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Application of chiral baryon-baryon interactions to light hypernuclei

Andreas Nogga, Forschungszentrum Jülich

Workshop on Strange Hadrons as Precision Tool for Strongly Interacting Systems, ECT*, Trento, May 13-17, 2024

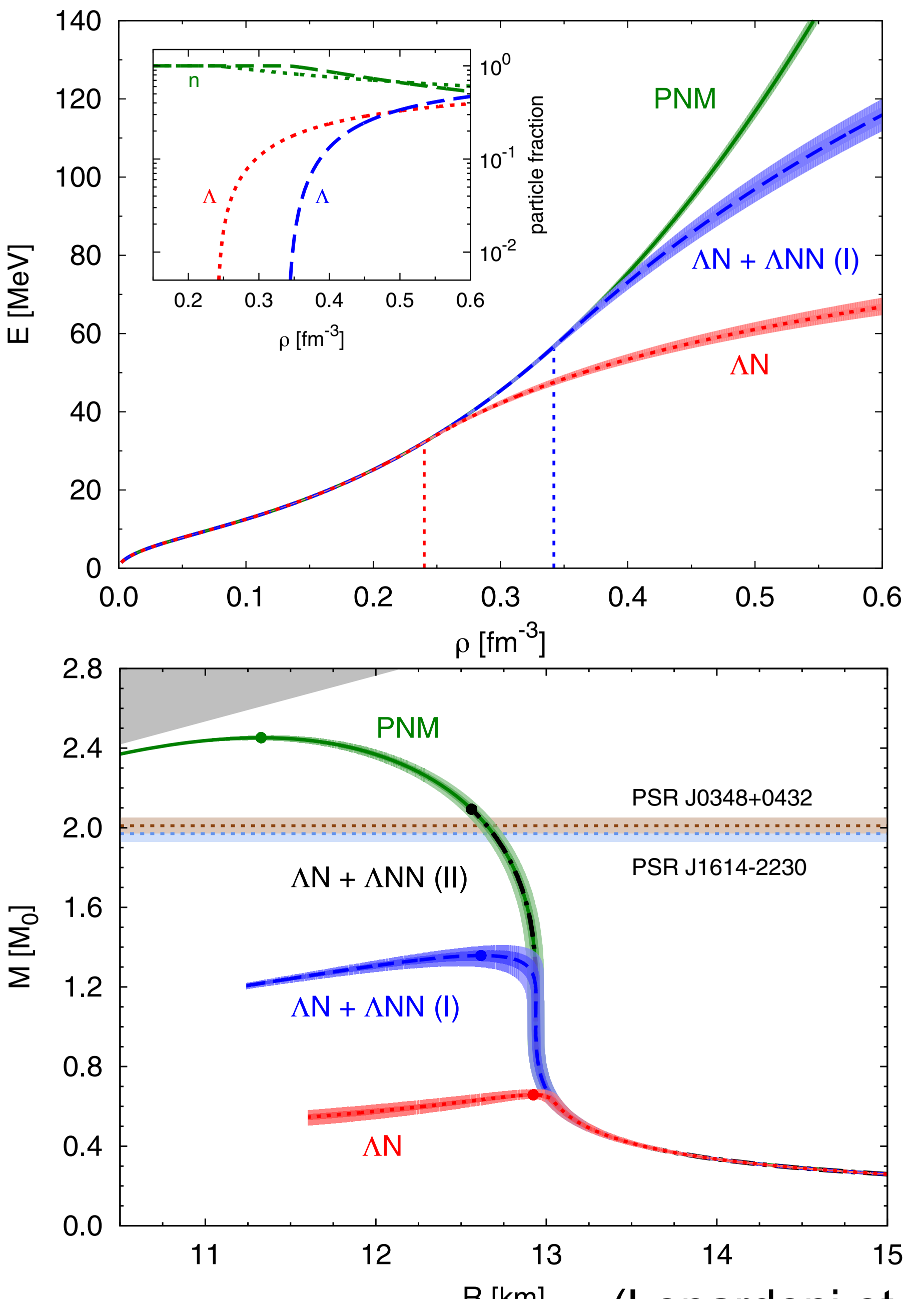
- Motivation
- YN and YY interactions
- SRG evolution of (hyper-)nuclear interactions
- Determination of CSB contact interactions and Λn scattering length
- Application to $A = 7$ and 8 hypernuclei
- Uncertainty of Λ separation energies and size of chiral 3BF contributions
- Chiral YNN interactions
- Conclusions & Outlook

in collaboration with Johann Haidenbauer, **Hoai Le**, Ulf Meißner

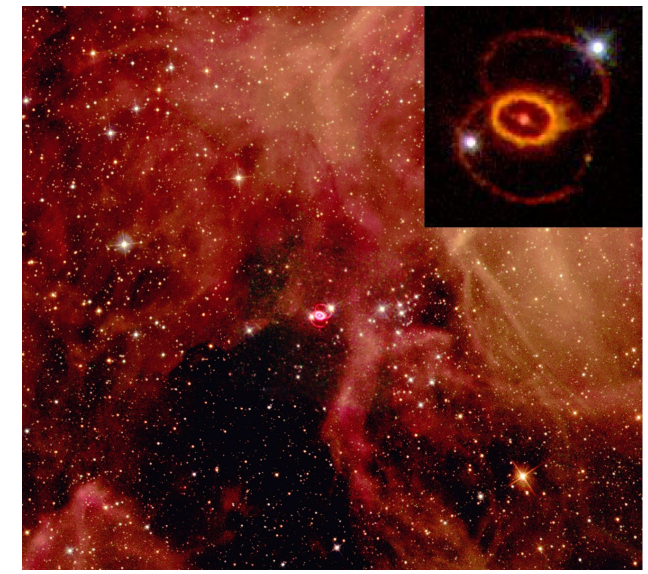
Hypernuclear interactions

Why is understanding hypernuclear interactions interesting?

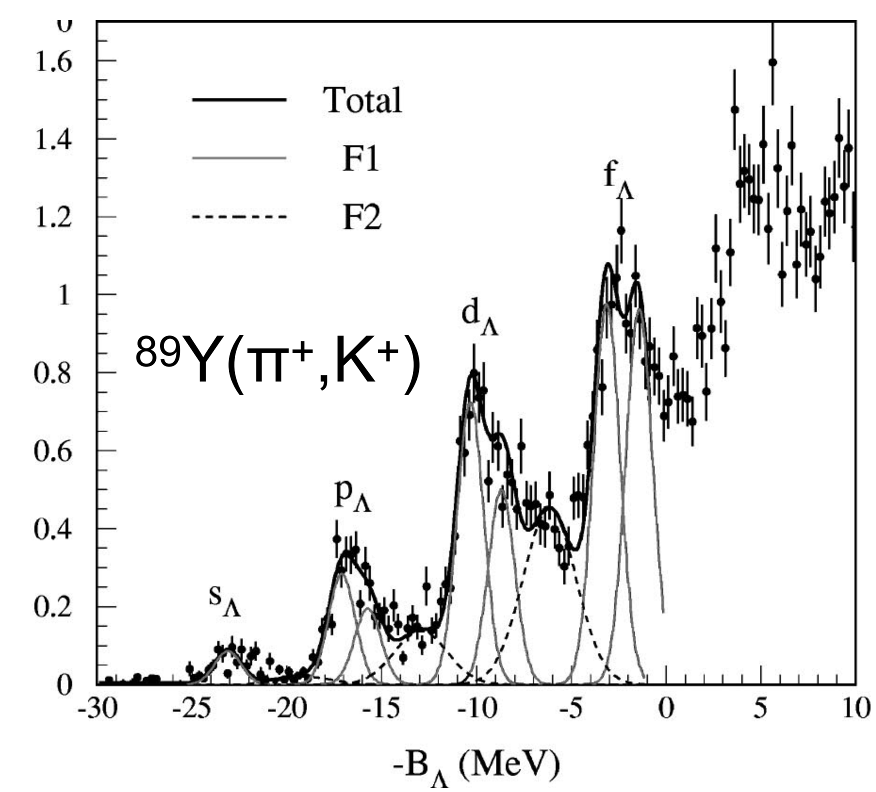
- *hyperon contribution to the EOS, neutron stars, supernovae*
- *"hyperon puzzle"*
- *Λ as probe to nuclear structure*
- ***flavor dependence of baryon-baryon interactions***



(Lonardoni et al. (2015))



(SN1987a, Wikipedia)



(Hotchi et al. (2001))

Hypernuclei

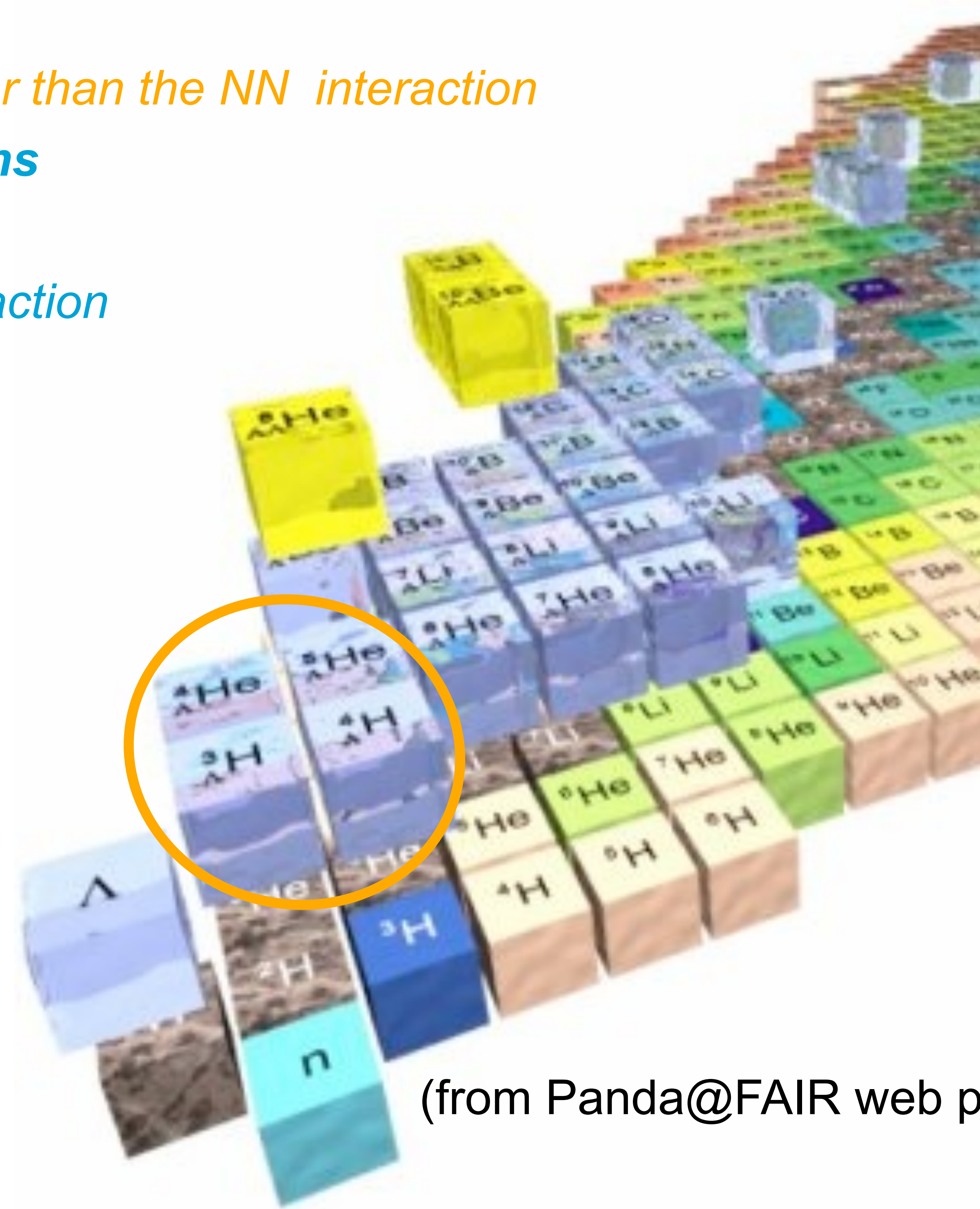
Only few YN data. Hypernuclear data provides additional constraints.

- *ΛN interactions are generally weaker than the NN interaction*
  - *naively:* ***core nucleus + hyperons***
  - *„separation energies“ are* ***quite*** *independent from NN(+3N) interaction*

- *no Pauli blocking of Λ in nuclei*
  - *good to study nuclear structure*
  - *even light hypernuclei exist in* ***several spin states***

- ***non-trivial constraints*** *on the YN interaction even from lightest ones*

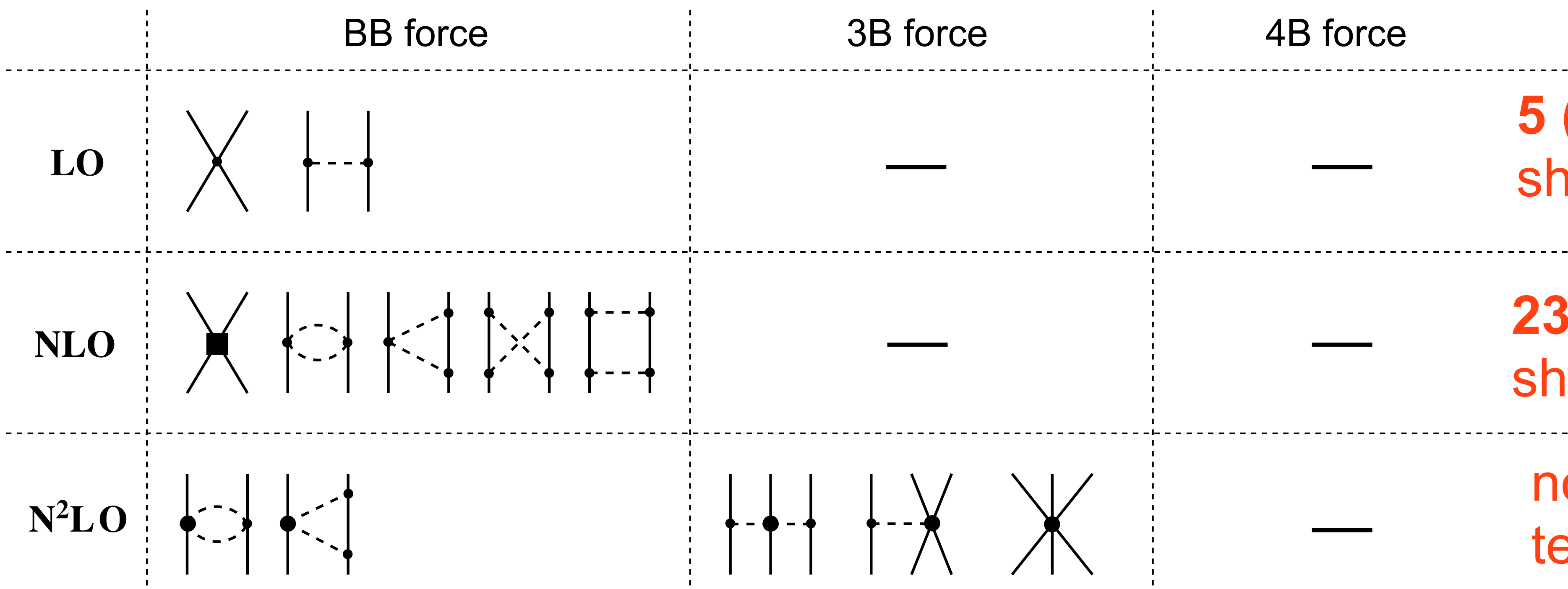
- *size of* ***YNN*** *interactions? need to include* ***Λ-Σ conversion****!*



(from Panda@FAIR web page)

Chiral NN & YN & YY interactions

EFT based approaches



5 (+1) NN/YN (YY) short range parameters

23(+5) NN/YN (YY) short range parameters

no additional contact terms in NN/YN (YY)

Chiral EFT implements **chiral symmetry of QCD** (adapted from Epelbaum, 2008)

- symmetries constrain exchanges of Goldstone bosons
- relations of two- and three- and more-baryon interactions
- breakdown scale $\approx 600-700\,\mathrm{MeV}$
- Semi-local momentum regularization (SMS) up to N²LO (for YN, YY within NRW Fair)

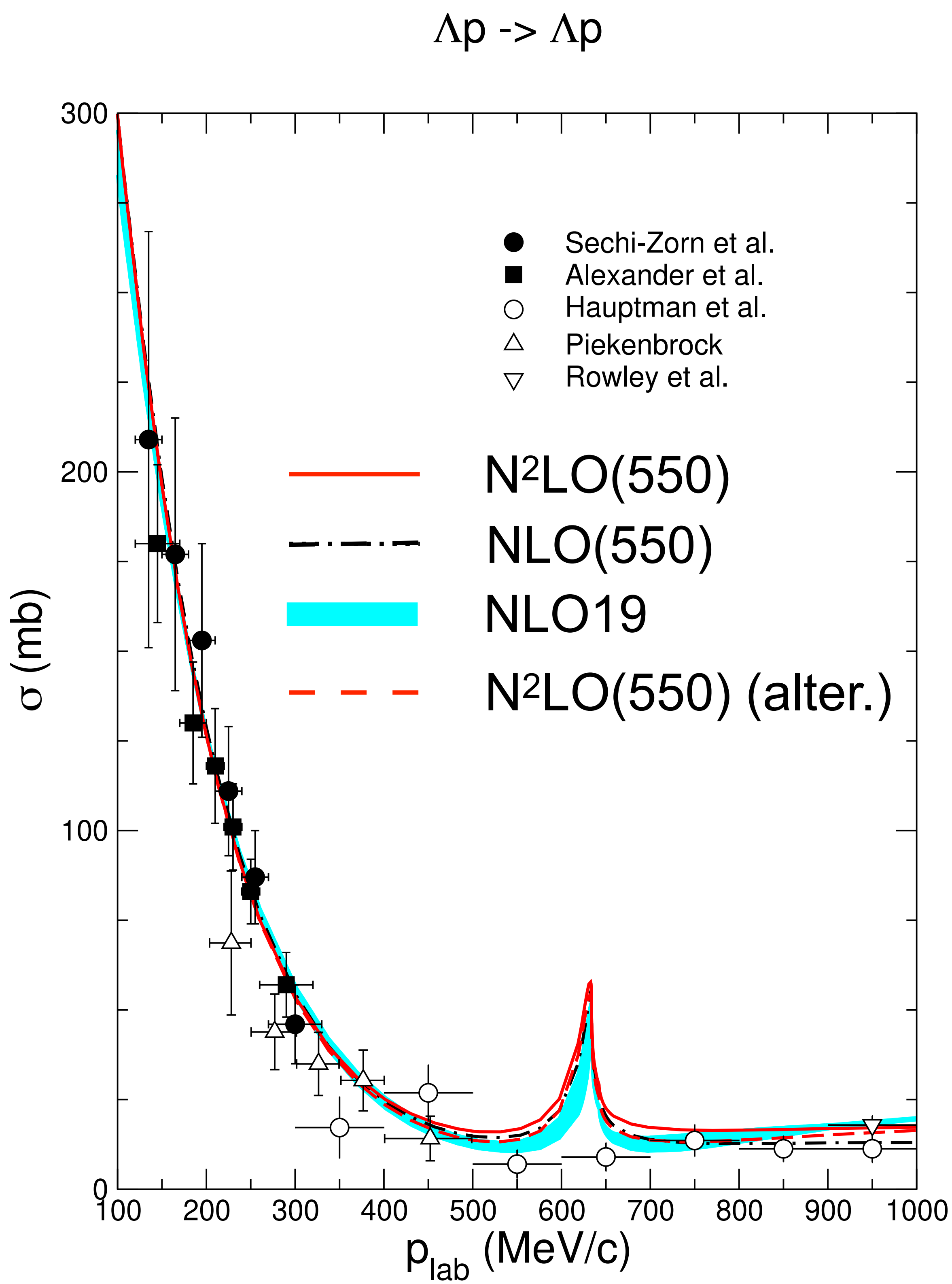
Retain flexibility to adjust to data due to counter terms

Regulator required — cutoff/different orders often used to estimate uncertainty

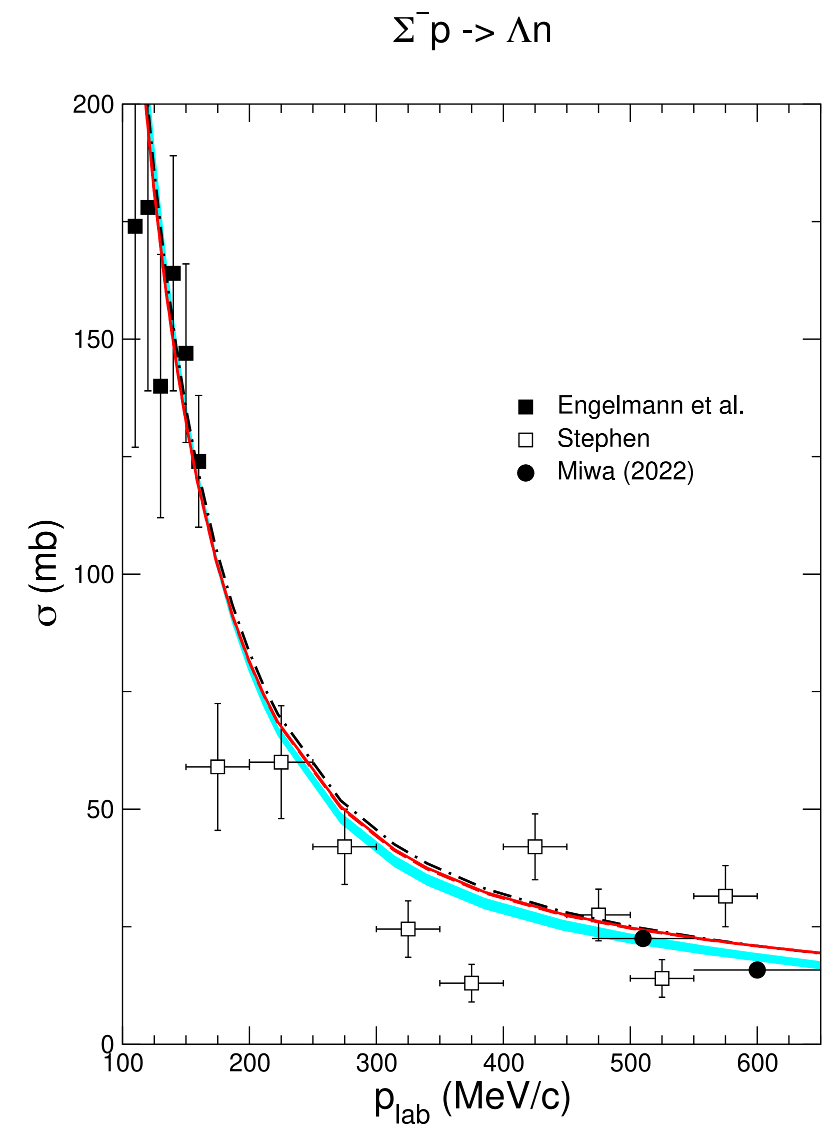
$\Lambda-\Sigma$ and $\Lambda\Lambda-\Sigma\Sigma-\Xi N$ **conversion** is explicitly included (3BFs only in N²LO)

SMS NLO/N²LO interaction

Selected results (show $\Lambda = 550\,\mathrm{MeV}$, others are very similar in quality)



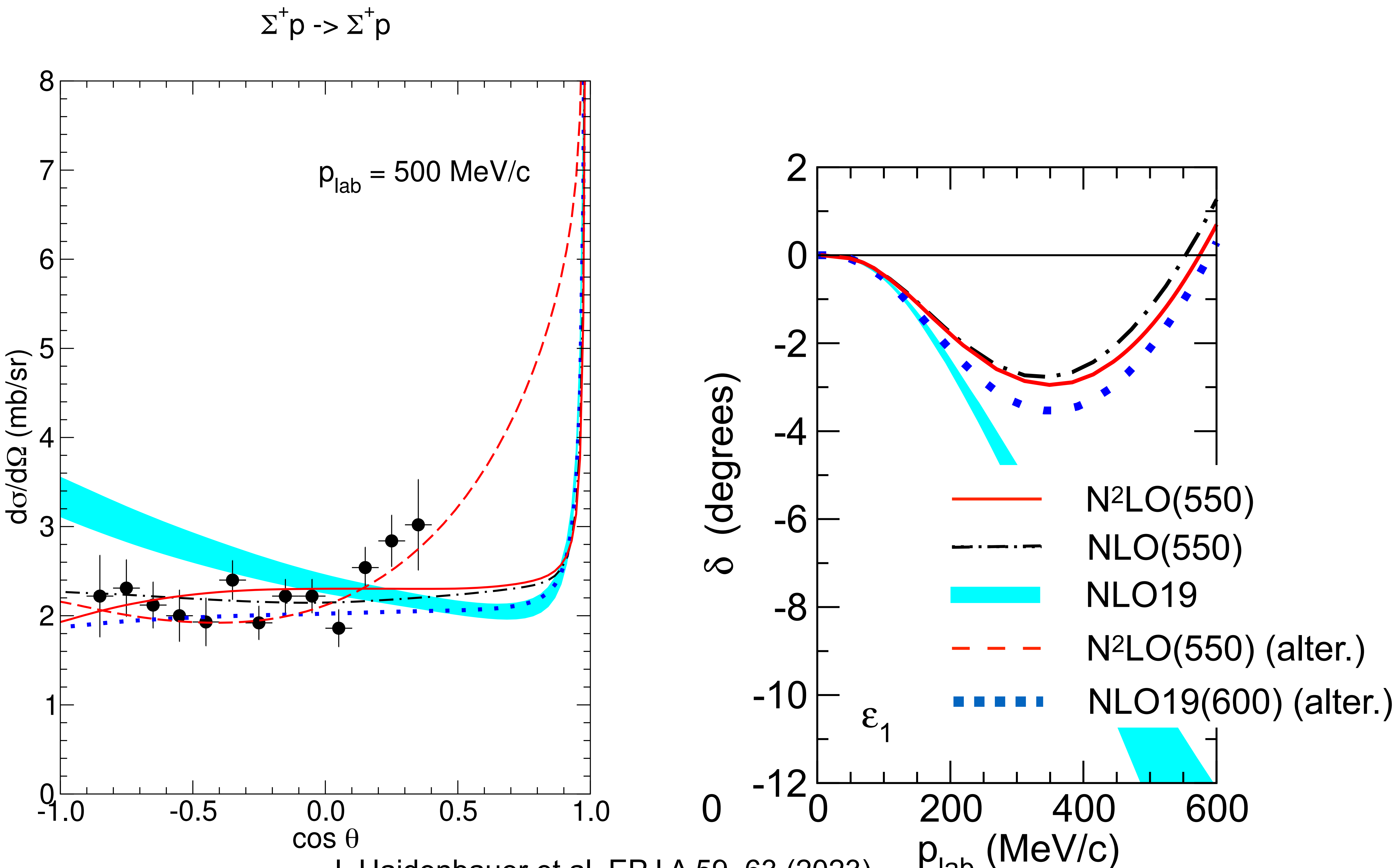
- most relevant cross sections very similar in NLO and N²LO
- similar to NLO19
- alternative fit (see later)



J. Haidenbauer et al. EPJ A 59, 63 (2023).

SMS NLO/N²LO interaction

new data (Miwa(2022)) at higher energies provides **new constraints**!



J. Haidenbauer et al. EPJ A 59, 63 (2023).

SRG interactions

Similarity renormalization group is by now a **standard tool** to obtain soft effective interactions for various many-body approaches (NCSM, coupled-cluster, MBPT, …)

Idea: perform a unitary transformation of the NN (and YN interaction) using a cleverly defined "generator"

$$\frac{dH_s}{ds} = [\, \underbrace{[T\,,\, H(s)]}_{\equiv \eta(s)},\, H(s)\,] \qquad H(s) = T + V(s)$$

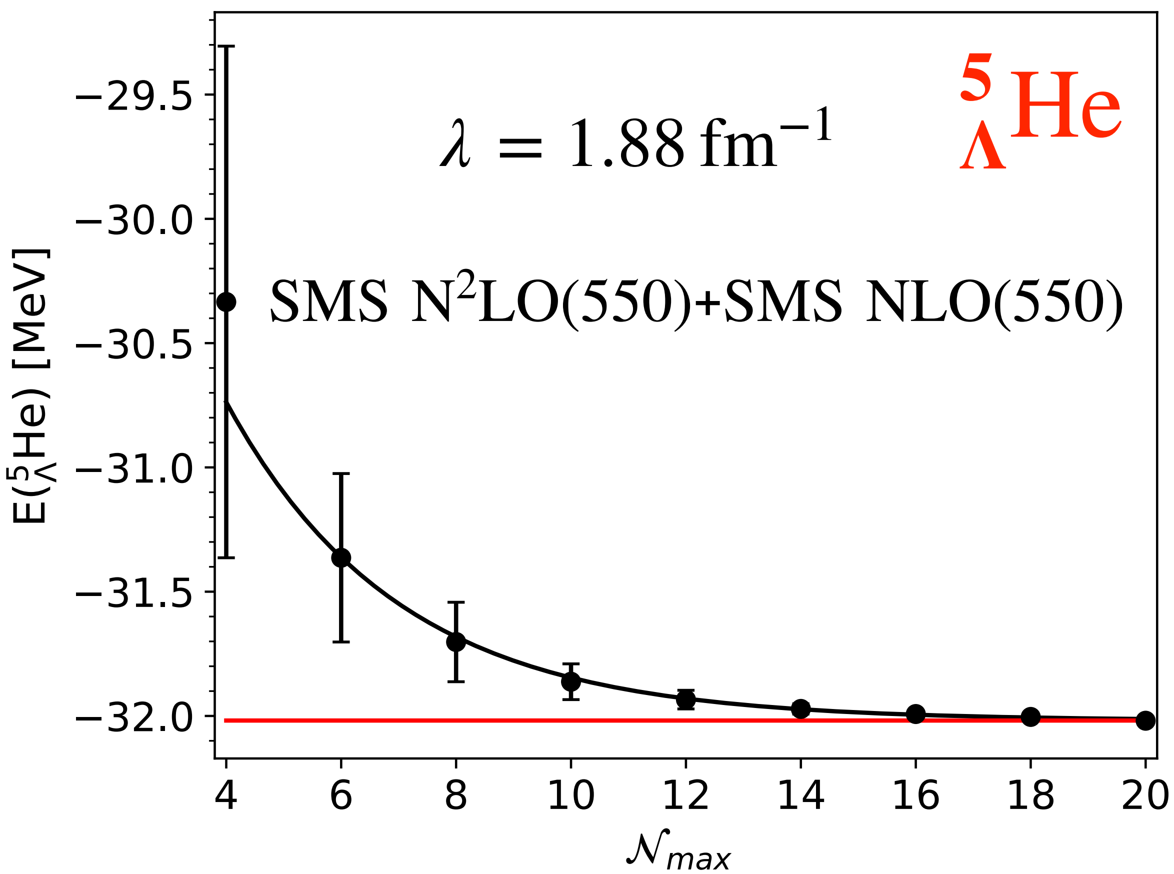
this choice of generator drives *V(s)* into a diagonal form in momentum space

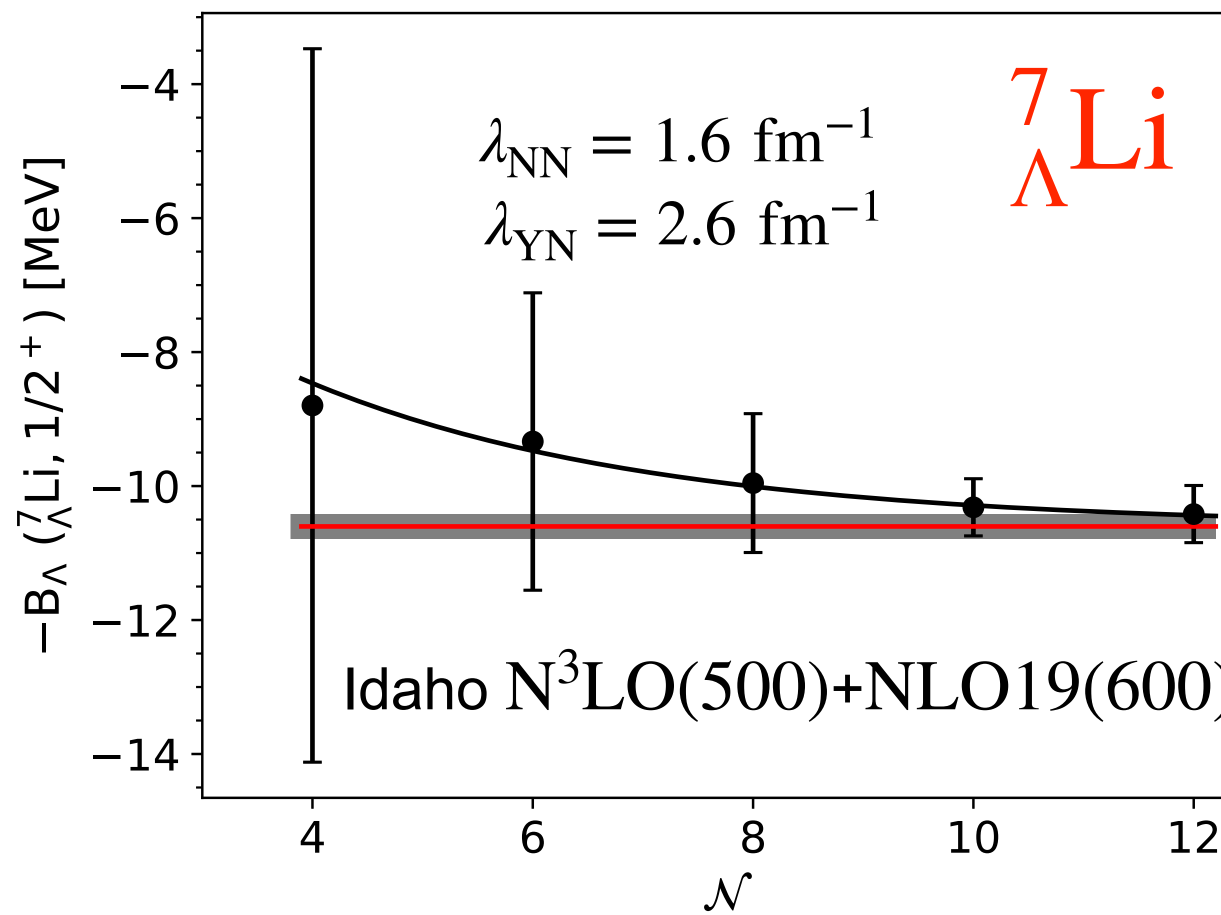
- *V(s)* will be **phase equivalent** to original interaction
- short range *V(s)* will change towards **softer interactions**
- Evolution can be restricted to **2-,3-, … body level** (approximation)
- $\lambda = \left(\frac{4\mu_{BN}^2}{s}\right)^{1/4}$ is a measure of the width of the interaction in momentum space
- **dependence** of results on λ or s is a measure for **missing terms**

(Bogner et al., 2007)

J-NCSM convergence

SRG evolution improves convergence





$$E\left({}^{5}_{\Lambda}\mathrm{He}\right) = -32.018 \pm 0.001\ \text{MeV}$$

$$E_\Lambda\left({}^{7}_{\Lambda}\mathrm{Li}\right) = 10.6 \pm 0.2\ \text{MeV}$$

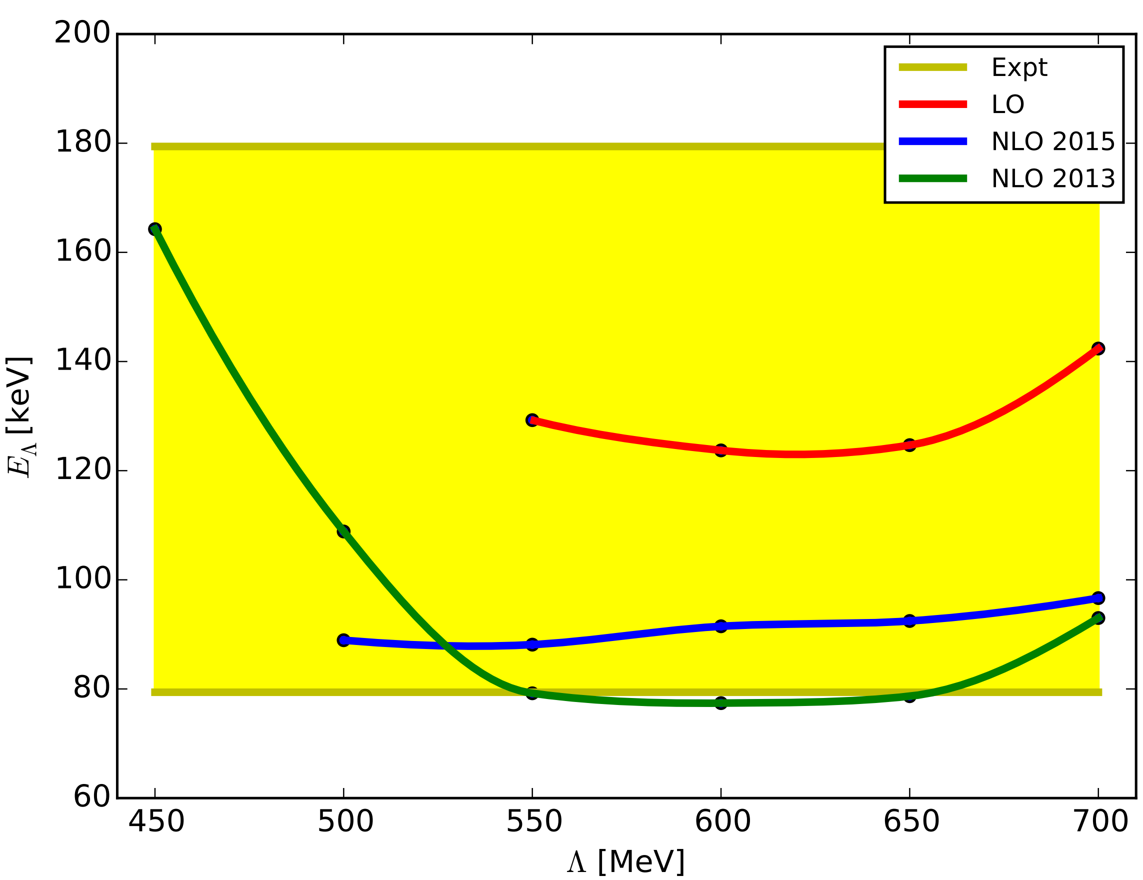
- for light nuclei and hypernuclei, the numerical uncertainty is negligible.
- for p-shell nuclei/hypernuclei, the uncertainty is visible
- extrapolation of separation energy can reduce uncertainty of this quantity

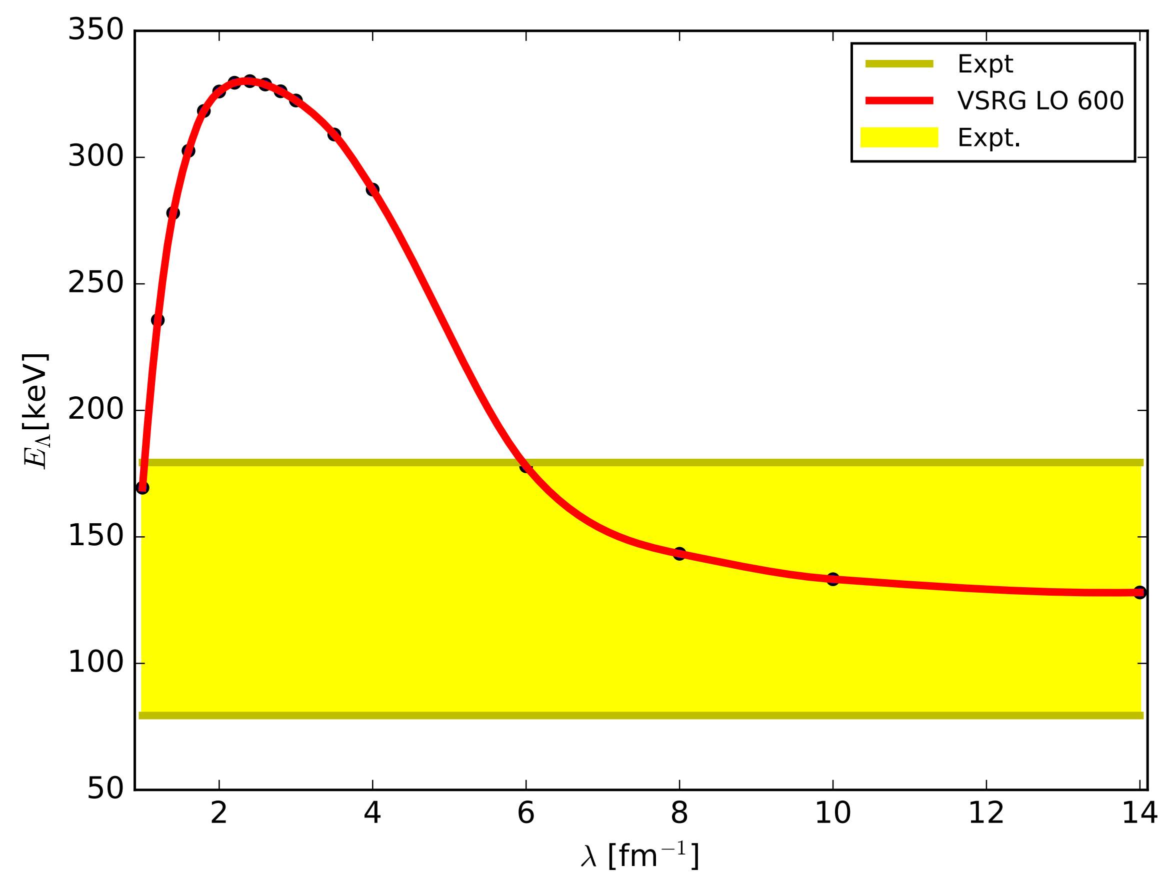
Induced 3BF …

SRG parameter dependence is significant when NN and YN interactions are evolved

→ missing 3N and YNN interactions

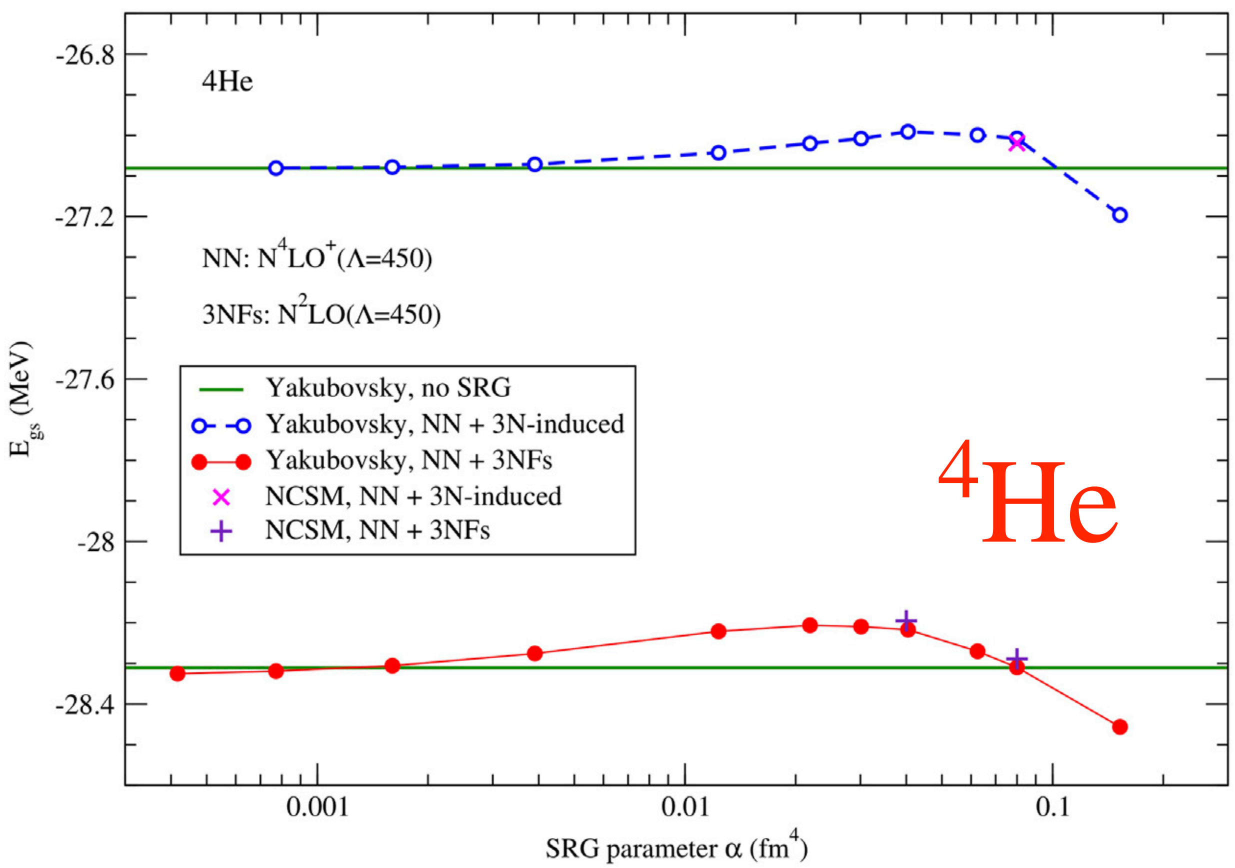
- 3NF is comparable to chiral 3NF
- YNN is larger than chiral YNN



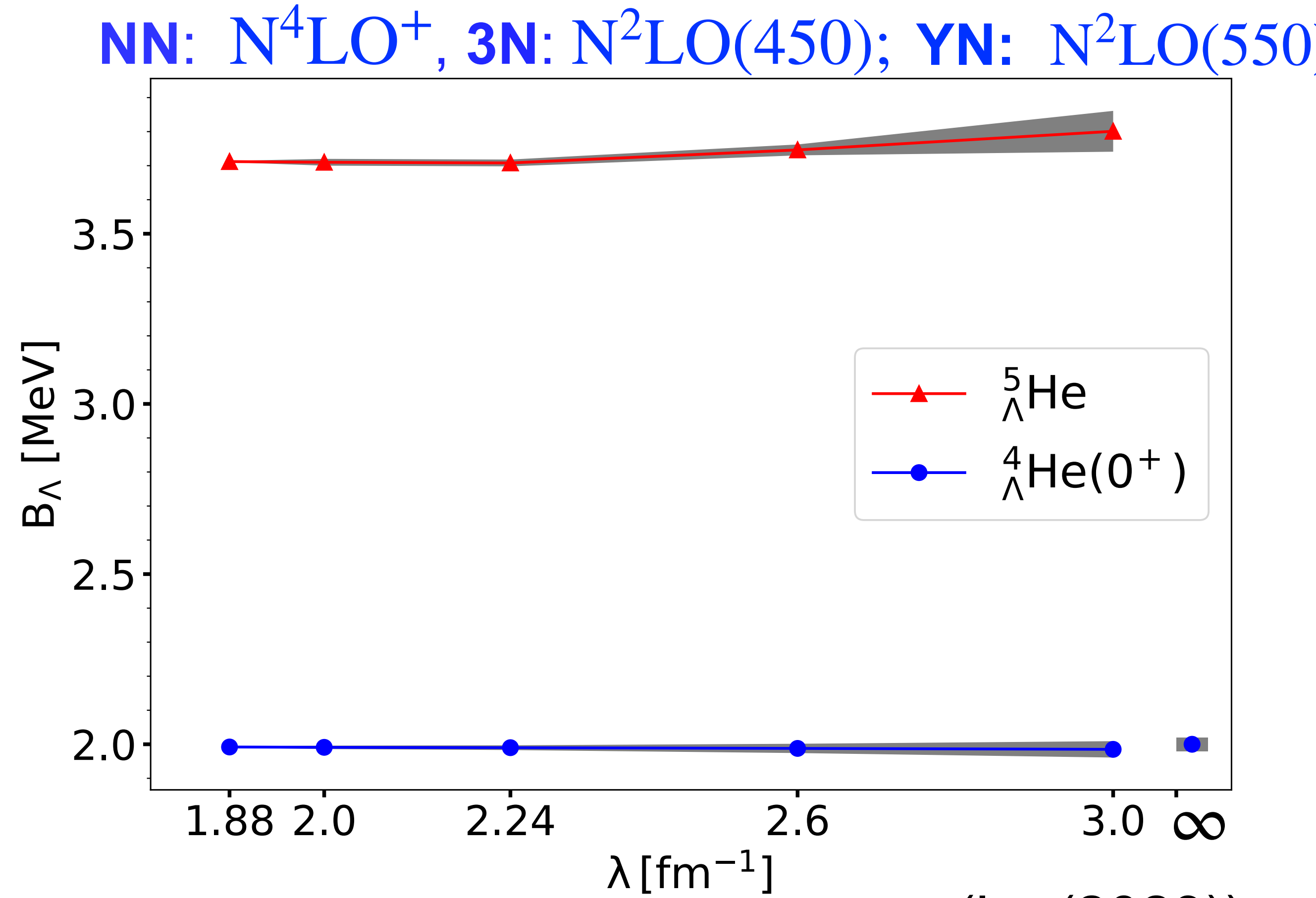


SRG dependence of results

- SRG-induced 3N and YNN interactions
- ^{4}He binding energies varies by $\approx 100 - 200$ keV (relevant in the future?)
- separation energies are even less dependent (YNNN forces small)



(Maris, Le, Nogga, Roth, Vary (2023))

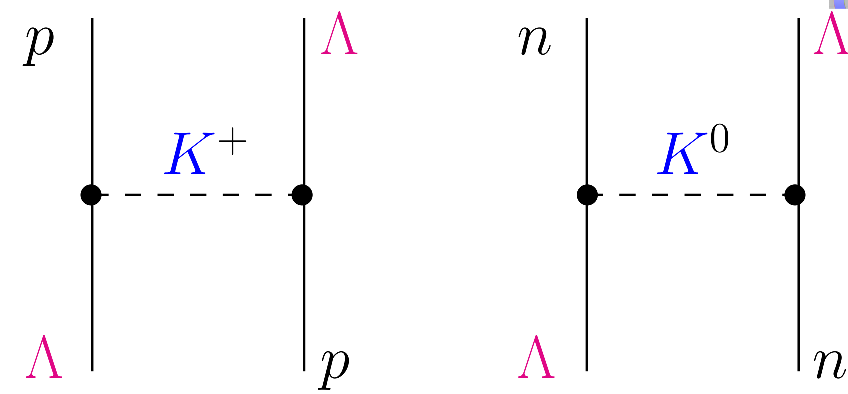


(Le (2023))

For **hypernuclei**, calculations based on SRG induced BB and 3B interactions are sufficiently accurate!

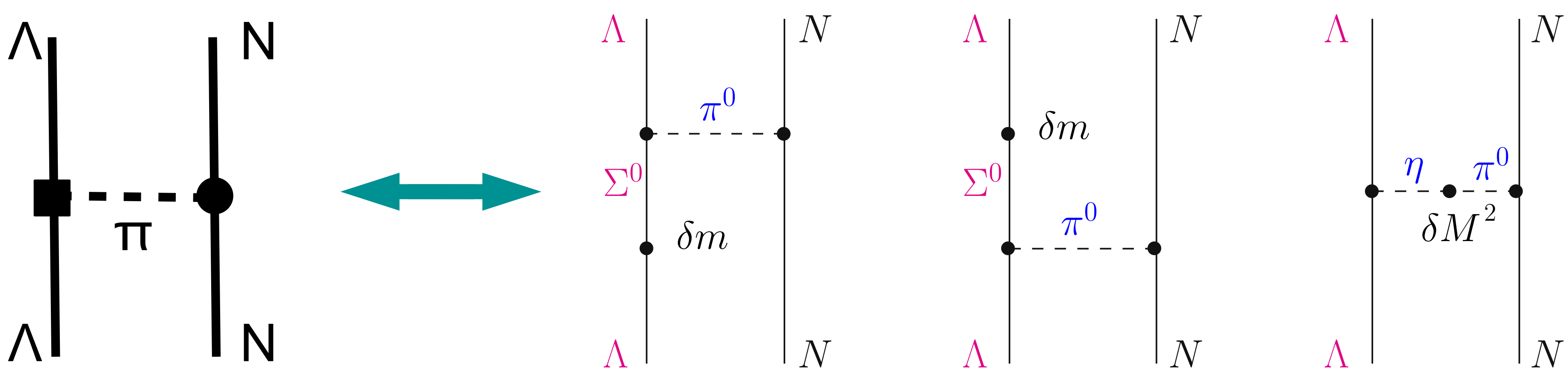
CSB contributions to YN interactions

- **formally leading** contributions:
  Goldstone boson mass difference
  - — very small due to the small relative difference of kaon masses

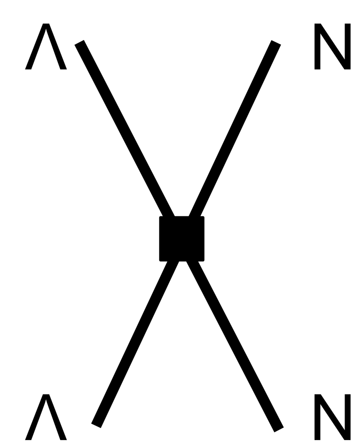


- **subleading but most important**
  - — effective CSB ΛΛπ coupling constant (Dalitz, van Hippel, 1964)

$$f_{\Lambda\Lambda\pi} = \left[-2\frac{\langle \Sigma^0 | \delta m | \Lambda \rangle}{m_{\Sigma^0} - m_\Lambda} + \frac{\langle \pi^0 | \delta M^2 | \eta \rangle}{M_\eta^2 - M_{\pi^0}^2} \right] f_{\Lambda\Sigma\pi} \approx (-0.0297 - 0.0106)\, f_{\Lambda\Sigma\pi}$$



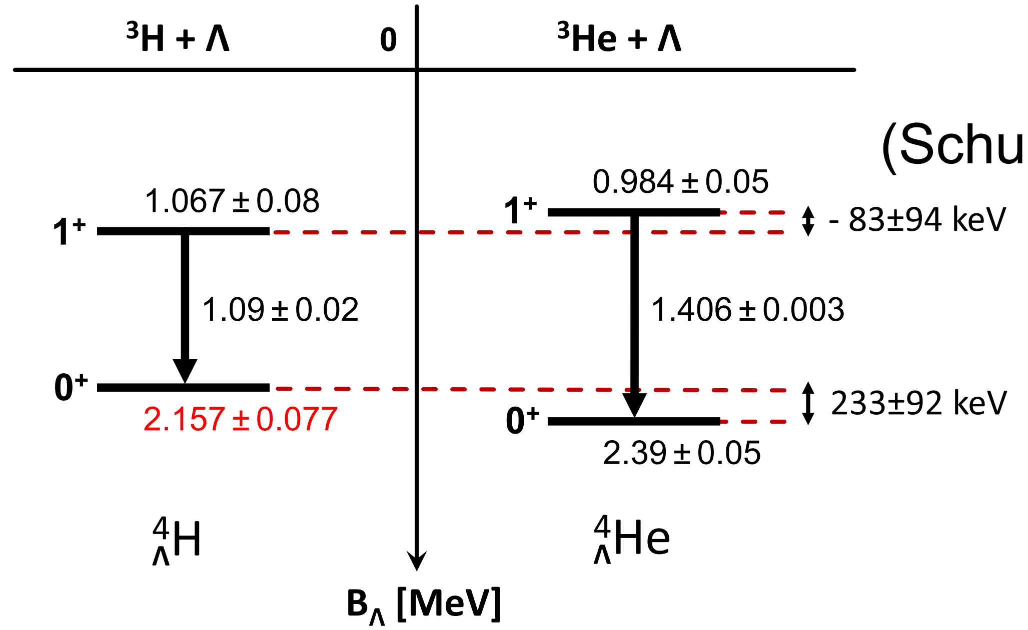
- **so far less considered, but equally important**
  - — CSB contact interactions (for singlet and triplet)



Aim: use A=4 hypernuclei to determine the two unknown CSB LECs and predict Λn scattering

(so far: NLO13 and NLO19)

Fit of contact interactions



(Schulz et al.,2016; Yamamoto, 2015)

- Adjust the two CSB contact interactions to one main scenario (**CSB1**)

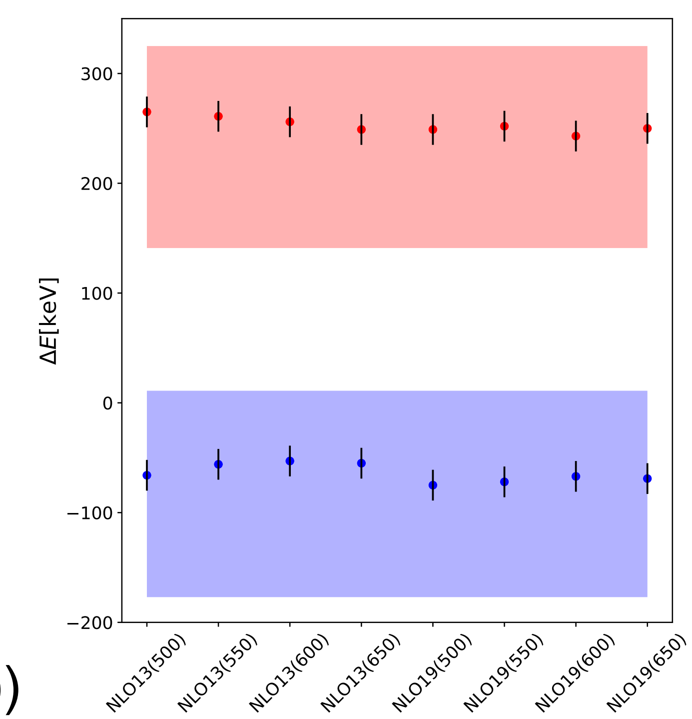
Λ	NLO13		NLO19	
	C_s^{CSB}	C_t^{CSB}	C_s^{CSB}	C_t^{CSB}
500	4.691×10^{-3}	-9.294×10^{-4}	5.590×10^{-3}	-9.505×10^{-4}
550	6.724×10^{-3}	-8.625×10^{-4}	6.863×10^{-3}	-1.260×10^{-3}
600	9.960×10^{-3}	-9.870×10^{-4}	9.217×10^{-3}	-1.305×10^{-3}
650	1.500×10^{-2}	-1.142×10^{-3}	1.240×10^{-2}	-1.395×10^{-3}

The values of the LECs are in 10^4 GeV^{-2}

- Size of LECs as expected by power counting

$$\frac{m_d - m_u}{m_u + m_d}\left(\frac{M_\pi}{\Lambda}\right)^2 C_{S,T} \approx 0.3 \cdot 0.04 \cdot 0.5 \cdot 10^4\,\mathrm{GeV}^{-2} \propto 6 \cdot 10^{-3} \cdot 10^4\,\mathrm{GeV}^{-2}$$

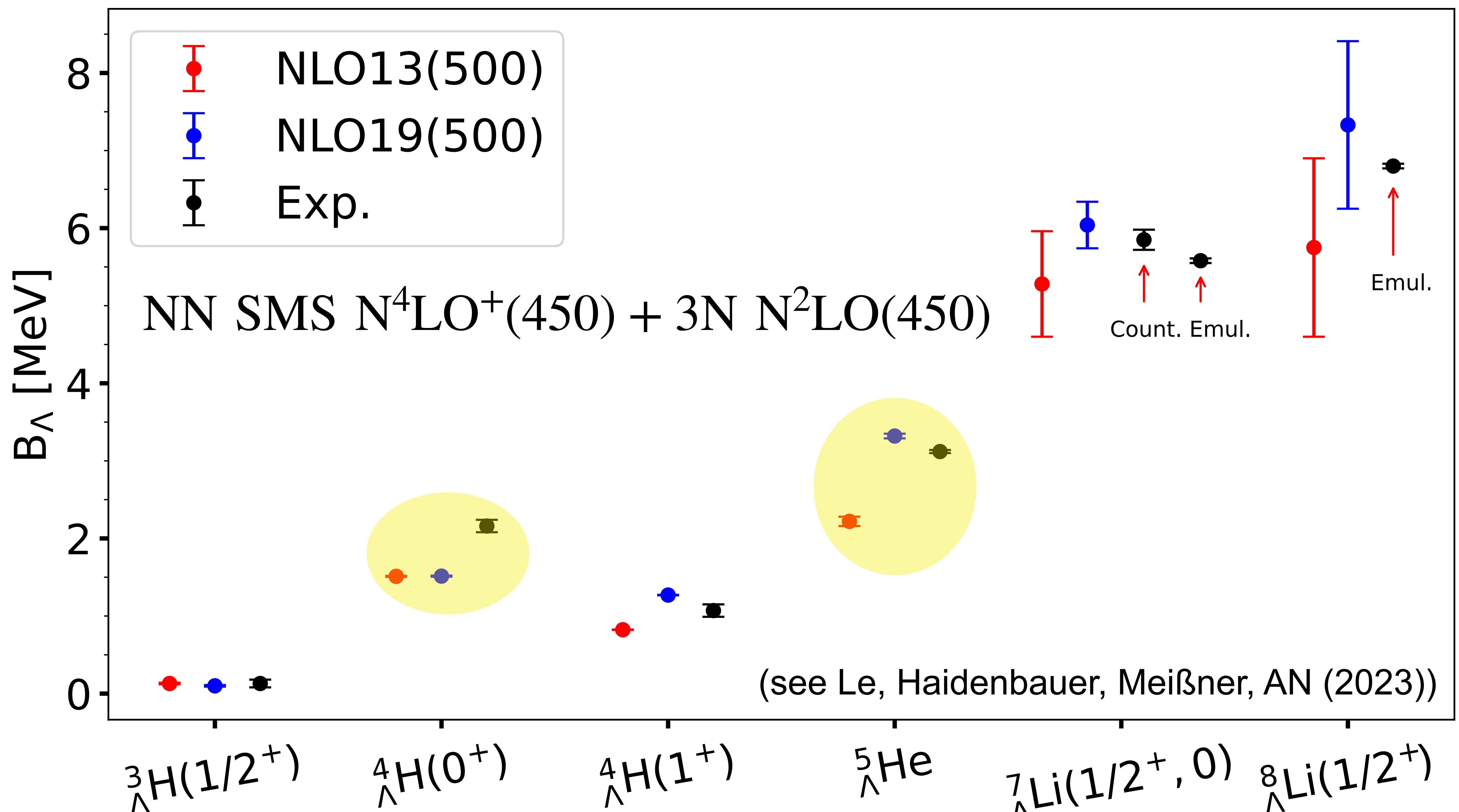
- Problem: large experimental uncertainty of experiment
- here only **fit to central values** to test theoretical uncertainties



(see Haidenbauer, Meißner, AN (2021))

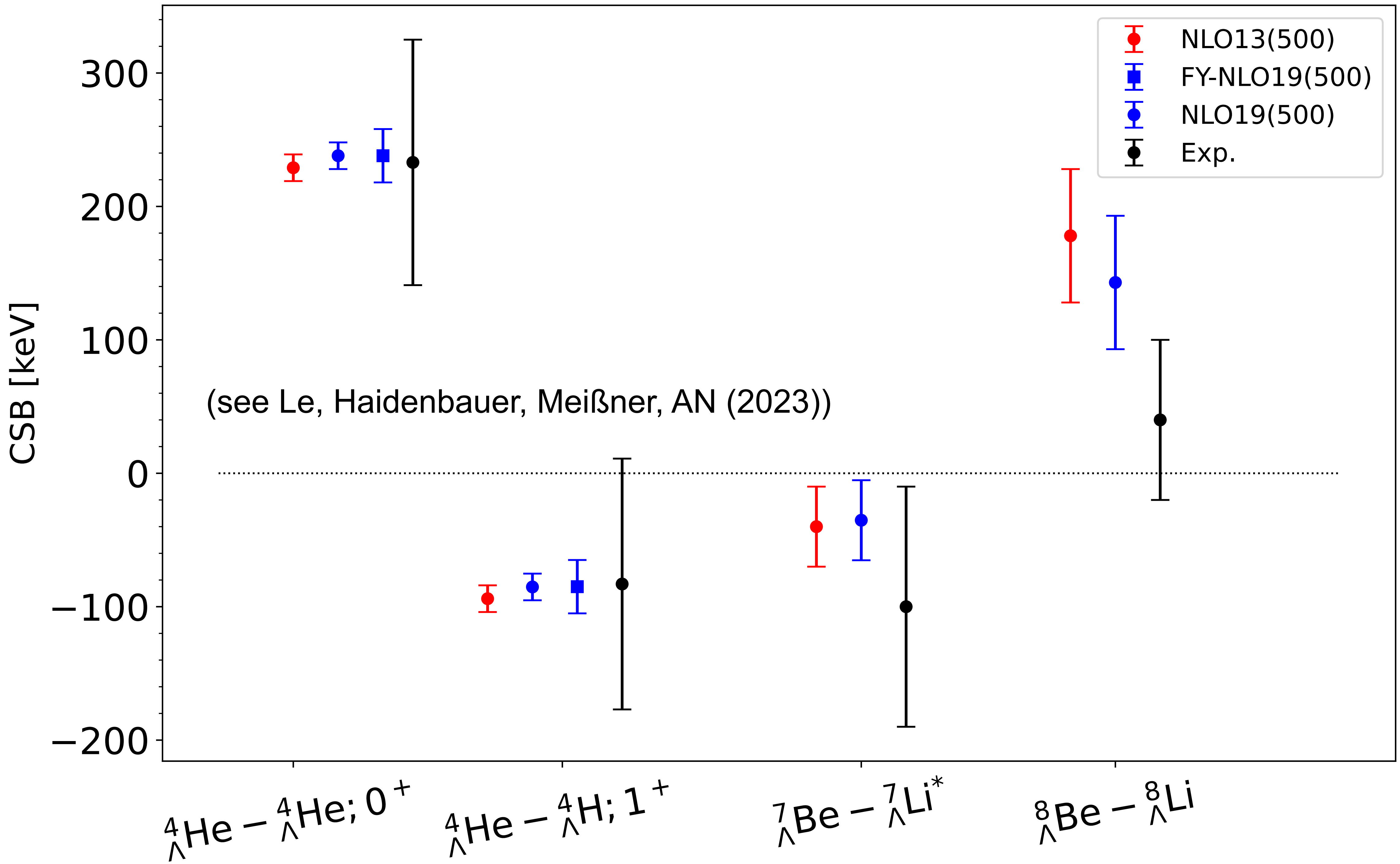
Application to $A = 7$ and 8

- YN interaction adjusted to the hypertriton — YNN is small
- based only on YN interactions: splitting for ${}^4_\Lambda\mathrm{H}$ is not well reproduced — YNN(?)
- NLO19 gives better results for ${}^5_\Lambda\mathrm{He}$ and heavier hypernuclei
  — accidentally small YNN interaction?
- uncertainties are numerical — no estimate of chiral uncertainties yet



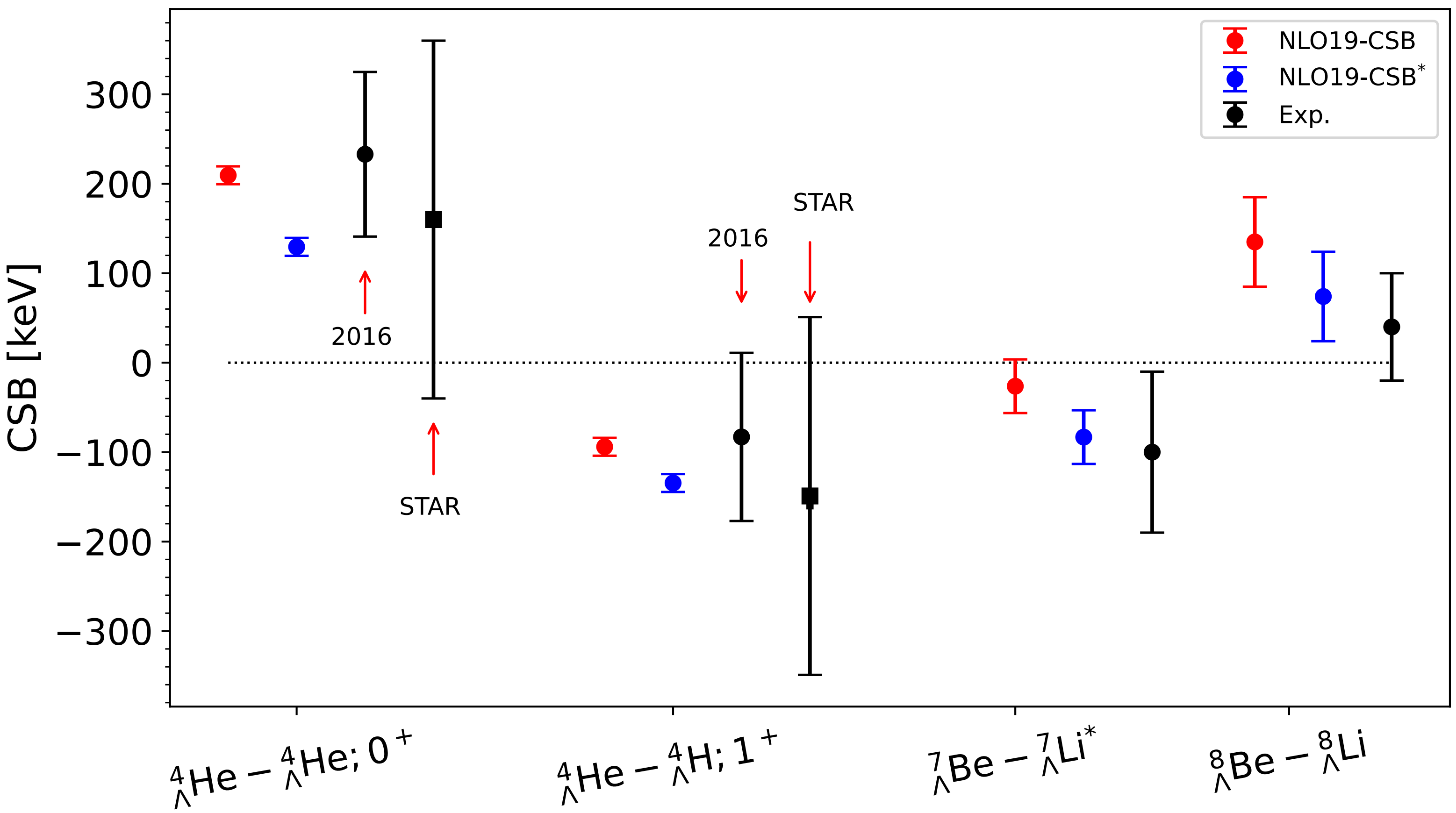
Application to $A = 7$ and 8

- CSB of singlet and triplet states interferes differently
- CSB still not fixed — experimental uncertainty is large
- scenario studied here is only **marginally consistent** with CSB in $A = 8$



New STAR data for $A = 4$ CSB

- fit to STAR data only
- only slight adjustment required
- improves description to p-shell CSB
- higher experimental accuracy is desirable
- good example of using hypernuclei to determine YN interactions



	NLO19(500)	CSB	CSB*
$a_s^{\Lambda p}$	-2.91	-2.65	-2.58
$a_s^{\Lambda n}$	-2.91	-3.20	-3.29
$\boldsymbol{\delta a_s}$	**0**	**0.55**	**0.71**
$a_t^{\Lambda p}$	-1.42	-1.57	-1.52
$a_t^{\Lambda n}$	-1.41	-1.45	-1.49
$\boldsymbol{\delta a_t}$	**-0.01**	**-0.12**	**-0.03**

(see Le, Haidenbauer, Meißner, AN (2023))

Uncertainty analysis to $A = 3$ to 5

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Order N²LO requires combination of chiral NN, YN, 3N and YNN interaction

Need calculation of separation energies (use Faddeev, Yakubovsky eq. or J-NCSM) and use **different orders** for uncertainty estimate.

Assuming a negligible numerical uncertainty and the following ansatz for the order by order convergence

$$X_K = X_{ref} \sum_{k=0}^{K} c_k \, Q^k \quad \text{where} \quad Q = M_\pi^{eff} / \Lambda_b \quad (X_{ref} \text{ LO, exp., max, ...})$$

a **Bayesian analysis** of the uncertainty is possible (see Melendez et al. 2017,2019)

Extracting c_k for $k \leq K$ from calculations and assuming identical probability distributions for c_k for $k > K$ the uncertainty is given by the distribution of

$$\delta X_K = X_{ref} \sum_{k=K+1}^{\infty} c_k \, Q^k$$

Numerical uncertainties negligible (carefully checked!).

Uncertainty due to missing higher orders is most relevant!

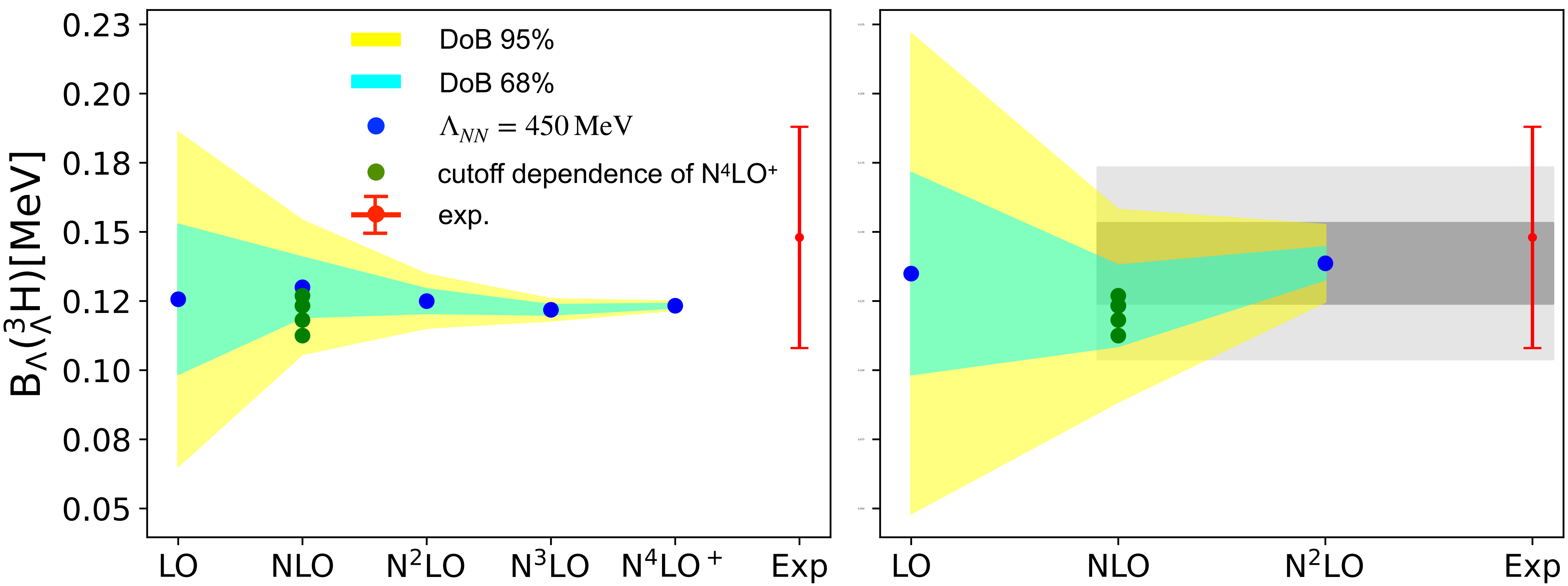
Application to ${}^{3}_{\Lambda}\mathrm{H}$

- Q, ν_0 and τ_0 are chosen using all available data (NN and YN convergence)
- uncertainties are extracted using c_k for NN or YN convergence
- use c_k of individual hypernuclei

→ individual uncertainties for NN and YN convergence for each separation energy

consistent with experimental data

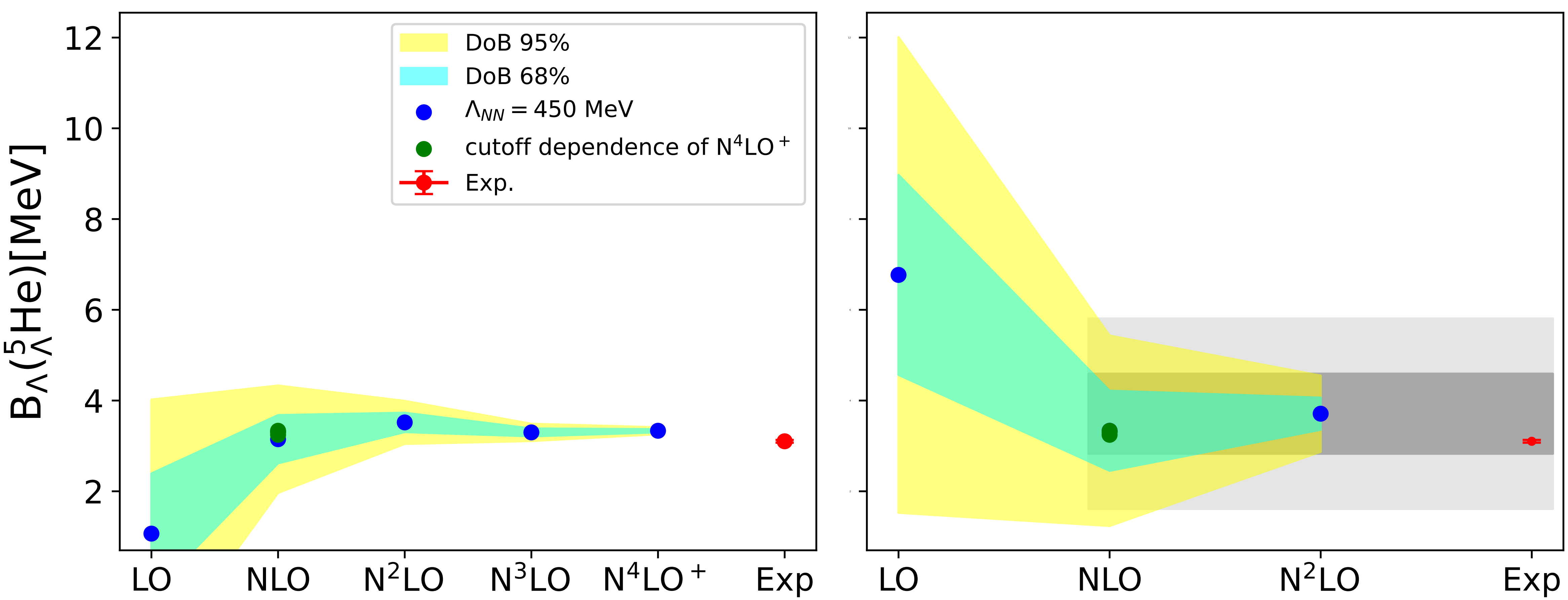
cutoff dependence always at least NLO (YNN missing!)



Application to ${}^{5}_{\Lambda}\mathrm{He}$ and summary

- without YNN: sizable uncertainties at $A = 4$ and 5
- $A = 3$ sufficiently accurate
- NN/YN dependence small at least for $A = 3$

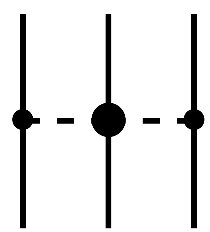
nucleus	$\Delta_{68}(NN)$	$\Delta_{68}(YN)$
${}^{3}_{\Lambda}\mathrm{H}$	0.011	0.015
${}^{4}_{\Lambda}\mathrm{He}\,(0^{+})$	0.157	0.239
${}^{4}_{\Lambda}\mathrm{He}\,(1^{+})$	0.114	0.214
${}^{5}_{\Lambda}\mathrm{He}$	0.529	0.881



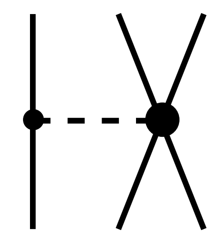
YNN (ΛNN) interactions

Leading 3BF with the usual topologies (see Petschauer et al., 2016 & 2017)

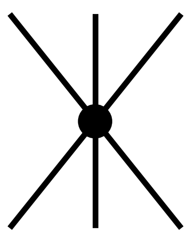
ChPT → all octet mesons contribute → only take π explicitly into account



2 LECs in ΛNN
(up to 10)



2 LECs in ΛNN
(up to 14)

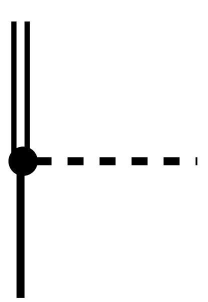


3 LECs in ΛNN
5 LECs in ΣNN + 1 Λ-Σ transition

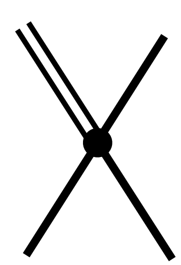
only few data → need to keep the # of LECs small

Decuplet baryons ($\Sigma^* \ldots$) enhances YNN partly to NLO (see Petschauer et al., 2017)

By decuplet saturation all LECs can be related to the following leading octet-decuplet transitions (Petschauer et al. , 2020)



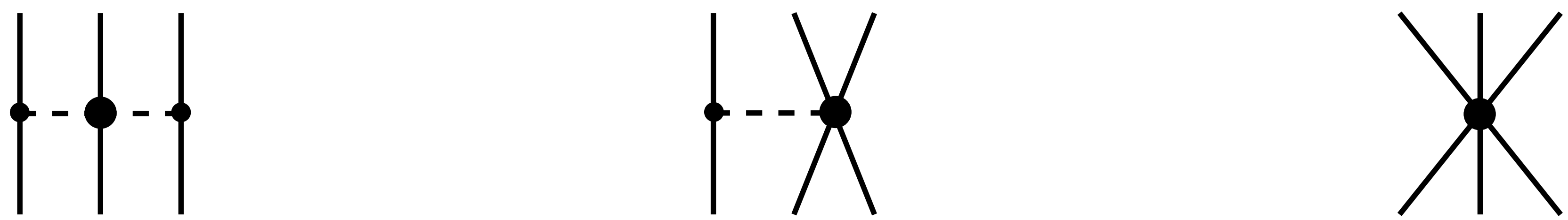
$\propto C = \frac{3}{4} g_A$



$\propto G_1, G_2$ → **reduction to 2 LECs**

YNN (ΛNN) interactions

Decuplet saturation relates all LECs to G_1 and G_2



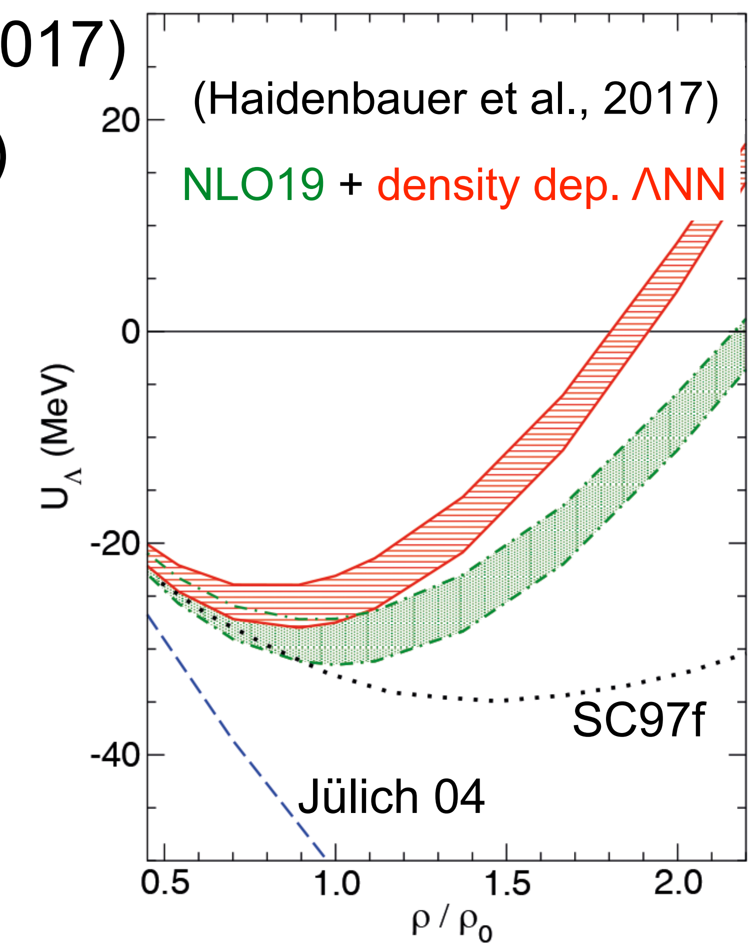
$\propto C^2$ $\quad\quad$ $\propto CG_1, CG_2$ $\quad\quad$ $\propto (G_1)^2, (G_2)^2, G_1G_2$

For ΛNN: $\propto C^2$ $\quad\quad$ $\propto C(G_1 + 3G_2)$ $\quad\quad$ $\propto (G_1 + 3G_2)^2$ $\quad$ 1 LEC

→ density dependent BB interactions (Petschauer et al., 2017)

→ application to nuclear matter (Haidenbauer et al., 2017)

neutron stars (Logoteta et al., 2019)



- contribution on the single particle potentials can be large
- realistic results seem to require partly
  cancelations of 2π and 1π exchange (sign of $G_1 + 3G_2$!)

Results for hypertriton of Kohno et al., 2023/2024 small

…but need correction … benchmark now successful

YNN (ΛNN) interactions

Trying to fit G_1/G_2 to ${}^4_\Lambda\mathrm{He/H}$ …

Recalculate 2π, 1π and contact terms of $\mathbf{\Lambda NN}$ using old **non-local** regularization

to compare to Kohno et al. (use fixed constant $G_1 = G_2 = \dfrac{1}{4f_\pi^{\,2}}$, $G_1 + 3G_2 = +\dfrac{1}{f_\pi^{\,2}}$)

→ ΛNN matrix elements ✔

Comparison of separation energies is ongoing ($\mathrm{SMS\ N^4LO^+(550)/N^2LO + NLO19}$):

	w/o YNN	$w/$ 2π	$w/$ $2\pi/1\pi$	$w/$ $2\pi/1\pi/\mathrm{ct}$
${}^3_\Lambda\mathrm{H}$ w/o ΣNN	0.080	0.151	0.215	0.208
${}^3_\Lambda\mathrm{H}$		0.241	0.564	0.549
${}^4_\Lambda\mathrm{He}(0^+)$	1.432	2.412		
${}^4_\Lambda\mathrm{He}(1^+)$	1.164	2.623		
${}^5_\Lambda\mathrm{He}$	3.174	7.139		

preliminary

Large contribution to all light hypernuclei (larger than estimate!)

— consistent description requires larger cancelation of 2π and 1π part

— contact terms neglible for ${}^3_\Lambda\mathrm{H}$

YNN (ΛNN) interactions

On the way to fit G_1/G_2 to ${}^{4}_{\Lambda}\mathrm{He/H}$ …

apply locally regularized YNN **including subtractions**

inclusion of ΣNN gives dependence on G_1/G_2 independently

Test results $(\mathrm{SMS\ N^4LO^+(550)/N^2LO + SMS\ NLO(550)})$:

	w/o YNN	$w/\ 2\pi$	$w/\ 2\pi/1\pi$	$w/\ 2\pi/1\pi/\mathrm{ct}$
${}^{3}_{\Lambda}\mathrm{H}$ w/o subtr	0.107	0.149		
${}^{3}_{\Lambda}\mathrm{H}$ only subtr		0.086		
${}^{3}_{\Lambda}\mathrm{H}$ ΛNN compl		0.124		
${}^{3}_{\Lambda}\mathrm{H}$		0.159	0.238	
${}^{4}_{\Lambda}\mathrm{He}(0^+)$	1.969	2.333		
${}^{4}_{\Lambda}\mathrm{He}(1^+)$	1.063	1.367		
${}^{5}_{\Lambda}\mathrm{He}$	3.247	4.294		

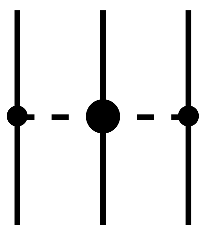
SMS regularization leads to much more **natural results.**

Consistent regularization of NN/3N and YN/YNN forces?

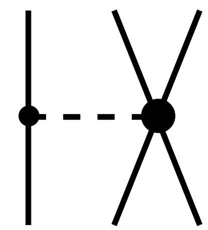
Sensitivity of hypernuclear binding to G_1/G_2 ?

YNN (ΛNN) interactions

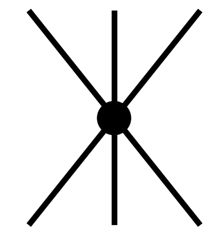
Decuplet approximation in YNN



$\propto C^2$

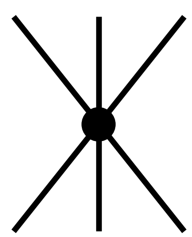


$\propto CG_1, CG_2$



$\propto (G_1)^2, (G_2)^2, G_1G_2$

+ ΛNN contact terms without decuplet constraints



ΛNN $\propto C_1', C_2', C_3'$

***ad hoc* choice to alter C_2:**

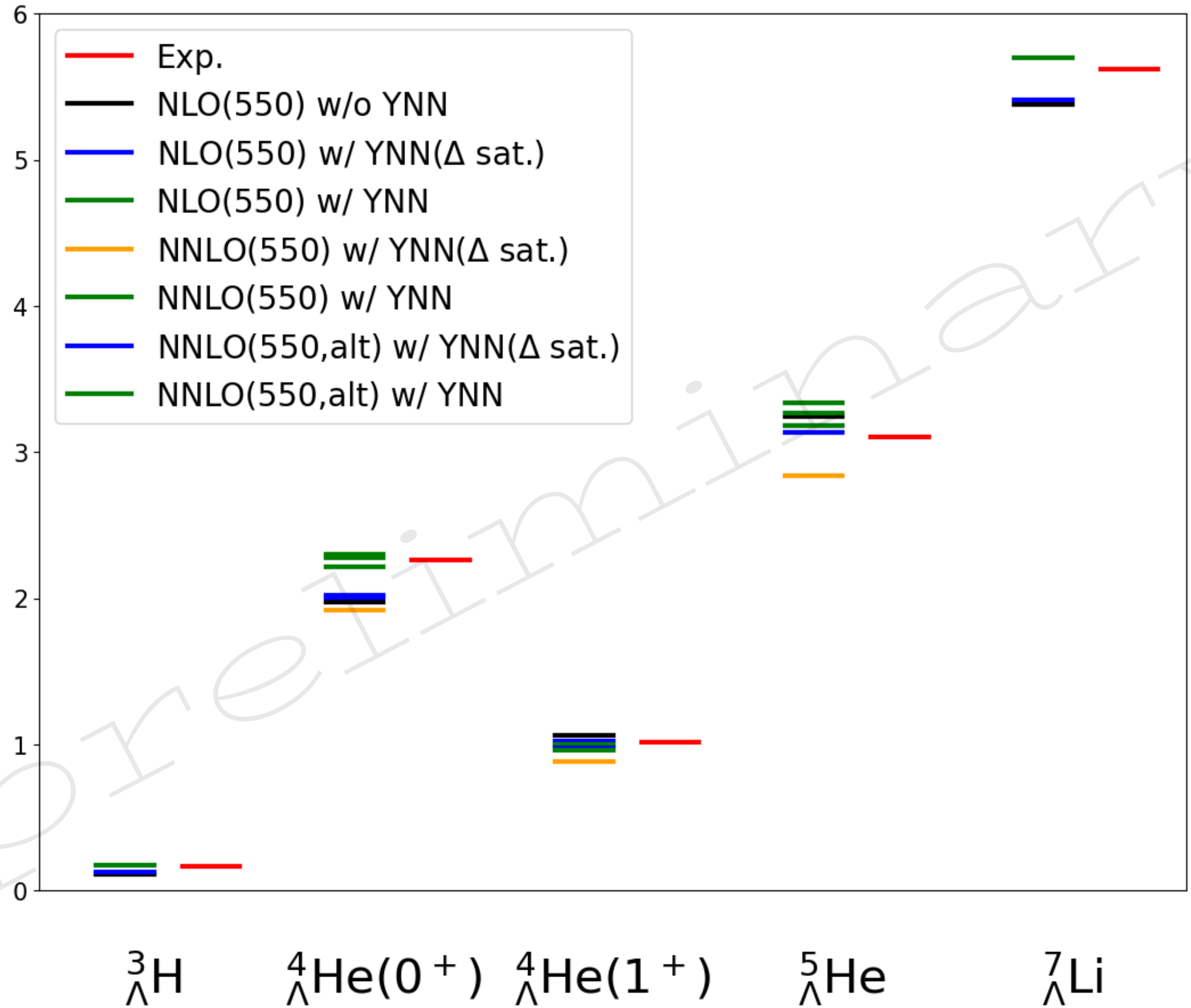
$$C_1' = C_3' = \frac{(G_1 + 3G_2)^2}{72\Delta}$$

$$V_{\Lambda\mathrm{NN}} = C_2{}' \, \vec{\sigma}_1 \cdot \left(\vec{\sigma}_2 + \vec{\sigma}_3\right) \left(1 - \vec{\tau}_2 \cdot \vec{\tau}_3\right)$$

$C_2' = 0$ → $C_2' = G_3$

C_2' introduces a spin dependent interaction in the most relevant particle channel

YNN fit



6
5
4
3
2
1
0
Exp.
NLO(550) w/o YNN
NLO(550) w/ YNN(Δ sat.)
NLO(550) w/ YNN
NNLO(550) w/ YNN(Δ sat.)
NNLO(550) w/ YNN
NNLO(550,alt) w/ YNN(Δ sat.)
NNLO(550,alt) w/ YNN
$^{3}_{\Lambda}$H
$^{4}_{\Lambda}$He(0^+)
$^{4}_{\Lambda}$He(1^+)
$^{5}_{\Lambda}$He
$^{7}_{\Lambda}$Li
preliminary

Conclusions & Outlook

- **YN (& YY) interactions not well understood**
  - *scarce YN data (almost no YY data)*
  - *more information necessary to solve "hyperon puzzle"*
- **Hypernuclei provide important constraints**
  - *CSB of ΛN scattering & ${}^{4}_{\Lambda}\mathrm{He}$ / ${}^{4}_{\Lambda}\mathrm{H}$*
  - ${}^{3}_{\Lambda}\mathrm{H}$ *is used to constrain the spin dependence*
  - *new experiments & analyses planned at J-PARC, MAMI, J-Lab, FAIR,…*
- **New SMS YN interactions**
  - *give an accurate description low energy YN data*
  - *order LO, NLO and N^{2}LO allow uncertainty quantification*
  - *have a non-unique determination of contact interactions (data necessary)*
- **Chiral 3BF** need to be included
  - *NLO uncertainty is sizable in $A = 4$ and beyond*
  - *chiral 3BFs are now available*
  - *regularization affects size of individual contributions*
  - *fitting to ${}^{4}_{\Lambda}\mathrm{He}$ in progress, preliminary results are available*
  - *relate to orders, re-perform uncertainty estimate in N^{2}LO*