Assorted aspects of the hyperon-nucleon interaction

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(Hoai Le, Ulf-G. Meißner, Andreas Nogga)

Outline

BB interaction in chiral effective field theory

Baryon-baryon interaction in SU(3) χEFT à la Weinberg (1990)

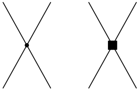
- Power counting: systematic improvement by going to higher order
- Possibility to derive two- and three-baryon forces and external current operators in a consistent way

- degrees of freedom: octet baryons (N, Λ, Σ, Ξ), pseudoscalar mesons (π, K, η)

- meson-exchange: fixed by the underlying chiral symmetry of QCD + SU(3)
- short-distance dynamics remains unresolved – represented by contact terms (involve low-energy constants (LECs)) that need to be determined from data)

$$V^{CT}_{B_1 B_2 \to B_1' B_2'} = \tilde{C}_\alpha + C_\alpha (p'^2 + p^2) \quad (C_\beta p'^2, \; C_\gamma p' p)$$

$$\alpha = {}^1S_0, {}^3S_1; \; \beta = {}^3S_1 - {}^3D_1; \; \gamma = {}^3P_0, {}^1P_1, {}^3P_1, {}^3P_2$$



ΛN-ΣN interaction:
LO: H. Polinder, J.H., U.-G. Meißner, NPA 779 (2006) 244
NLO13: J.H., S. Petschauer, N. Kaiser, U.-G. Meißner, A. Nogga, W. Weise, NPA 915 (2013) 24
NLO19: J.H., U.-G. Meißner, A. Nogga, EPJA 56 (2020) 91
N^{2}LO: J.H. et al., in preparation, arXiv:2208.13542

Spin dependence of the ΛN interaction

Experiments: only integrated cross sections (G. Alexander et al.; B. Sechi-Zorn et al.)
(angular distribution of events)
No information on spin dependence (singlet, triplet)

$$\sigma_{\Lambda N} \propto \frac{1}{4}|T^s_{\Lambda N}|^2 + \frac{3}{4}|T^t_{\Lambda N}|^2$$

s-shell hypernuclei (Herndon & Tang, PR 153 (1967) 1091):

$$^3_\Lambda\mathrm{H}: \quad \tilde{V}_{\Lambda N} \approx \frac{3}{4}V^s_{\Lambda N} + \frac{1}{4}V^t_{\Lambda N} \qquad ^4_\Lambda\mathrm{He}\ (0^+): \quad \tilde{V}_{\Lambda N} \approx \frac{1}{2}V^s_{\Lambda N} + \frac{1}{2}V^t_{\Lambda N}$$

$$^4_\Lambda\mathrm{He}\ (1^+): \quad \tilde{V}_{\Lambda N} \approx \frac{1}{6}V^s_{\Lambda N} + \frac{5}{6}V^t_{\Lambda N} \qquad ^5_\Lambda\mathrm{He}: \quad \tilde{V}_{\Lambda N} \approx \frac{1}{4}V^s_{\Lambda N} + \frac{3}{4}V^t_{\Lambda N}$$

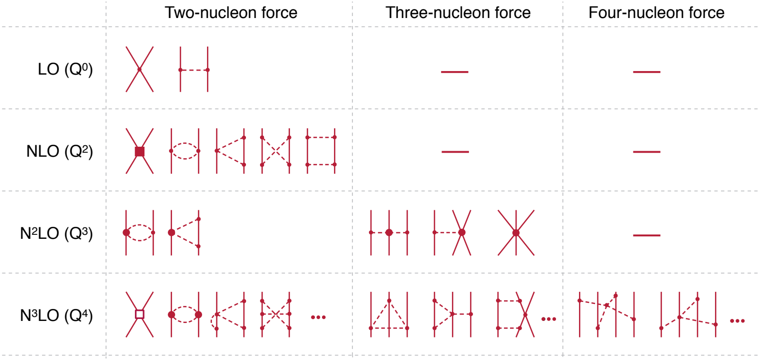
Jülich-Bonn group:

- use $\sigma_{\Lambda p}$ and $^3_\Lambda\mathrm{H}$ separation energy (130 ± 50 keV) to fix relative strength of singlet/triplet interaction

  Three-body forces?
- estimate of 3BF contribution (e.g. from power counting): $\Delta B_\Lambda < 50$ keV
- experimental uncertainty ≥ 50 keV

$\Rightarrow$ direct experimental information on strength of ΛN singlet/triplet interaction is needed

3- and many-body forces in chiral EFT (E. Epelbaum)



number of independent LECs in the three-body force:

NNN: one-pion exchange 3NF (c_D), contact term (c_E)

Λ*NN*: one-pion exchange 3BF (2 LECs), contact term (3 LECs)
(decuplet saturation (NLO): 1 (Λ*NN*) + 1 (Σ*NN*) LECs)

(two-pion exchange 3BF is fixed from chiral symmetry + SU(3))

ab initio no-core shell model (NCSM)

Basic idea: use harmonic oscillator states and soft interactions

- m-scheme uses single particle states (center-of-mass motion not separated)
- antisymmetrization for nucleons easily performed (Slater determinant)
- larger dimensions (applications to *p*-shell hypernuclei by Wirth & Roth)

Jacobi-NCSM

- uses relative (Jacobi) coordinates (Hoai Le et al., EPJA 56 (2020) 301)
- explicit separation of center-of-mass motion possible
- antisymmetrization for nucleons difficult but feasible for $A \leq 9$
- small dimensions

Soft interactions: Similarity renormalization group (SRG) (unitary transformation)

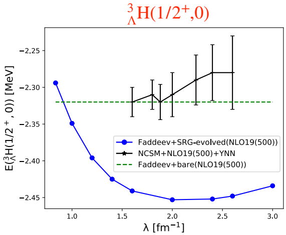
$$\frac{dH(s)}{ds} = [[T, H(s)], H(s)] \qquad H(s) = T + V(s) \qquad V(s) : V^{NN}(s), V^{YN}(s)$$

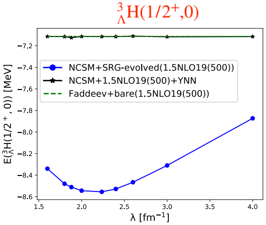
- Flow equations are solved in momentum space
- parameter (cutoff) $\lambda = \left(4\mu_{BN}^2/s\right)^{1/4}$ is a measure of the width of the interaction in momentum space
- $V(s)$ is phase equivalent to original interaction
- transformation leads to induced 3BFs, 4BFs, ...

  (induced 3BFs included in the work of Wirth & Roth and in our recent studies)
  (induced 4BFs are most likely very small)

A=3-5 Λ hypernuclei with SRG-induced YNN force

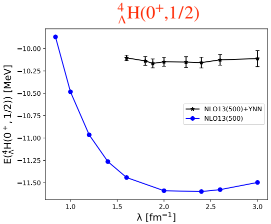
Hoai Le, arXiv:2210.02860 (HYP2022)

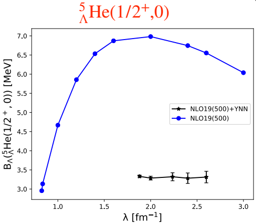




NN:SMS N^{4}LO+(450)

3N: N^{2}LO(450)





$\Rightarrow$ contributions of SRG-induced $YNNN$ forces are negligible

(R. Wirth, R. Roth, PRL 117 (2016); PRC 100 (2019))

Separation energies for A=3-8 Λ hypernuclei (MeV)

- NLO13 and NLO19 are practically phase equivalent ($\chi^2 \approx 16$ for 36 YN data points)
- NLO13 characterized by a stronger ΛN-ΣN coupling potential ($^3S_1$-$^3D_1$)

	$^3_\Lambda$H [Faddeev]	$^4_\Lambda$H(0^+)	$^4_\Lambda$H(1^+)	$^5_\Lambda$He	$^7_\Lambda$Li	$^8_\Lambda$Li
NLO13	0.135	1.55 ± 0.01	0.82 ± 0.01	2.22 ± 0.06	5.28 ± 0.68	5.75 ± 1.08
NLO19	0.100	1.51 ± 0.01	1.27 ± 0.01	3.32 ± 0.03	6.04 ± 0.30	7.33 ± 1.15
Exp.	0.13 ± 0.05	2.16 ± 0.08	1.07 ± 0.08	3.12 ± 0.02	5.85 ± 0.13	6.80 ± 0.03
	0.41 ± 0.12 [S]				5.58 ± 0.03	
	0.072 ± 0.063 [A]					

NN: SMS N^{4}LO+(450) + 3NF: N^{2}LO(450) + SRG-induced YNN force
[S] ... STAR Collaboration, [A] ... ALICE Collaboration

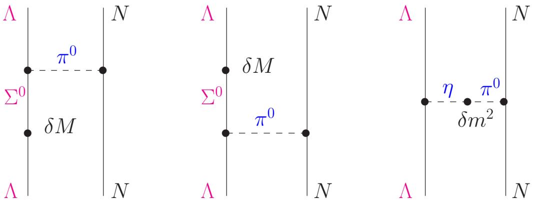
NLO19 (500): $^4_\Lambda$H(1^+), $^5_\Lambda$He, $^7_\Lambda$Li fairly well described
NLO13 (500) underestimates the separation energies

clear signal for (missing) chiral YNN forces:

in (standard) chiral EFT 3BFs appear at N^{2}LO
with decuplet saturation at NLO (LECs: 1 ΛNN + 1 ΣNN)

$\rightarrow$ could be fixed from separation energies of, e.g.,
$^4_\Lambda$H (0^+, 1^+) or $^4_\Lambda$H (0^+, 1^+), $^5_\Lambda$He

Charge symmetry breaking in the ΛN interaction



CSB due to $\Lambda - \Sigma^0$ mixing leads to a long-ranged contribution to the ΛN interaction (R.H. Dalitz & F. von Hippel, PL 10 (1964) 153)

Strength can be estimated from the electromagnetic mass matrix:

$$\langle \Sigma^0 | \delta M | \Lambda \rangle = [M_{\Sigma^0} - M_{\Sigma^+} + M_p - M_n]/\sqrt{3}$$

$$\langle \pi^0 | \delta m^2 | \eta \rangle = [m^2_{\pi^0} - m^2_{\pi^+} + m^2_{K^+} - m^2_{K^0}]/\sqrt{3}$$

$$f_{\Lambda\Lambda\pi} = \left[-2\frac{\langle \Sigma^0 | \delta M | \Lambda \rangle}{M_{\Sigma^0} - M_\Lambda} + \frac{\langle \pi^0 | \delta m^2 | \eta \rangle}{m^2_\eta - m^2_{\pi^0}}\right] f_{\Lambda\Sigma\pi}$$

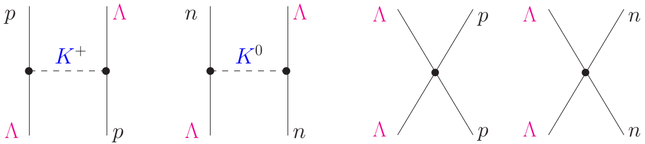
latest PDG mass values $\Rightarrow$

$$f_{\Lambda\Lambda\pi} \approx (-0.0297 - 0.0106)\, f_{\Lambda\Sigma\pi} \approx -0.0403\, f_{\Lambda\Sigma\pi}$$

CSB in chiral EFT

CSB (CIB) in χEFT: worked out for *pp*, *nn* (and *np*) scattering

Walzl, Meißner, Epelbaum, NPA 693 (2001) 663; Epelbaum, Glöckle, Meißner, NPA 747 (2005) 362
J. Friar et al., PRC 68 (2003) 024003
LØ: Coulomb interaction, m_{π^0}-$m_{\pi^\pm}$ in OPE   NLØ: isospin breaking in $f_{NN\pi}$, leading-order contact terms

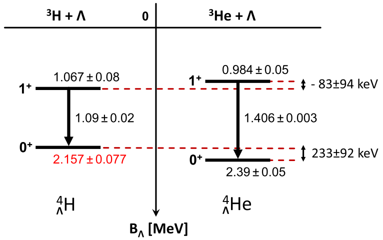


$NN\ ^1S_0$: $a_{pp} - a_{nn} \approx 1.5$ fm
mostly due to short-range forces (ρ^0-ω mixing, a_1^0-f_1 mixing)

Faddeev-Yakubovsky calculation for NLO13 and NLO19 interactions with CSB forces including contact terms:
(J.H., U.-G. Meißner, A. Nogga, FBS 62 (2021) 105)

Charge symmetry breaking in ${}^4_\Lambda$H-${}^4_\Lambda$He

- $\Delta B(0^+) = B^{0^+}_\Lambda({}^4_\Lambda\mathrm{He}) - B^{0^+}_\Lambda({}^4_\Lambda\mathrm{H})$
  $= 233 \pm 92$ keV
- $\Delta B(1^+) = B^{1^+}_\Lambda({}^4_\Lambda\mathrm{He}) - B^{1^+}_\Lambda({}^4_\Lambda\mathrm{H})$
  $= -83 \pm 94$ keV



adjust CSB contact terms to ΔB's

(Schulz et al., 2016; Yamamoto et al., 2015)

(fm // keV)	$a_s^{\Lambda p}$	$a_s^{\Lambda n}$	$a_t^{\Lambda p}$	$a_t^{\Lambda n}$	$\Delta B(0^+)$	$\Delta B(1^+)$
NLO19(500)	-2.649	-3.202	-1.580	-1.467	249	-75
NLO19(550)	-2.640	-3.205	-1.524	-1.407	252	-72
NLO19(600)	-2.632	-3.227	-1.473	-1.362	243	-67
NLO19(650)	-2.620	-3.225	-1.464	-1.365	250	-69

CSB in singlet (${}^1S_0$) much larger than in triplet (${}^3S_1$)
practically independent of cutoff; same results for NLO13
without CSB interaction: $a_s^{\Lambda p} \approx a_s^{\Lambda n} \approx -2.9$ fm
with CSB interaction: $\Delta a_s = a_s^{\Lambda p} - a_s^{\Lambda n} \approx 0.62 \pm 0.08$ fm; $\Delta a_t \approx -0.10 \pm 0.02$ fm

Charge symmetry breaking in A=7-8 Λ-hypernuclei

Hoai Le, J.H., U.-G. Meißner, A. Nogga, arXiv:2210.03387

(in keV)		ΔT	$\Delta V_{\rm NN}$	$\Delta V_{\rm YN}$ $^1S_0$	$^3S_1$	total	ΔB
$^7_\Lambda$Be-$^7_\Lambda$Li*	NLO13	8	-24	-49	26	-24	-40 (30)
	NLO19	6	-41	-43	42	9	-35 (30)
	Hiyama		-70			200	150
	Gal	3	-70			50	-17
	experiment						-100 ± 90
$^7_\Lambda$Li*-$^7_\Lambda$He	NLO13	7	-14	-49	26	-24	-31 (30)
	NLO19	5	-21	-38	37	-1	-16 (30)
	Hiyama		-80			200	130
	Gal	2	-80			50	-28
	experiment						-20 ± 230
$^8_\Lambda$Be-$^8_\Lambda$Li	NLO13	12	7	100	56	159	178 (50)
	NLO19	6	-11	62	79	147	143 (50)
	Hiyama		40				160
	Gal	11	-81			119	49
	experiment						40 ± 60

experimental results are taken from E. Botta et al., NPA 960 (2017) 165
A. Gal, PLB 744 (2015) 352 (shell model); E. Hiyama et al., PRC 80 (2009) 054321 (cluster model)

CSB: $A = 7$ results are comparable with experiment; $A = 8$ too large

Λd scattering

Λd scattering experiments are practically impossible
however, one can study the Λd system as final-state interaction:

- Heavy ion collisions
  Λd correlations measured in Ni+Ni collisions
  FOPI Collaboration (Norbert Herrmann, 2012)

- $K^- A \to A' \Lambda d$
  Λd invariant mass spectrum
  FINUDA Collaboration, 2007

  $K^- {}^4\mathrm{He} \to n \Lambda d$:
  KEK-PS E549 Collaboration, 2007
  AMADEUS Collaboration (C. Curceanu, O. Vazquez Doce, 2012-14)

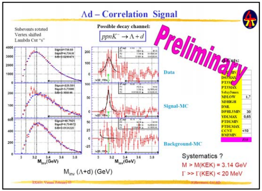
- $pd \to K^+ \Lambda d$
  Λd invariant mass spectrum
  COSY, Jülich, 2012 – but not yet analyzed

- Λd two-particle momentum correlations in pp collisions
  ALICE Collaboration

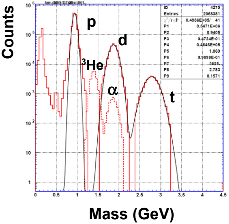
Λd – correlations

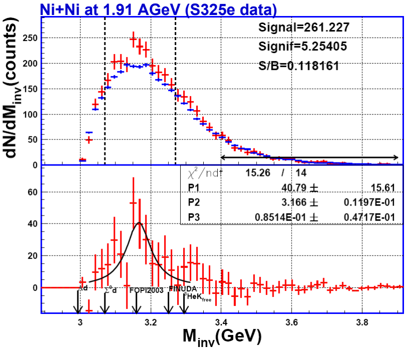
K. Wisniewski

N. Herrmann, EXA2005



Improvement (2003→2008): PID





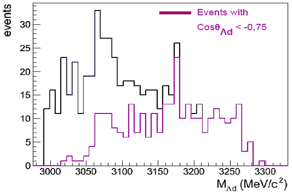
FOPI 2003 and 2008 data are consistent,
Inconsistent with cusp (Σ – d – threshold) and FINUDA.

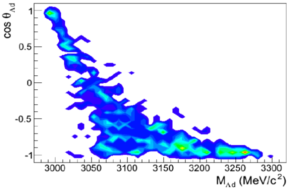
K^{-} ^{4}He (Oton Vazquez Doce (KLOE), 2014)

KLOE data: Λd, Λt events

Λd sample in ^{4}He

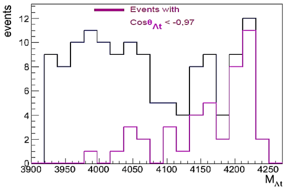
572 events

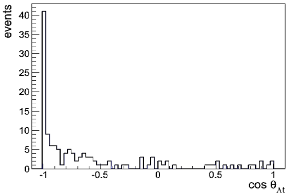




Λt sample in ^{4}He

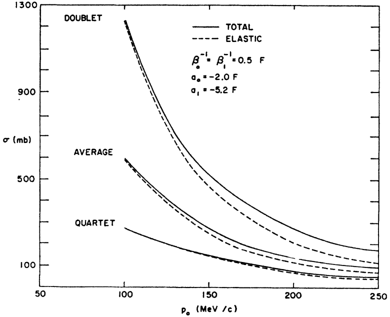
134 events





Information on Λd scattering

Faddeev calculations by Hetherington and Schick, PR 139 (1965) B 1164



$\sigma_{\Lambda d} = \frac{1}{3}\sigma_{1/2} + \frac{2}{3}\sigma_{3/2}$

$\Rightarrow$ doublet Λd S-wave dominates near threshold

hypertriton: $B_E = 2.354 \pm 50$ MeV [Λpn] 0.130 ± 50 MeV [Λd]

$\not\pi$EFT (H.-W. Hammer, NPA 705 (2002) 173) $\Rightarrow$ $a_{1/2} = 16.8^{+4.4}_{-2.4}$ fm, $r_{3/2} = 1.3 \pm 0.3$ fm

however, H & S: $a_s^{\Lambda N} = -2.0$ fm, $a_t^{\Lambda N} = -0.52$ fm

$\Rightarrow$ nowadays: $a_s^{\Lambda N} = -(2.5 \sim 2.9)$ fm, $a_t^{\Lambda N} = -(1.5 \sim 1.7)$ fm

Λd scattering lengths

in the spin-doublet S-wave:

Cobis et al., JPG 23 (1997) 401	H.-W. Hammer, NPA 705 (2002) 173 ($\not\pi$EFT)
$a_{1/2} = 16.3^{+4.0}_{-2.1}$ fm $\quad r_{1/2} = 3.2$ fm	$a_{1/2} = 16.8^{+4.4}_{-2.4}$ fm $\quad r_{1/2} = 2.3 \pm 0.3$ fm

(see also Hildenbrand/Hammer, PRC 100 (2019) 034002 + Erratum: $a_{1/2} = 15.4^{+4.3}_{-2.2}$ fm)

Bethe formula:

$$\frac{1}{a_{1/2}} = \gamma - \frac{1}{2} r_{1/2}\,\gamma^2, \qquad B_\Lambda = \frac{\gamma^2}{2\mu_{\Lambda d}}, \qquad B_\Lambda = 0.13 \pm 0.05 \text{ MeV}$$

in the spin-quartet S-wave:

M. Schäfer et al., PLB 808 (2020) 135614 ($\not\pi$EFT)				
	$a_s^{\Lambda N}$	$a_t^{\Lambda N}$	$a_{3/2}$ (fm)	$r_{3/2}$
Alexander B	-1.80 fm	-1.60 fm	-17.3 fm	3.6 fm
NSC97f	-2.60 fm	-1.71 fm	-10.8 fm	3.8 fm
χEFT (NLO)	-2.91 fm	-1.54 fm	-7.5 fm	3.6 fm

3BF: 3 ΛNN LECs fitted to ${}^3_\Lambda$H, ${}^4_\Lambda$H (0^+, 1^+)

Two-particle correlation function $C(k)$

- Koonin-Pratt formalism

$$C(k) \simeq 1 + \int_0^\infty 4\pi r^2\, dr\, S_{12}(\mathbf{r}) \left[|\psi(k,r)|^2 - |j_0(kr)|^2 \right]$$

(spherical Gaussian source with radius R: $S_{12}(\mathbf{r}) = \exp(-r^2/4R^2)/(2\sqrt{\pi}R)^3$)

- Lednicky-Lyuboshitz model

replace full wave function by its asymptotic form: $\psi(k,r) \approx j_0(kr) + f(k)\frac{\exp(ikr)}{r}$

$$\int_0^\infty 4\pi r^2 dr\, S_{12}(r) \left[|\psi(k,r)|^2 - |j_0(kr)|^2 \right] \approx \frac{|f(k)|^2}{2R^2} F(r_0) + \frac{2\mathrm{Re} f(k)}{\sqrt{\pi}R} F_1(x) - \frac{\mathrm{Im} f(k)}{R} F_2(x)$$

$f(k) = (\mathcal{S}(k) - 1)/2ik$ ... scattering amplitude ($\mathcal{S}$ ... S-matrix)

$\Rightarrow$ replace by effective-range expansion: $f(k) \approx 1/(-\frac{1}{a} + r_0 k^2/2 - ik)$

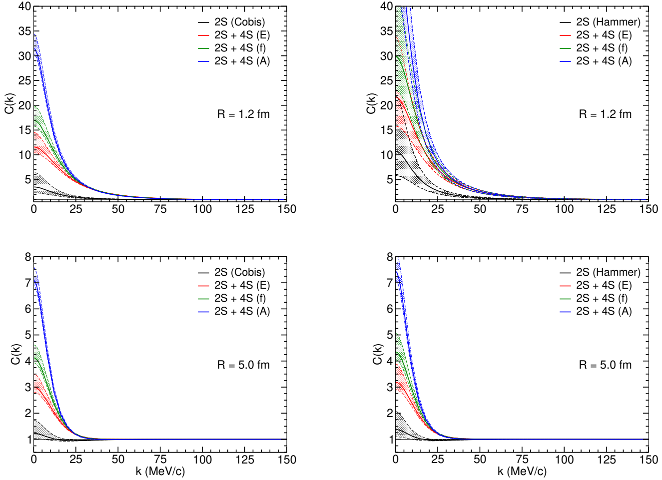
$F(r_0) = 1 - r_0/(2\sqrt{\pi}R)$ ... correction to wave function

$F_1(x) = \int_0^x dt\, e^{t^2-x^2}/x, \quad F_2(x) = (1 - e^{-x^2})/x, \quad x = 2kR$

⋆ valid when the range of the interaction is smaller than the source size

⋆ if $f(k)$ (scattering length a) is large, the first term dominates
$\rightarrow$ result depends strongly on (the corrections to) the wave function

Results with Lednicky-Lyuboshitz formula



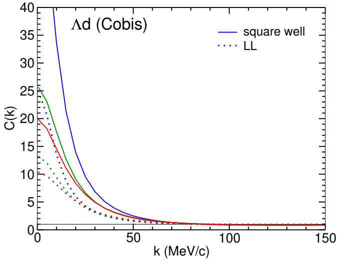
spin-doublet *S*-wave (2S) fixed from A. Cobis (left) or H.-W. Hammer (right)

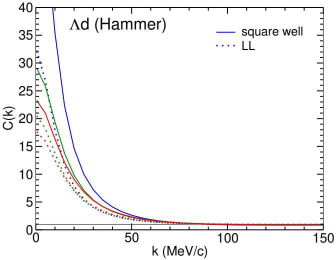
spin-quartet *S*-wave (4S) fixed from $\not{\pi}$EFT results based on χEFT, NSC97f or Alexander ΛN scattering lengths

bands represent the uncertainty in the ${}^{3}_{\Lambda}$H separation energy (Jurič et al., 1973)

(J.H., PRC 102 (2020) 034001)

Square well potential versus Lednicky-Lyuboshitz (LL)





- $R = 1.2$ fm: large difference between LL and wave-function calculation
- $R = 5$ fm (not shown): difference between results from LL and for a square well potential is small

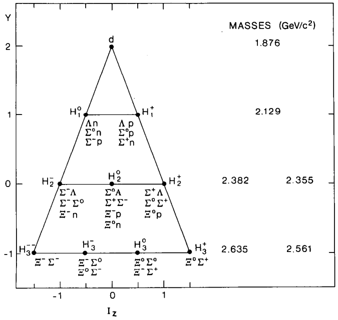
- Λd: strongly attractive (large scattering lengths in both spin channels)

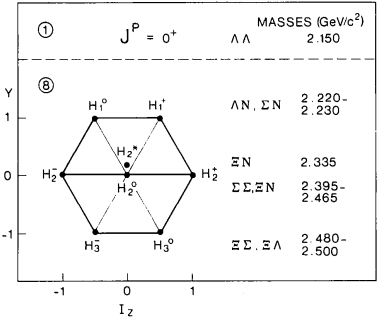
$\rightarrow$ LL formula is unreliable for a quantitative analysis

also for effective Λd potentials further tests are required

investigations based on three-body calculations of Λd are desirable

Strange dibaryons





R.J. Oakes, PR 131 (1963) 2239

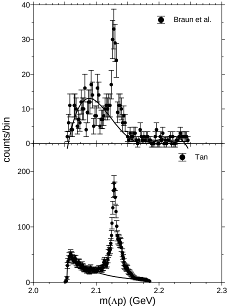
SU(3) flavor symmetry $\{10^*\}$
strange partners of the deuteron

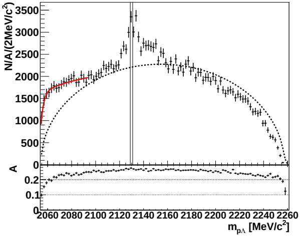
R.L. Jaffe, PRL 38 (1977) 195

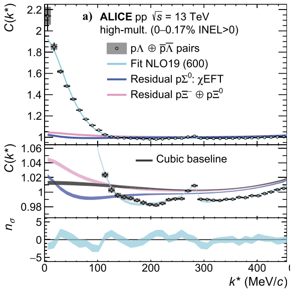
MIT quark bag model

Experimental evidence for threshold structure

$M_{\Sigma^+} + M_n = 2128.97$ MeV $\quad M_{\Sigma^0} + M_p = 2130.87$ MeV







$K^- d \to \pi^- \Lambda p$

T.T. Tan, PRL 7 (1969) 395
O. Braun et al., NPB 124 (1977) 45

$pp \to K^+ \Lambda p$

M. Röder et al., EPJA 49 (2013) 157

Λp corr. in pp coll.

S. Acharya [ALICE Coll.], PLB 833 (2022) 137272

"ordinary" threshold effect? bound state? virtual state ($np\ ^1S_0$) ?

χ^2 for $\Sigma^- p$ and $\Sigma^+ p$ data

ΛN result near ΣN threshold is primarily constrained by (20) near-threshold $\Sigma^- p$ data

reaction	NLO13				NLO19				Jülich '04	NSC97f (ND)
	500	550	600	650	500	550	600	650		
$\Sigma^- p \to \Lambda n$	3.7	3.9	4.1	4.4	4.7	4.7	4.0	4.4	8.3	3.9 (4.3)
$\Sigma^- p \to \Sigma^0 n$	6.1	5.8	5.8	5.7	5.5	5.5	6.0	5.7	6.4	6.0 (5.5)
$\Sigma^- p \to \Sigma^- p$	2.0	1.8	1.9	1.9	3.0	2.9	2.2	1.9	1.6	2.3 (3.6)
$\Sigma^+ p \to \Sigma^+ p$	0.3	0.4	0.5	0.3	0.3	0.4	0.4	0.3	0.1	0.2 (0.1)
r_R	0.1	0.2	0.1	0.2	1.1	0.7	0.1	0.5	53.6	0.0 (0.9)
total χ^2	12.2	12.0	12.3	12.5	14.6	14.2	12.7	12.8	70 [16.4]	12.4 (14.4)

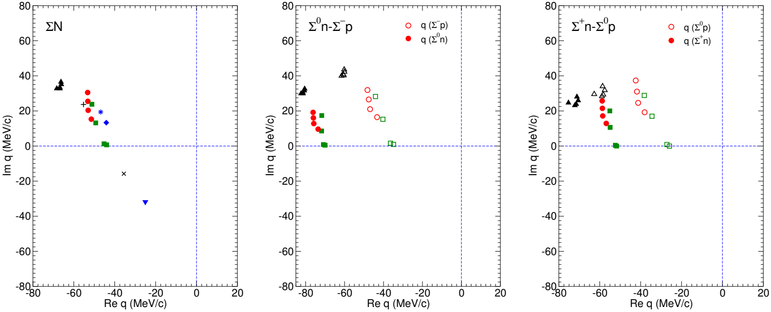
$$\left(r_R = \frac{1}{4}\,\frac{\sigma_s(\Sigma^- p \to \Sigma^0 n)}{\sigma_s(\Sigma^- p \to \Lambda n) + \sigma_s(\Sigma^- p \to \Sigma^0 n)} + \frac{3}{4}\,\frac{\sigma_t(\Sigma^- p \to \Sigma^0 n)}{\sigma_t(\Sigma^- p \to \Lambda n) + \sigma_t(\Sigma^- p \to \Sigma^0 n)} \right)$$

best description of near-threshold ΣN data: NLO13, NLO19 (600,650), NSC97a-f
$\Rightarrow \chi^2 = 12 - 13$

J.H., U.-G. Meißner, Chin. Ph. C 45 (2021) 9 $\Rightarrow$ search for ΣN poles in complex plane

Poles in the complex $q_{\Sigma N}$ plane

- 2nd quadrant (sheet II, bt): unstable bound state
- 3rd quadrant (sheet IV, tb): inelastic virtual state



● NLO13 ■ NLO19 ▲ Nijmegen NSC97b-f ▼ Jülich '04 × Nijmegen ND (1977)

NLO13: E = 2131.90 − i1.39 ··· 2131.25 − i3.01 MeV
NLO19: E = 2131.73 − i1.11 ··· 2131.35 − i0.00 MeV
NSC97: E = 2133.04 − i3.80 ··· 2133.79 − i3.53 MeV

Thresholds: $\Sigma^+ n$ (2128.97) $\Sigma^0 p$ (2130.87)

⇒ bound state (dibaryon) – but above threshold!

Hadronic level shifts in $\Sigma^- p$

Deser-Trueman formula:

$$\Delta E_S + \mathrm{i}\,\frac{\Gamma_S}{2} \quad = \quad -\frac{2}{\mu_{\Sigma p}\, r_B^3}\, a_S^{sc} \left(1 - \frac{a_S^{sc}}{r_B}\beta\right)$$

r_B ... Bohr radius ... 51.4 fm; $\quad \beta = 2(1 - \Psi(1)) \approx 3.1544$
$\mu_{\Sigma p}$ ... $\Sigma^- p$ reduced mass
a^{sc} ... Coulomb-distorted $\Sigma^- p$ scattering length

Carbonell, Richard, Wycech, ZPA 343 (1992) 343 ($\bar{p}p$):
works well once Coulomb and p-n mass difference is taken into account

NOTE:
different sign conventions for scattering lengths in ΣN ($\bar{N}N$) and $\bar{K}N$!
$\Delta E < 0 \Leftrightarrow$ repulsive shift

Hadronic level shifts in $\Sigma^- p$ (in eV)

	NLO13				NLO19				Jülich '04	NSC97f
Λ (MeV)	500	550	600	650	500	550	600	650		
$E_{^1S_0}$	-248	-231	-146	-106	-249	-234	-146	-107	-130	-498
$\Gamma_{^1S_0}$	1401	1391	1357	1317	1471	1455	1381	1309	1788	1809
$E_{^3S_1}$	-1286	-1256	-1211	-1159	-944	-942	-1210	-1141	+884	-825
$\Gamma_{^3S_1}$	2338	2514	2657	2865	3506	3406	2620	2975	4782	2605
E_{1S}	-1026	-1000	-945	-896	-770	-765	-944	-882	+630	-743
Γ_{1S}	2104	2233	2332	2478	2997	2918	2310	2558	4034	2406

antiprotonic atoms: $E_{1S} \approx -720$ eV, $\Gamma_{1S} \approx 1100$ eV

K^- atoms: $E_{1S} \approx -280$ eV, $\Gamma_{1S} \approx 540$ eV

$\Rightarrow$ width Γ noticeably larger for $\Sigma^- p$

threshold of the neutral “partner” channel ($\Sigma^0 n$) is slightly below the one of $\Sigma^- p$

$\bar{p}p$ and $K^- p$: corresponding channels ($\bar{n}n$ and $\bar{K}^0 n$) are slightly above

Summary

Hyperon-nucleon interaction constructed within chiral EFT

- Approach is based on a modified Weinberg power counting, analogous to applications for NN scattering
- The potential (contact terms, pseudoscalar-meson exchanges) is derived imposing $SU(3)_f$ constraints
- $S = -1$ dibaryon: strong evidence for its existence
  – but not as ideal textbook (Breit-Wigner type) resonance

Hypernuclei

- three-body forces: should be small for ($^{3}_{\Lambda}\mathrm{H}$) or moderate ($^{4}_{\Lambda}\mathrm{H}$, $^{4}_{\Lambda}\mathrm{He}$, $^{5}_{\Lambda}\mathrm{He}$)
  needs to be quantified/confirmed by explicit inclusion of 3BFs
- charge-symmetry breaking in $^{4}_{\Lambda}\mathrm{H} - {}^{4}_{\Lambda}\mathrm{He}$
  can be reproduced when taking into account the full leading CSB potential within chiral EFT
- charge-symmetry breaking in $A = 7 - 8$ Λ-hypernuclei
  predicted CSB splitting for $^{7}_{\Lambda}$Be, $^{7}_{\Lambda}$Li*, $^{7}_{\Lambda}$He is in line with experiments
  CSB splitting for $^{8}_{\Lambda}$Be, $^{8}_{\Lambda}$Li is overestimated

Λd momentum correlation function

- Could provide more insight into the spin dependence of the ΛN interaction
- however, elaborate (Faddeev-type) calculations might be needed

Backup slides

Contact terms for YN – partial-wave projected

spin-momentum structure up to NLO

$$
\begin{aligned}
V(^1S_0) &= \tilde{C}_{^1S_0} + C_{^1S_0}(p^2 + p'^2) \\
V(^3S_1) &= \tilde{C}_{^3S_1} + C_{^3S_1}(p^2 + p'^2) \\
V(\alpha) &= C_\alpha \, p \, p' \qquad \alpha \triangleq {}^1P_1,\ {}^3P_0,\ {}^3P_1,\ {}^3P_2 \\
V(^3D_1 - {}^3S_1) &= C_{^3S_1 - {}^3D_1} \, p'^2
\end{aligned}
$$

$$
\begin{aligned}
V(^1P_1 - {}^3P_1) &= C_{^1P_1 - {}^3P_1} \, p \, p' \\
V(^3P_1 - {}^1P_1) &= C_{^3P_1 - {}^1P_1} \, p \, p'
\end{aligned}
$$

(antisymmetric spin-orbit force: $(\vec{\sigma}_1 - \vec{\sigma}_2) \cdot (\vec{q} \times \vec{k})$)

- $\tilde{C}_\alpha$, C_α ... low-energy constants (LECs)
- need to be fixed by a fit to (NN, YN, ...) data

$SU(3)$ structure for scattering of two octet baryons $\rightarrow$

$8 \otimes 8 = 1 \oplus 8_a \oplus 8_s \oplus 10^* \oplus 10 \oplus 27$

BB interaction can be given in terms of LECs corresponding to the $SU(3)_f$ irreducible representations: C^1, C^{8a}, C^{8s}, C^{10^*}, C^{10}, C^{27}

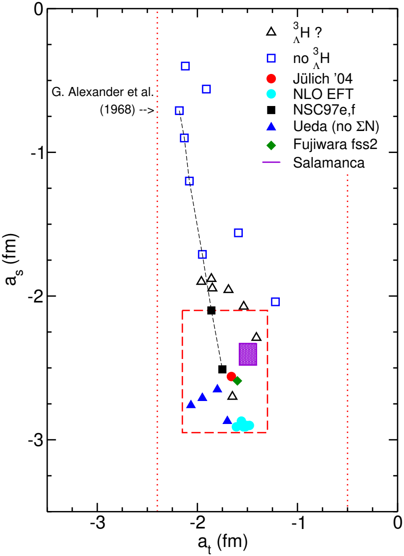
	Channel	I	V_α	V_β	$V_{\beta\rightarrow\alpha}$
$S = 0$	$NN \rightarrow NN$	0	–	$C^{10^*}_\beta$	–
	$NN \rightarrow NN$	1	C^{27}_α	–	–
$S = -1$	$\Lambda N \rightarrow \Lambda N$	$\frac{1}{2}$	$\frac{1}{10}\left(9C^{27}_\alpha + C^{8s}_\alpha\right)$	$\frac{1}{2}\left(C^{8a}_\beta + C^{10^*}_\beta\right)$	$-C^{8sa}$
	$\Lambda N \rightarrow \Sigma N$	$\frac{1}{2}$	$\frac{3}{10}\left(-C^{27}_\alpha + C^{8s}_\alpha\right)$	$\frac{1}{2}\left(-C^{8a}_\beta + C^{10^*}_\beta\right)$	$-3C^{8sa}$
					C^{8sa}
	$\Sigma N \rightarrow \Sigma N$	$\frac{1}{2}$	$\frac{1}{10}\left(C^{27}_\alpha + 9C^{8s}_\alpha\right)$	$\frac{1}{2}\left(C^{8a}_\beta + C^{10^*}_\beta\right)$	$3C^{8sa}$
	$\Sigma N \rightarrow \Sigma N$	$\frac{3}{2}$	C^{27}_α	C^{10}_β	–

$\alpha = {}^1S_0, {}^3P_0, {}^3P_1, {}^3P_2$, $\beta = {}^3S_1, {}^3S_1 - {}^3D_1, {}^1P_1$

No. of contact terms: LO: 2 (NN) + 3 (YN) + 1 (YY)
NLO: 7 (NN) + 11 (YN) + 4 (YY)

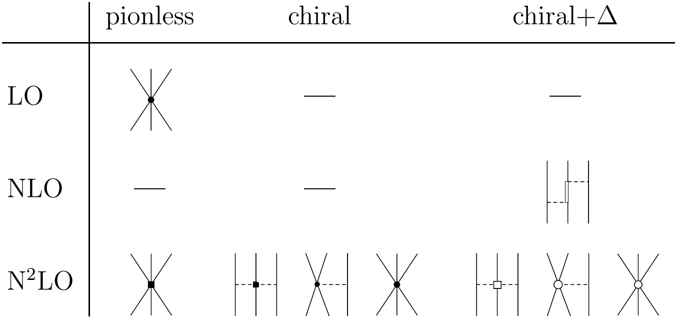
(No. of spin-isospin channels in NN+YN: 10 $S = -2, -3, -4$: 27)

ΛN scattering lengths versus Hypertriton ($^3_\Lambda$H)



G. Alexander et al., PR 173 (1968) 1452: $a_s = -1.8^{+2.3}_{-4.2}$ fm, $a_t = -1.6^{+1.1}_{-0.8}$ fm

3-body forces strongly scheme dependent!



different degrees of freedom in the effective field theory

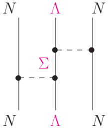
- different counting schemes
- different hierarchy of 3BFs

(Hammer, Nogga, Schwenk, Rev. Mod. Phys. 85 (2013) 197)

Estimation of 3BFs based on NLO results

- ${}^{3}_{\Lambda}$H
  (a) cutoff variation: ΔE_Λ (3BF) $\leq$ 50 keV
  (b) "3BF" from ΛN-ΣN coupling:

  switch off ΛN-ΣN coupling
  in Faddeev-Yakubovsky equations:
  ΔE_Λ (3BF) $\approx$ 10 keV
  expect similar ΔE_Λ from $\Sigma^*(1385)$ excitation



  (c) ^{3}H: 3NF $\sim Q^3\,|\langle V_{NN}\rangle|_{^3\mathrm{H}} \sim$ 650 keV
  ($|\langle V_{NN}\rangle|_{^3\mathrm{H}} \sim$ 50 MeV; $Q \sim m_\pi/\Lambda_b$; $\Lambda_b \simeq$ 600 MeV)
  ${}^{3}_{\Lambda}$H: $|\langle V_{\Lambda N}\rangle|_{^3_\Lambda\mathrm{H}} \sim$ 3 MeV $\rightarrow \Delta E_\Lambda$ (3BF) $\approx Q^3\,|\langle V_{\Lambda N}\rangle|_{^3_\Lambda\mathrm{H}} \simeq$ 40 keV

  Note: root-mean-square radius of ${}^{3}_{\Lambda}$H: $\sqrt{\langle r^2\rangle} \approx$ 5 fm
  (deuteron: $\sqrt{\langle r^2\rangle} \approx$ 2 fm)
  $\Rightarrow$ most of the time Λ and two Ns are outside of the range of a standard 3BF!

- ${}^{4}_{\Lambda}$H, ${}^{4}_{\Lambda}$He
  (a) cutoff variation: ΔE_Λ (3BF) $\approx$ 200 keV (0^+) and $\approx$ 300 keV (1^+)
  (b) "3BF" from ΛN-ΣN coupling:
  ΔE_Λ (3BF) $\approx$ 230 – 340 keV (0^+), $\approx$ 150 – 180 keV (1^+)

${}^{3}_{\Lambda}$H and ${}^{4}_{\Lambda}$H(He) calculations with explicit inclusion of 3BFs are planned for the future

Hypernuclei studies based on chiral EFT potentials

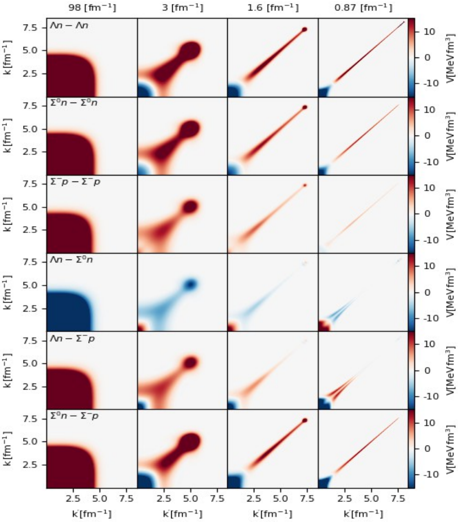
Goal: perform few- and many-body calculations that take into account the full complexity of the underlying YN interaction (tensor coupling, ΛN-ΣN coupling, ...) in a consistent framework

- Faddeev-Yakubovsky calculations:
  feasible only up to $A = 4$: ${}^{3}_{\Lambda}\mathrm{H}$, ${}^{4}_{\Lambda}\mathrm{H}$ (0^+), ${}^{4}_{\Lambda}\mathrm{H}$ (1^+)
  enough hypernuclei to fix 3BF LEC up to NLO (decuplet saturation)
  not enough hypernuclei to fix 3BF LECs up to N^{2}LO
  so far no (explicit) 3BFs included
  (Andreas Nogga, Jülich)
- No-core shell model (NCSM)
  calculations for LO interaction
  hypernuclei up to ${}^{13}_{\Lambda}\mathrm{C}$ have been considered
  (Wirth & Roth, PRL 117 (2016) 182501, PRC 100 (2019) 044313)
  so far no (explicit) 3BFs included

  calculations for NLO interaction
  hypernuclei up to ${}^{7}_{\Lambda}\mathrm{Li}$ have been considered
  (Hoai Le, PhD thesis, Jülich 2020)
  so far no 3BFs included
- ... other potentials, other groups, other methods

SRG applied to the YN interaction

Hoai Le, PhD thesis, University of Bonn 2020



$^1S_0$, NLO19 ($\Lambda = 650$ MeV)

CSB in A=7 iso-triplet ${}^7_\Lambda$He, ${}^7_\Lambda$Li*, ${}^7_\Lambda$Be

	NLO19(500)	NLO13(500)	Exp.[2] emulsion	Exp.[2] counter
${}^7_\Lambda$Be	5.54 ± 0.22	4.30 ± 0.47	5.16 ± 0.08	**?**
${}^7_\Lambda$Li*	$\mathbf{5.64 \pm 0.28}$	4.42 ± 0.58	5.26 ± 0.03	$\mathbf{5.53 \pm 0.13}$
${}^7_\Lambda$He	$\mathbf{5.64 \pm 0.27}$	4.39 ± 0.54		$\mathbf{5.55 \pm 0.1}$

NN:SMS N^{4}LO+(450)

+3N: N^{2}LO(450)

+SRG-induced YNN

Separation energies in A=7 isotriplet

	YN	ΔT	ΔNN	ΔYN $^1S_0$	ΔYN $^3S_1$	ΔYN total	ΔE_Λ^{pert}
(${}^7_\Lambda$Be, ${}^7_\Lambda$Li*)	NLO13	6.8	-24	-1.0	0	0	-17.2(30)
	CSB1	**7.8**	**-24**	**-49.3**	**25.5**	**-24**	**-40.2(30)**
	NLO19	5.8	-40	-0.6	0	0	-34.2(30)
	CSB1	**5.8**	**-41**	**-43.1**	**42.1**	**-0.3**	**-35.2(30)**
	Gal[1]	3	-70			50	-17
	Exp[2]						-100 ± 90

[1] **A. Gal PLB 744 (2015)**

[2] **E. Botta et al., NPA 960 (2017)**

- NLO19 predicts rather well separation energies of the A=7 isotriplet
- CSB: NLO13 & NLO19 results are comparable with experiment

(Hoai Le, J.H., U.-G. Meißner, A. Nogga, arXiv:2210.03387)

CSB in A=8 iso-doublet ${}^{8}_{\Lambda}$Li, ${}^{8}_{\Lambda}$Be

	λ_{YN}	${}^{8}_{\Lambda}$Be	${}^{8}_{\Lambda}$Li
NLO13	0.765	5.56 ± 0.25	5.57 ± 0.30
NLO19	0.823	7.15 ± 0.10	7.17 ± 0.10
Hiyama et al.		6.72	6.80
Exp. emulsion		6.84 ± 0.05	6.80 ± 0.03
Exp. **counter**		**?**	**?**

Separation energies in A=8 doublet, computed at λ that reproduces $B_\Lambda({}^{5}_{\Lambda}\mathrm{He})$

YN	ΔT	ΔNN	ΔYN ${}^{1}S_0$	ΔYN ${}^{3}S_1$	ΔYN total	ΔE_Λ^{pert}
NLO13	12.2	8	-2.1	0	-4.0	16.2(50)
CSB1	**11.9**	**7**	**99.8**	**55.5**	**158.8**	**177.7(50)**
NLO19	6.6	-11	-0.9	0	-1.9	-6.3(50)
CSB1	**6.3**	**-11**	**62**	**79.1**	**147.3**	**142.6(50)**
Hiyama[1]						160
Gal[2]	11	-81			119	49
Exp[3]						40 ± 60

NN:SMS N^{4}LO+(450)

+3N: N^{2}LO(450)

+SRG-induced YNN

[1] **E. Hiyama et al., PRC 80 (2009)**

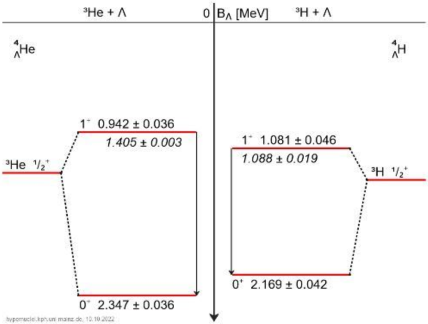
[2] **A. Gal PLB 744 (2015)**

[3] **E. Botta et al., NPA 960 (2017)**

- NLO13 underestimates and NLO19 overestimates separation energies
- CSB: NLO13 & NLO19 results are too large compared to experiment

Charge symmetry breaking - Mainz 2022

https://hypernuclei.kph.uni-mainz.de/



- $\Delta B(0^+) = B_\Lambda^{0^+}({}^4_\Lambda\mathrm{He}) - B_\Lambda^{0^+}({}^4_\Lambda\mathrm{H})$
  $= 178 \pm 55$ keV
- $\Delta B(1^+) = B_\Lambda^{1^+}({}^4_\Lambda\mathrm{He}) - B_\Lambda^{1^+}({}^4_\Lambda\mathrm{H})$
  $= -139 \pm 58$ keV

STAR Collaboration, PLB 834 (2022) 137449:
$\Delta B(0^+) = 160 \pm 140$ keV, $\Delta B(1^+) = -160 \pm 140$ keV