Hyperon-nucleon interaction and light hypernuclei

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(Hoai Le, Ulf-G. Meißner, Andreas Nogga)

Outline

1. Introduction
2. YN interaction in chiral effective field theory
3. Light Λ hypernuclei
4. Summary

Hyperon physics - recent developments

- Role of hyperons in neutron stars (“hyperon puzzle”)
  Neutron stars with masses $\geq 2M_\odot \Rightarrow$ stiff equation of state (EoS)
  With increasing density $n \to \Lambda \Rightarrow$ softening of the EoS
  $\Rightarrow$ Conventional explanations of observed mass-radius relation fail
- New measurements of Λp cross sections by the CLAS Collaboration at JLab
  New extended measurements of ΣN observables in the E40 experiment at J-PARC
  differential cross sections for $\Sigma^+ p$, $\Sigma^- p$
- Measurements of two-particle momentum correlation functions by the STAR, HADES, and ALICE Collaborations
  (Λp, $\Lambda\Lambda$, $\Xi^- p$, ...)
- HAL QCD: Lattice QCD simulations for YN interactions for quark masses close to the physical point ($M_\pi \approx 145$ MeV)
- Progress in *ab initio* methods like no-core shell model (NCSM)
  microscopic calculations of hypernuclei up to $A \geq 10$
- Nuclear lattice effective field theory including the Λ hyperon

BB interaction in chiral effective field theory

Baryon-baryon interaction in SU(3) χEFT à la Weinberg (1990)

Advantages:

- Power counting
  systematic improvement by going to higher order
- Possibility to derive two- and three-baryon forces and external current operators in a consistent way

- degrees of freedom: octet baryons (*N*, Λ, Σ, Ξ), pseudoscalar mesons (π, *K*, η)

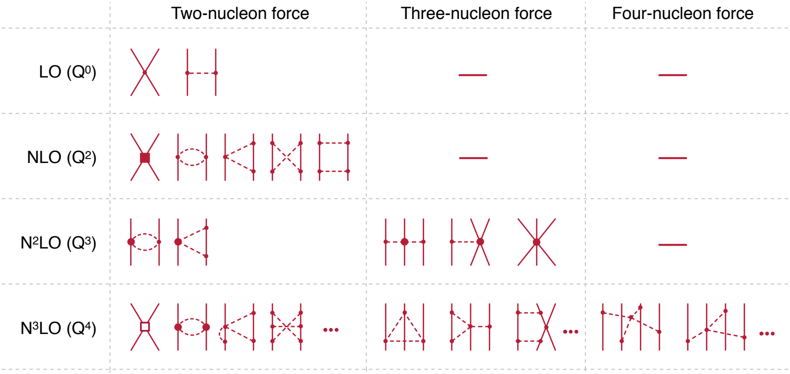
- pseudoscalar-meson exchanges
- contact terms – represent unresolved short-distance dynamics
  involve low-energy constants (LECs) that need to be fixed
  by a fit to data

Λ*N*-Σ*N* interaction
LO: H. Polinder, J.H., U.-G. Meißner, NPA 779 (2006) 244
NLO13: J.H., S. Petschauer, N. Kaiser, U.-G. Meißner, A. Nogga, W. Weise, NPA 915 (2013) 24
NLO19: J.H., U.-G. Meißner, A. Nogga, EPJA 56 (2020) 91
SMS NLO, NNLO: J.H., U.-G. Meißner, A. Nogga, H.Le, EPJA 59 (2023) 63

(*BB* systems with strangeness $S = -1$ to -6)

Extension of chiral EFT interaction up to NNLO

(Nucleon-nucleon forces in chiral EFT (E. Epelbaum))

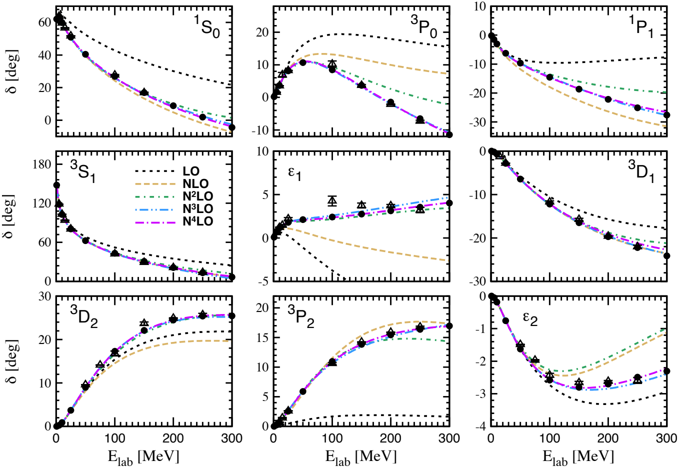


N^{2}LO: no new (additional) LECs in the two-body sector

leading-order three-body forces (3BFs)

NN interaction in chiral EFT

Semilocal momentum-space (SMS) regularized chiral *NN* potential

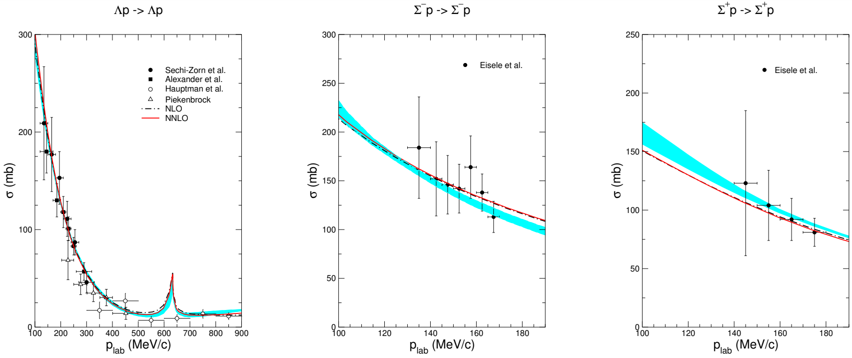


(Reinert, Krebs, Epelbaum, EPJA 54 (2018) 86) [up to N^{4}LO (N^{4}LO$^+$) !!]

LO to NLO: drastic change in all partial waves

NLO to N^{2}LO: changes mostly in *P*-waves and higher partial waves

Results for SMS chiral YN interactions



SMS YN potentials up to NLO, NNLO (with $\Lambda = 550$ MeV)

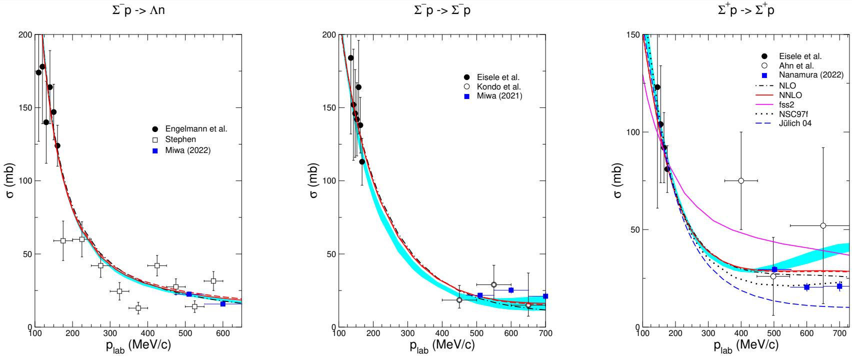
(J.H., U.-G. Meißner, A. Nogga, H. Le, EPJ A 59 (2023) 63)

NLO19: J.H., U.-G. Meißner, A. Nogga, EPJA 56 (2020) 91

quality of the fit – total χ^2 (36 data points):
NLO19(600): 16.0 SMS NLO: 15.2 SMS NNLO: 15.6

cross sections dominated by *S*-waves (are already well described at NLO)
→ (as expected) practically no change when going to NNLO

Results for SMS YN interactions



integrated cross sections at higher energies not included in the fitting process!

$\Sigma^+ p \to \Sigma^+ p$ and $\Sigma^- p \to \Sigma^- p$ cross sections:

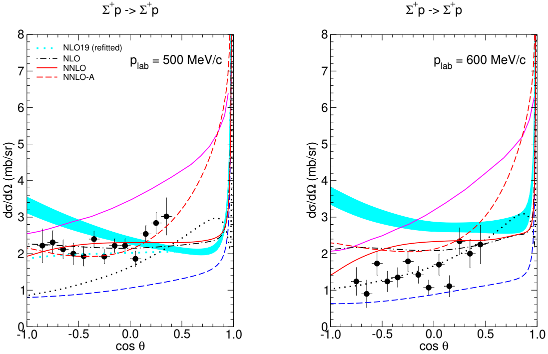
$$\sigma = \frac{2}{\cos\theta_{\rm max} - \cos\theta_{\rm min}} \int_{\cos\theta_{\rm min}}^{\cos\theta_{\rm max}} \frac{d\sigma(\theta)}{d\cos\theta} d\cos\theta$$

$\cos\theta_{\rm min} = -0.5;\ \cos\theta_{\rm max} = 0.5$

fss2 ... Fujiwara et al. (constitutent quark model) Jülich 04, Nijmegen NSC97f ... meson-exchange potentials

Results for SMS YN interactions

$\Sigma^+ p$ (T. Nanamura et al., PTEP 2022 (2022) 093D01)



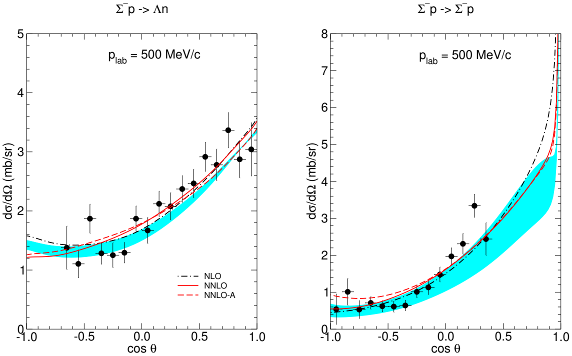
LECs in the $^1S_0$, $^3S_1$-$^3D_1$ fixed from low-energy cross sections

SMS NLO: LECs in ^3P-waves taken over from NN fit (RKE)
(strict SU(3) symmetry: $V_{NN} \equiv V_{\Sigma^+ p}$ in the $^1S_0$, $^3P_{0,1,2}$ partial waves!)

SMS NNLO: LECs in P-waves fitted to the E40 data (two examples)!

data for $(550 \leq p \leq 650)$ MeV/c are overestimated (influence of $\Lambda p \pi^+$ threshold?)

Results for SMS YN interactions



$\Sigma^- p \to \Lambda n$: quite well reproduced by NLO19 (NLO13) and SMS YN potentials
$\Sigma^- p \to \Sigma^- p$: behavior at forward angles remains unclear

$\Sigma^- p$ and $\Sigma^- p \to \Lambda n$ data for $(550 \leq p \leq 650)$ MeV/c are reproduced with comparable quality

- no unique determination of all P-wave LECs possible
- one needs data from additional channels (Λp, $\Sigma^- p \to \Sigma^0 n$, ...)
- one needs additional differential observables (polarizations, ...)

Hypernuclei within the NCSM

ab initio no-core shell model (NCSM)

Basic idea: use harmonic oscillator states and soft interactions

- m-scheme uses single particle states (center-of-mass motion not separated)
- antisymmetrization for nucleons easily performed (Slater determinant)
- larger dimensions (applications to *p*-shell hypernuclei by Wirth & Roth)

Jacobi-NCSM

- uses relative (Jacobi) coordinates (Hoai Le et al., EPJA 56 (2020) 301)
- explicit separation of center-of-mass motion possible
- antisymmetrization for nucleons difficult but feasible for $A \leq 9$
- small dimensions

Soft interactions: Similarity renormalization group (SRG) (unitary transformation)

$$\frac{dH(s)}{ds} = [[T, H(s)], H(s)] \qquad H(s) = T + V(s) \qquad V(s) : V^{NN}(s), V^{YN}(s)$$

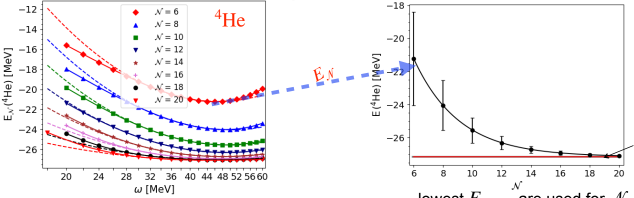
- Flow equations are solved in momentum space
- parameter (cutoff) $\lambda = \left(4\mu_{BN}^2/s\right)^{1/4}$ is a measure of the width of the interaction in momentum space
- $V(s)$ is phase equivalent to original interaction
- transformation leads to induced 3BFs, 4BFs, ...

  (induced 3BFs included in the work of Wirth & Roth and in our recent studies)
  (induced 4BFs are most likely very small)

Procedure

slide from Hoai Le:

- **extrapolation of energies:**
  - $E_b(\omega,\mathcal{N}) = E_{\mathcal{N}} + \kappa(\log(\omega) - \log(\omega_{opt}))^2$
  - $E_{\mathcal{N}} = E_\infty + Ae^{-b\mathcal{N}}$



NN: SMS $N^4LO^+(450)$

$\lambda = 7\,\mathrm{fm}^{-1}$

$E_{FY} = -27.15 \pm 0.02$ MeV

$E_\infty = -27.146 \pm 0.062$ MeV

$\boxed{\delta E = E_\infty - E_{\mathcal{N}_{max}}}$

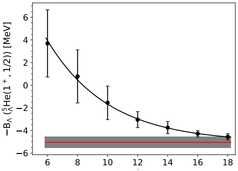
- lowest $E_{\mathcal{N},\omega_{opt}}$ are used for $\mathcal{N}$-space extrapolation ✓
- estimated uncertainties are rather conservative

- **extrapolation of Λ separation energies:** $B_\Lambda = E_{nucl} - E_{hyp}$
  - strong correlations between $E_{nucl}(\mathcal{N})$, $E_{hypnucl}(\mathcal{N})$

$$\rightarrow \quad B_{\Lambda,\mathcal{N}} = E_{nucl}(\mathcal{N}) - E_{hypnucl}(\mathcal{N})$$

$$\boxed{B_{\Lambda,\mathcal{N}} = B_{\Lambda,\infty} + A_1 e^{-b_1\mathcal{N}}}$$

HL, J. Haidenbauer, U.-G. Meißner, A. Nogga EPJA 56 (2020)



YN: SMS $N^2LO(550)$

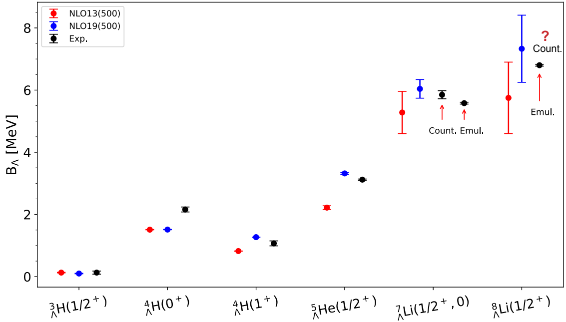
$\lambda_{YN} = 7\,\mathrm{fm}^{-1}$

Results for $B_\Lambda (A \leq 8)$

Hoai Le et al., PRC 107 (2023) 024002

- **NLO13** and **NLO19** are almost **phase equivalent** in the 2-body sector
- **NLO13** characterised by a stronger $\Lambda N - \Sigma N$ transition potential (especially in $^3S_1$)

→ **manifest in higher-body observables** **(J. Haidenbauer et al. NPA 915 (2019))**



NN:SMS N^4LO+(450)

+3N: N^2LO(450)

+YN: NLO13,19(CSB)

+SRG-induced YNN

Experiment:

M. Agnello et al. PLB 681

M. Juric NPB 52(1973)

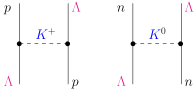
→ $^4_\Lambda$H(1^+), $^5_\Lambda$He, $^7_\Lambda$Li, $^8_\Lambda$Li are fairly well described by **NLO19**;

NLO13 underestimates separation energies

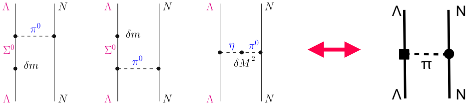
signal for (missing) chiral YNN forces

Charge symmetry breaking in the ΛN interaction

J.H., U.-G. Meißner, A. Nogga, FBS 62 (2021) 105



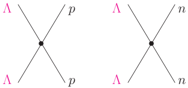
- leading contributions are small $\left((m_{K^0} - m_{K^\pm})/m_K \ll 1\right)$
- sub-leading contributions are more important:
  - effective CSB $\Lambda\Lambda\pi$ coupling constant



$$f_{\Lambda\Lambda\pi} = \left[-2\frac{\langle\Sigma^0|\delta m|\Lambda\rangle}{m_{\Sigma^0} - m_\Lambda} + \frac{\langle\pi^0|\delta M^2|\eta\rangle}{M_\eta^2 - M_{\pi^0}^2} \right] f_{\Lambda\Sigma\pi}$$

(Dalitz, van Hippel, 1964)

- two contact interactions C_s^{CSB}, C_t^{CSB} **adjusted to A=4 CSB**



$$\Delta B_\Lambda({}^4_\Lambda\text{He} - {}^4_\Lambda\text{He};\ 0^+) = \mathbf{233 \pm 92}\ \text{keV}$$

$$\Delta B_\Lambda({}^4_\Lambda\text{He} - {}^4_\Lambda\text{He};\ 1^+) = \mathbf{-83 \pm 94}\ \text{keV}$$

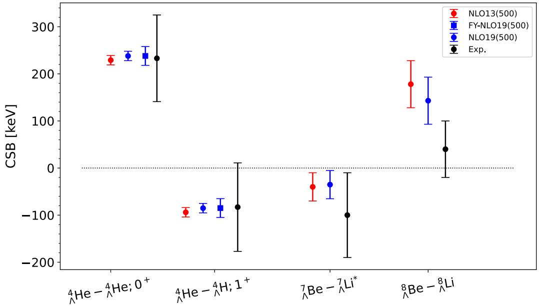
(Schulz et al. (2016); Yamamoto et al. (2015))

(fm//keV)	$a_s^{\Lambda p}$	$a_s^{\Lambda n}$	δa_s	$a_t^{\Lambda p}$	$a_t^{\Lambda n}$	δa_t
NLO19(**500**) no CSB	**-2.91**	**-2.91**	**0**	**-1.42**	**-1.41**	**-0.01**
CSB(**500**)	**-2.65**	**-3.20**	**0.55**	**-1.58**	**-1.47**	**-0.11**
CSB(550)	-2.64	-3.21	0.57	-1.52	-1.41	-0.11
CSB(600)	-2.63	-3.23	0.6	-1.47	-1.36	-0.09
CSB(650)	-2.62	-3.23	0.61	-1.46	-1.37	-0.09

→ $a(\Lambda n)$ **is independent of cutoff and of YN potentials**

CSB results for A=4,7,8 hypernuclei

Hoai Le, J.H., U.-G. Meißner, A. Nogga, PRC 107 (2023) 024002



NN:SMS N^{4}LO+(450)

+3N: N^{2}LO(450)

+YN: NLO13,19(CSB)

+SRG-induced YNN

- NLO13 & NLO19 CSB results for A=7 **are comparable to experiment.**
- two potentials predict a somewhat **larger CSB in A=8 doublet as compared to experiment**

→ ‣ experimental CS splitting for A=8 could be **larger than 40 ± 60 keV?**

- CSB estimate for $A = 4$ too large? different spin-dependence?

Consider new Star measurement

STAR Collaboration (M. Abdallah et al.), PLB 834 (2022) 137449

Recent Star measurement suggests somewhat different CSB in A=4:

$$\Delta E(1^+) = B_\Lambda({}^4_\Lambda\mathrm{He}, 1^+) - B_\Lambda({}^4_\Lambda\mathrm{H}, 1^+)$$

$$= -83 \pm 94 \text{ keV} \quad \Rightarrow (\text{CSB})$$

$$= \mathbf{-160 \pm 140 \pm 100} \text{ keV} \quad \Rightarrow (\text{CSB*})$$

$$\Delta E(0^+) = B_\Lambda({}^4_\Lambda\mathrm{He}, 0^+) - B_\Lambda({}^4_\Lambda\mathrm{H}, 0^+)$$

$$= 233 \pm 92 \text{ keV} \quad \Rightarrow (\text{CSB})$$

$$= \mathbf{160 \pm 140 \pm 100} \text{ keV} \quad \Rightarrow (\text{CSB*})$$

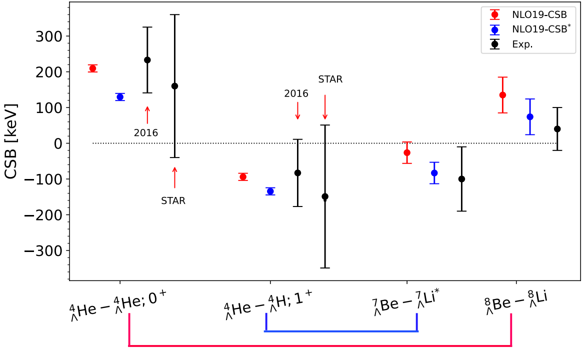
* STAR Collaboration PLB 834 (2022)

	NLO19(500)	CSB	CSB*
$a_s^{\Lambda p}$	-2.91	-2.65	-2.58
$a_s^{\Lambda n}$	-2.91	-3.20	-3.29
$\boldsymbol{\delta a_s}$	**0**	**0.55**	**0.71**
$a_t^{\Lambda p}$	-1.42	-1.57	-1.52
$a_t^{\Lambda n}$	-1.41	-1.45	-1.49
$\boldsymbol{\delta a_t}$	**-0.01**	**-0.12**	**-0.03**

→ $\delta a(^1S_0)$ **increases** while $\delta a(^3S_1)$ **decreases**

→ **How does the STAR measurement affect the predictions of CSB in A=7,8 multiplets ?**

Impact of Star measurement on CSB in A=7,8



NN:SMS N^{4}LO+(450)

+YN: NLO13,19(CSB)

$$\lambda_{NN} = 1.6\,\mathrm{fm}^{-1}$$

$$\lambda_{YN}^{opt} = 0.823\,\mathrm{fm}^{-1}$$

$$B_\Lambda(^5_\Lambda\mathbf{He}, \lambda_{YN}^{opt}) = B_\Lambda(^5_\Lambda\mathbf{He},\mathrm{3BFs})$$

- **CSB* fit predicts reasonable CSB in both A=7 and A=8 systems**
- CSB in A=4($\mathbf{0}^+$) and A=8, and in A=4($\mathbf{1}^+$) and A=7 are correlated

Separation energies for A=3-8 Λ hypernuclei (MeV)

- NLO13(19), SMS NLO,N^{2}LO are phase equivalent ($\chi^2 \approx 16$ for 36 YN data points)
- NLO13 characterized by a stronger ΛN-ΣN coupling potential (${}^3S_1$-${}^3D_1$)

	${}^3_\Lambda\mathrm{H}$ [Faddeev]	${}^4_\Lambda\mathrm{He}(0^+)$	${}^4_\Lambda\mathrm{He}(1^+)$	${}^5_\Lambda\mathrm{He}$	${}^7_\Lambda\mathrm{Li}$	${}^8_\Lambda\mathrm{Li}$
NLO13	0.090	1.48 ± 0.02	0.58 ± 0.02	2.22 ± 0.06	5.28 ± 0.68	5.75 ± 1.08
NLO19	0.091	1.46 ± 0.02	1.06 ± 0.02	3.32 ± 0.03	6.04 ± 0.30	7.33 ± 1.15
SMS NLO	0.124	2.10 ± 0.02	1.10 ± 0.02	3.34 ± 0.01		
SMS N^{2}LO	0.139	2.02 ± 0.02	1.25 ± 0.02	3.71 ± 0.01		
Exp.*	0.148 ± 0.04	2.347 ± 0.036	0.942 ± 0.036	3.102 ± 0.03	5.85 ± 0.13 5.58 ± 0.03	6.80 ± 0.03

NN: SMS N^{4}LO+(450) + 3NF: N^{2}LO(450) + SRG-induced YNN force

NLO19 (600): ${}^4_\Lambda\mathrm{He}(1^+)$, ${}^5_\Lambda\mathrm{He}$, ${}^7_\Lambda\mathrm{Li}$ fairly well described
NLO13 (600) underestimates the separation energies
SMS NLO,N^{2}LO (550): ${}^4_\Lambda\mathrm{He}(0^+, 1^+)$, ${}^5_\Lambda\mathrm{He}$ fairly well described

chiral YNN forces appear at N^{2}LO → ΛNN: 5 LECs
with decuplet saturation at NLO (LECs: 1 ΛNN + 1 ΣNN)
→ could be fixed from separation energies of, e.g.,
${}^4_\Lambda\mathrm{He}$ (0^+, 1^+) or ${}^4_\Lambda\mathrm{He}$ (0^+, 1^+), ${}^5_\Lambda\mathrm{He}$

* Chart of Hypernuclides *https://hypernuclei.kph.uni-mainz.de/*

Uncertainty quantification

- Uncertainty for a given observable $X(p)$:

  (Epelbaum, Krebs, Meißner, EPJA 51 (2015) 53)

  (S. Binder et al. [LENPIC coll.], PRC 93 (2016) 044002)
- estimate uncertainty via
  - the expected size of higher-order corrections
  - the actual size of higher-order corrections

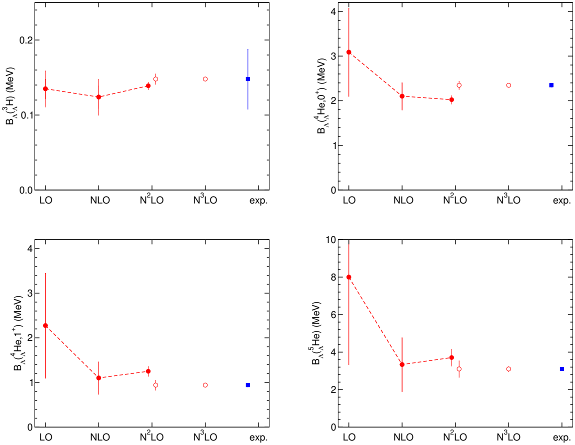
$$\begin{aligned}
\Delta X^{LO} &= Q^2|X^{LO}| \qquad (X^{NLO} \approx Q^2 X^{LO}) \\
\Delta X^{NLO} &= \max\left(Q^3|X^{LO}|, Q^1|\delta X^{NLO}|\right) ; \quad \delta X^{NLO} = X^{NLO} - X^{LO} \\
\Delta X^{N^2LO} &= \max\left(Q^4|X^{LO}|, Q^2|\delta X^{NLO}|, Q^1|\delta X^{N^2LO}|\right) ; \quad \delta X^{N^2LO} = X^{N^2LO} - X^{NLO} \\
\Delta X^{N^3LO} &= \max\left(Q^5|X^{LO}|, Q^3|\delta X^{NLO}|, Q^2|\delta X^{N^2LO}|, Q^1|\delta X^{N^3LO}|\right) ; \quad \delta X^{N^3LO} = X^{N^3LO} - X^{N^2LO}
\end{aligned}$$

- expansion parameter Q is defined by

$$Q = \max\left(\frac{p}{\Lambda_b}, \frac{m_\pi}{\Lambda_b}\right) ; \qquad p \ldots \Lambda p \text{ on} - \text{shell momentum}$$

Λ_b ... breakdown scale → $\Lambda_b = 500 - 600$ MeV [for $R = 0.8 - 1.2$ fm] (EKM, 2015)

Estimate of truncation error (preliminary!)



- filled symbols: actual estimates for SMS LO, NLO, N^2LO YN potentials
- opaque symbols: anticipated results when YNN 3BFs are included

- ${}^3_\Lambda$H: used as constraint! Conclusions on true uncertainty are not possible
- Q: $Q = M_\pi^{eff}/\Lambda_b \approx 200/650$ (Epelbaum et al., for light nuclei)

Summary

Hyperon-nucleon interaction within chiral EFT

- ΛN-ΣN interaction within semilocal momentum-space regularized chiral EFT
  confirm our previous YN results (up to NLO) based on a nonlocal regulator
  successful extension to NNLO
  new $\Sigma^{\pm}p$ differential cross sections around $p_{lab} \approx 500$ MeV/c can be described
  unique determination of the P-waves is not yet possible

Hypernuclei

- three-body forces: should be small for (${}^{3}_{\Lambda}\mathrm{H}$) or moderate (${}^{4}_{\Lambda}\mathrm{H}$, ${}^{4}_{\Lambda}\mathrm{He}$, ${}^{5}_{\Lambda}\mathrm{He}$)
  needs to be quantified/confirmed by explicit inclusion of 3BFs
- charge-symmetry breaking in ${}^{4}_{\Lambda}\mathrm{H} - {}^{4}_{\Lambda}\mathrm{He}$
  can be reproduced when taking into account the full leading CSB potential within chiral EFT
- charge-symmetry breaking in $A = 7 - 8$ Λ-hypernuclei
  predicted CSB splitting for ${}^{7}_{\Lambda}$Be, ${}^{7}_{\Lambda}$Li*, ${}^{7}_{\Lambda}$He is in line with experiments
  CSB splitting for ${}^{8}_{\Lambda}$Be, ${}^{8}_{\Lambda}$Li is overestimated

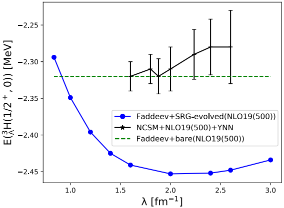
Λp momentum correlation functions

- ALICE measurement: indications that the Λp is possibly somewhat weaker than what the cross section data from the 1960ies suggest (D. Mihaylov, M. Korwieser)

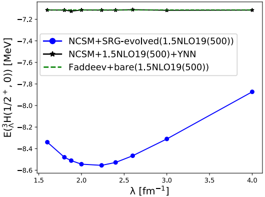
backup slides

A=3-5 Λ hypernuclei with SRG-induced YNN force

Hoai Le, EPJ Web Conf. 271 (2022) 01004 (HYP2022)



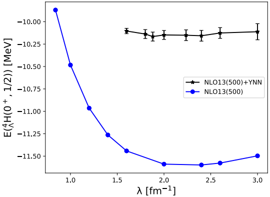
$^{3}_{\Lambda}\mathrm{H}(1/2^{+},0)$



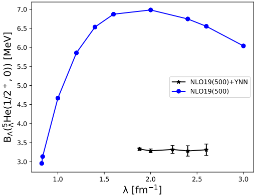
$^{3}_{\Lambda}\mathrm{H}(1/2^{+},0)$

NN:SMS N^{4}LO+(450)

3N: N^{2}LO(450)



$^{4}_{\Lambda}\mathrm{H}(0^{+},1/2)$

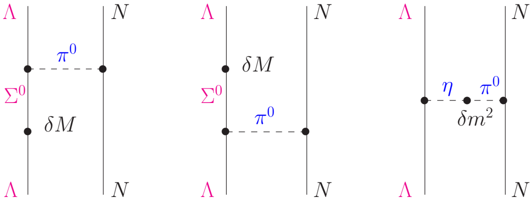


$^{5}_{\Lambda}\mathrm{He}(1/2^{+},0)$

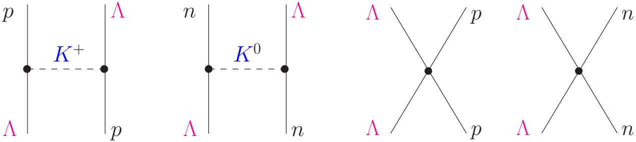
⇒ contributions of SRG-induced $YNNN$ forces are negligible

(R. Wirth, R. Roth, PRL 117 (2016); PRC 100 (2019))

Charge symmetry breaking in the ΛN interaction



CSB due to $\Lambda - \Sigma^0$ mixing: long-ranged contribution to the ΛN interaction
(R.H. Dalitz & F. von Hippel, PL 10 (1964) 153)

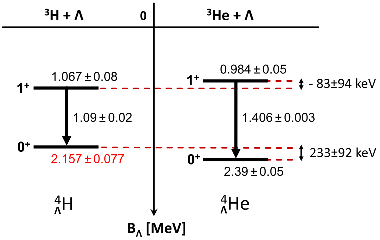


CSB in chiral EFT: additional short-range contributions ⇒ contact terms
(NN: Epelbaum, Glöckle, Meißner, NPA 747 (2005) 362; etc.)

J.H., U.-G. Meißner, A. Nogga, FBS 62 (2021) 105

Charge symmetry breaking in $^4_\Lambda$H-$^4_\Lambda$He

- $\Delta E(0^+) = E_\Lambda^{0^+}(^4_\Lambda\text{He}) - E_\Lambda^{0^+}(^4_\Lambda\text{H})$
  $= 233 \pm 92$ keV
- $\Delta E(1^+) = E_\Lambda^{1^+}(^4_\Lambda\text{He}) - E_\Lambda^{1^+}(^4_\Lambda\text{H})$
  $= -83 \pm 94$ keV



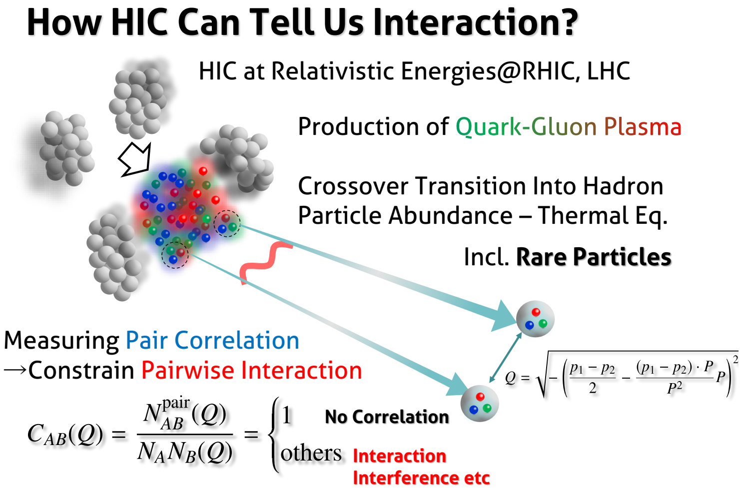
adjust CSB contact terms to ΔE's

(Schulz et al., 2016; Yamamoto et al., 2015)

(fm // keV)	$a_s^{\Lambda p}$	$a_s^{\Lambda n}$	$a_t^{\Lambda p}$	$a_t^{\Lambda n}$	$\Delta E(0^+)$	$\Delta E(1^+)$
NLO19(500)	-2.649	-3.202	-1.580	-1.467	249	-75
NLO19(550)	-2.640	-3.205	-1.524	-1.407	252	-72
NLO19(600)	-2.632	-3.227	-1.473	-1.362	243	-67
NLO19(650)	-2.620	-3.225	-1.464	-1.365	250	-69

CSB in singlet ($^1S_0$) much larger than in triplet ($^3S_1$)
practically independent of cutoff; same results for NLO13
without CSB: $a_s^{\Lambda p} \approx a_s^{\Lambda n} \approx -2.9$ fm

Two-particle correlation function (Kenji Morita)



How HIC Can Tell Us Interaction?

HIC at Relativistic Energies@RHIC, LHC

Production of Quark-Gluon Plasma

Crossover Transition Into Hadron
Particle Abundance – Thermal Eq.

Incl. **Rare Particles**

Measuring Pair Correlation
→Constrain Pairwise Interaction

$$Q = \sqrt{-\left(\frac{p_1 - p_2}{2} - \frac{(p_1 - p_2)\cdot P}{P^2}P\right)^2}$$

$$C_{AB}(Q) = \frac{N_{AB}^{\text{pair}}(Q)}{N_A N_B(Q)} = \begin{cases} 1 & \textbf{No Correlation} \\ \text{others} & \textbf{Interaction} \\ & \textbf{Interference etc} \end{cases}$$

Two-particle correlation function

Koonin-Pratt formalism

Correlation function for identical particles ($\Lambda\Lambda$, $\Sigma^+\Sigma^+$, ...)

$$C(k) \simeq 1 - \frac{1}{2}\exp(-4k^2R^2) + \frac{1}{2}\int_0^\infty 4\pi r^2\, dr\, S_{12}(\mathbf{r}) \left[|\psi(k,r)|^2 - |j_0(kr)|^2 \right]$$

Correlation function for non-identical particles (Λp, $\Xi^- p$, $K^- p$, ...)

$$C(k) \simeq 1 + \int_0^\infty 4\pi r^2\, dr\, S_{12}(\mathbf{r}) \left[|\psi(k,r)|^2 - |j_0(kr)|^2 \right]$$

Extension to multi-channel problem

$$|\psi(k,r)|^2 \to \sum_\beta \omega_\beta |\psi_{\beta\alpha}(k_\alpha, r)|^2$$

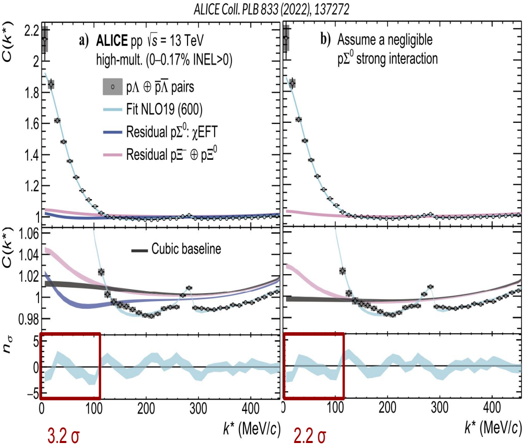
$$C_\alpha(k_\alpha) \simeq 1 + \sum_\beta \omega_\beta \int_0^\infty 4\pi r^2\, dr\, S_\beta(\mathbf{r}) \left[\left|\psi_{\beta\alpha}(k_\alpha, r)\right|^2 - \delta_{\beta\alpha}\, |j_0(k_\alpha r)|^2 \right]$$

$\sum_\beta$ ... over all two-body intermediate states that couple to α

ω_β ... weights of the various components (often put to 1)

assume a static and spherical Gaussian source with radius R:
$S_{12}(\mathbf{r}) = \exp(-r^2/4R^2)/(2\sqrt{\pi}R)^3$

Λp momentum correlation function at $\sqrt{s} = 13$ TeV


ALICE Coll. PLB 833 (2022), 137272

ALICE Collaboration: *pp* collisions at 13 TeV (S. Acharya et al., PLB 833 (2022) 137272)

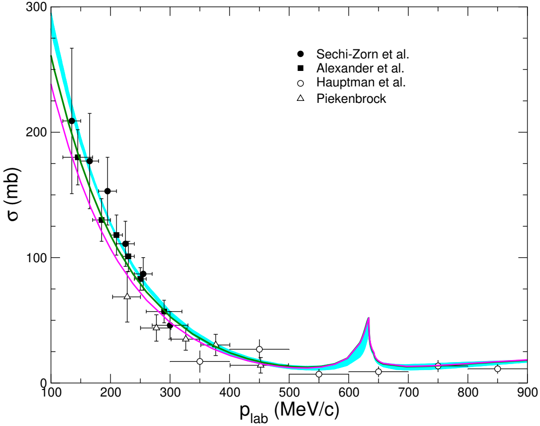
⇒ prediction of NLO19 is fairly well in line with data

sensitive to the assumption about the contribution of the $\Sigma^0 p$ feed-down

Λp : Slightly weaker energy dependence? Reduced overall strength?

Mihaylov & Gonzalez (arXiv:2305.08441): $a_t = -1.15 \pm 0.07$ fm

Reduced strength of the ΛN interaction in the $^3S_1$ state



NLO19(600) is used as starting point

$$a_t = -1.41\ \text{fm} \quad \Rightarrow \quad a_t = -1.30\ \text{fm} \quad [-1.15\ \text{fm}]$$

$$\chi^2 = 2.09 \quad \Rightarrow \quad \chi^2 = 3.45 \quad [7.14]\ (\text{Sechi}-\text{Zorn})$$

$$\chi^2 = 1.29 \quad \Rightarrow \quad \chi^2 = 1.15 \quad [6.00]\ (\text{Alexander})$$

$$n_\sigma = 3.2 \quad \Rightarrow \quad n_\sigma = 2.2\ (\text{with residual } \Sigma^0 p \text{ interaction included})$$

(reduction in the $^1S_0$ state is limited since we want/need the $^3_\Lambda$H to be bound!)

$SU(3)$ structure of contact terms for BB

$SU(3)$ structure for scattering of two octet baryons $\rightarrow$
$8 \otimes 8 = 1 \oplus 8_a \oplus 8_s \oplus 10^* \oplus 10 \oplus 27$

BB interaction can be given in terms of LECs corresponding to the $SU(3)_f$ irreducible representations: C^1, C^{8a}, C^{8s}, C^{10^*}, C^{10}, C^{27}

	Channel	I	V_α	V_β	$V_{\beta\rightarrow\alpha}$
$S = \ \ 0$	$NN \rightarrow NN$	0	$-$	$C^{10^*}_\beta$	$-$
	$NN \rightarrow NN$	1	C^{27}_α	$-$	$-$
$S = -1$	$\Lambda N \rightarrow \Lambda N$	$\frac{1}{2}$	$\frac{1}{10}\left(9C^{27}_\alpha + C^{8_s}_\alpha\right)$	$\frac{1}{2}\left(C^{8_a}_\beta + C^{10^*}_\beta\right)$	$-C^{8sa}$
	$\Lambda N \rightarrow \Sigma N$	$\frac{1}{2}$	$\frac{3}{10}\left(-C^{27}_\alpha + C^{8_s}_\alpha\right)$	$\frac{1}{2}\left(-C^{8_a}_\beta + C^{10^*}_\beta\right)$	$-3C^{8sa}$
					C^{8sa}
	$\Sigma N \rightarrow \Sigma N$	$\frac{1}{2}$	$\frac{1}{10}\left(C^{27}_\alpha + 9C^{8_s}_\alpha\right)$	$\frac{1}{2}\left(C^{8_a}_\beta + C^{10^*}_\beta\right)$	$3C^{8sa}$
	$\Sigma N \rightarrow \Sigma N$	$\frac{3}{2}$	C^{27}_α	C^{10}_β	$-$

$\alpha = {}^1S_0, {}^3P_0, {}^3P_1, {}^3P_2$, $\quad \beta = {}^3S_1, {}^3S_1 - {}^3D_1, {}^1P_1$

No. of contact terms: LO: $\quad 2\ (NN) + \ \ 3\ (YN) + 1\ (YY)$
NLO: $7\ (NN) + 11\ (YN) + 4\ (YY)$

(No. of spin-isospin channels in NN+YN: 10 $\qquad S = -2, -3, -4$: 27)

Contact terms for YN – partial-wave projected

spin-momentum structure up to NLO

$$
\begin{aligned}
V(^1S_0) &= \tilde{C}_{^1S_0} + C_{^1S_0}(p^2 + p'^2) \\
V(^3S_1) &= \tilde{C}_{^3S_1} + C_{^3S_1}(p^2 + p'^2) \\
V(\alpha) &= C_\alpha \, p \, p' \qquad \alpha \hat{=} \, ^1P_1, \ ^3P_0, \ ^3P_1, \ ^3P_2 \\
V(^3D_1 - \, ^3S_1) &= C_{^3S_1 - \, ^3D_1} \, p'^2
\end{aligned}
$$

$$
\begin{aligned}
V(^1P_1 - \, ^3P_1) &= C_{^1P_1 - \, ^3P_1} \, p \, p' \\
V(^3P_1 - \, ^1P_1) &= C_{^3P_1 - \, ^1P_1} \, p \, p'
\end{aligned}
$$

(antisymmetric spin-orbit force: $(\vec{\sigma}_1 - \vec{\sigma}_2) \cdot (\vec{q} \times \vec{k})$)

- $\tilde{C}_\alpha$, C_α ... low-energy constants (LECs)
- need to be fixed by a fit to (NN, YN, ...) data

chiral YN potential up to NNLO

Reinert, Krebs, Epelbaum, EPJA 54 (2018) 86

- Λ: 350 – 550 MeV ... 450 MeV give best results

YN interaction: approximate SU(3) flavor symmetry
$m_\pi = 138$ MeV, $m_K = 495$ MeV, $m_\eta = 547$ MeV

want to keep effects from SU(3) symmetry breaking generated by the single-meson exchange contributions
⇒ Λ: 500 – 600 MeV

two-meson exchange contributions: πK, $\pi\eta$, ... are represented by contact terms

⇒ some SU(3) symmetry breaking in the YN LECs

(S. Petschauer, N. Kaiser, NPA 916 (2013) 1)

$$V^{cont} = \tilde{C}^\alpha + C^\alpha(p^2 + p'^2) + C^\chi(m_K^2 - m_\pi^2)$$

$\tilde{C}^\alpha$, C^α, $\alpha = \{27\}, \{10^*\}, \{10\}, \{8_s\}, \{8_a\}, \{1\}$, ... "regular" contact terms in SU(3) chiral EFT
C_i^χ: SU(3) symmetry breaking contact terms

(in NLO13 and NLO19 ΛN-ΣN potentials we assumed that $C_i^\chi = 0$)

chiral YN potential up to NNLO

adopt the framework of Reinert, Krebs, Epelbaum, EPJA 54 (2018) 86:

“Semilocal momentum-space regularized (SMS) chiral NN potentials”

- employ a regulator that minimizes artifacts from cutoff Λ

nonlocal cutoff $(\vec{q} = \vec{p}\,' - \vec{p})$

$$V_{1\pi}^{\rm reg} \propto \frac{e^{-\frac{p'^4+p^4}{\Lambda^4}}}{\vec{q}^{\,2}+m_\pi^2} \to \frac{1}{\vec{q}^{\,2}+m_\pi^2}\left[1-\frac{p'^4+p^4}{\Lambda^4}+\mathcal{O}(\Lambda^{-8})\right]$$

local cutoff:

$$V_{1\pi}^{\rm reg} \propto \frac{e^{-\frac{\vec{q}^{\,2}+m_\pi^2}{\Lambda^2}}}{\vec{q}^{\,2}+m_\pi^2} \to \frac{1}{\vec{q}^{\,2}+m_\pi^2} - \frac{1}{\Lambda^2} + \frac{\vec{q}^{\,2}+m_\pi^2}{\Lambda^4} + \ldots$$

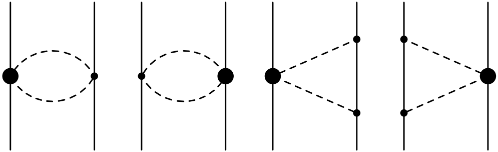
does not affect long-range physics at any order in the $1/\Lambda^2$ expansion

applicable to 2π exchange too:

$$V_{2\pi} = \frac{2}{\pi}\int_{2m_\pi}^{\infty} \mu d\mu \frac{\rho(\mu)}{\vec{q}^{\,2}+\mu^2} \to V_{2\pi}^{\rm reg} = e^{-\frac{\vec{q}^{\,2}}{2\Lambda^2}} \frac{2}{\pi}\int_{2m_\pi}^{\infty} \mu d\mu \frac{\rho(\mu)}{\vec{q}^{\,2}+\mu^2} e^{-\frac{\mu^2}{2\Lambda^2}} \quad + \ldots$$

chiral YN interaction up to NNLO

- no new *BB* contact terms (no additional LECs) enter
- sub-leading meson-baryon vertices enter at NNLO



πN: fixed from calculating pion-nucleon scattering in chiral perturbation theory
sub-leading (up to Q^2) πN LECs: $c_1 = -0.74;\ c_3 = -3.61;\ c_4 = 2.44$

(cf. RKE 2018)

$\pi\Lambda$, $\pi\Sigma$, $\pi\Lambda \leftrightarrow \pi\Sigma$:
involve additional LECs: d_1, d_2, d_3, b_D, b_F, b_0, b_1, b_2, b_3, b_4
fixed from resonance saturation via decuplet baryons ($\Sigma^*(1385)$)

(cf. Petschauer et al., NPA 957 (2017) 347)

Coupled channels Lippmann-Schwinger Equation

$$\begin{aligned} T^{\nu'\nu,J}_{\rho'\rho}(p',p) &= V^{\nu'\nu,J}_{\rho'\rho}(p',p) \\ &+ \sum_{\rho'',\nu''}\int_0^\infty \frac{dp''p''^2}{(2\pi)^3}\, V^{\nu'\nu'',J}_{\rho'\rho''}(p',p'')\frac{2\mu_{\rho''}}{p^2-p''^2+i\eta}\, T^{\nu''\nu,J}_{\rho''\rho}(p'',p) \end{aligned}$$

ρ', ρ = ΛN, ΣN ($\Lambda\Lambda$, ΞN, $\Lambda\Sigma$, $\Sigma\Sigma$)

LS equation is solved for particle channels (in momentum space)

Coulomb interaction is included via the Vincent-Phatak method

SMS: A nonlocal regulator is applied to the contact terms

$$V^{\nu'\nu,J}_{\rho'\rho}(p',p) \rightarrow f^\Lambda(p')\,V^{\nu'\nu,J}_{\rho'\rho}(p',p)f^\Lambda(p); \quad f^\Lambda(p) = e^{-(p/\Lambda)^2}$$

consider values $\Lambda = 500$ - 600 MeV [guided by *NN*, achieved χ^2]

NLO19 (NLO13): A a nonlocal regulator is applied to the whole potential

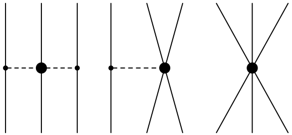
$$V^{\nu'\nu,J}_{\rho'\rho}(p',p) \rightarrow f^\Lambda(p')\,V^{\nu'\nu,J}_{\rho'\rho}(p',p)f^\Lambda(p); \quad f^\Lambda(p) = e^{-(p/\Lambda)^4}$$

with values $\Lambda = 500$ - 650 MeV

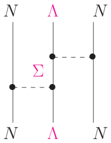
Three-body forces

- SU(3) χEFT 3BFs at NNLO (S. Petschauer et al., PRC 93 (2016) 014001)
- however, 5 LECs for ΛNN 3BF alone! (only 2 LECs for NNN)

SU(3) χEFT :

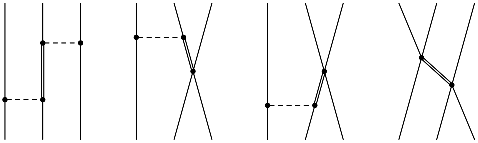


(at LO!) :



solve coupled channel (ΛN-ΣN) Faddeev-Yakubovsky equations:
$\Rightarrow$ ΛNN “3BF” from Σ coupling is automatically included

- 3BFs with inclusion of decuplet baryons (S. Petschauer et al., NPA 957 (2017) 347)



estimate ΛNN 3BF based on the $\Sigma^*(1385)$ excitation (appear at NLO!)

- only 1 LEC for ΛNN (2 LECs for YNN in general)

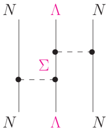
Estimation of 3BFs based on NLO results

- ${}^{3}_{\Lambda}\mathrm{H}$

  (a) cutoff variation: ΔE_Λ (3BF) ≤ 50 keV

  (b) "3BF" from ΛN-ΣN coupling:

  switch off ΛN-ΣN coupling
  in Faddeev-Yakubovsky equations:
  ΔE_Λ (3BF) ≈ 10 keV
  expect similar/smaller ΔE_Λ from $\Sigma^*(1385)$ excitation



  (c) ${}^{3}\mathrm{H}$: 3NF $\sim Q^3\,|\langle V_{NN}\rangle|_{{}^{3}\mathrm{H}} \sim 650$ keV

  ($|\langle V_{NN}\rangle|_{{}^{3}\mathrm{H}} \sim 50$ MeV; $Q \sim m_\pi/\Lambda_b$; $\Lambda_b \simeq 600$ MeV)

  ${}^{3}_{\Lambda}\mathrm{H}$: $|\langle V_{\Lambda N}\rangle|_{{}^{3}_{\Lambda}\mathrm{H}} \sim 3$ MeV $\to \Delta E_\Lambda$ (3BF) $\approx Q^3\,|\langle V_{\Lambda N}\rangle|_{{}^{3}_{\Lambda}\mathrm{H}} \simeq 40$ keV

- ${}^{4}_{\Lambda}\mathrm{H}$, ${}^{4}_{\Lambda}\mathrm{He}$

  (a) cutoff variation: ΔE_Λ (3BF) ≈ 200 keV (0^+) and ≈ 300 keV (1^+)

  (b) "3BF" from ΛN-ΣN coupling:

  ΔE_Λ (3BF) $\approx 230 - 340$ keV (0^+), $\approx 150 - 180$ keV (1^+)

${}^{3}_{\Lambda}\mathrm{H}$ and ${}^{4}_{\Lambda}\mathrm{H}$(He) calculations with explicit inclusion of 3BFs utilizing the decuplet saturation are planned for the future