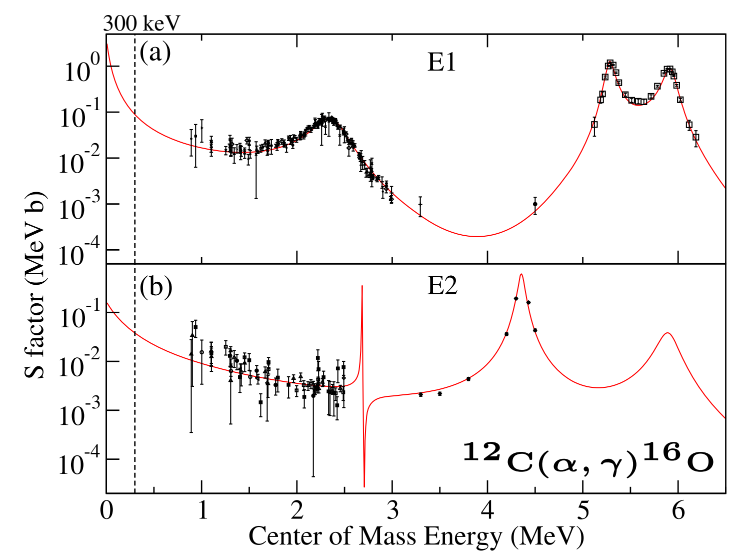
Our goal: Ab initio nuclear structure & reactions

- Nuclear structure:
  - ⋆ limits of stability
  - ⋆ 3-nucleon forces
  - ⋆ alpha-clustering
  - ⋆ EoS & neutron stars
  - ⋮



© National Nuclear Data Center

- Nuclear reactions, nuclear astrophysics:
  - ⋆ alpha-particle scattering
  - ⋆ triple-alpha reaction
  - ⋆ alpha-capture on carbon
  - ⋮



de Boer et al, Rev. Mod. Phys. **89** (2017) 035007

The nucleus as a quantum laboratory

- The nucleus can be used to test the weak interactions and fundamental symmetries

$\hookrightarrow$ Experimental verification of P-violation

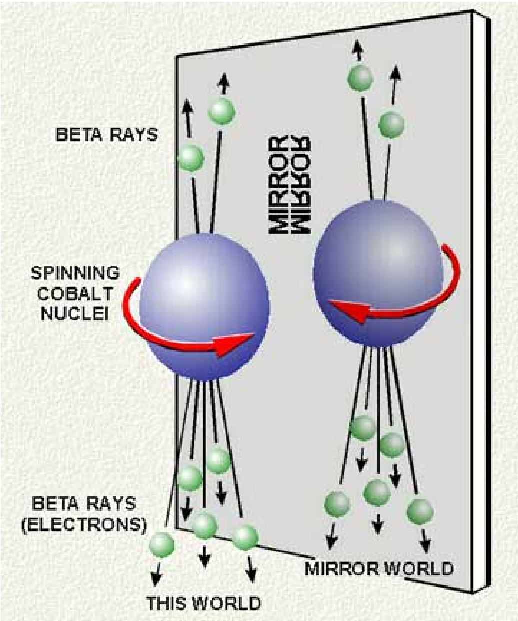
Wu et al. (1957)

$\hookrightarrow$ Hadronic PV in nuclear reactions & decays

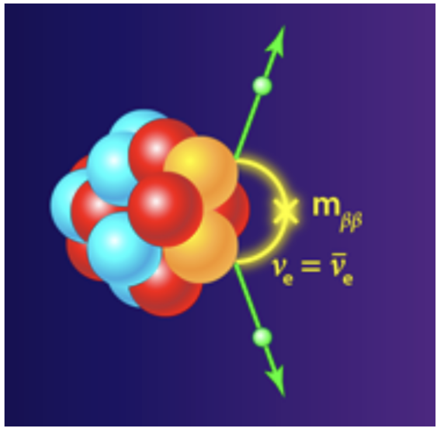
$\hookrightarrow$ Precise determination of V_{us} from superallowed $0^+ \to 0^+$ β-decays

Hardy, Towner (1973) and on-going

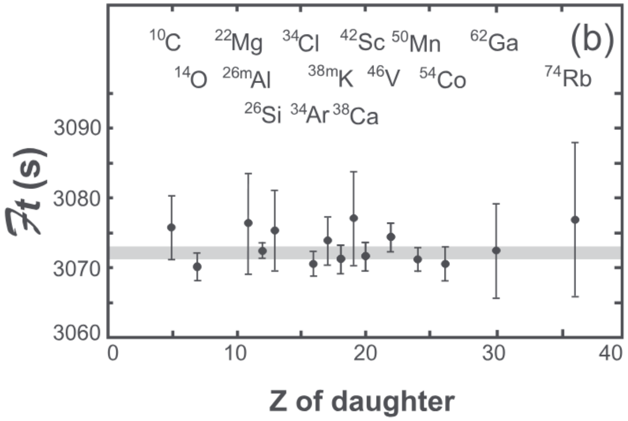
$\hookrightarrow$ Elucidating the nature of the neutrino: if $0\nu2\beta$ deacy is measured $\to$ Majorana nature



@APS



@APS



Nuclear many-body methods – a primer

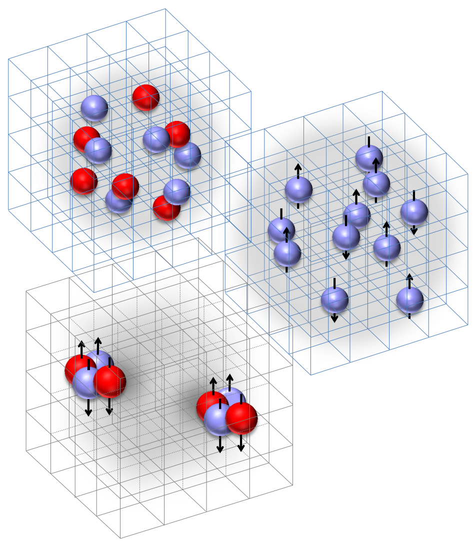
- Solve the Schrödinger equation for A nucleons in a given nucleus:

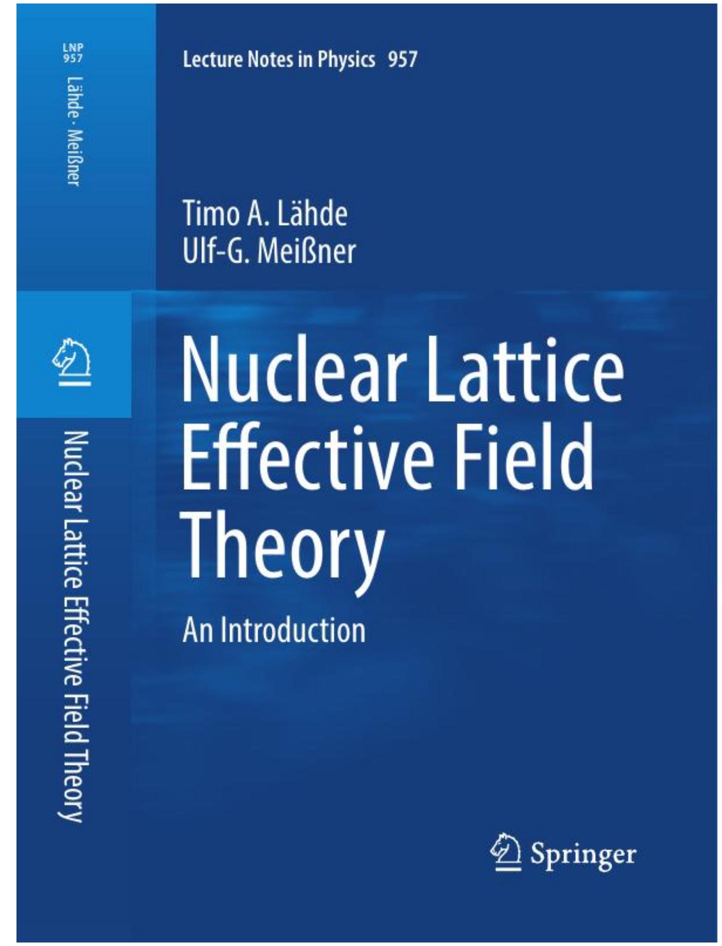
$$\left(-\sum_{i=1}^{A}\frac{\nabla_i^2}{2m_N}+V\right)|\Psi\rangle = E|\Psi\rangle\ , \qquad V = V_{\rm NN}+V_{\rm NNN}+\ldots$$

- a variety of classical many-body approaches:
  - — the shell-model (independent particles) Goeppert Mayer, Jensen (1949)  1963
  - — the deformed shell-model (rotational bands) Nilsson (1957)
  - — collective excitations (deformations, vibrations) Bohr, Mottelson (1958)  1975
  - — coupled-cluster approach (correlations) Koester, Kümmel (1958)
  - — density-functional approach (correlations) Kohn, Sham (1963)  1998
  - — and various others....

$\hookrightarrow$ all have limitations, we want to do better (**exact** solutions w/ **modern** forces)

Chiral EFT on a lattice





T. Lähde & UGM

Nuclear Lattice Effective Field Theory - An Introduction

Springer Lecture Notes in Physics **957** (2019) 1 - 396

More on EFTs

- Much more details on EFTs in light quark physics:

Effective Field Theories

AUTHORS:

Ulf-G Meißner, Rheinische Friedrich-Wilhelms-Universität Bonn and Forschungszentrum Jülich

Akaki Rusetsky, Rheinische Friedrich-Wilhelms-Universität Bonn

DATE PUBLISHED: August 2022

AVAILABILITY: Available

FORMAT: Hardback

ISBN: 9781108476980

Rate & review

https://www.cambridge.org/de/academic/subjects/physics/theoretical-physics-and-mathematical-physics/effective-field-theories

Nuclear lattice effective field theory (NLEFT)

Frank, Brockmann (1992), Koonin, Müller, Seki, van Kolck (2000) , Lee, Schäfer (2004), . . .
Borasoy, Krebs, Lee, UGM, Nucl. Phys. **A768** (2006) 179; Borasoy, Epelbaum, Krebs, Lee, UGM, Eur. Phys. J. **A31** (2007) 105

- *new method* to tackle the nuclear many-body problem

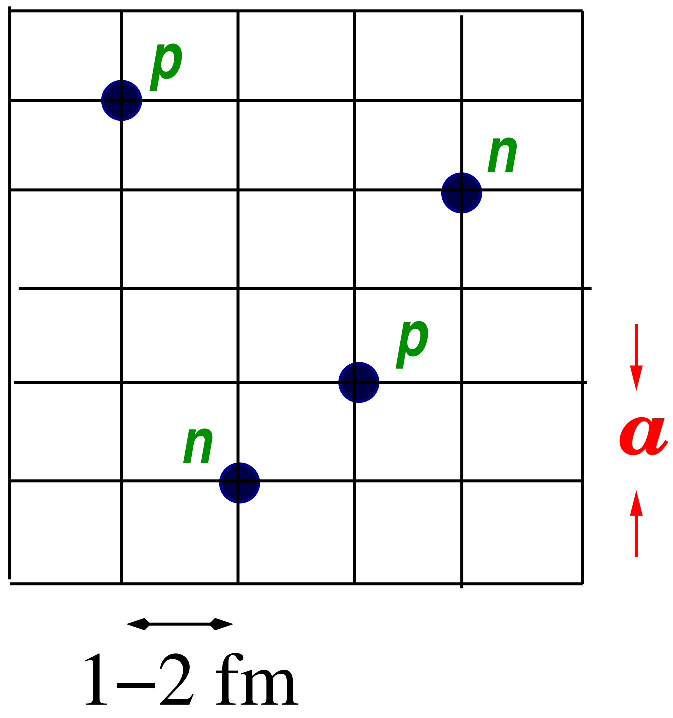
- discretize space-time $V = L_s \times L_s \times L_s \times L_t$:
  nucleons are point-like particles on the sites

- discretized chiral potential w/ pion exchanges
  and contact interactions + Coulomb

  → see Epelbaum, Hammer, UGM, Rev. Mod. Phys. **81** (2009) 1773

- typical lattice parameters

$$p_{\max} = \frac{\pi}{a} \simeq 315 - 630 \text{ MeV [UV cutoff]}$$



- strong suppression of sign oscillations due to approximate Wigner SU(4) symmetry

  E. Wigner, Phys. Rev. **51** (1937) 106; T. Mehen et al., Phys. Rev. Lett. **83** (1999) 931; J. W. Chen et al., Phys. Rev. Lett. **93** (2004) 242302

- physics independent of the lattice spacing for $a = 1 \ldots 2$ fm

  Alarcon, Du, Klein, Lähde, Lee, Li, Lu, Luu, UGM, EPJA **53** (2017) 83; Klein, Elhatisari, Lähde, Lee, UGM, EPJA **54** (2018) 121

Transfer matrix method

- Correlation–function for A nucleons: $Z_A(\tau) = \langle \Psi_A | \exp(-\tau H) | \Psi_A \rangle$

  Euclidean time

  with Ψ_A a Slater determinant for A free nucleons
  [or a more sophisticated (correlated) initial/final state]

- Transient energy

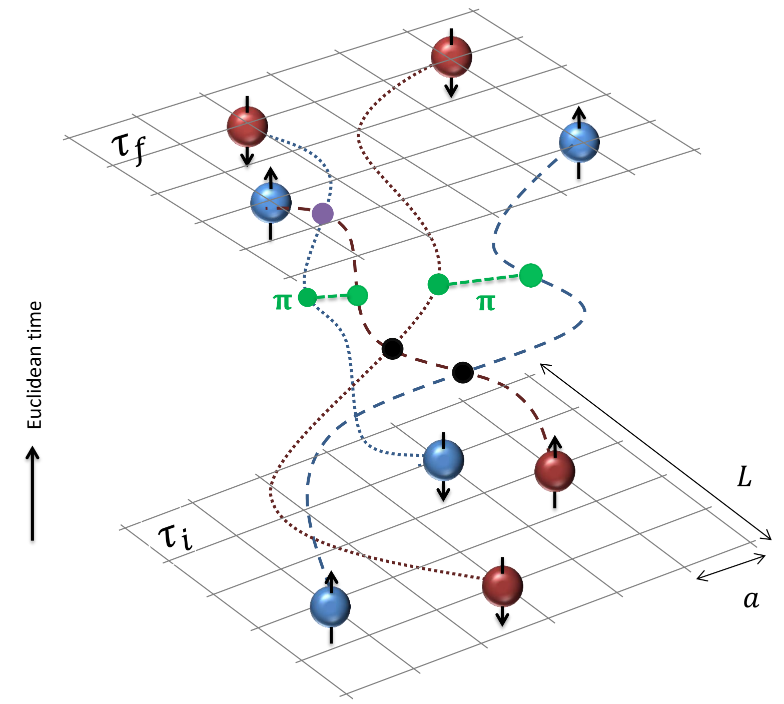
$$E_A(\tau) = -\frac{d}{d\tau} \ln Z_A(\tau)$$

$\rightarrow$ ground state: $E_A^0 = \lim_{\tau\to\infty} E_A(\tau)$

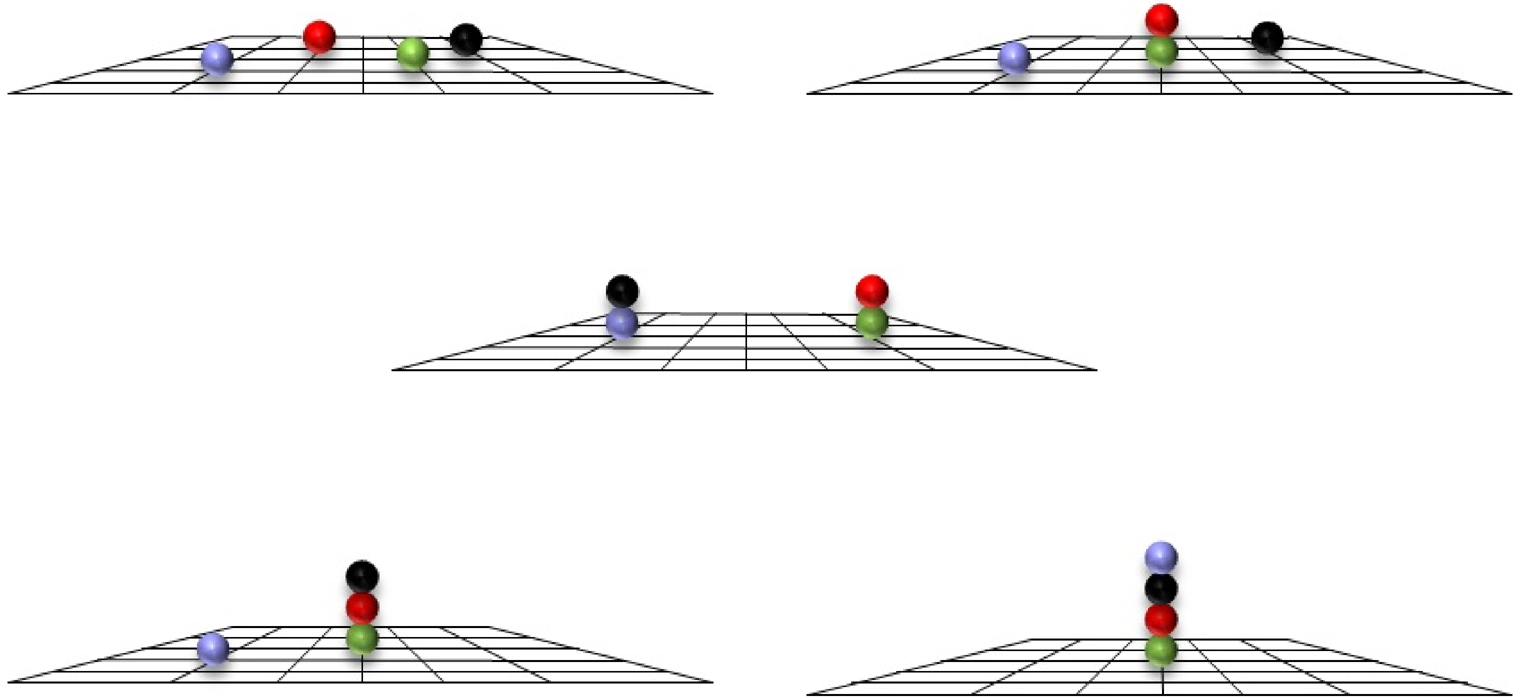
- Excited states: $Z_A(\tau) \rightarrow Z_A^{ij}(\tau)$, diagonalize,

  e.g. $0_1^+, 0_2^+, 0_3^+, \ldots$ in ^{12}C

- Other operators (radii, ...)



Configurations



⇒ all *possible* configurations are sampled

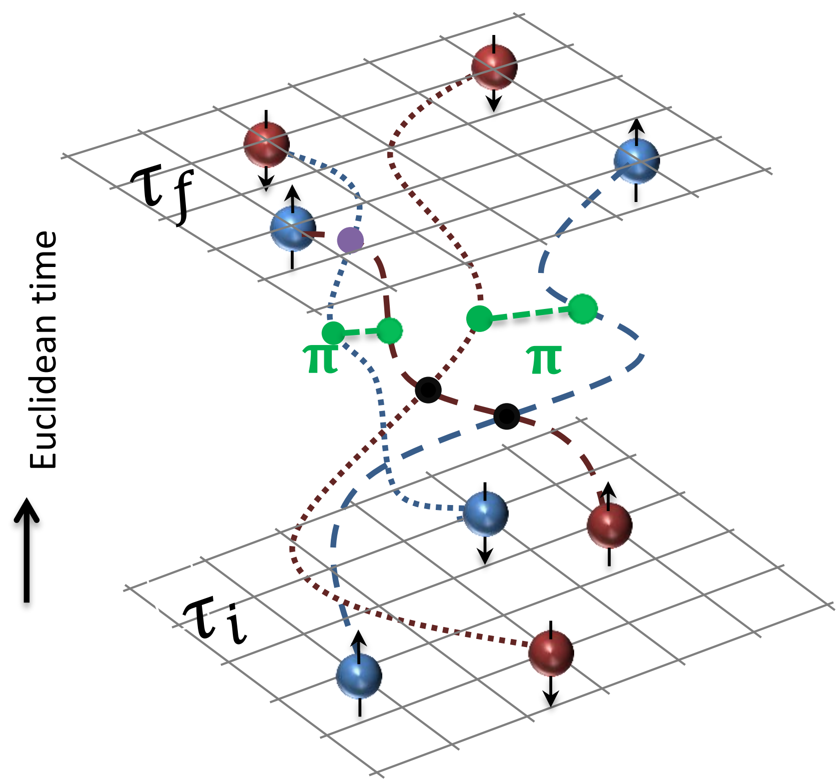
⇒ preparation of *all possible* initial/final states

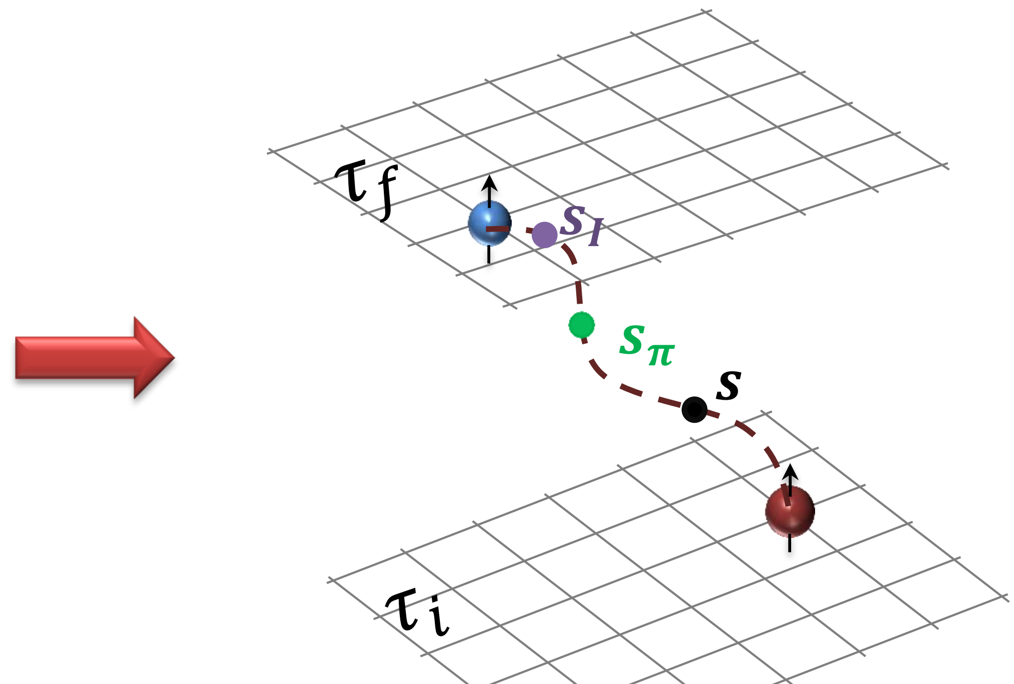
⇒ *clustering* emerges *naturally*

Auxiliary field method

- Represent interactions by auxiliary fields (Gaussian quadrature):

$$\exp\left[-\frac{C}{2}\left(N^\dagger N\right)^2\right] = \sqrt{\frac{1}{2\pi}}\int ds \exp\left[-\frac{s^2}{2}+\sqrt{C}\, s\left(N^\dagger N\right)\right]$$

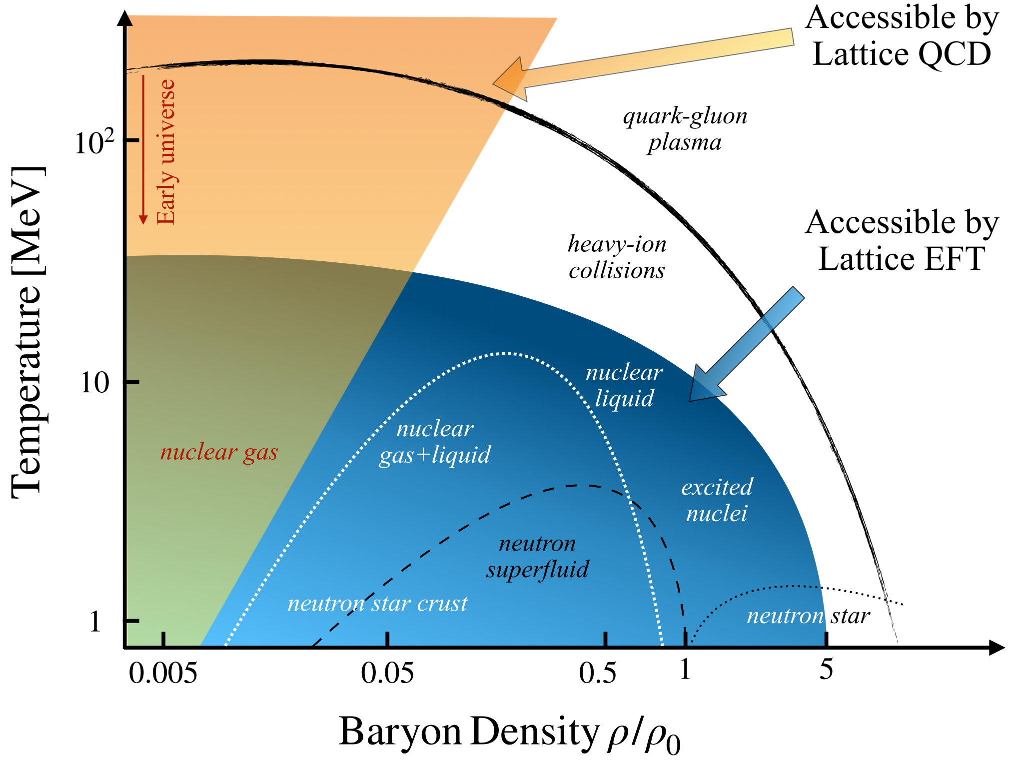




optimally suited for parallel computing!

Comparison to lattice QCD

LQCD (quarks & gluons)	NLEFT (nucleons & pions)
relativistic fermions	non-relativistic fermions
renormalizable th'y	EFT
continuum limit	no continuum limit
(un)physical masses	physical masses
Coulomb - difficult	Coulomb - easy
high T/small ρ	small T/nuclear densities
sign problem severe	sign problem moderate



- For nuclear physics, NLEFT is the far better methodology!

Computational equipment

- Present = JUWELS (modular system) + FRONTIER + ...

The minimal nuclear interaction: Foundations

A minimal nuclear interaction

- Basic problem: Straightforward application of chiral EFT forces leads to problems when one goes beyond light nuclei (e.g. the radius problem)

- Main idea: Construct a minimal nuclear interactions that reproduces the ground state properties of light nuclei, medium-mass nuclei, and neutron matter simultaneously with no more than a few percent error in the energies and charge radii

- This can be achieved by making use of Wigner's SU(4) spin-isospin symmetry

Wigner, Phys. Rev. **C 51** (1937) 106

- If the nuclear Hamiltonian does not depend on spin and isospin, then it is obviously invariant under SU(4) transformations [really $U(4) = U(1) \times SU(4)$]:

$$N \to UN\,, \quad U \in SU(4)\,, \quad N = \begin{pmatrix} p \\ n \end{pmatrix}$$

$$N \to N + \delta N\,, \quad \delta N = i\epsilon_{\mu\nu}\sigma^\mu\tau^\nu\, N\,, \quad \sigma^\mu = (1, \sigma_i)\,, \quad \tau^\mu = (1, \tau_i)$$

Remarks on Wigner's SU(4) symmetry

- Wigner SU(4) spin-isospin symmetry in the context of pionless nuclear EFT

  ↪ large scattering lengths — Mehen, Stewart, Wise, Phys. Rev. Lett. **83** (1999) 931

- Wigner SU(4) spin-isospin symmetry is particularly beneficial for NLEFT

  ↪ suppression of sign oscillations — Chen, Lee, Schäfer, Phys. Rev. Lett. **93** (2004) 242302

  ↪ provides a very much improved LO action when smearing is included

  Lu, Li, Elhatisari, Lee, Epelbaum, UGM, Phys. Lett. B **797** (2019) 134863

- Initimately related to α-clustering in nuclei

  ↪ cluster states in ^{12}C like the famous Hoyle state

  Epelbaum, Krebs, Lee, UGM, Phys. Rev. Lett. **106** (2011) 192501

  ↪ nuclear physics is close to a quantum phase transition

  Elhatisari et al., Phys. Rev. Lett. **117** (2016) 132501

Essential elements for nuclear binding

Lu, Li, Elhatisari, Epelbaum, Lee, UGM, Phys. Lett. B **797** (2019) 134863 [arXiv:1812.10928]

- Highly SU(4) symmetric LO action without pions, only **four** parameters

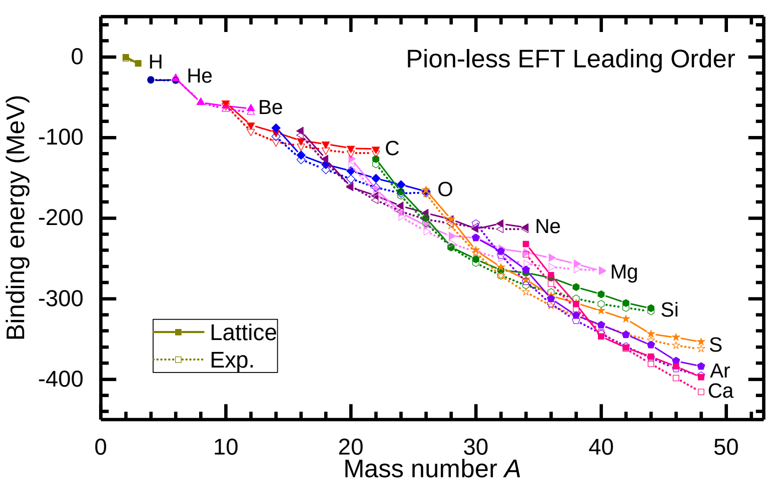
$$H_{\mathrm{SU(4)}} = H_{\mathrm{free}} + \frac{1}{2!} C_2 \sum_n \tilde{\rho}(n)^2 + \frac{1}{3!} C_3 \sum_n \tilde{\rho}(n)^3$$

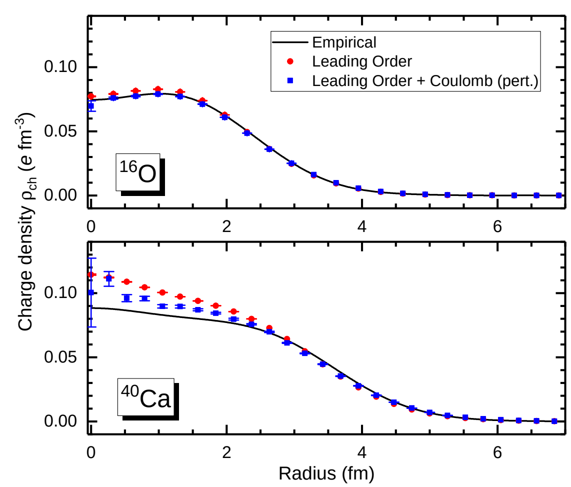
$$\tilde{\rho}(n) = \sum_i \tilde{a}_i^\dagger(n)\tilde{a}_i(n) + s_L \sum_{|n'-n|=1} \sum_i \tilde{a}_i^\dagger(n')\tilde{a}_i(n')$$

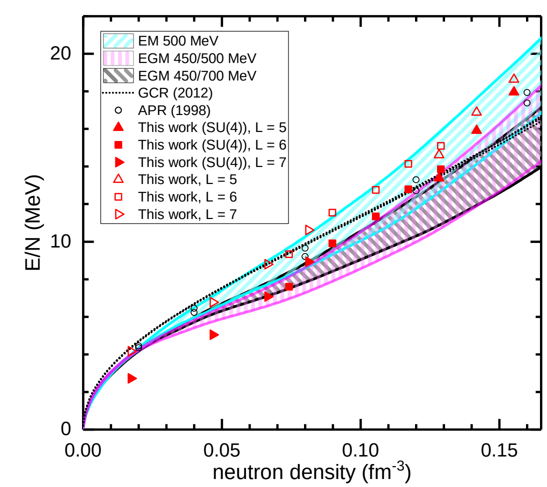
$$\tilde{a}_i(n) = a_i(n) + s_{NL} \sum_{|n'-n|=1} a_i(n')$$

s_L controls the locality of the interactions, s_{NL} the non-locaity of the smearing

$\rightarrow$ describes binding energies, radii, charge densities and the EoS of neutron matter







The minimal nuclear interaction: Applications

Wigner's SU(4) symmetry and the carbon spectrum

- Study of the spectrum (and other properties) of ^{12}C

  $\hookrightarrow$ spin-orbit splittings are known to be weak

  Hayes, Navratil, Vary, Phys. Rev. Lett. **91** (2003) 012502 Johnson, Phys. Rev. C **91** (2015) 034313

  $\hookrightarrow$ start with cluster and shell-model configurations $\to$ next slide

- Fit the four parameters:

  C_2, C_3 – ground state energies of ^{4}He and ^{12}C

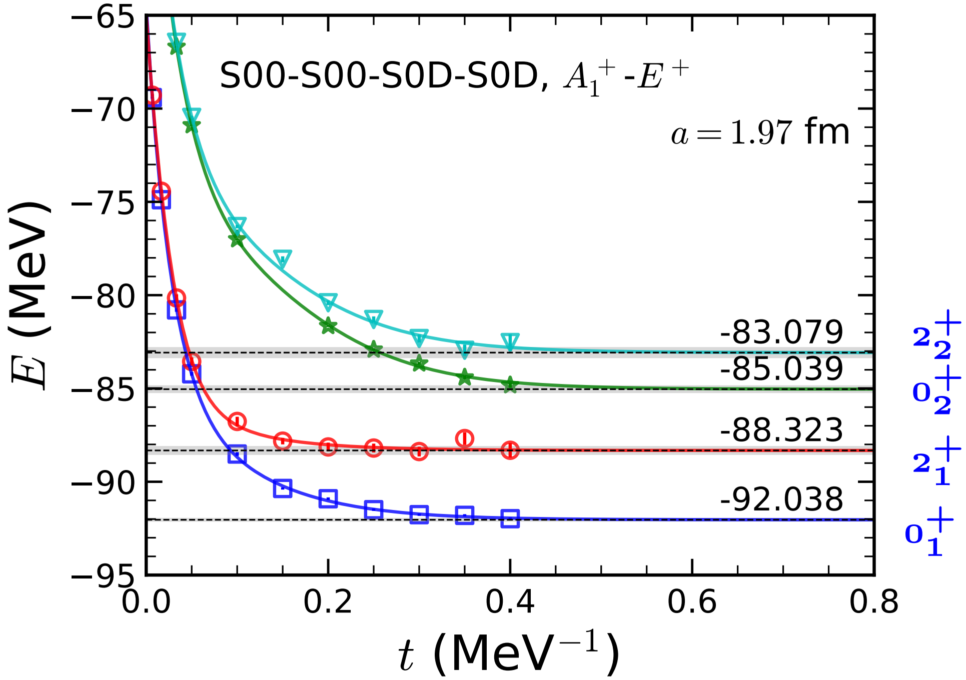
  s_L – radius of ^{12}C around 2.4 fm

  s_{NL} – best overall description of the transition rates

- Calculation of em transitions

  requires coupled-channel approach

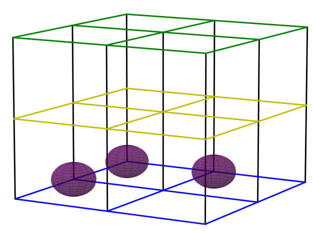
  e.g. 0^+ and 2^+ states



Configurations

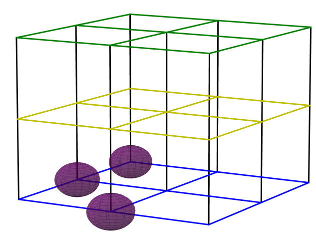
- Cluster and shell model configurations

S1



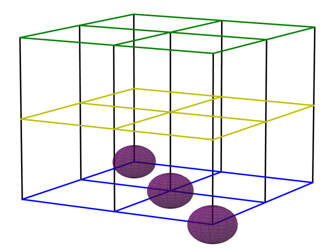
— isoscele right triangle

S2



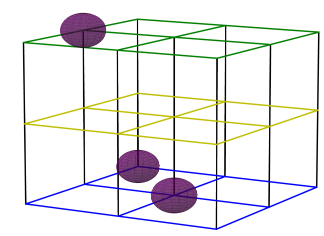
— “bent-arm” shape

S3



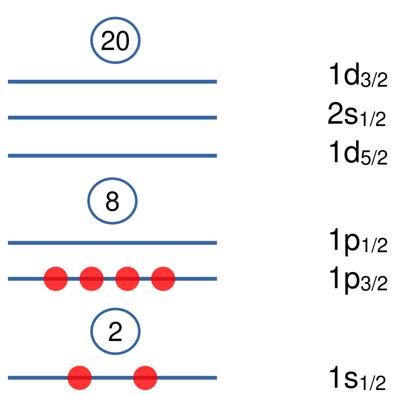
— linear diagonal chain

S4

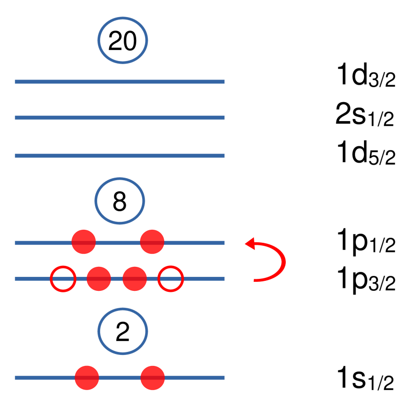


— acute isoscele triangle

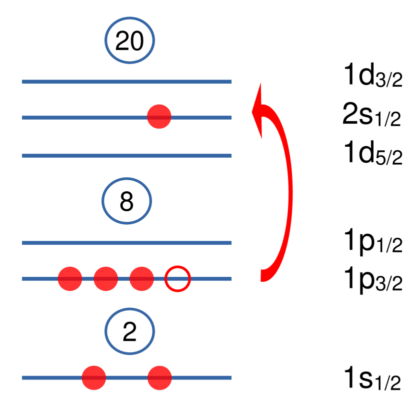
Gaussian wave packets
$w = 1.7 - 2.1$ fm



— ground state $|0\rangle$



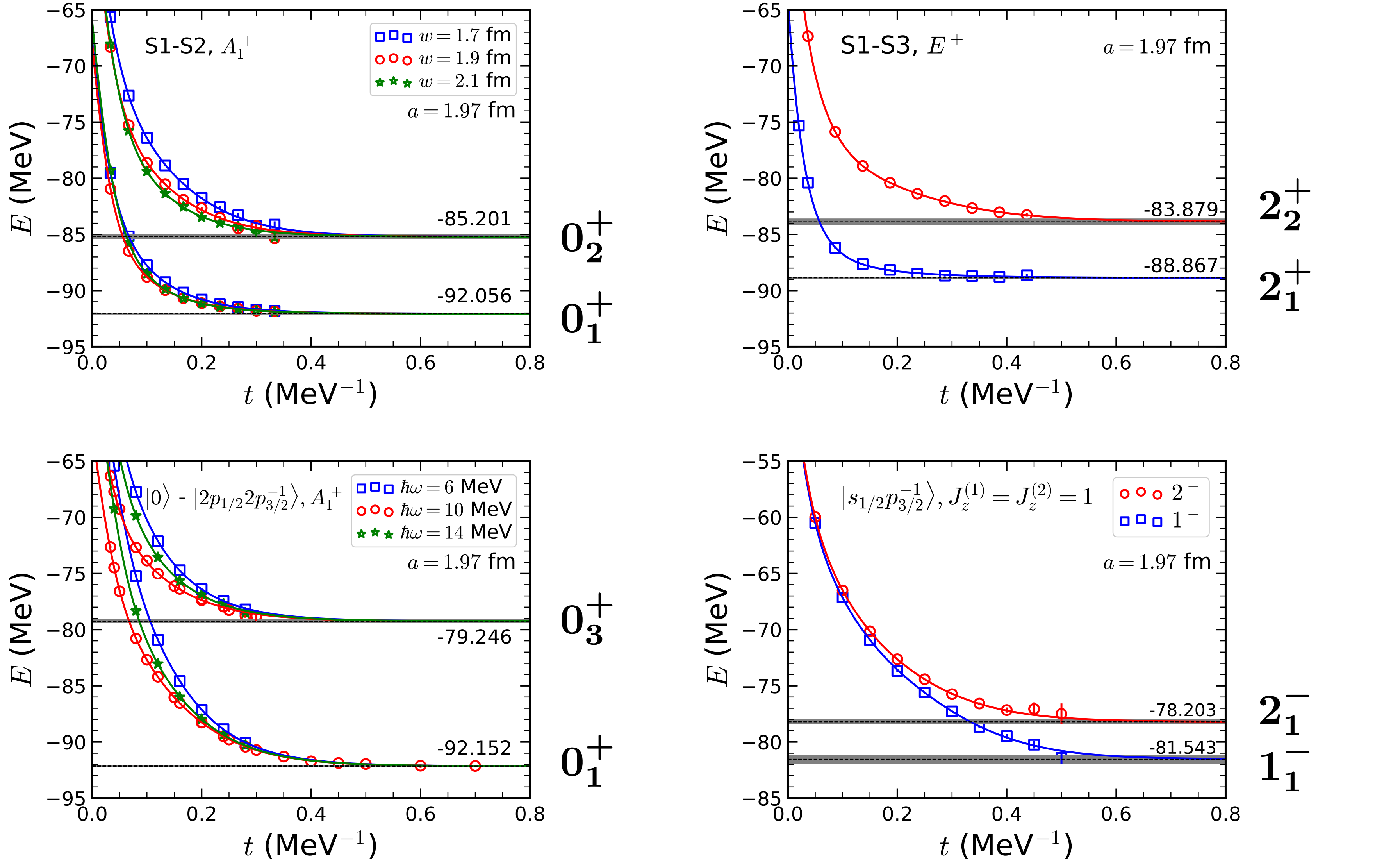
— $2p$-$2h$ state, $J_z = 0$



— $1p$-$1h$ state, $J_z^{(1)} = J_z^{(2)} = 1$

Transient energies

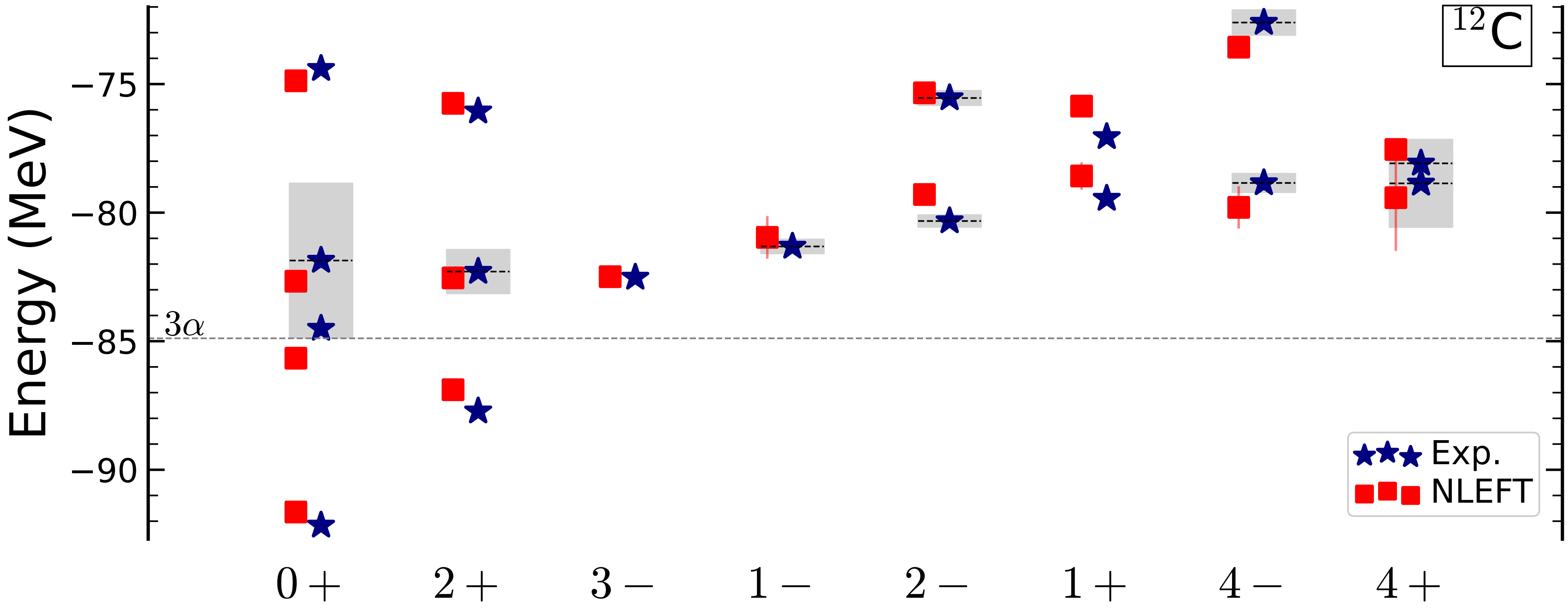
- Transient energies from cluster and shell-model configurations



Spectrum of ^{12}C

Shen, Elhatisari, Lähde, Lee, Lu, UGM, Nature Commun. **14** (2023) 2777

- Improved description when 3NFs are included, amazingly good

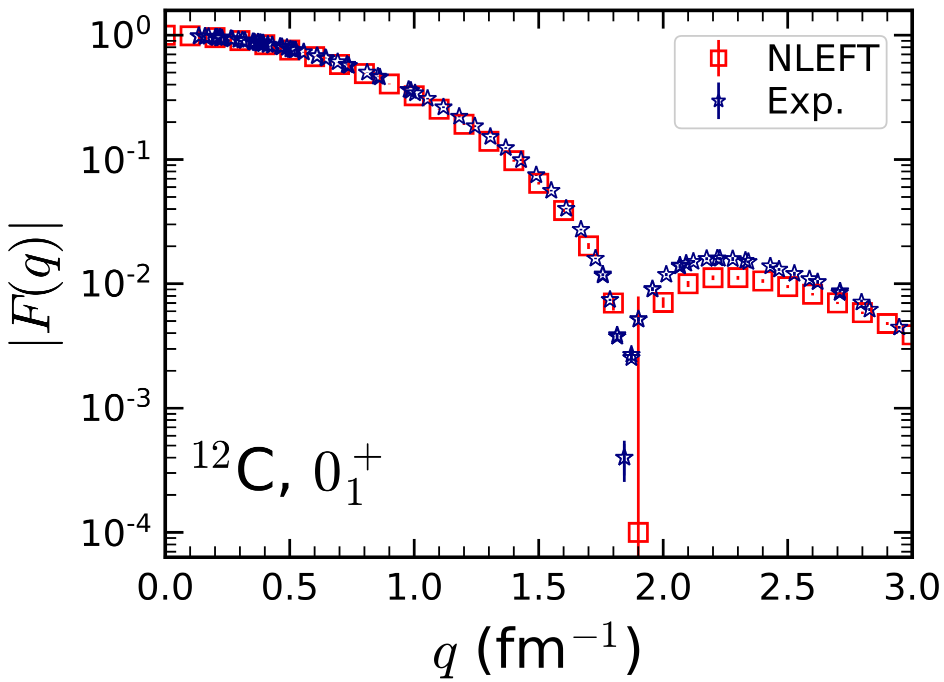


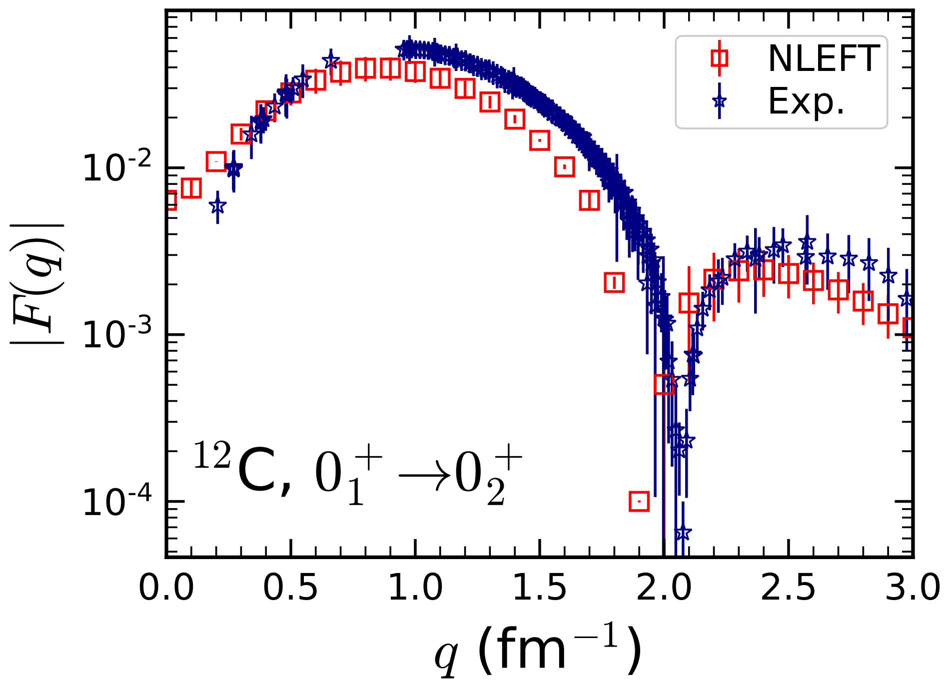
→ solidifies earlier NLEFT statements about the structure of the $\mathbf{0_2^+}$ and $\mathbf{2_2^+}$ states

Electromagnetic properties

Shen, Elhatisari, Lähde, Lee, Lu, UGM, Nature Commun. **14** (2023) 2777

- Form factors and transition ffs [essentially parameter-free]:



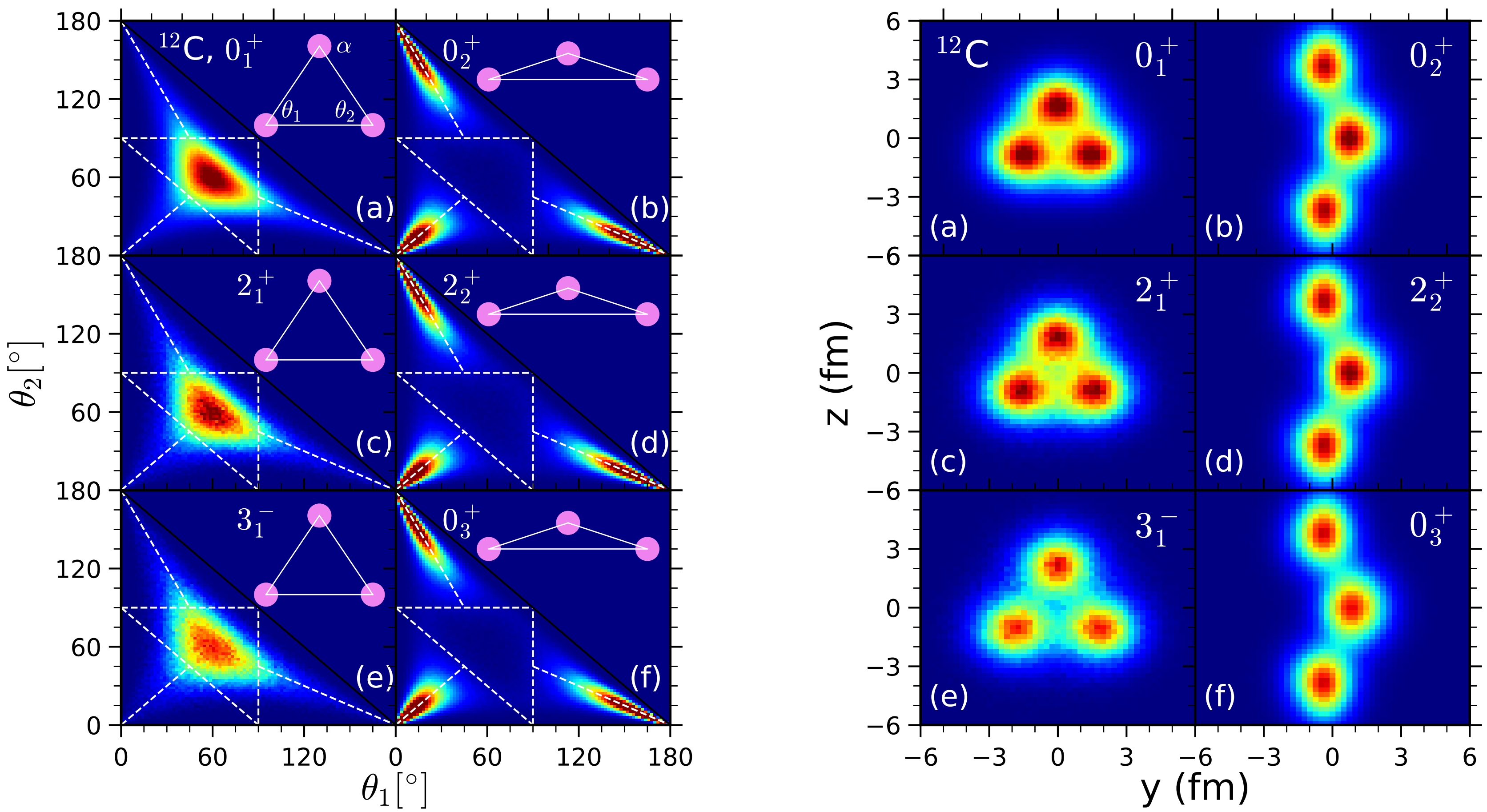


Sick, McCarthy, Nucl. Phys. A **150** (1970) 631
Strehl, Z. Phys. **234** (1970) 416
Crannell et al., Nucl. Phys. A **758** (2005) 399

Chernykh et al., Phys. Rev. Lett. **105** (2010) 022501

Emergence of geometry

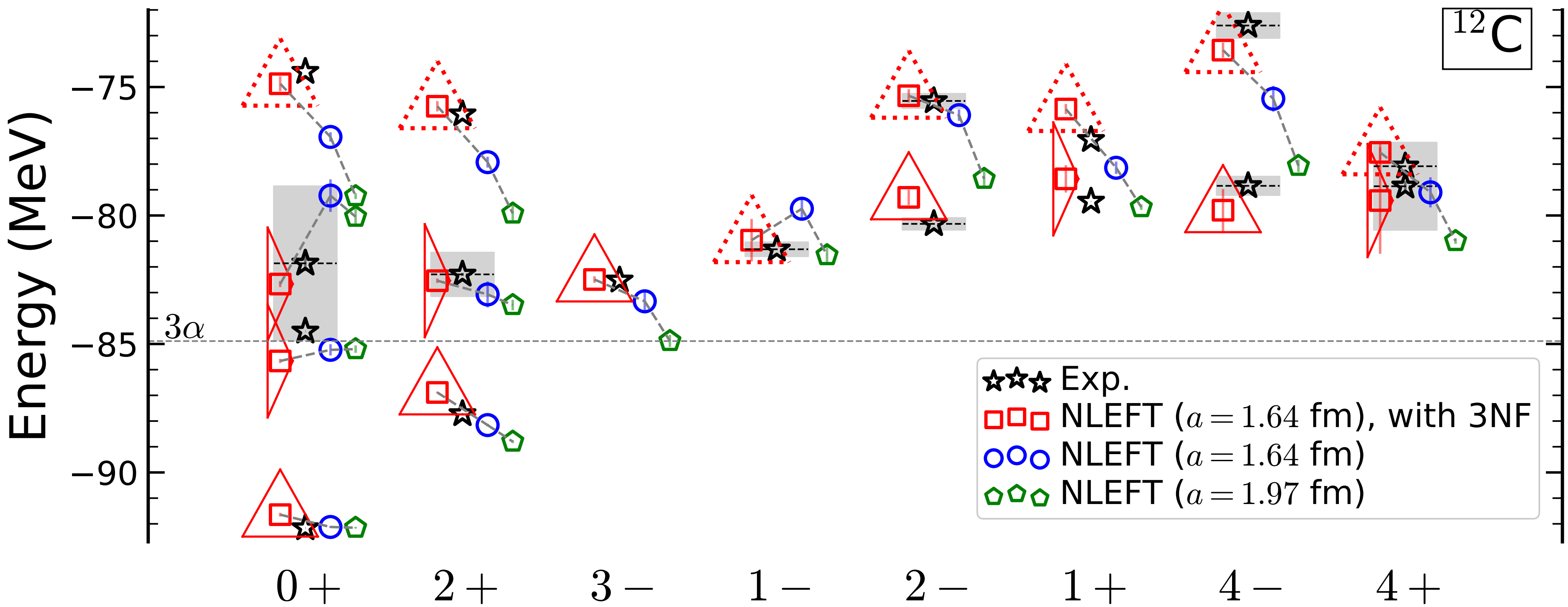
- Use the pinhole algorithm to measure the distribution of α-clusters/matter:



- equilateral & obstuse triangles $\rightarrow$ $\mathbf{2^+}$ states are excitations of the $\mathbf{0^+}$ states

Emergence of duality

- ^{12}C spectrum shows a cluster/shell-model duality

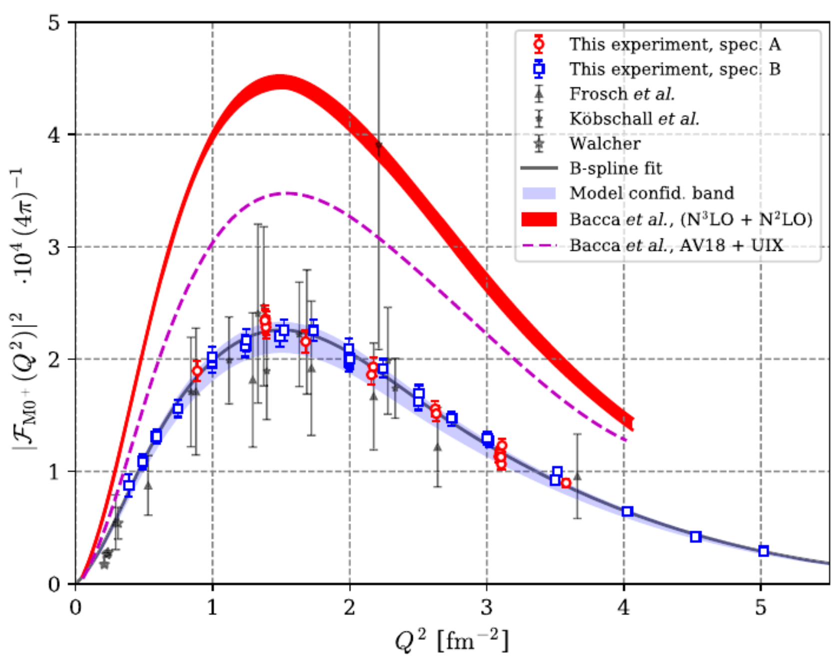


- dashed triangles: strong 1p-1h admixture in the wave function

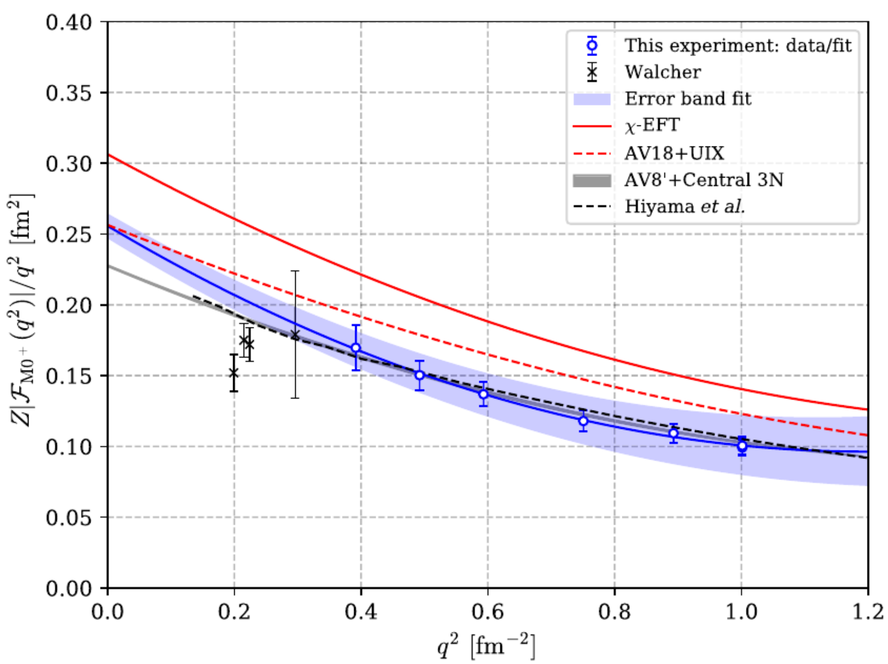
The ^{4}He form factor puzzle

- Recent Mainz measurements of $F_{M0}(0_2^+ \to 0_1^+)$ appear to be in stark disagreement with *ab initio* nuclear theory Kegel et al., Phys. Rev. Lett. **130** (2023) 152502

- Monopole transition ff
- low-momentum expansion



[calculations from 2013]



$\Rightarrow$ **A low-energy puzzle for nuclear forces?**

Ab initio calculation of the ^{4}He transition form factor

UGM, Shen, Elhatisari, Lee, Phys. Rev. Lett. **132** (2024) 062501 [2309.01558 [nucl-th]]

- Use the essential elements action, **all parameters fixed!**

- Calculate the transition ff and its low-energy expansion form the transition density

$$\rho_{\rm tr}(r) = \langle 0_1^+ | \hat{\rho}(\vec{r}) | 0_2^+ \rangle$$

$$F(q) = \frac{4\pi}{Z} \int_0^\infty \rho_{\rm tr}(r) j_0(qr) r^2 dr = \frac{1}{Z} \sum_{\lambda=1}^{\infty} \frac{(-1)^\lambda}{(2\lambda+1)!} q^{2\lambda} \langle r^{2\lambda} \rangle_{\rm tr}$$

$$\frac{Z|F(q^2)|}{q^2} = \frac{1}{6} \langle r^2 \rangle_{\rm tr} \left[1 - \frac{q^2}{20} \mathcal{R}_{\rm tr}^2 + \mathcal{O}(q^4) \right]$$

$$\mathcal{R}_{\rm tr}^2 = \langle r^4 \rangle_{\rm tr} / \langle r^2 \rangle_{\rm tr}$$

- The first excited state sits in the continuum & close to the ^3H-p threshold

  $\hookrightarrow$ use large volumes $L = 10, 11, 12$ or $L = 13.2$ fm, 14.5 fm, 15.7 fm

  $\hookrightarrow$ the lattice spacing is fixed to $a = 1.32$ fm, corresponding $\Lambda = \pi/a = 465$ MeV

The first excited state

- 3 coupled channels with $\mathbf{0}^+$ q.n's $\rightarrow$ accelerates convergence as $L_t \rightarrow \infty$
- Shell-model wave functions (4 nucleons in $1s_{1/2}$, twice 3 in $1s_{1/2}$ and 1 in $2s_{1/2}$)

L [fm]	$E(0_1^+)$ [MeV]	$E(0_2^+)$ [MeV]	ΔE [MeV]
13.2	$-28.32(3)$	$-8.37(14)$	$0.28(14)$
14.5	$-28.30(3)$	$-8.02(14)$	$0.42(14)$
15.7	$-28.30(3)$	$-7.96(9)$	$0.40(9)$

$\hookrightarrow$ statistical and large-L_t errors

$\hookrightarrow$ agreement w/ experiment: $E(0_1^+) = 28.3$ MeV, $\Delta E = 0.4$ MeV

$\hookrightarrow$ ΔE consistent w/ no-core Gamov shell model (no 3NFs)

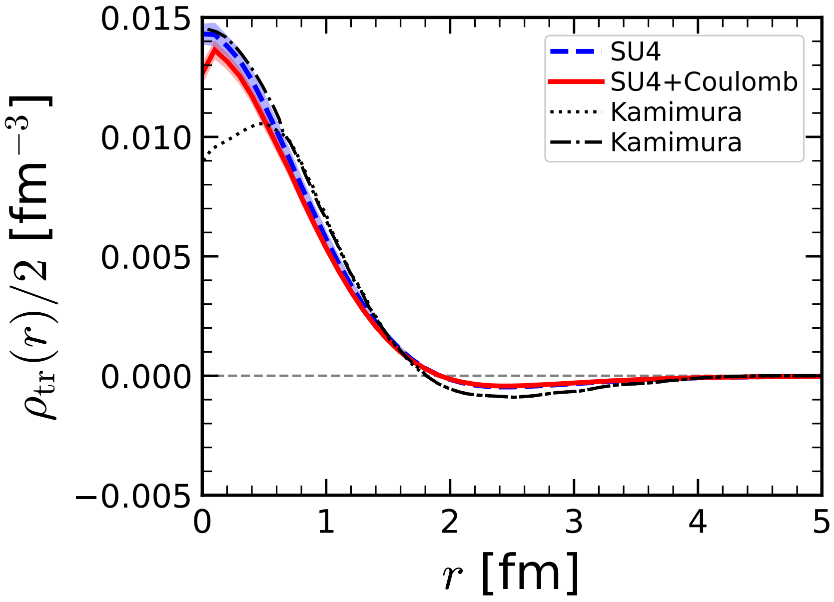
Michel, Nazarewicz, Ploszajczak, Phys. Rev. Lett. **131** (2023) 242502

$\hookrightarrow$ consistent w/ recent work of the Pisa group (continuum)

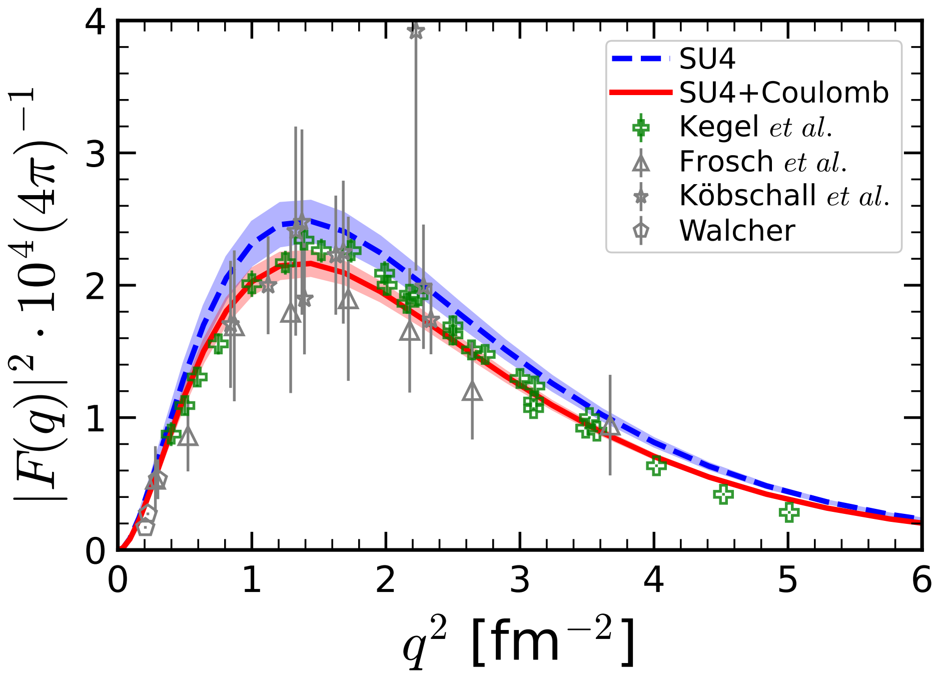
Viviani, Kievsky, Marcucci, Girlanda, Few Body Syst. **65** (2024) 74

The transition form factor

- Transition charge density



- Transition form factor



$\hookrightarrow$ agrees with the reconstructed one from Kamimura PTEP **2023** (2023) 071D01

$\hookrightarrow$ very small central depletion (no zero)

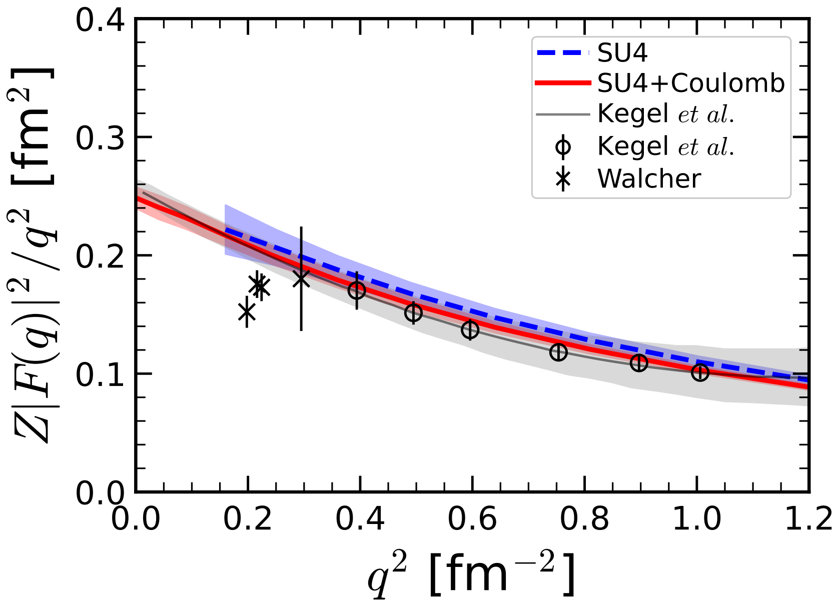
$\hookrightarrow$ excellent description of the data

$\hookrightarrow$ Coulomb required plus smaller uncertainty (improved signal)

$\hookrightarrow$ 3NFs important!

The transition form factor II

- Small momentum expansion



	$\langle r^2\rangle_{\rm tr}$ [fm^2]	$\mathcal{R}_{\rm tr}$ [fm]
Experiment	1.53 ± 0.05	4.56 ± 0.15
Th (AV8'+ centr. 3N)*	1.36 ± 0.01	4.01 ± 0.05
Th (AV18 + UIX)	1.54 ± 0.01	3.77 ± 0.08
Th (NLEFT)	1.49 ± 0.01	4.00 ± 0.04

*Hiyama, Gibson, Kamimura, PRC **70** (2004) 031001

↪ Also consistent description of the low-energy data

↪ **No puzzle** to the nuclear forces!

↪ Can be improved using N3LO action + wave function matching

Elhatisari et al., Nature **630** (2024) 59

Chiral Interactions at N3LO: Foundations

Towards precision calculations of heavy nuclei

- Groundbreaking work (Hoyle state, α-α scattering, ...) done at N2LO

  $\hookrightarrow$ precision limited, need to go to N3LO

- Two step procedure:

  1) Further improve the LO action

  $\hookrightarrow$ minimize the sign oscillations

  $\hookrightarrow$ minimize the higher-body forces

  $\hookrightarrow$ essentially done $\surd$ $\longrightarrow$ as just discussed

  2) Work out the corrections to N3LO

  $\hookrightarrow$ first on the level of the NN interaction $\surd$

  $\hookrightarrow$ new important technique: **wave function matching** $\surd$

  $\hookrightarrow$ second for the spectra/radii/... of nuclei (first results) $\surd$

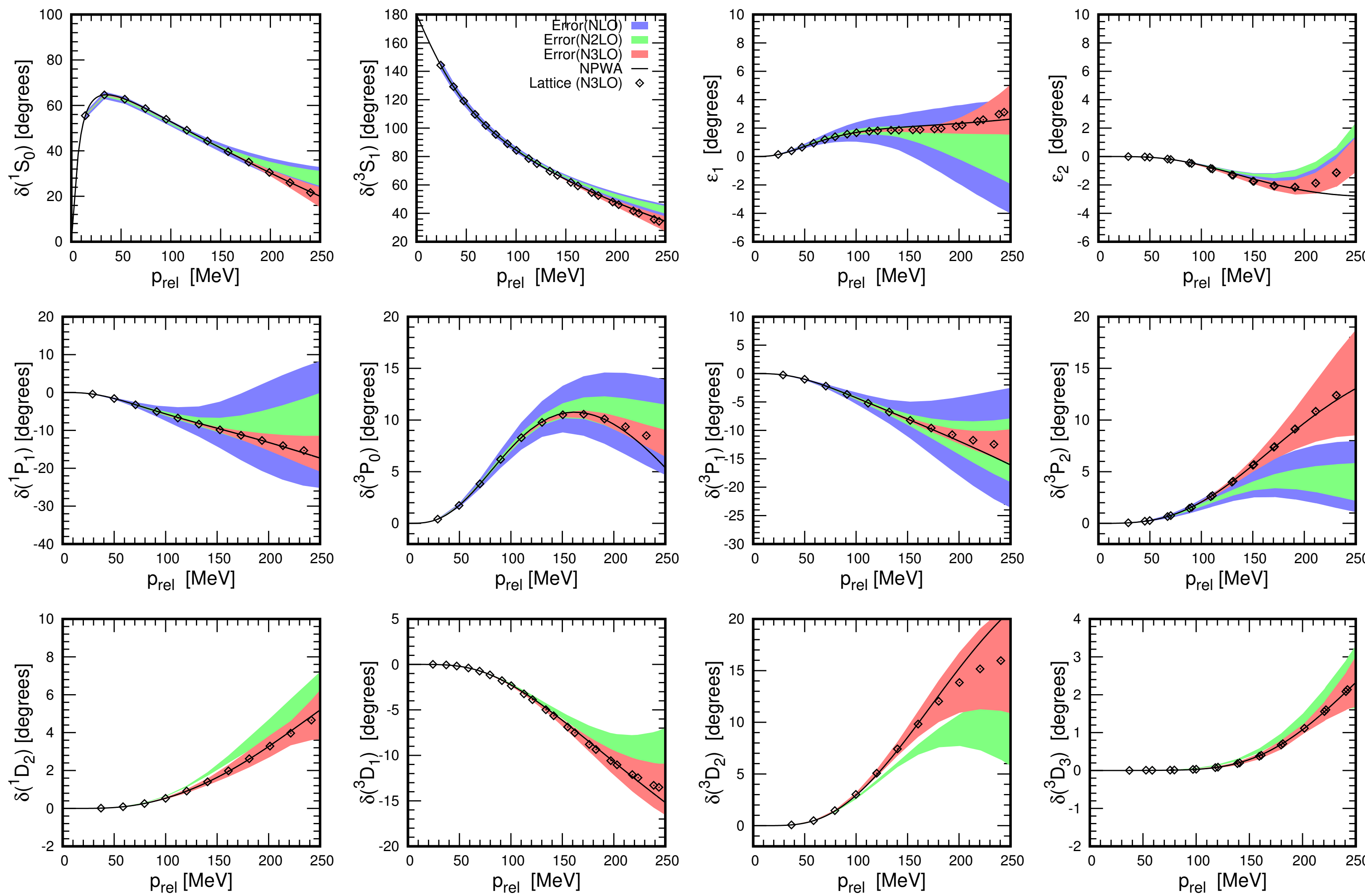
  $\hookrightarrow$ third for nuclear reactions/astrophysics (first results) $\surd$

NN interaction at N3LO

Li et al., Phys. Rev. C **98** (2018) 044002; Phys. Rev. C **99** (2019) 064001

- np phase shifts including uncertainties for $a = 1.32$ fm (cf. Nijmegen PWA)

NLO
N2LO
N3LO

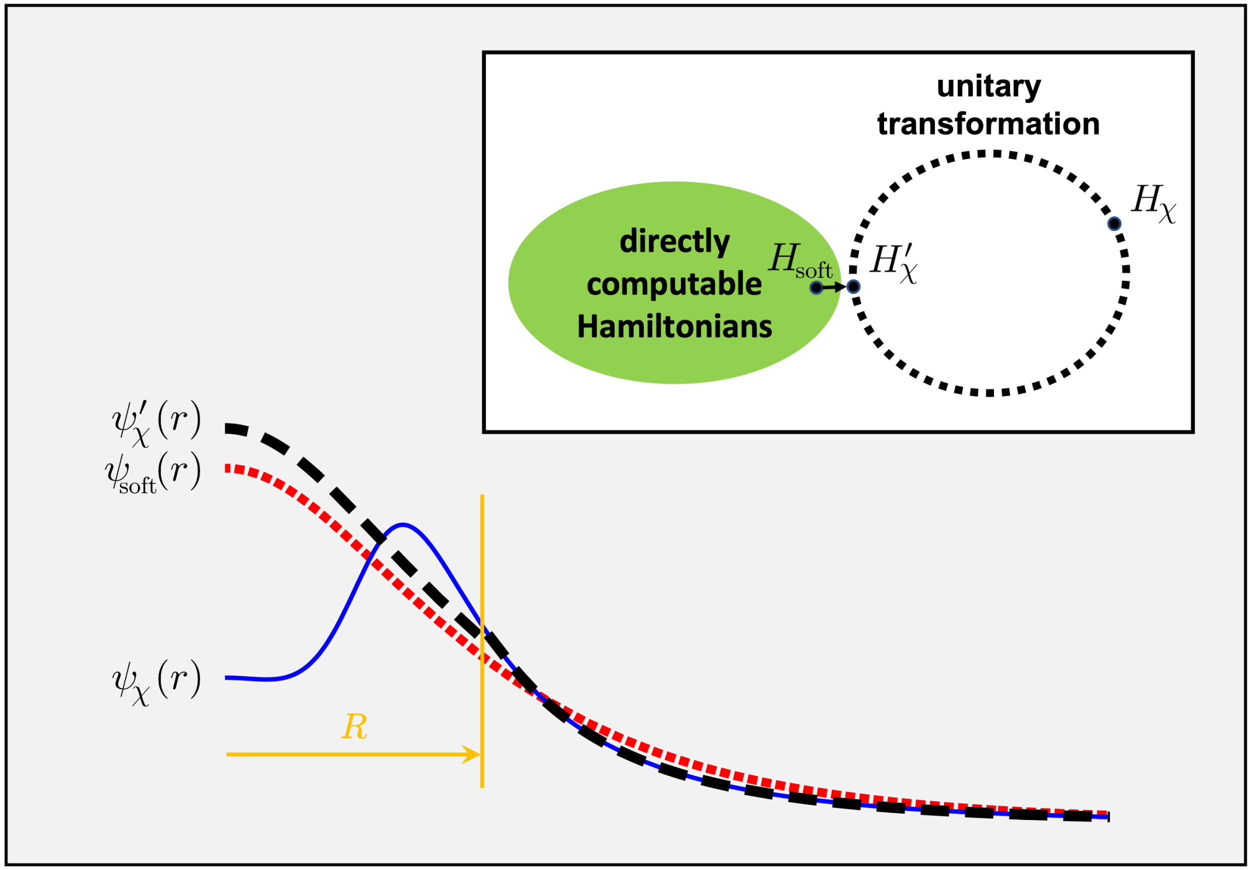


uncertainty estimates á la Epelbaum, Krebs, UGM,
Eur. Phys. J. A **51** (2015) 53

Wave function matching

Elhatisari et al., Nature **630** (2024) 59 [arXiv:2210.17488 [nucl-th]]

- Graphical representation of w.f. matching



- W.F. matching is a “Hamiltonian translator”:
  eigenenergies from $\boldsymbol{H_1}$ but w.f. from $\boldsymbol{H_2 = U^\dagger H_1 U}$

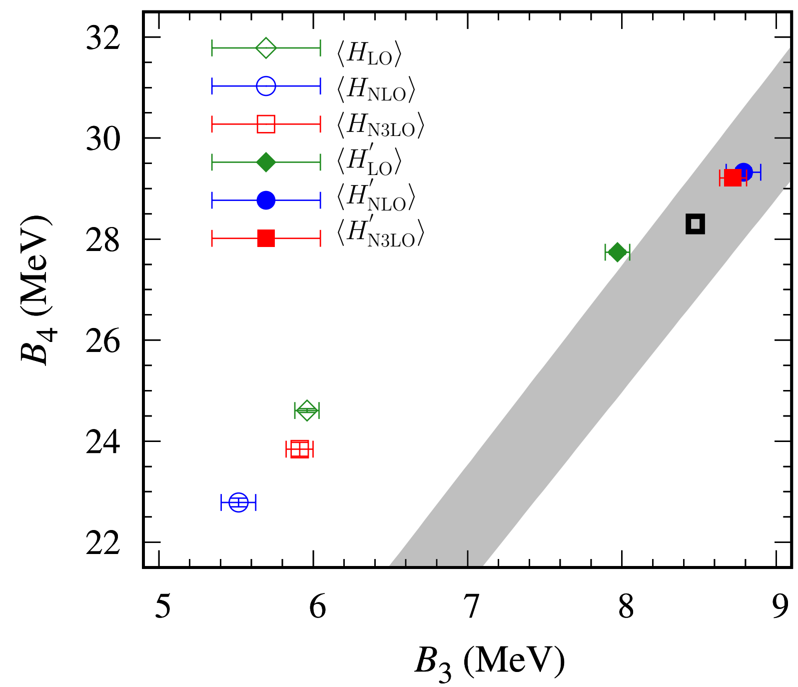
Chiral Interactions at N3LO: Applications to nuclear structure

Wave function matching for light nuclei

Elhatisari et al., Nature **630** (2024) 59 [arXiv:2210.17488 [nucl-th]], L. Bovermann, PhD thesis

- W.F. matching for the light nuclei

Nucleus	$B_{\rm LO}$ [MeV]	$B_{\rm N3LO}$ [MeV]	Exp. [MeV]
$E_{\chi,\rm d}$	1.79	2.21	2.22
$\langle\psi^0_{\rm soft}\,\vert\, H_{\chi,\rm d}\,\vert\psi^0_{\rm soft}\rangle$	0.45	0.62	
$\langle\psi^0_{\rm soft}\,\vert\, H'_{\chi,\rm d}\,\vert\psi^0_{\rm soft}\rangle$	1.65	2.01	
$\langle\psi^0_{\rm soft}\,\vert\, H_{\chi,\rm t}\,\vert\psi^0_{\rm soft}\rangle$	5.96(8)	5.91(9)	8.48
$\langle\psi^0_{\rm soft}\,\vert\, H'_{\chi,\rm t}\,\vert\psi^0_{\rm soft}\rangle$	7.97(8)	8.72(9)	
$\langle\psi^0_{\rm soft}\,\vert\, H_{\chi,\alpha}\,\vert\psi^0_{\rm soft}\rangle$	24.61(4)	23.84(14)	28.30
$\langle\psi^0_{\rm soft}\,\vert\, H'_{\chi,\alpha}\,\vert\psi^0_{\rm soft}\rangle$	27.74(4)	29.21(14)	



- reasonable accuracy for the light nuclei

- Tjon-band recovered with H'_χ

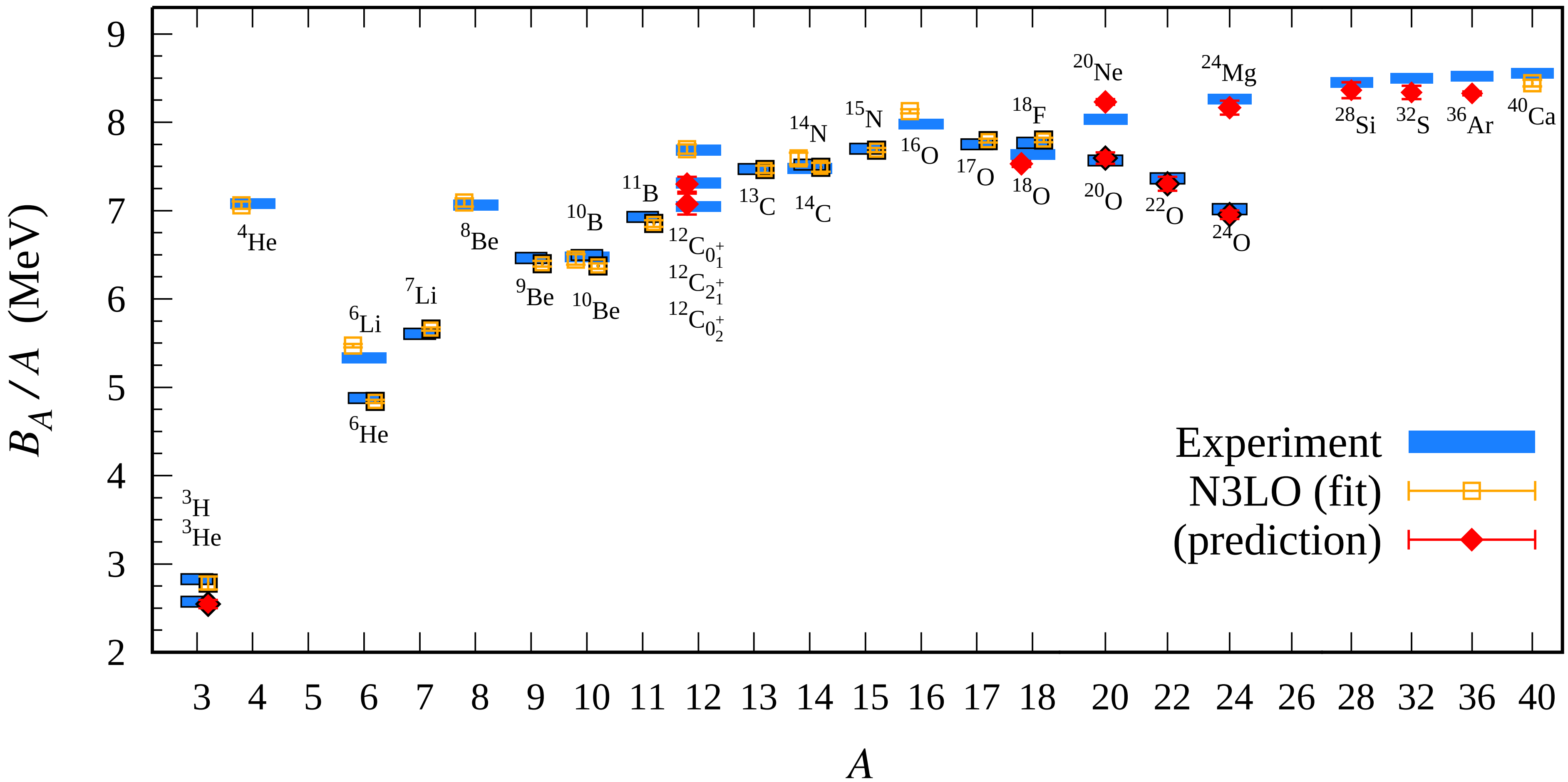
  Platter, Hammer, UGM, Phys. Lett. B **607** (2005) 254

$\hookrightarrow$ now let us go to larger nuclei....

Nuclei at N3LO

- Binding energies of nuclei for $a = 1.32$ fm: Determining the 3NF LECs

Elhatisari et al., Nature **630** (2024) 59 [arXiv:2210.17488 [nucl-th]]

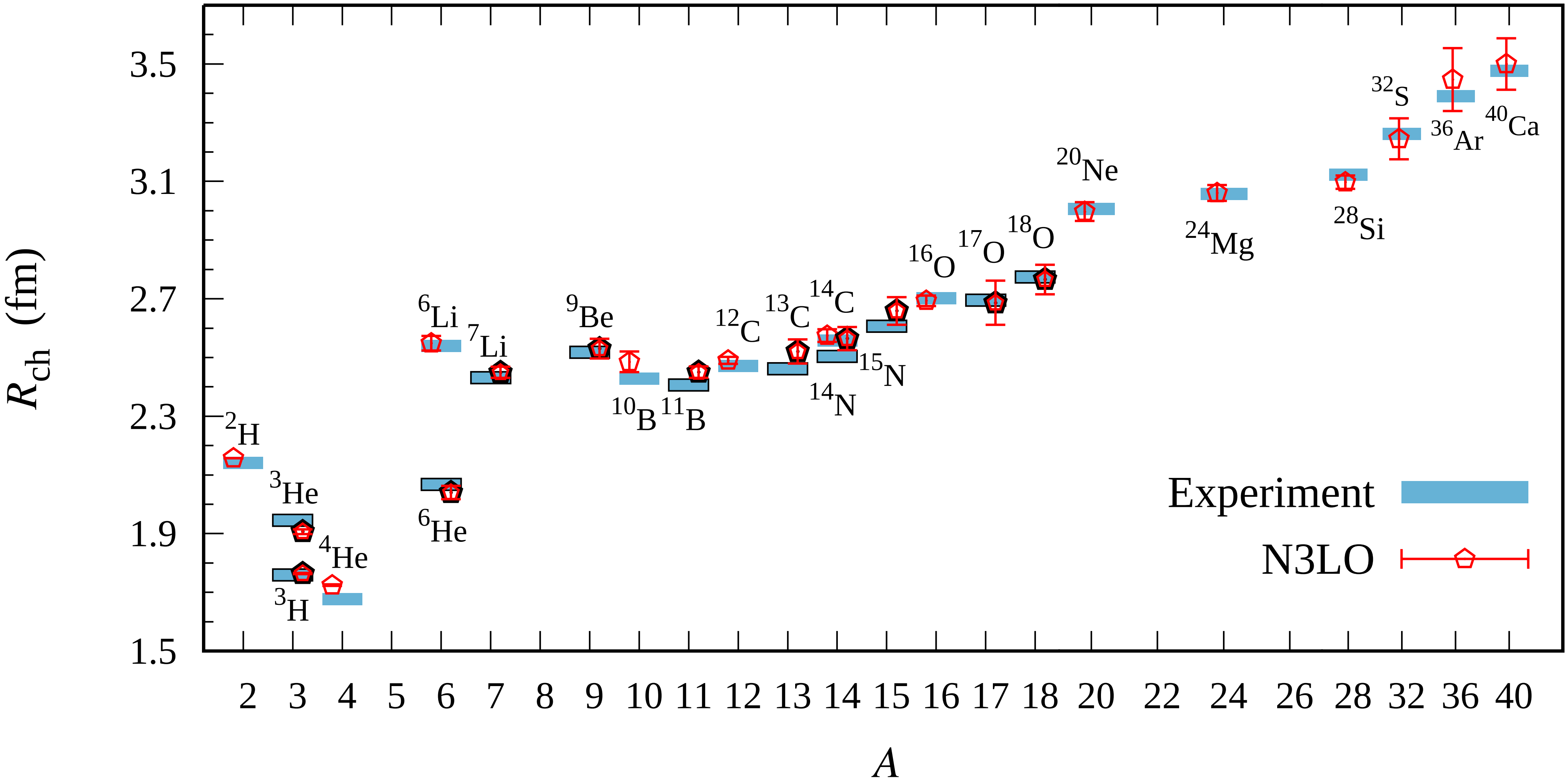


$\longrightarrow$ excellent starting point for precision studies

Prediction: Charge radii at N3LO

Elhatisari et al., Nature **630** (2024) 59 [arXiv:2210.17488 [nucl-th]]

- Charge radii ($a = 1.32$ fm, statistical errors can be reduced)

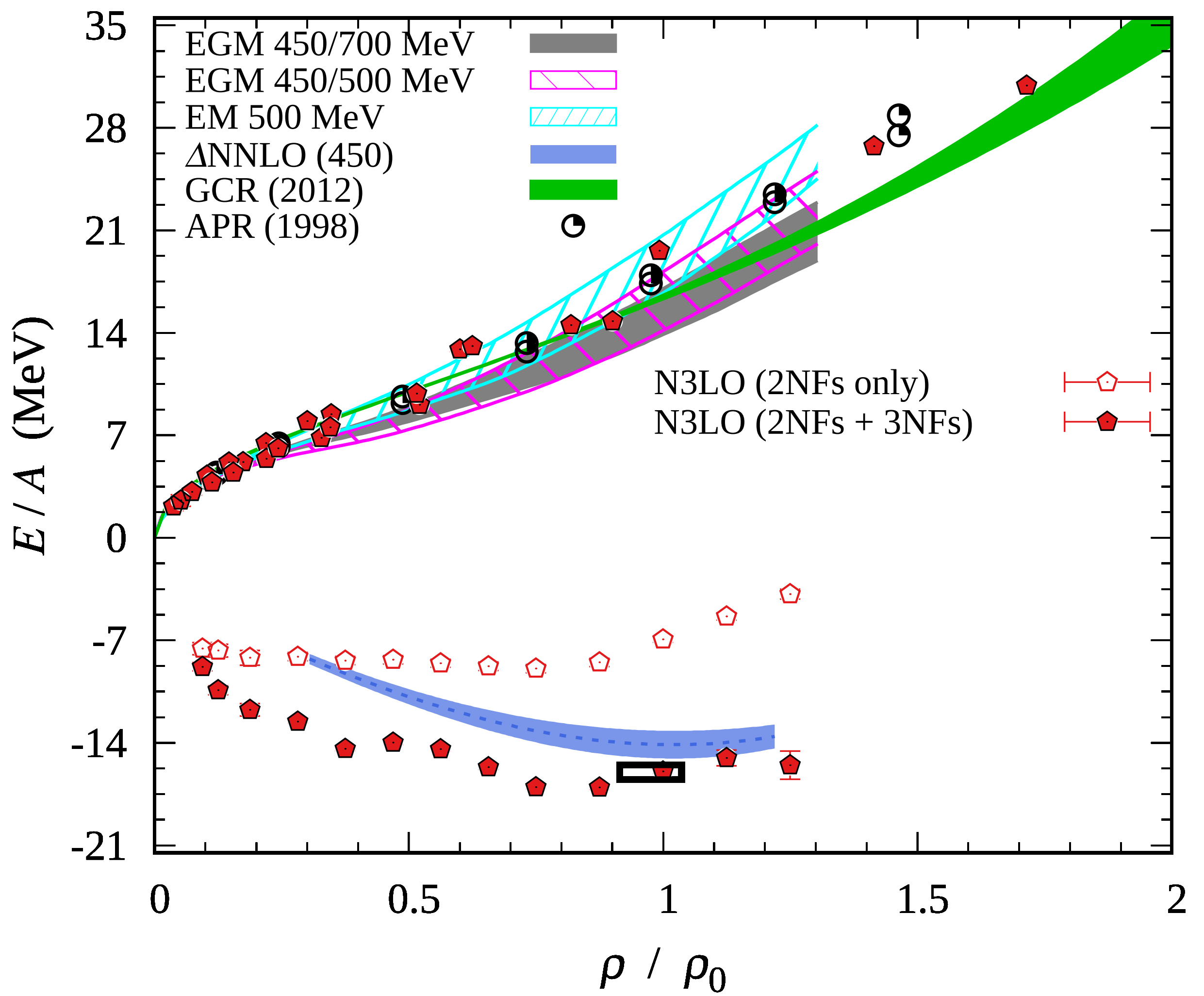


$\hookrightarrow$ no radius problem!

Prediction: Neutron & nuclear matter at N3LO

Elhatisari et al., Nature **630** (2024) 59 [arXiv:2210.17488 [nucl-th]]

- EoS of pure neutron matter & nuclear matter ($a = 1.32$ fm)

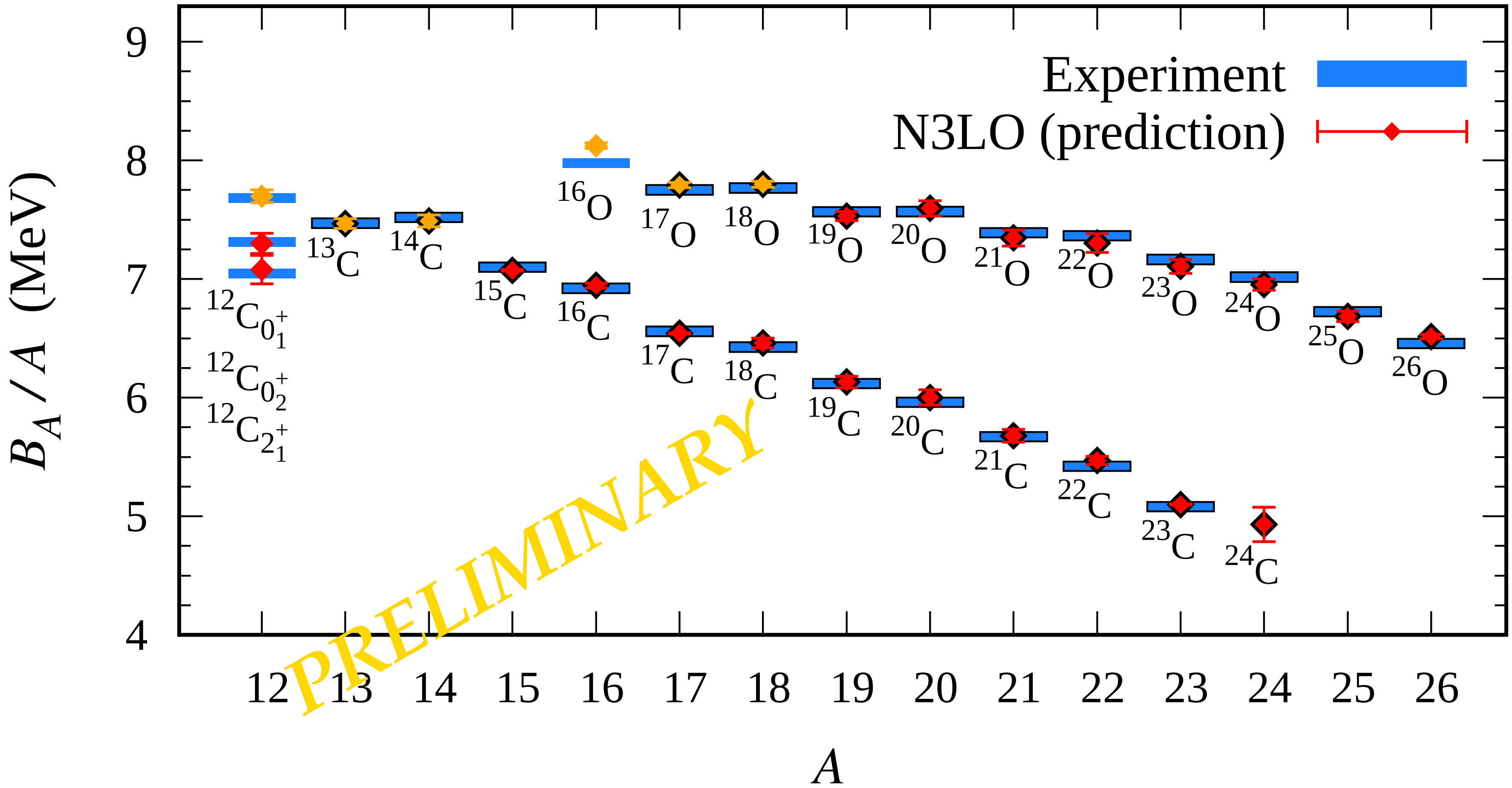


$\hookrightarrow$ can be improved using twisted b.c.'s

Prediction: Isotope chains of carbon & oxyen

NLEFT collaboration, in progress

- Towards the neutron drip-line in carbon and oxygen:

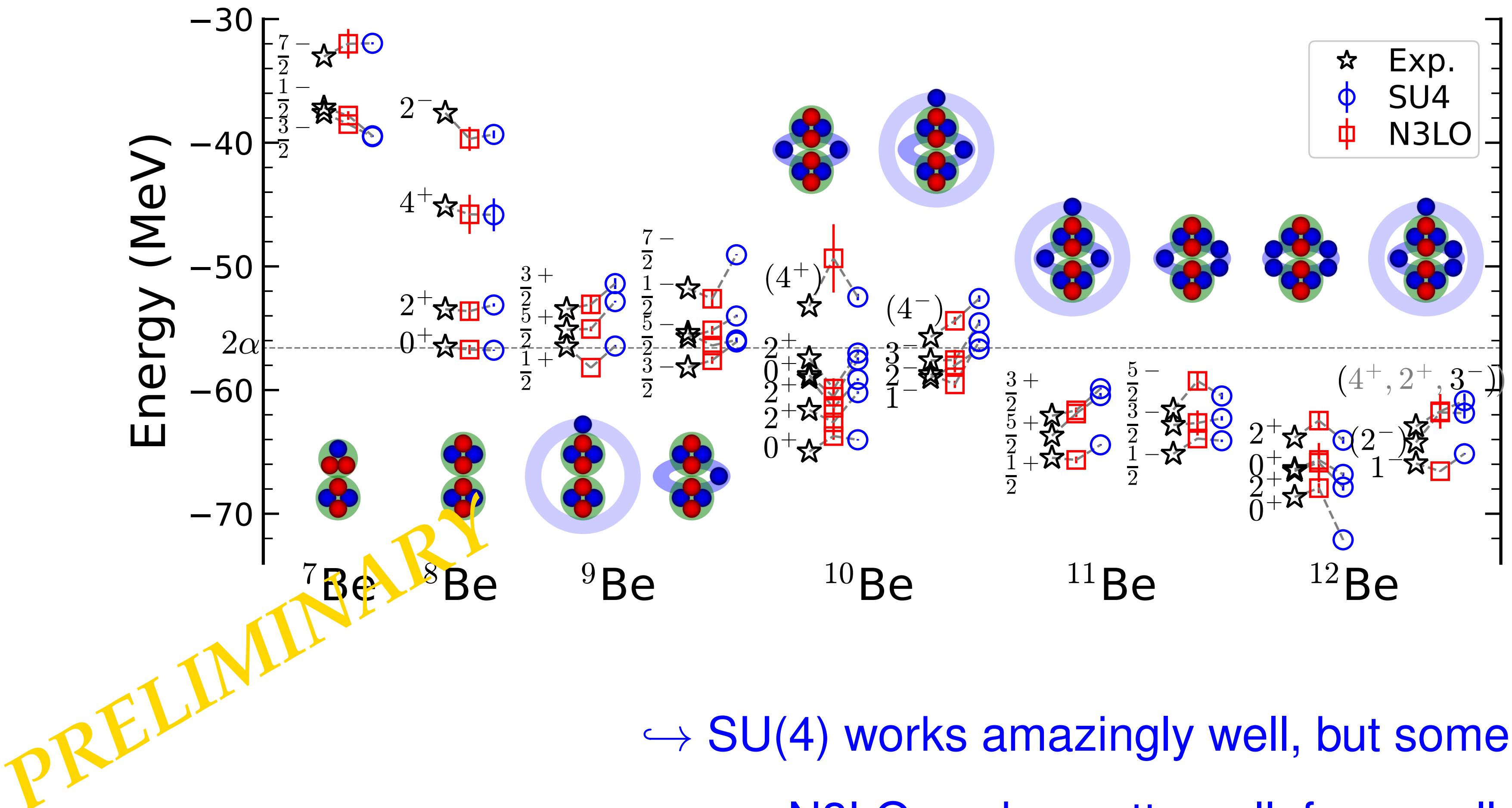


$\hookrightarrow$ 3NFs of utmost importance for the n-rich isotopes!

Prediction: Be isotopes

Shen, ..., NLEFT collaboration, in progress

- Systematic study of the Be isotopes & their em transitions:



$\hookrightarrow$ SU(4) works amazingly well, but some deviations

$\hookrightarrow$ N3LO works pretty well, few small deviations

Triton β-decay

Why triton β-decay?

- Triton β-decay: $\boxed{{}^3\mathbf{H} \to {}^3\mathbf{He} + e^- + \bar{\nu}_e}$

- Simplest nuclear β-decay, serves as a precision testbed:

  $\hookrightarrow$ precision 3N wave functions (various methods: FY, HH, ...)

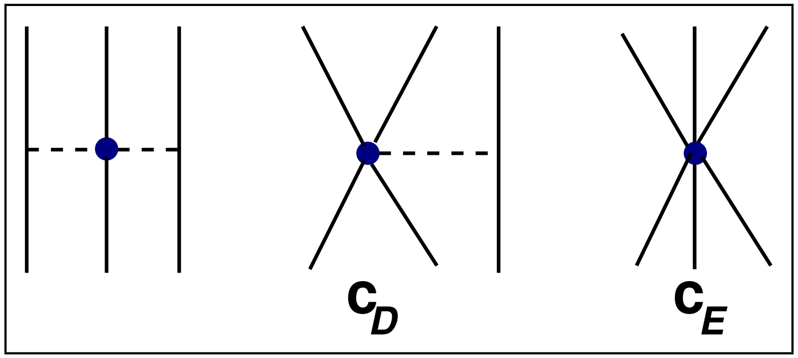
  $\hookrightarrow$ meson-exchange current corrections required (consistency?)

- Test of the axial-vector current in nuclei

- In terms of nuclear EFT:

  $\hookrightarrow$ allows to pin down LECs c_D, c_E of the leading chiral 3NF w/ $E({}^3\mathrm{He})$

  Gazit et al. (2009)



- Let us take this decay as a first step towards studying weak decays in NLEFT

Formalism for triton β-decay

- Master formula for the triton lifetime $t_{1/2}$:

$$(1+\delta_R)\, t_{1/2}\, f_V = \frac{K/G_V^2}{\langle \mathsf{F} \rangle^2 + \frac{f_A}{f_V}\, g_A^2\, \langle \mathsf{GT} \rangle^2}$$

with $f_A = 2.8506 \cdot 10^{-6}$, $f_V = 2.8355 \cdot 10^{-6}$ Fermi functions

$K/G_V^2 = 2\pi^3 \ln 2/(m_e^5 G_V^2) = 6145.5(11)$, $\delta_R = 1.9\%$ Hardy, Towner, ...

$(1+\delta_R)\, t_{1/2}\, f_V = 1132.1(25)\,\mathrm{s}$ comparative half-lifetime

Akulov et al. (2005), Baroni et al. (2017)

- Nuclear structure enters via the Fermi and Gamov-Teller M.E.s:

$$\langle \mathsf{F} \rangle = \sum_{n=1}^{3} \langle {}^3\mathrm{He} \| \tau_{n,+} \| {}^3\mathrm{H} \rangle \ , \qquad \langle \mathsf{GT} \rangle = \sum_{n=1}^{3} \langle {}^3\mathrm{He} \| \sigma_n \tau_{n,+} \| {}^3\mathrm{H} \rangle$$

$= 0.9998$ (AV18+Urbana-IX) $\qquad = 1.6497(23)$

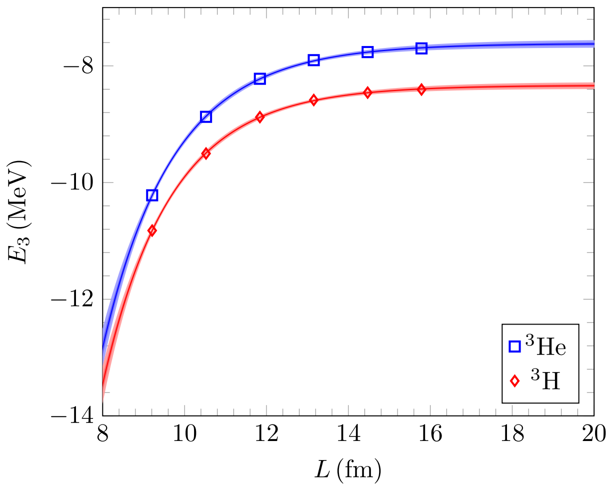
NLEFT calculation for triton β-decay

- Perturbative treatment not sufficient

$\hookrightarrow$ fully non-perturbative calculation

$$E(^3\mathrm{H}) = 8.33(2)\ \mathrm{MeV}$$

$$E(^3\mathrm{He}) = 7.62(2)\ \mathrm{MeV}$$



- 3NFs: $V_{c_D} = f_D\left(V_{c_D}^{(0)} + V_{c_D}^{(1)} + V_{c_D}^{(2)}\right)$

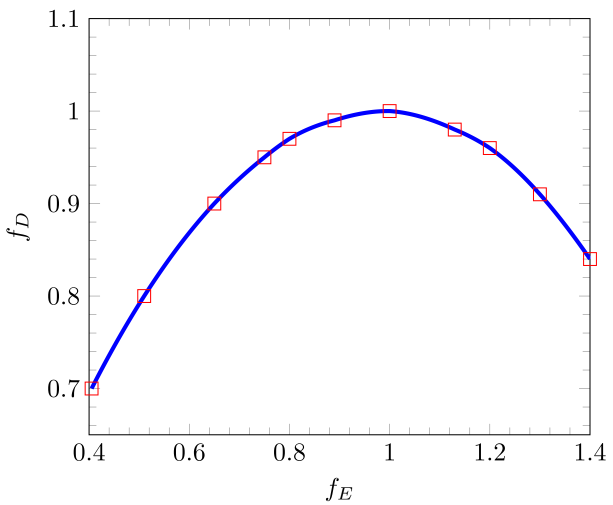
$$V_{c_E} = f_E\left(V_{c_E}^{(0)} + V_{c_E}^{(1)} + V_{c_E}^{(2)} + V_{c_E}^{(l)} + V_{c_E}^{(t)}\right)$$

$\hookrightarrow$ smeared representation, more LECs

Elhatisari et al. (2024)

$\hookrightarrow$ more non-linear correlation between the 2 topologies

Epelbaum et al. (2002), Gazit et al. (2009)



- Work at $L = 12$ fm due to computational & methodological constraints

NLEFT calculation for triton β-decay

- **Consistent** calculation of the M.E.s

- Fermi matrix element:

$$\langle \mathbf{F} \rangle = \mathbf{0.999571(48)}$$

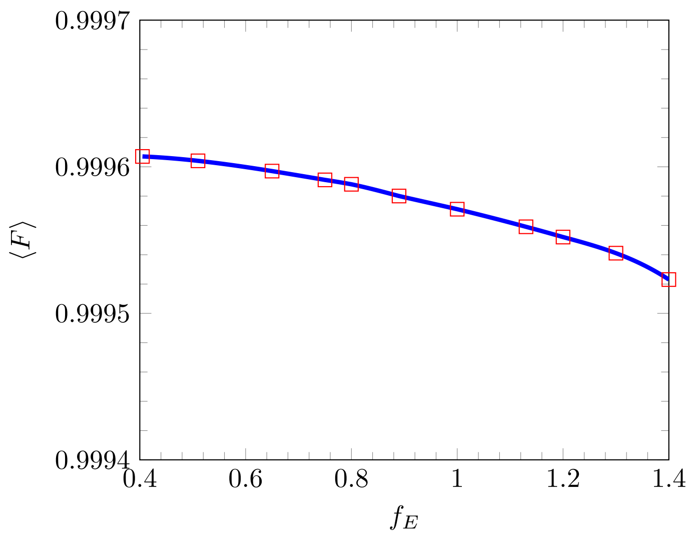
- Gamov-Teller matrix element:

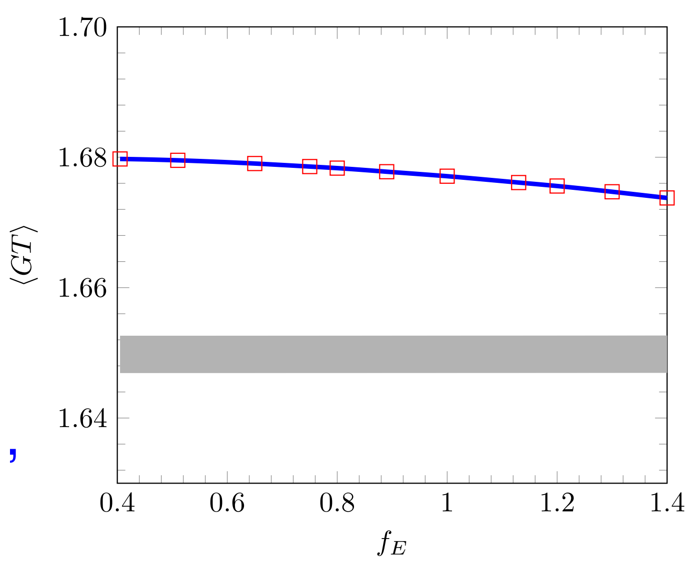
$$\langle \mathbf{GT} \rangle = \mathbf{1.6770(34)}$$

- Triton lifetime:

$$(1 + \delta_R)\, t_{1/2}\, f_V = 1102.1(2)\,\mathrm{s}$$

$\hookrightarrow$ Neccessary improvements: currents at N3LO, infinite volume extrapolation formulas for ⟨**F**⟩ and ⟨**GT**⟩, refinement of the 3NFs (seen e.g. in n-α scattering)





Summary & outlook

- Nuclear lattice simulations: a new quantum many-body approach
  - $\to$ based on the successful continuum nuclear chiral EFT
  - $\to$ a number of highly visible results already obtained

- Recent developments
  - $\to$ highly improved LO action based on Wigner's SU(4)
    - $\hookrightarrow$ a number of interesting application (^{12}C, ^{4}He,...)
    - $\hookrightarrow$ towards the neutron matter EoS at high denstities
  - $\to$ NN interaction at N3LO w/ wave function matching
    - $\hookrightarrow$ first promising results for nuclear structure and scattering
    - $\hookrightarrow$ weak decay studies are under investigation, here triton β-decay
    - $\hookrightarrow$ towards the holy grail of nuclear astrophysics:

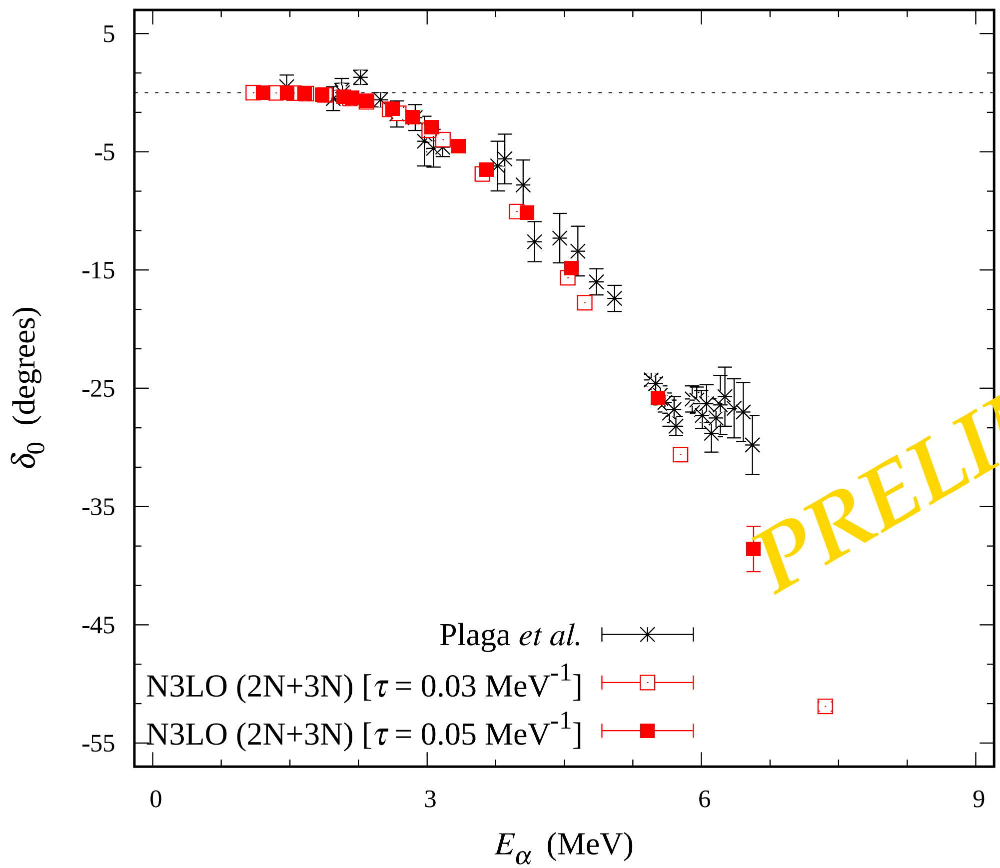
      $\mathbf{^{12}C(\alpha,\gamma)^{16}O}$ at stellar energies

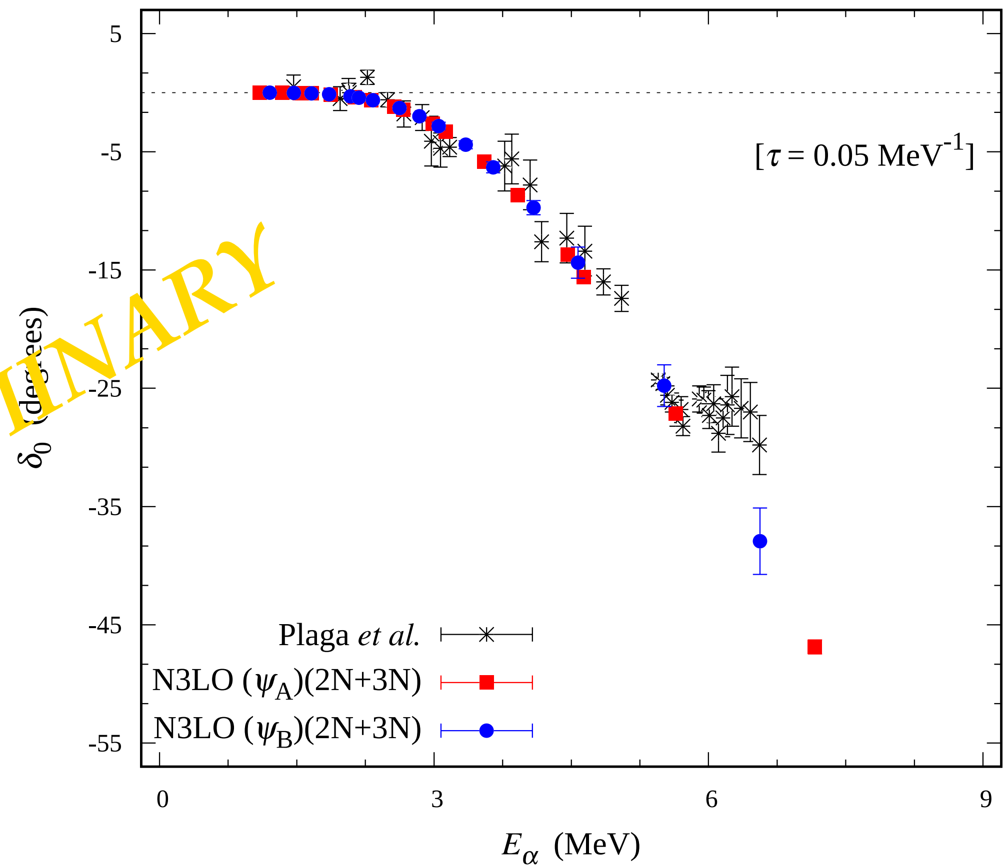
Scattering: Alpha-carbon scattering at N3LO

Elhatisari, Hildenbrand, UGM, ... NLEFT, in progress

- Use the APM, first step for the holy grail of nuclear astrophysics

  $\hookrightarrow$ different Euclidean times & different initial states





Plaga et al., Nucl. Phys. A **465** (1987) 291

$\psi_A \sim {}^{16}\text{O}\,,\ \ \psi_B \sim {}^{12}\text{C} + {}^{4}\text{He}$

SPARES

The minimal nuclear interaction: Extension to hyper-nuclei

The minimal interaction with strangeness I

Tong, Elhatisari, UGM, in progress

- Baryon-baryon interaction (consider nucleons and Λ's plus non-local smearing):

$$V_{\Lambda N} = c_{N\Lambda} \sum_{\vec{n}} \tilde{\rho}(\vec{n})\tilde{\xi}(\vec{n}) + c_{\Lambda\Lambda} \frac{1}{2} \sum_{\vec{n}} \left[\tilde{\xi}(\vec{n})\right]^2$$

$$\tilde{\rho}(\vec{n}) = \sum_{i,j=0,1} \tilde{a}^\dagger_{i,j}(\vec{n})\, \tilde{a}_{i,j}(\vec{n}) + s_{\rm L} \sum_{|\vec{n}-\vec{n}'|^2=1} \sum_{i,j=0,1} \tilde{a}^\dagger_{i,j}(\vec{n}')\, \tilde{a}_{i,j}(\vec{n}')$$

$$\tilde{\xi}(\vec{n}) = \sum_{i=0,1} \tilde{b}^\dagger_{i}(\vec{n})\, \tilde{b}_{i}(\vec{n}) + s_{\rm L} \sum_{|\vec{n}-\vec{n}'|^2=1} \sum_{i=0,1} \tilde{b}^\dagger_{i}(\vec{n}')\, \tilde{b}_{i}(\vec{n}')$$

- Three-baryon forces (consider nucleons and Λ's, no non-local smearing):

Petschauer, Kaiser, Haidenbauer, UGM, Weise, Phys. Rev. C **93** (2016) 014001

$$V_{NN\Lambda} = c_{NN\Lambda} \frac{1}{2} \sum_{\vec{n}} [\rho(\vec{n})]^2\, \xi(\vec{n})\ , \qquad V_{N\Lambda\Lambda} = c_{N\Lambda\Lambda} \frac{1}{2} \sum_{\vec{n}} \rho(\vec{n})\ [\xi(\vec{n})]^2$$

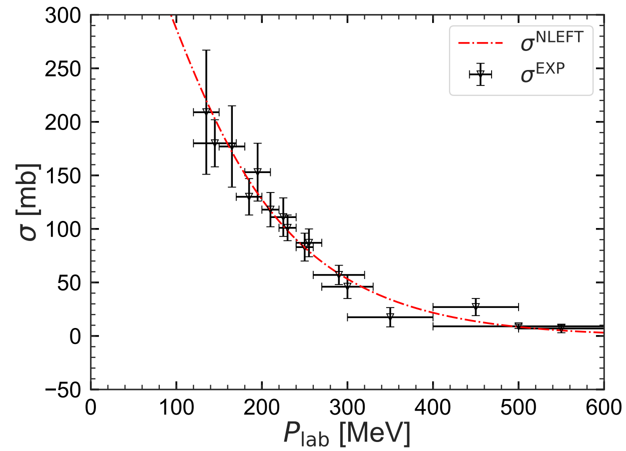
$\hookrightarrow$ must determine 4 LECs! [smearing parameters from the nucleon sector]

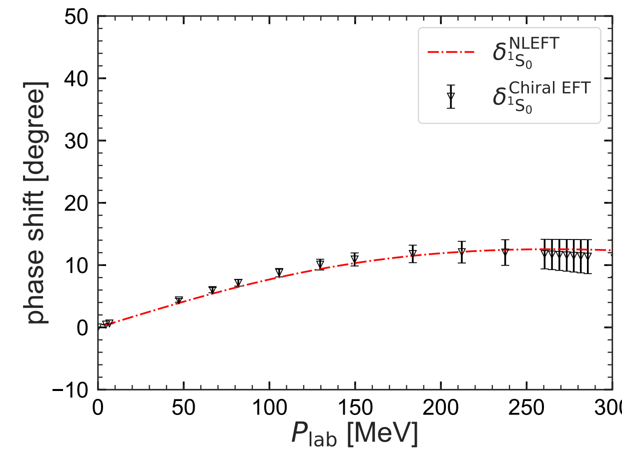
$\hookrightarrow$ first time that the $\Lambda\Lambda N$ three-body force is included

The minimal interaction with strangeness II

Tong, Elhatisari, UGM, in progress

- Two-body LECs from scattering data (ΛN)
  & chiral EFT phase shift ($\Lambda\Lambda$)





- Three-body LECs from the separation energies of Λ and $\Lambda\Lambda$ hyper-nuclei:

$$B_\Lambda({}^A_\Lambda Z) = E({}^{A-1}Z) - E({}^A_\Lambda Z)$$

$$B_{\Lambda\Lambda}({}^A_{\Lambda\Lambda} Z) = E({}^{A-2}Z) - E({}^A_{\Lambda\Lambda} Z)$$

Nucleus	NLEFT [MeV]	Exp. [MeV]
${}^5_\Lambda$He	**3.10(9)**	**3.10(3)**
${}^9_\Lambda$Be	**6.64(13)**	**6.61(7)**
${}^{13}_\Lambda$C	**11.71(14)**	**11.80(16)**
${}^6_{\Lambda\Lambda}$He	**6.96(9)**	**6.91(16)**
${}^{10}_{\Lambda\Lambda}$Be	**14.35(13)**	**14.70(40)**

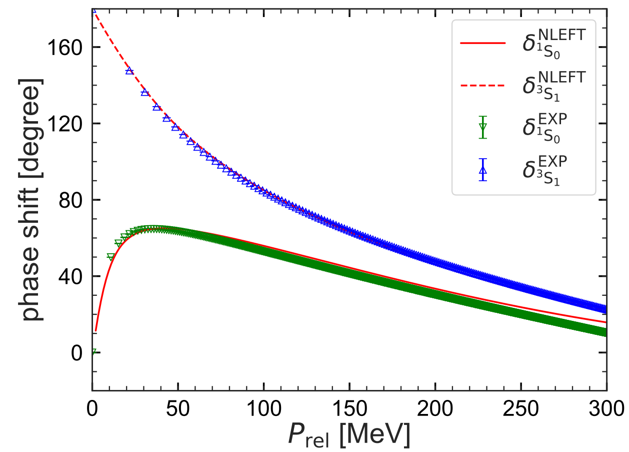
$\hookrightarrow$ this defines our EoS of hyper-nuclear matter called **HMN(I)**

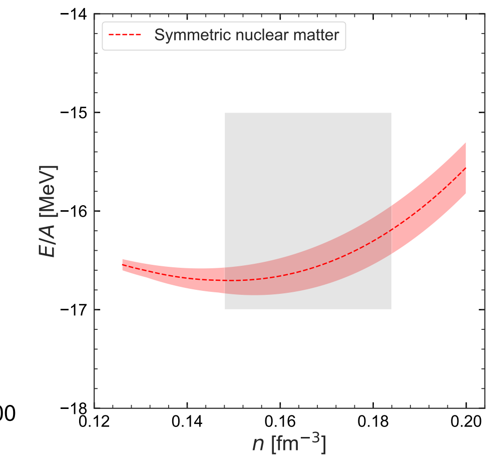
The minimal nuclear interaction: EoS & neutron star properties

Pure neutron matter

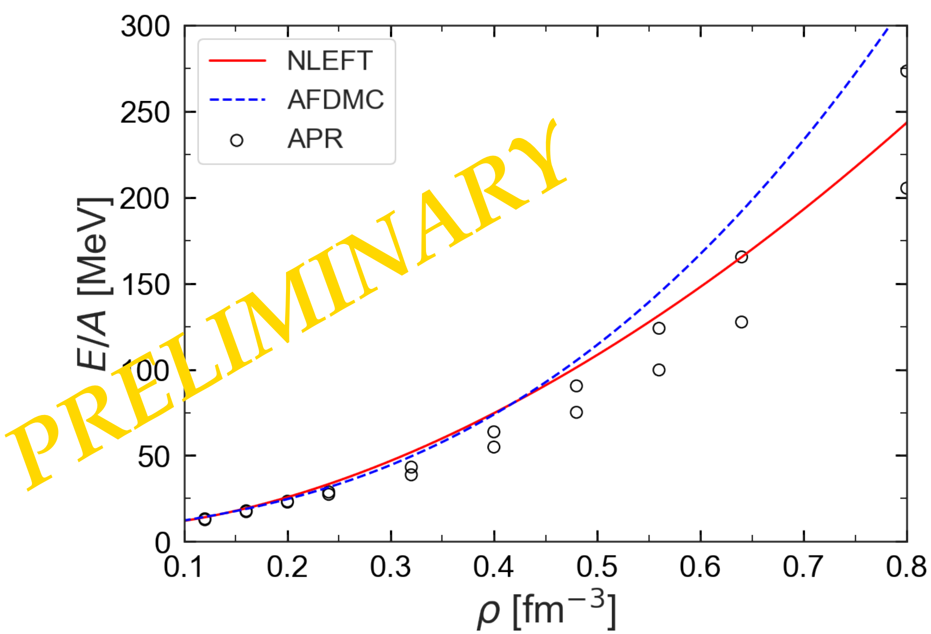
Tong, Elhatisari, UGM, in progress

- Input: S-wave phase shifts (2N)
  & symmetric nuclear matter (3N)
- Note: extension of the minimal
  interaction (leading SU(4) breaking)





⇒ Output: Pure neutron matter (PNM) EoS



– comparable to the renowned APR EoS

Akmal, Pandharipande, Ravenhall, Phys. Rev. C **58** (1998) 1804

– less stiff than the recent AFDMC one

Gandolfi et al., Eur. Phys. J. A **50** (2014) 10

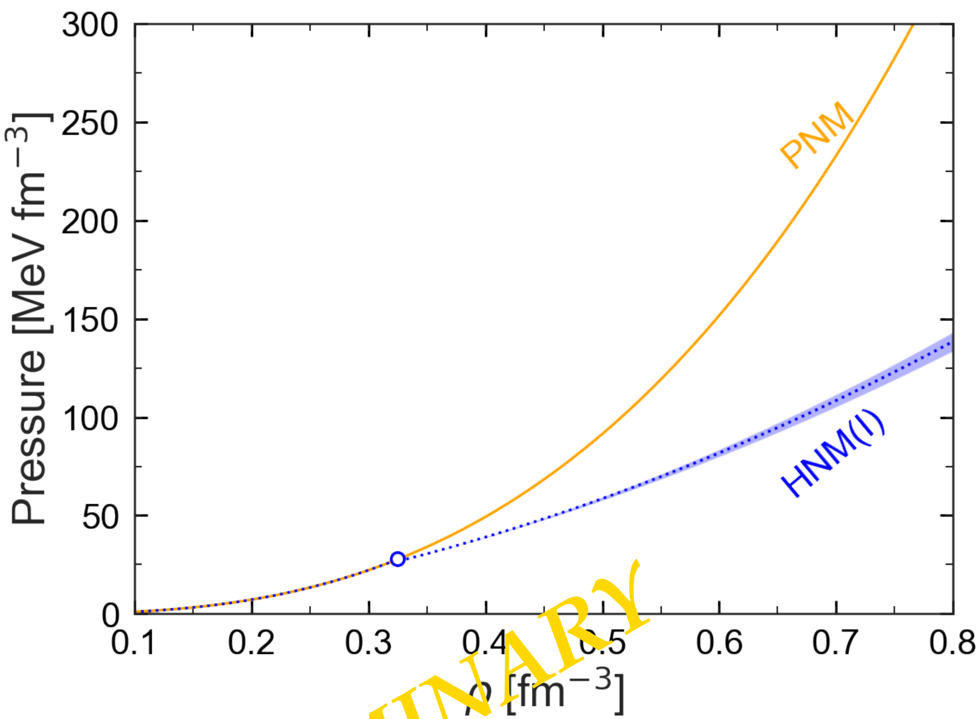
→ work out consequences for
neutron stars based on this PNM EoS

Neutron star properties

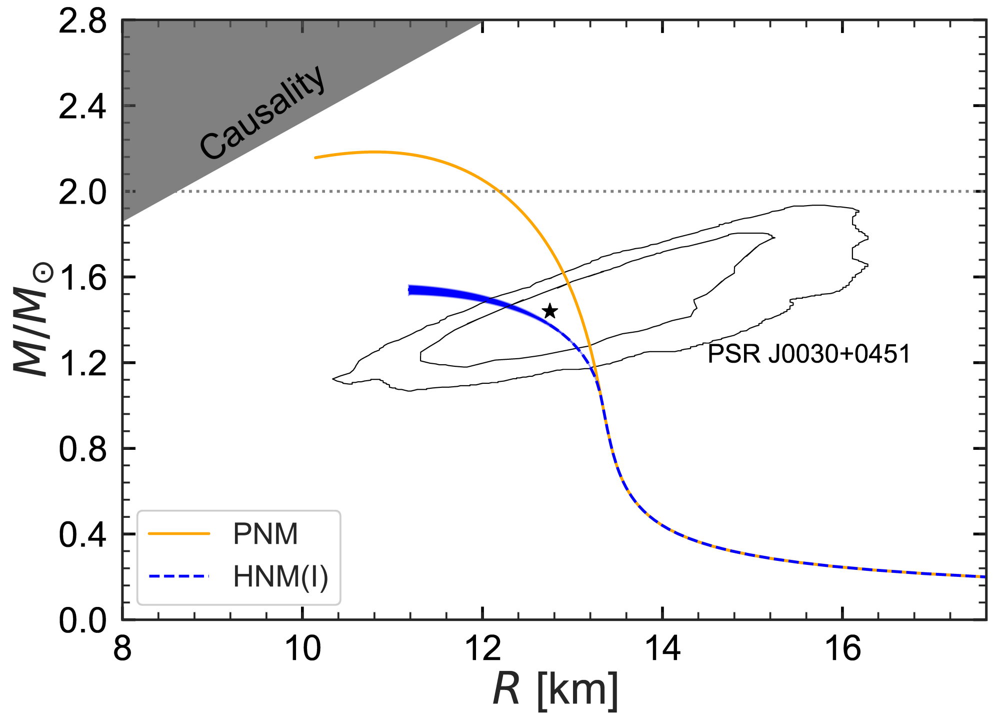
Tong, Elhatisari, UGM, in progress

- Now solve the TOV equations for the PNM and HNM(I) EoSs:

  - EoS (PNM and HMN(I))



  - Mass-radius relation



- Maximum neutron star mass: $\boldsymbol{M_{\rm max} = 2.18(1)\, M_\odot}$ for PNM

  $\boldsymbol{M_{\rm max} = 1.54(2)\, M_\odot}$ for HNM(I) $\rightarrow$ need repulsion

EoS of hyper-neutron matter

Tong, Elhatisari, UGM, in progress

- Not surprisingly, we need more repulsion [as in the pure neutron matter case]

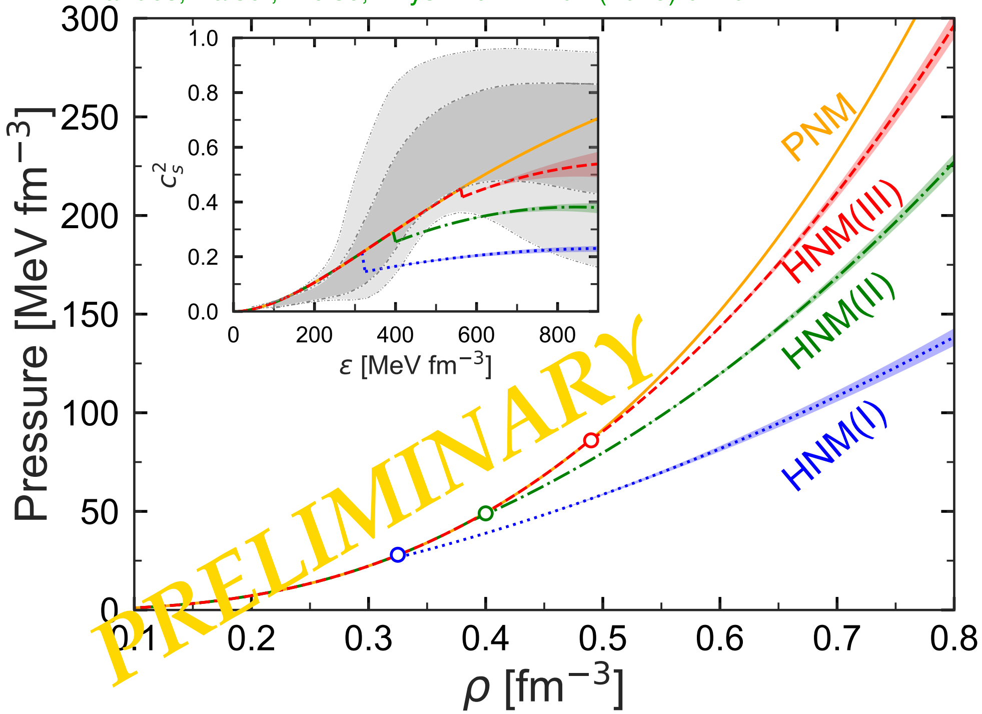
  $\hookrightarrow$ this will move the threshold of $\mu_\Lambda = \mu_n$ up

  $\hookrightarrow$ take $M_{\mathbf{max}}$ as data point: $M_{\mathbf{max}} = \mathbf{1.9} M_\odot$ for HNM(II)

  $M_{\mathbf{max}} = \mathbf{2.1} M_\odot$ for HNM(III)

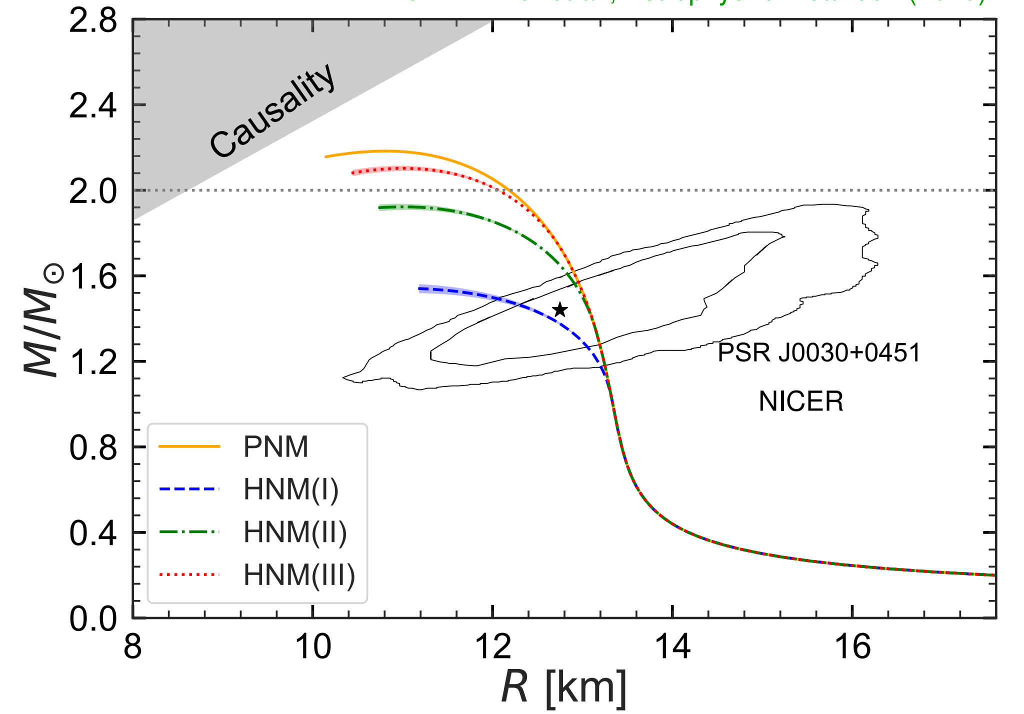
- EoS & speed of sound

Brandes, Kaiser, Weise, Phys. Rev. D **107** (2023) 014011



- Mass-radius relation

NICER: Miller et al., Astrophys. J. Lett. **887** (2019) L24



Finite temperature physics

- Just two teasers for finite T calculations

↪ talks by Bing-Nan Lu and Dean Lee

PHYSICAL REVIEW LETTERS **125,** 192502 (2020)

Ab Initio Nuclear Thermodynamics

Bing-Nan Lu,[1] Ning Li,[1] Serdar Elhatisari,[2] Dean Lee,[1] Joaquín E. Drut,[3] Timo A. Lähde,[4] Evgeny Epelbaum,[5] and Ulf-G. Meißner[6,4,7]

[1]*Facility for Rare Isotope Beams and Department of Physics and Astronomy, Michigan State University, East Lansing, Michigan 48824, USA*
[2]*Faculty of Engineering, Karamanoglu Mehmetbey University, Karaman 70100, Turkey*
[3]*Department of Physics and Astronomy, University of North Carolina, Chapel Hill, North Carolina 27599-3255, USA*
[4]*Institute for Advanced Simulation, Institut für Kernphysik, and Jülich Center for Hadron Physics, Forschungszentrum Jülich, D-52425 Jülich, Germany*
[5]*Ruhr-Universität Bochum, Fakultät für Physik und Astronomie, Institut für Theoretische Physik II, D-44780 Bochum, Germany*
[6]*Helmholtz-Institut für Strahlen- und Kernphysik and Bethe Center for Theoretical Physics, Universität Bonn, D-53115 Bonn, Germany*
[7]*Tbilisi State University, 0186 Tbilisi, Georgia*

(Received 11 April 2020; revised 6 August 2020; accepted 29 September 2020; published 3 November 2020)

We propose a new Monte Carlo method called the pinhole trace algorithm for *ab initio* calculations of the thermodynamics of nuclear systems. For typical simulations of interest, the computational speedup relative to conventional grand-canonical ensemble calculations can be as large as a factor of one thousand. Using a leading-order effective interaction that reproduces the properties of many atomic nuclei and neutron matter to a few percent accuracy, we determine the location of the critical point and the liquid-vapor coexistence line for symmetric nuclear matter with equal numbers of protons and neutrons. We also present the first *ab initio* study of the density and temperature dependence of nuclear clustering.

Phys. Lett. B 850 (2024) 138463

Contents lists available at ScienceDirect

Physics Letters B

journal homepage: www.elsevier.com/locate/physletb

ELSEVIER

Letter

Ab initio study of nuclear clustering in hot dilute nuclear matter

Zhengxue Ren[a,b,*], Serdar Elhatisari[c,b], Timo A. Lähde[a,d], Dean Lee[e], Ulf-G. Meißner[b,a,f]

[a] *Institut für Kernphysik, Institute for Advanced Simulation and Jülich Center for Hadron Physics, Forschungszentrum Jülich, D-52425 Jülich, Germany*
[b] *Helmholtz-Institut für Strahlen- und Kernphysik and Bethe Center for Theoretical Physics, Universität Bonn, D-53115 Bonn, Germany*
[c] *Faculty of Natural Sciences and Engineering, Gaziantep Islam Science and Technology University, Gaziantep 27010, Turkey*
[d] *Center for Advanced Simulation and Analytics (CASA), Forschungszentrum Jülich, D-52425 Jülich, Germany*
[e] *Facility for Rare Isotope Beams and Department of Physics and Astronomy, Michigan State University, East Lansing, MI 48824, USA*
[f] *Tbilisi State University, 0186 Tbilisi, Georgia*

A R T I C L E I N F O

Editor: A. Schwenk

A B S T R A C T

We present a systematic *ab initio* study of clustering in hot dilute nuclear matter using nuclear lattice effective field theory with an SU(4)-symmetric interaction. We introduce a method called light-cluster distillation to determine the abundances of dimers, trimers, and alpha clusters as a function of density and temperature. Our lattice results are compared with an ideal gas model composed of free nucleons and clusters. Excellent agreement is found at very low density, while deviations from ideal gas abundances appear at increasing density due to cluster-nucleon and cluster-cluster interactions. In addition to determining the composition of hot dilute nuclear matter as a function of density and temperature, the lattice calculations also serve as benchmarks for virial expansion calculations, statistical models, and transport models of fragmentation and clustering in nucleus-nucleus collisions.

- new pinhole trace algorithm

↪ liquid-vapor phase transition

↪ location of the critical point

- new light cluster distillation method

↪ abundances of dimers, trimers, tetramers

↪ benchmark for virial calculations

Chiral Interactions at N3LO: Applications to scattering

Scattering: Methods I

- The time-honored Lüscher approach: Lüscher, Commun. Math. Phys. **105** (1986) 153; Nucl. Phys. B **354** (1991) 531

  Phase shifts from the volume dependence of the energy levels

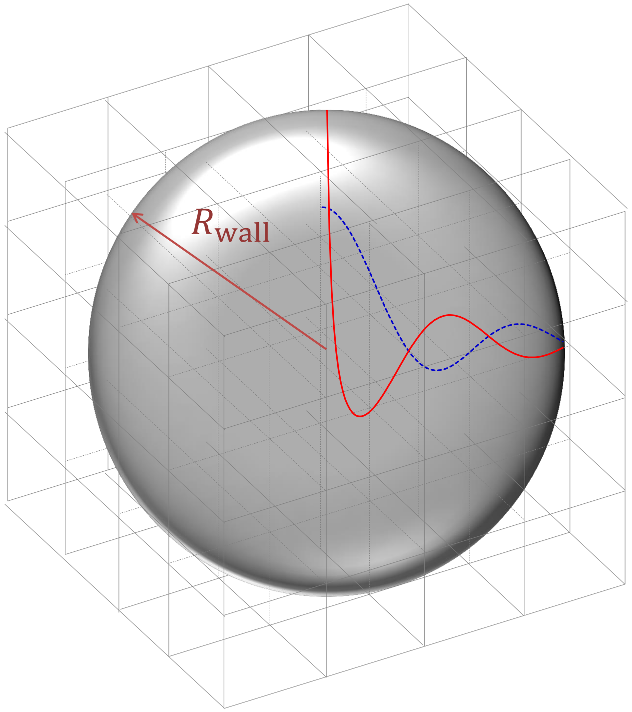
$\hookrightarrow$ works in many cases, problems w/ partial-wave mixing and cluster-cluster scattering

- Spherical wall technique:

  impose spherical b.c.'s on the lattice

  Carlson et al., Nucl. Phys. A **424** (1984) 47; Borasoy et al., Eur. Phys. J. A **34** (2007) 185

$\hookrightarrow$ not too small lattices, partial-wave mixing under control



- Improved spherical wall method:

  Lu, Lähde, Lee, UGM, Phys. Lett. B **760** (2016) 309

  – perform angular momentum projection

  – impose an auxiliary potential behind $\boldsymbol{R}_{\rm wall}$

  $\hookrightarrow$ much improved precision

Scattering: Methods II

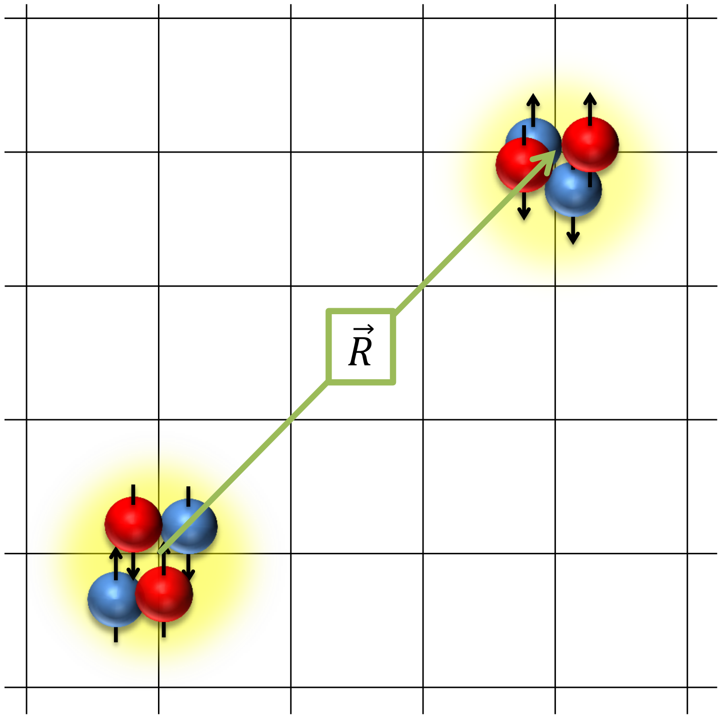
- Adiabatic projection method :

Rupak, Lee, Phys. Rev. Lett. **111** (2013) 032502; Pine, Lee, Rupak, Eur. Phys. J. A **49** (2013) 151; Elhatisari et al., Eur. Phys. J. A **52** (2016) 174; ....

- Construct a low-energy effective theory for clusters

- Use initial states parameterized by the relative separation between clusters

$$|\vec{R}\rangle = \sum_{\vec{r}} |\vec{r} + \vec{R}\rangle \otimes \vec{r}$$



- project them in Euclidean time with the chiral EFT Hamiltonian H

$$|\vec{R}\rangle_\tau = \exp(-H\tau)|\vec{R}\rangle$$

$\to$ "dressed cluster states" (polarization, deformation, Pauli)

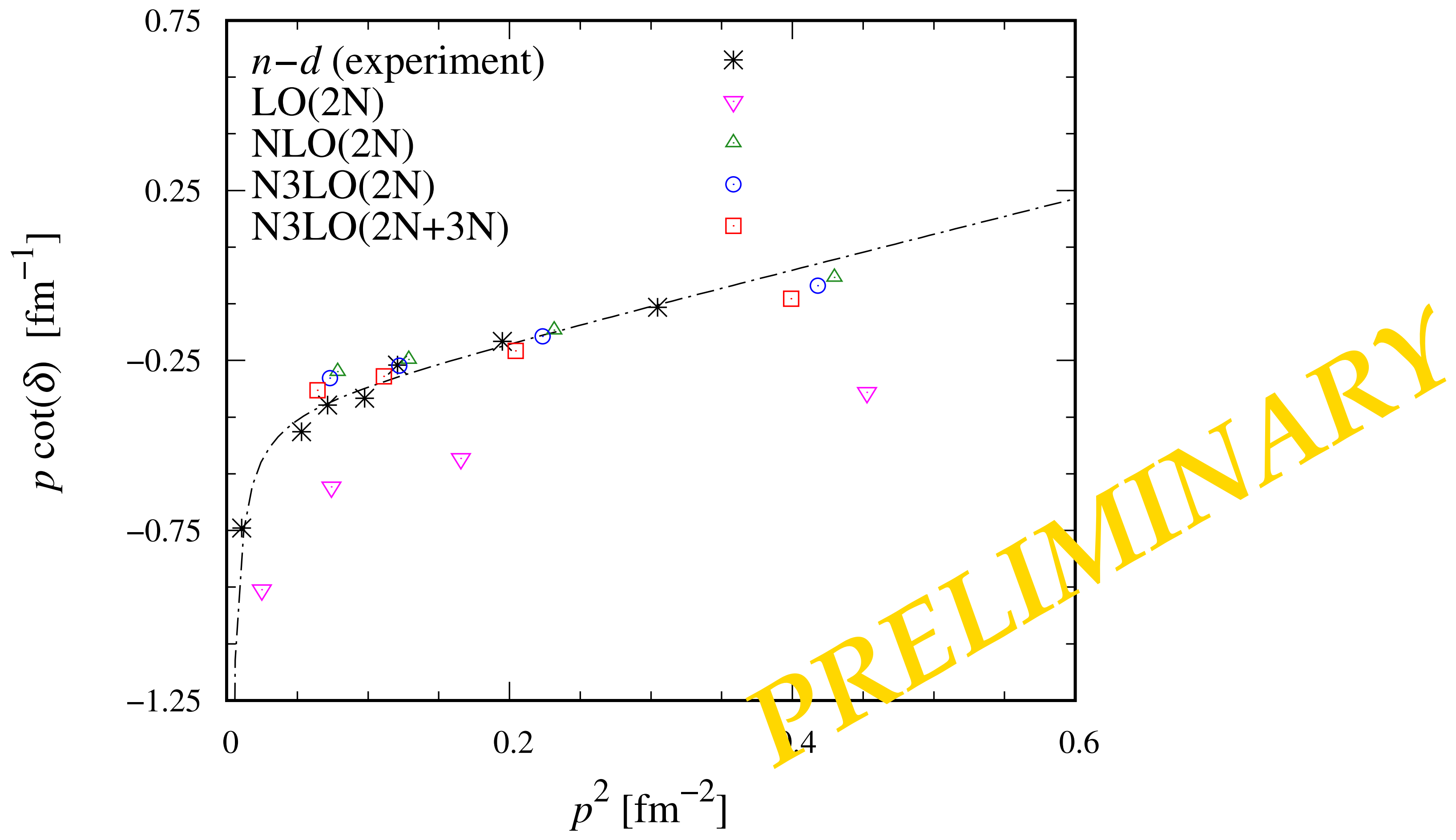
- Adiabatic Hamiltonian (requires norm matrices)

$$[H_\tau]_{\vec{R}\vec{R}'} = {}_\tau\langle\vec{R}|H|\vec{R}'\rangle_\tau$$

Scattering: Neutron-deuteron scattering at N3LO

Elhatisari, Hildenbrand, UGM, in progress

- Use Lüscher's method to calculate spin doublet n-d scattering

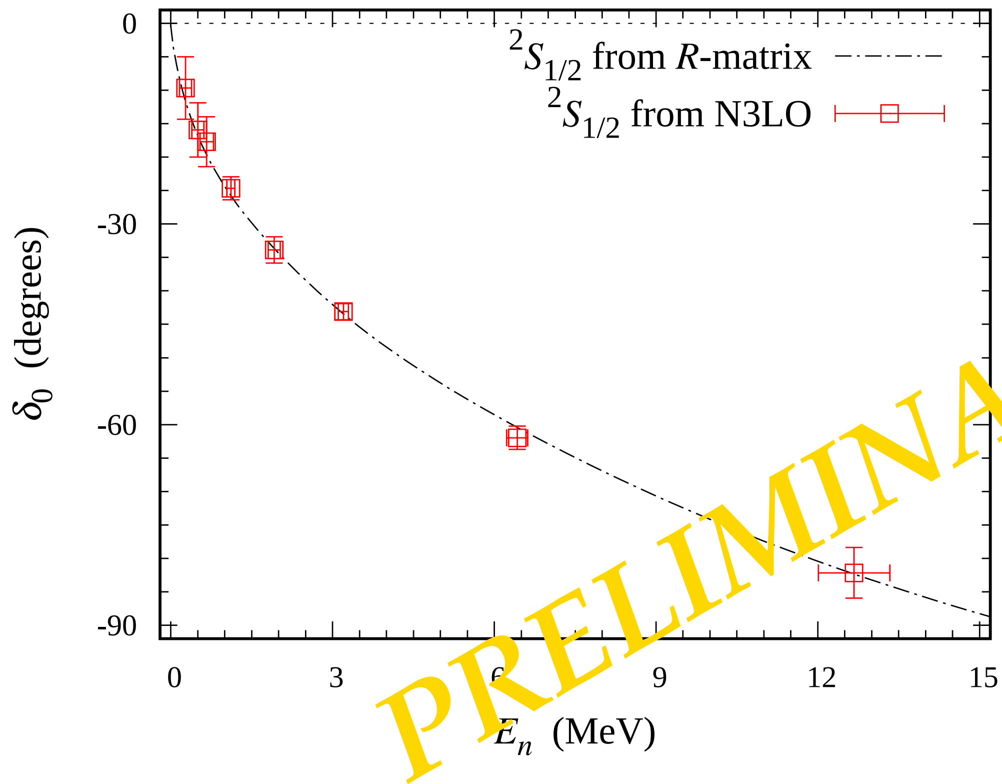


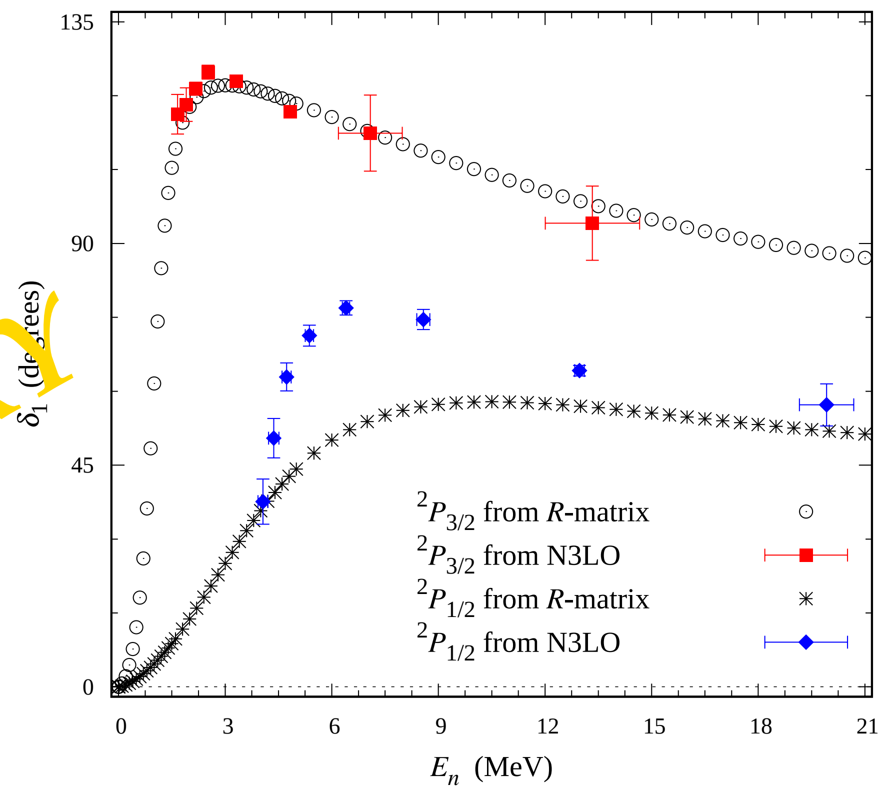
$\hookrightarrow$ shows good convergence

Scattering: Neutron-alpha scattering at N3LO

Elhatisari, Hildenbrand, UGM, in progress

- Use Lüscher's method to calculate n-α scattering





- R-matrix results from G. Hale, private communication

$\hookrightarrow$ Some fine-tuning of three-body forces for $^2P_{1/2}$ needed